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Research Article

Modeling of Sand and Crude Oil Flow in Horizontal Pipes during Crude Oil Transportation

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Some oil and gas reservoirs are often weakly consolidated making them liable to sand intrusion. During upstream petroleum production operations, crude oil and sand eroded from formation zones are often transported as a mixture through horizontal pipes up to the well heads and between well heads and flow stations. The sand transported through the pipes poses serious problems ranging from blockage, corrosion, abrasion, and reduction in pipe efficiency to loss of pipe integrity. A mathematical description of the transport process of crude oil and sand in a horizontal pipe is presented in this paper. The model used to obtain the mathematical description is the modified form of Doan et al. (1996 and 2000) models. Based on the necessity to introduce a sand deposit concentration term in the mass conservation equation, an additional equation for solid phase was derived. Difference formulae were generated having applied Fick's equation for diffusion to the mass conservation equations since diffusion is one of the transport mechanisms. Mass and volume flow rates of oil were estimated. The new model, when tested with field data, gave 85% accuracy at the pipe inlet and 97% accuracy at the exit of the pipe.

1. Introduction

During upstream petroleum production practices, rock oils from reservoirs are often transported as a mixture with sand up to the well heads and from the well heads to flow stations. At the head of the wells, horizontal transmission lines with or without screens transport the residual sand in the oil from feeder lines to flow stations. The entrained sand may deposit on the walls of the pipe due to pressure drop causing problems such as abrasion, corrosion, pipe blockage, reduction in flow area, loss in pipe integrity, and most importantly low output from the lines [1]. Sand exclusion measures (sand screens, sand filters, and gravel packs) used hitherto are somewhat laborious and expensive [2]; hence, it is necessary to search for an alternative solution to the problem such as using a mathematical model. Popoola et al. [3] discussed corrosion problems and mitigation of corrosion during oil and gas production. In this paper, about eight commonly encountered corrosion types as they relate to oil and gas production were mentioned alongside methods of controlling them. Amongst the methods suggested are materials selection, injection of inhibitors, the application of protective coatings, corrosion monitoring and inspection, and cathodic protection. To date, a model approach to sand corrosion control is yet to be established; however, various fluid-particle flow models were reviewed so as to make an apt choice. The paper of Srdjan et al. [4] established an internal corrosion prediction model for multiphase flow in a pipeline where a comprehensive CO_2/H_2S flow model was applied in order to predict the effects of H_2S, water entrainment, corrosion inhibition, and localized attack on a pipeline. The model was validated using experimental data where effect of trace amount of H_2S on corrosion rate in the absence of iron sulfide scales and the effects at the onset of iron sulfide scale formations were evaluated and measured at pH values less than 5 and equal to 6, at temperature of 20–80°C, pressure of 1 to 7.7 bars, and conditions of $T = 60°C$ and 7.7 bars, respectively. Van et al. [5] gave a numerical sensitivity analysis of the Wilson two-three-layer models for fully and partially stratified flows. They confirmed the validity of

the two-layer model for partially stratified flows but the three-layer model was found suitable for bed load motion where friction is significant. Patankar and Joseph [6] in their work showed the validation of a developed numerical scheme with experiments using a bimodal suspension in a sedimentation column. The model was used to estimate sedimentation rates using two simulations with different grid sizes, parcel number, and time steps. Frederic et al. [7] modeled the settling of solid particles embedded in a viscous fluid flowing under gravity through a narrower section of a pipe. They studied the effect of particle shape on relaxation time for both disk and rectangular shaped particles. Glowinsky et al. [8] model is useful for the direct numerical simulation of three-dimensional fluidization and sedimentation phenomena. The model suits well the Newtonian and non-Newtonian incompressible viscous flows past moving rigid bodies.

Doan et al. [1] model represents a simulation approach of sand deposition inside a horizontal well. Although the model includes channel height, it can also account for the effect of oil viscosity and particle size on the transport process. It is also suitable for calculating fluid and particles concentration and quantifying fluid delivery but cannot simulate the turbulent transport of oil and sand. Huang et al. [9] focused on the motion of a two-dimensional circular cylinder in Couette and Poiseuille flows of a viscoelastic fluid. Both neutrally buoyant particles and nonneutrally buoyant particles were considered.

Joseph [10] developed a general model for particulate flows. The model incorporates only two types of forces in its phase equations: interaction and viscous forces. The model is suitable for quantifying fluid delivery and can handle a wide range of particle loading and types. Doan et al. [11] model is a simulation of the movement of sand and crude oil inside a horizontal well. Two fluids of different viscosities were considered and the relationship between viscosity, Reynolds number, drag coefficient, and interaction coefficient was determined. The model does not consider the effect of eddies which makes it unsuitable for turbulent transport of crude oil and sand. Therefore, this paper seeks to cover the gap in knowledge by modifying the aforementioned Doan et al. models thus describing a new model for laminar and turbulent transport of sand and crude oil in a horizontal pipe between the head of a well and its flow station.

2. Model Modification

The Doan et al. [1, 11] models were developed for the case of sand and oil flow in an oil well and this informed why they were chosen from the reviewed models for application, other reasons being the inclusion of parameters such as sand and crude oil concentration terms, solid and liquid phase pressures, solid and liquid densities, liquid and solid interaction forces, kinematic pressure, liquid and solid concentrations, solid and liquid velocities, and liquid viscosity among others. The model represented by (1)–(4) was subsequently modified by assuming that all other components (asphaltenes, resins, and olefins) were dissolved in the oil at the flow conditions while taking effect of eddies into account.

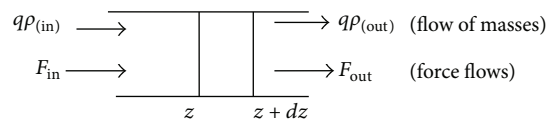

Figure 1: A typical pipeline system.

2.1. The Doan et al. Model. Consider

$$\frac{\partial}{\partial t}(\phi) + \frac{\partial}{\partial z}(\phi w_s) = 0 \quad \text{(mass conservation equation for solid phase-suspension)}, \tag{1}$$

$$\frac{\partial}{\partial t}(\varepsilon) + \frac{\partial}{\partial z}(\varepsilon w_f) = 0 \quad \text{(mass conservation equation for fluid phase-suspension)}, \tag{2}$$

$$\frac{\partial}{\partial t}(\phi w_s) + \frac{\partial}{\partial z}(\phi w_s w_s) = -(\phi g) - \frac{\phi}{\rho_s}\frac{\partial P_s}{\partial z} + \frac{\beta}{\rho_s}(w_f - w_s) - \frac{P_k}{\rho_s}\frac{\partial \phi}{\partial z} \quad \text{(force equation for Solid phase)}, \tag{3}$$

$$\frac{\partial}{\partial t}(\varepsilon w_f) + \frac{\partial}{\partial z}(\varepsilon w_f w_f) = -(\varepsilon g) - \frac{\varepsilon}{\rho_f}\frac{\partial P_f}{\partial z} + \frac{\beta}{\rho_f}(w_f - w_s) \quad \text{(force equation for fluid phase)}. \tag{4}$$

2.2. Model Development and Modification. Considering Figure 1, where a mixture of incompressible crude oil and sand flows through an element of length dz within a pipe of length L, the conservation equations can be generated as follows:

mass or force flow into the system − mass or force flow out of the system = time rate of change of mass or momentum in the system.

This can be expressed mathematically as

$$M_{in} - M_{out} = \frac{\partial M}{\partial t}. \tag{5}$$

2.3. Model Assumptions. Sand and oil do not mix. Hence, their mixing coefficients are ignored. The particles are spherical and are of uniform size so that the same buoyancy effect will be experienced by particles of the same size within a region of flow and pipe section. The oil is Newtonian. The flow is isothermal. Sand-oil suspension behaves as a continuum; that is, sand particles behave like fluid and have a fluidized velocity; hence, sand molecules are not taken as a discrete entity but are in a continuous phase. The fluid-particle

interaction force and particle-particle interaction forces are of significance. Sand particles movement is as a result of the surrounding liquid phase and pressure forces which are inherent as a result of inability of the liquid to stay when it offers resistance to shear. Gravity force is due to particles weight in the carrying medium. Buoyancy force is interpreted as fluid-particle interaction force since liquid molecules are being displaced by descending solid particles acted upon by gravity and inertia force which keeps a body in its state of rest or maintains its motion while moving and kinematic pressure which is a particle-particle interaction force. The deposit is considered entirely of sand phase; that is, other components of the oil that may add to the weight of sand, that is, resins and olefins or those that have tendencies of being deposited such as asphaltenes, are all considered soluble in the oil under the flow conditions. The pipe wall appeared somewhat smooth; hence, surface roughness of the pipe was ignored. Diffusion is one of the major controlling mechanisms of fluid-particle transport. A coming paper will describe the effect of mechanisms such as Euler and Froude numbers on the flow behaviour.

2.4. Application of Taylor's Series. Taylor's series expansion formula was applied at the inlet and exit portions of the pipe to obtain the mass and force balance equations. This includes a third equation for solid phase and a sand deposit concentration term.

Taylor's series expansion formula is as follows:

$$f(z+dz) = f(z) + \frac{dzf'(z)}{1!} + \frac{|dz|^2 f''(z)}{2!}. \tag{6}$$

Truncating at the 2nd term gives

$$f(z+dz) = f(z) + \frac{dzf'(z)}{1!}. \tag{7}$$

Since

$$f(z) = \sigma q_s \rho_s \tag{8}$$

then $dzf'(z)/1!$ implies

$$dzf'(z) = dzf'(\sigma q_s \rho_s). \tag{9}$$

2.5. The New Model. Consider

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial z}(\phi w_s) = 0, \tag{10}$$

$$\frac{\partial \sigma}{\partial t} + \frac{\partial}{\partial z}(\sigma w_s) = 0, \tag{11}$$

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial}{\partial z}(\varepsilon w_f) = 0, \tag{12}$$

$$\begin{aligned} &\frac{\partial}{\partial t}(\Psi w_s) + \left(\frac{\partial}{\partial z}(\Psi w_s w_s)\right) \\ &\quad = -(\Psi g) - \frac{\Psi}{\rho_s}\frac{\partial P_s}{\partial z} + \frac{\beta}{\rho_s}(w_f - w_s) - \frac{P_k}{\rho_s}\frac{\partial \Psi}{\partial z}, \end{aligned} \tag{13}$$

$$\begin{aligned} &\frac{\partial}{\partial t}(\varepsilon w_f) + \left(\frac{\partial}{\partial z}(\varepsilon w_f w_f)\right) \\ &\quad = -(\varepsilon g) - \frac{\varepsilon}{\rho_f}\frac{\partial P_f}{\partial z} + \frac{\beta}{\rho_f}(w_f - w_s). \end{aligned} \tag{14}$$

The new model as compared to the Doan et al. [1, 11] models shows that (11) is an additional equation for solid phase while (13) is the modified form of (3) because it includes a total sand concentration term, that is, Ψ.

2.6. Model Calibration. Correlations were used to obtain constants such as molecular and eddy diffusivities. The correlations used include the following.

(i) Correlation for Evaluation of Molecular Diffusivity. Consider

$$D = \frac{1}{6} * d * u', \tag{15}$$

where D = molecular diffusivity (coefficient of diffusion), d = diameter of particle, u' = average velocity of the entire mixture of sand and oil $((w_s + w_f)/2)$, and w_s and w_f = nominal sand and fluid velocities, respectively.

As explained in Doan et al. [1], fluid phase nominal velocities of 5.0 cm/s and 6.8 cm/s correspond to an average production rate of 110 m^3/day and 150 m^3/day, respectively. Equation (16) gives the flow rate of the mixture:

$$Q\ (\text{Flow rate}) = U\ (\text{velocity}) * A\ (\text{cross-sectional area}), \tag{16}$$

which implies $Q \propto U$:

$$802.5\ \text{m}^3/\text{day} \longrightarrow w_m \quad (\text{mix velocity}). \tag{17}$$

The sand and oil velocities were however scaled in order to avoid numerical instability. Now, by calculation, we obtain the nominal field velocity for a production rate of 802.5 m^3/day:

$$\begin{gathered} 802.5\ \text{m}^3/\text{day} \longrightarrow w_m, \\ 150\ \text{m}^3/\text{day} \longrightarrow 6.8\ \text{m/s}, \\ 110\ \text{m}^3/\text{day} \longrightarrow 5.0\ \text{m/s} \quad \text{which implies} \\ \left(\frac{802.5 - 150}{150 - 110}\right) = \left(\frac{w_m - 6.8}{6.8 - 5.0}\right) \\ : .w_m = 30.04\ \text{m/s}. \end{gathered} \tag{18}$$

Sand nominal velocity is assumed to be 90% of mix velocity, so, sand nominal velocity

$$\begin{aligned}(w_s) &= 0.9 * 30.04\,\text{m/s} = 27.04\,\text{m/s},\\ d &= 0.05\,\text{m},\\ \phi w_s\,(\text{sand velocity}) &= 0.06 * 27.04\,\text{m/s} = 1.6224\,\text{m/s},\\ \varepsilon w_f\,(\text{fluid velocity}) &= 0.94 * 30.04\,\text{m/s} = 28.34\,\text{m/s},\\ :.D &= \frac{1}{6} * 0.05 * \frac{28.34 + 1.6224}{2} = 0.1248\,\text{m}^2/\text{s}.\end{aligned} \tag{19}$$

But,

$$\begin{aligned}Na &= D\frac{\partial C}{\partial z}\\ :.D &\propto \frac{1}{C}\ \left(\text{molecular diffusivity is inversely proportional to change in concentration}\right),\\ \Longrightarrow DC &= k\\ \Longrightarrow D_1C_1 &= D_2C_2,\end{aligned} \tag{20}$$

where D_1 = diffusivity associated with 100% concentration, D_2 = diffusivity associated with 6% concentration, C_1 = mix concentration, C_2 = sand concentration, $D_1 = 0.2378\,\text{m}^2/\text{s}$, $C_1 = 100\%$, $D_2 = D_e$, and $C_2 = 6\%$.

If $D_2 = D_e$, then

$$D_e\ (\text{effective diffusivity}) = \frac{0.1248}{0.06} = 2.08\,\text{m}^2/\text{s}. \tag{21}$$

(ii) Correlation for Eddy Diffusivities. Based on the work of Escobedo and Mansoori [12], within the limit of the sublaminar layer of fluid, $r^+ \le 5$ and the eddy diffusivity was evaluated using

$$\varepsilon_1 = \left(\frac{(r^+)}{11.15}\right)^3 * \upsilon, \tag{22}$$

where r^+ = dimensionless radial distance, $r < r^+ < r^\infty$, and by averaging we have $(r \to r^\infty)/2 = (0 + 5)/2 = 2.5$.

The reason for averaging is because solid particles were assumed to be of the same shape and size and, for such particles in a region of flow, it is easy to evaluate the mean mix velocity and hence the mean dimensionless radial distance.

But,

$$\upsilon = \frac{\mu}{\rho_f}, \tag{23}$$

where υ = kinematic viscosity, μ = dynamic viscosity, and ρ_f = fluid density. Consider

$$\begin{aligned}\mu &= 0.0971\,\text{kg/m}\cdot\text{s},\\ \rho_f &= 784.43\,\text{kg/m}^3,\\ \varepsilon_1 &= \left(\frac{(2.5)}{11.15}\right)^3 * \frac{0.0971}{984.43} = 0.000001395\,\text{m}^2/\text{s}.\end{aligned} \tag{24}$$

For the Buffer region,

$$\varepsilon_2 = \left(\left(\frac{(r^+)}{11.4}\right)^2 - 0.1923\right)\upsilon. \tag{25}$$

Here, $5 \le r^+ \le 30$:

$$r^+ = \frac{5 + 30}{2} = 17.5 \quad (\text{calculated average value}). \tag{26}$$

The $\varepsilon_3 = (((17.5)/11.4)^2 - 0.1923) * 0.0972/784.43 = 0.000267893\,\text{m}^2/\text{s}$.

In the turbulent core region,

$$\varepsilon_3 = \frac{(0.4r^+)}{1} * \upsilon. \tag{27}$$

Here, $r^+ \ge 30$.

Considering the whole range, $r^+ \le 30$ and $r^+ \ge 30$, the least value for r^+ that can be obtained within the turbulent region is 30. Hence, an average value was arbitrarily obtained. If the entire radial distance lies between 0 and 100, for particles in a region of flow, the average value for the turbulent core should be 50, but, other average values obtained have decimal parts of 0.5; hence, this value was reduced by 0.5

$$\varepsilon_3 = \left(\frac{(0.4 * 49.5)}{1} * \frac{0.0971}{784.43}\right) = 0.002450926\,\text{m}^2/\text{s}. \tag{28}$$

Now

$$\begin{aligned}\varepsilon_T &= \varepsilon_1 + \varepsilon_2 + \varepsilon_3\\ &= 0.000001395 + 0.000267893 + 0.002450926\\ &= 0.00272\,\text{m}^2/\text{s},\\ D_T &= D_e + \varepsilon_T = (2.08 + 0.00272)\ \text{m}^2/\text{s} = 2.08272\,\text{m}^2/\text{s}.\end{aligned} \tag{29}$$

Consider total diffusivity = sum of molecular and eddy diffusivities.

2.7. Closure Problem Resolution (Ensuring Zero Degree of Freedom)

Note. The new model would not have a solution because it consists of five equations with eight unknown variables (ϕ, σ, ϕ', ε, w_s, $w_f P_s$, P_f). However, three constitutive equations were introduced in order to resolve the closure problem. They are

$$\begin{aligned}&\text{(i)}\ \ \sigma + \phi = \Psi,\\ &\text{(ii)}\ \ \Psi + \varepsilon = 1,\ \text{and}\\ &\text{(iii)}\ \ P_s = P_i - P.\end{aligned} \tag{30}$$

Considering Taylor's series expansion form of the force equation for solid phase and substituting $P_s = P_i - P$, results are generated from simulation.

Table 1: Values and variables used in the model.

Parameter	Field value	Scaled value
Sand and oil nominal velocities	(27.04 & 30.04) cm/s	(27.04 & 30.04) cm/s
Choke size	20%	20%
Base sediment and water	14.64%	14.64%
Tubing oil volume	802.52 m^3/d	802.52 m^3/d
Tubing head temperature	95°C	95°C
Tubing bottom temperature	80.33°C	80.33°C
Tubing head pressure	245.7 bars	245.7 bars
Produced water flow rate	182.6 m^3/d	182.6 m^3/d
Sand diameter	150–200 microns	0.05 m
Mass flow rate of sand	$5.44E-05$ g/s	544 g/s
Sand density	1705.44 kg/m^3	1705.44 kg/m^3
Oil viscosity	0.0971 kg/m·s	0.0971 kg/m·s
Pipe diameter	5.44 inches (0.14 m)	0.10 m

Table 2: Similarities and differences between the new model and Doan et al.'s model.

Serial number	Condition	Doan et al. models	The modified Doan et al. model
(1)	Mass transport	1 solid phase equation + 1 liquid phase equation without eddies	2 solid phase equations + 1 liquid phase equation which include eddy parameter
(2)	Momentum transport	Including a fluid parameter and suspension parameter for sand	Including a fluid parameter and suspension and deposition parameters for sand
(3)	Mathematical solution	Considering molecular diffusivity term in its solution	Including both molecular and eddy diffusivity terms in the solution
(4)	Number of equations	(4)	(5)

2.8. Finite Difference Method. Difference formulae were generated by first applying Fick's equation for diffusion to the mass conservation equations in order to proffer solution to the model in terms of oil recovery:

$$\phi_i^{,l+1} = \lambda\left(\phi_i^{,l+1} - 2\phi_i^{,l} + \phi_{i-1}^{,l}\right) + \phi_i^{,l} \quad \text{(solid phase)},$$
$$\varepsilon_i^{l+1} = \lambda\left(\varepsilon_i^{l+1} - 2\varepsilon_i^{l} + \varepsilon_{i-1}^{l}\right) + \varepsilon_i^{l} \quad \text{(fluid phase)}, \tag{31}$$

where

$$\phi' = \phi w_s,$$
$$\varepsilon' = \varepsilon w_f, \qquad \lambda = -\frac{D_T \Delta t}{2\Delta z^2}, \tag{32}$$

where D_T = total diffusivity, Δt = time change, and Δz = change in axial distance.

3. Model Validation

The simulation runs were carried out using data in Table 1 within boundary conditions; that is, sand concentrations at the inlet and outlet are 0.06 and 0.03, respectively, while oil concentrations are 0.94 and 0.97 at the pipe inlet and exit, respectively:

gross oil in barrels per day = 7,419 bbl/d,

net oil in barrels per day = 6,082 bbl/d,

1 barrel of oil = 0.158987 m^3/d.

After 24 hrs, the simulation gave inlet concentration of 0.94 and exit concentration of 0.931495.

Mass flow rate of oil = volume flow rate of oil ∗ density of oil:

$$\frac{\text{measured value} - \text{calculated value}}{\text{measured value}} * 100\% = \%\ \text{error}. \tag{33}$$

The data provided were well substituted into Taylor's formulae obtained for the conservation equations in order to determine the inlet and outlet mass and volume flow rates alongside their corresponding errors. At the inlet and outlet, the model gave an accuracy of 85% and 97%, respectively, for compared values (i.e., measured against the calculated value) of mass flow rates of oil while the compared outlet mass and volume flow rates of oil yielded 97% accuracy each.

4. Results

Table 2 shows similarities and contrasts between the Doan et al. models and the new model.

Table 3: Field values and calculated mass and volume flow rates of oil.

Position	Measured value	Calculated value	% error
Inlet mass flow rate of oil	10.47 kg/s	9.144 kg/s	−14.86%
Inlet volume flow rate of oil	0.0117 m^3/s	0.0137 m^3/s	−3.08%
Outlet mass flow rate of oil	8.79 kg/s	9.061 kg/s	−14.6%
Outlet volume flow rate of oil	0.0112 m^3/s	0.01155 m^3/s	−3.13%

5. Discussion of Results

The new model includes a third equation and a deposition term and incorporates the effect of eddies in its difference formulae as contained in Table 2. The additional equation, the deposition term, and the incorporation of eddies are useful for estimating the sand deposit concentration within the pipe and to take care of the forces imposed on the particles by the convectional currents of the turbulent stream. At the end of 24 hrs, the oil influx at the pipe inlet in barrels per day was 7,419 bbl/d. The equivalent mass flow rate of oil is 10.47 kg/s. From the simulation, the calculated mass flow rate of oil is 9.144 kg/s yielding an error of 14.86% when compared with the measured value as shown in Table 3. The estimated error confirms that the model's accuracy in terms of quantifying oil influx is about 85%. Also, at the inlet, the measured and calculated oil volume flow rates, 0.0117 m^3/s and 0.0137 m^3/s, respectively, give a difference of 0.002 m^3/s whose error estimate is −14.6%. This error estimate reveals that the model is 85% accurate in terms of quantifying oil volume inflow. Considering the pipe exit, the measured and calculated mass flow rates of oil are 8.79 kg/s and 9.06 kg/s, respectively, which give a difference of 0.371 kg/s with a corresponding error of −3.08% while the measured and calculated volume flow rates are 0.0112 m^3/s and 0.01155 m^3/s, respectively, giving a difference of 0.00035 m^3/s with a corresponding error of −3.13% (see Table 3). The exit estimates for both cases (mass and volume flow rate) prove the new model to be about 97% accurate. The difference in percent accuracies between the inlet and exit may be due to back-push or drawback on the stream at the elbow joint where the stream strikes the pipe before it goes in. Also, crude oil contains gases (compressible) which may cause a change in density of the stream. Since (mass) $m = v * \rho$ (product of volume and density), it implies that higher volume flows correspond to reduced densities and vice versa. Furthermore, the model predictions from the mass conservation equations reveal that the new model is valid and suitable for turbulent transport operations of crude oil and sand in a horizontal pipe.

6. Conclusions

The following conclusions were offered for this study.

(i) A model has been developed that describes laminar and turbulent transport of sand and oil through horizontal pipes.

(ii) The model's accuracy reveals that the new model can be used to quantify oil recovery.

(iii) The model can serve as an alternative sand management tool.

Nomenclature

A: Cross-sectional area (m^2)
g: Gravitational acceleration ($m\,s^{-2}$)
P_f: Fluid phase pressure ($kg\,m^{-1}\,s^{-2}$)
P_k: Kinematic pressure ($kg\,m^{-1}\,s^{-2}$)
P_s: Solid phase pressure ($kg\,m^{-1}\,s^{-2}$)
q_f: Volume flow rate of oil ($m^3\,s^{-1}$)
q_s: Volume flow rate of sand ($m^3\,s^{-1}$)
t: Time (hrs or s)
v_m: Volume of mix symbol adopted is ideal (m^3)
w_f: Oil velocity ($m\,s^{-1}$)
w_s: Sand velocity ($m\,s^{-1}$)
z: Axial distance (m)
L: Pipe length (m)
β: Fluid-particle interaction coefficient ($kg\,m^3\,s^{-1}$)
Δz: Change in length (m)
ε: Oil concentration (volume fraction) (–)
ϕ: Suspended sand concentration (volume fraction) (–)
ρ_f: Oil density (kg/m^3)
ρ_s: Sand density (kg/m^3)
σ: Sand deposit concentration (–)
F_{in}: Force flow into system ($kg\,m/s^2$)
F_{out}: Force flow out of system ($kg\,m/s^2$).

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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