Research Article

Generation Method of Multipiecewise Linear Chaotic Systems Based on the Heteroclinic Shil'nikov Theorem and Switching Control

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Based on the heteroclinic Shil'nikov theorem and switching control, a kind of multipiecewise linear chaotic system is constructed in this paper. Firstly, two fundamental linear systems are constructed via linearization of a chaotic system at its two equilibrium points. Secondly, a two-piecewise linear chaotic system which satisfies the Shil'nikov theorem is generated by constructing heteroclinic loop between equilibrium points of the two fundamental systems by switching control. Finally, another multipiecewise linear chaotic system that also satisfies the Shil'nikov theorem is obtained via alternate translation of the two fundamental linear systems and heteroclinic loop construction of adjacent equilibria for the multipiecewise linear system. Some basic dynamical characteristics, including divergence, Lyapunov exponents, and bifurcation diagrams of the constructed systems, are analyzed. Meanwhile, computer simulation and circuit design are used for the proposed chaotic systems, and they are demonstrated to be effective for the method of chaos anticontrol.

1. Introduction

In 1994, Schiff et al. first put forward the concept of chaos anticontrol [1], which involves discrete-time and continuous-time system for the chaos anticontrol. In the study of chaos anticontrol for continuous-time system, a lot of progress has been made and some methods of generating chaotic and hyperchaotic systems were found, such as the methods of time-delay feedback to chaos [2–4], topological conjugate to chaos [5], pulse control to chaos [6], parameter perturbation control to hyperchaos [7–9], and state feedback control to hyperchaos [10–13]. However, these methods of chaos anticontrol, which are based on the parameter “try,” numerical simulation, and the Lyapunov index calculation, rely on the experience, lacking more theoretical basis. Therefore, it is still a challenge to construct a general approach of generating chaotic system.

Shil'nikov theorem is a beneficial tool to analyze the chaotic behavior of a nonlinear system [14], and it is also one of the most common decision theorems to determine whether chaos exists or not.

In 2004, Zhou et al. firstly constructed a three-dimensional autonomous chaotic system which has only one equilibrium point [15] by using the homoclinic orbit Shil'nikov theorem. Then, Li and Chen proposed a method to construct a two-piecewise linear chaotic system by using the heteroclinic Shil'nikov theorem [16, 17]. Recently, Yu et al. also proposed a method for constructing multipiecewise linear chaotic systems [18, 19]. However, the above studies lack further theoretical analysis and appropriate experimental verification.

In this paper, we use the heteroclinic Shil'nikov theorem to construct a kind of chaotic systems with both nonlinearities of two-piecewise linear and multipiecewise linear based

on switching control method. Some basic dynamical characteristics of the constructed systems are analyzed theoretically and verified experimentally. Results of simulations and circuit experiments demonstrate the validity for the method of chaos anticontrol.

2. Review of the Shil'nikov Theorem

Consider the following three-dimensional autonomous dynamical system:

$$\frac{dx}{dt} = f(x), \quad t \in R, \quad x \in R^3, \quad (1)$$

where $f : R^3 \rightarrow R^3$ belongs to C^r ($r \geq 2$) and $x_e \in R^3$ is a hyperbolic saddle focus; that is, the eigenvalues of Jacobian $J = Df(x_e)$ are γ and $\sigma \pm j\omega$, where $\gamma\sigma < 0$, $\omega \neq 0$, γ, σ , and ω are all real constants.

Theorem 1 (the heteroclinic Shil'nikov theorem) (see [14]). *Given the three-dimensional autonomous system shown in (1), let x_{e1} and x_{e2} be two distinct equilibrium points for (1). Suppose the following:*

- (1) Both x_{e1} and x_{e2} are saddle foci that satisfy the Shil'nikov inequality $|\gamma_i| > |\sigma_i| > 0$ ($i = 1, 2$) with the further constraint $\gamma_1\gamma_2 > 0$ or $\sigma_1\sigma_2 > 0$.
- (2) There is a heteroclinic loop, H_l , in which it joins x_{e1} to x_{e2} and is made up of two heteroclinic orbits H_i ($i = 1, 2$).

Both the original system (1) and its perturbed varieties exhibit the Smale horseshoe chaos.

In the following study, we will use the switching controller to construct piecewise linear chaotic systems based on this theorem.

3. Two Fundamental Linear Systems

For simplicity, we use the well-known Lorenz system to generate fundamental linear systems which are used to construct the piecewise linear chaotic systems.

The Lorenz system is described by

$$\begin{aligned} \dot{x} &= -a(x - y), \\ \dot{y} &= bx - xz - y, \\ \dot{z} &= -cz + xy, \end{aligned} \quad (2)$$

where $a = 10$, $b = 55$, and $c = 5/3$. The equilibrium points of (2) are given as

$$\begin{aligned} &O(0, 0, 0), \\ &P^+ \left(\sqrt{c(b-1)}, \sqrt{c(b-1)}, (b-1) \right), \\ &P^- \left(-\sqrt{c(b-1)}, -\sqrt{c(b-1)}, (b-1) \right). \end{aligned} \quad (3)$$

From (3), we can see that the equilibrium points P^+ and P^- have certain symmetry. By linearizing (2) at the

equilibrium points P^+ and P^- , the linear equations are obtained, respectively, as follows:

$$\begin{aligned} &\begin{pmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{z}_1 \end{pmatrix} \\ &= \begin{pmatrix} -a & a & 0 \\ 1 & -1 & -\sqrt{c(b-1)} \\ \sqrt{c(b-1)} & \sqrt{c(b-1)} & -c \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \quad (4) \\ &= J_1 X_1, \end{aligned}$$

$$\begin{aligned} &\begin{pmatrix} \dot{x}_2 \\ \dot{y}_2 \\ \dot{z}_2 \end{pmatrix} \\ &= \begin{pmatrix} -a & a & 0 \\ 1 & -1 & \sqrt{c(b-1)} \\ -\sqrt{c(b-1)} & -\sqrt{c(b-1)} & -c \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} \quad (5) \\ &= J_2 X_2. \end{aligned}$$

Equations (4) and (5) are the required fundamental linear systems which are used to construct a piecewise linear chaotic system. The equilibrium points of the two linear systems are $O_1(0, 0, 0)$ and $O_2(0, 0, 0)$, and the corresponding eigenvalues are: $\gamma_1 = \gamma_2 = -14.0640$ and $\sigma_1 \pm j\omega_1 = \sigma_2 \pm j\omega_2 = 0.6987 \pm j11.2915$. It can be seen that the equilibrium points $O_1(0, 0, 0)$ and $O_2(0, 0, 0)$ are all the saddle foci of index 2 and satisfy $|\gamma_i| > |\sigma_i| > 0$ ($i = 1, 2$). According to the heteroclinic Shil'nikov theorem, there is chaos in the sense of Smale horseshoe as long as the equilibrium points of system (4) and system (5) are connected with the heteroclinic loop via a certain way.

The eigenvectors corresponding to the equilibrium points $O_1(0, 0, 0)$ and $O_2(0, 0, 0)$ of systems (4) and (5) are, respectively,

$$\begin{aligned} \mu_1 &= \begin{pmatrix} 0.8539 \\ -0.3470 \\ -0.3879 \end{pmatrix}, \\ \eta_1 &= \begin{pmatrix} -0.2421 \\ 0.0617 \\ -0.7236 \end{pmatrix} \pm j \begin{pmatrix} 0.2840 \\ 0.5773 \\ 0 \end{pmatrix}, \\ \mu_2 &= \begin{pmatrix} 0.8539 \\ -0.3470 \\ 0.3879 \end{pmatrix}, \\ \eta_2 &= \begin{pmatrix} -0.2421 \\ 0.0617 \\ 0.7236 \end{pmatrix} \pm j \begin{pmatrix} 0.2840 \\ 0.5773 \\ 0 \end{pmatrix}. \end{aligned} \quad (6)$$

At point $O_1(0,0,0)$, the mathematical expressions of the one-dimensional stable manifold $E^S(O_1)$ and the two-dimensional unstable manifold $E^U(O_1)$ are given as

$$\begin{aligned} E^S(O_1): \quad \frac{x}{l_1} &= \frac{y}{m_1} = \frac{z}{n_1}, \\ E^U(O_1): \quad A_1x + B_1y + C_1z &= 0, \end{aligned} \quad (7)$$

where the direction vectors of the stable manifold $E^S(O_1)$ are $l_1 = 0.8539$, $m_1 = -0.3470$, and $n_1 = -0.3879$ and the direction vectors of the unstable manifold $E^U(O_1)$ are given by $A_1 = 0.4177$, $B_1 = -0.2055$, and $C_1 = -0.1573$.

Similarly, the space straight line $E^S(O_2)$ and the space plane $E^U(O_2)$, which, respectively, correspond to the one-dimensional stable manifolds and the two-dimensional unstable manifolds of equilibrium $O_2(0, 0, 0)$ are described as

$$\begin{aligned} E^S(O_2): \quad \frac{x}{l_2} &= \frac{y}{m_2} = \frac{z}{n_2}, \\ E^U(O_2): \quad A_2x + B_2y + C_2z &= 0, \end{aligned} \quad (8)$$

where the direction vectors of the stable manifold $E^S(O_2)$ are $l_2 = 0.8539$, $m_2 = -0.3470$, and $n_2 = 0.3879$ and those of unstable manifold $E^U(O_2)$ are $A_2 = -0.4177$, $B_2 = 0.2055$, and $C_2 = -0.1573$.

4. Construction of a Two-Piecewise Linear Chaotic System

Next, we will study the existence conditions of the heteroclinic loop connecting the equilibrium points of system (4) and system (5), choosing the controller as $F(x, y, z)$ and the switching plane as $S = \{(x, y, z) \mid x = 0\}$.

After translational transform for equilibrium points $O_1(0, 0, 0)$ and $O_2 = (0, 0, 0)$, systems (4) and (5) can be changed into the following form:

$$\begin{aligned} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} &= \begin{pmatrix} -a & a & 0 \\ 1 & -1 & -\sqrt{c(b-1)} \\ \sqrt{c(b-1)} & \sqrt{c(b-1)} & -c \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \\ &- F(x, y, z) = J_1 \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - F(x, y, z) \right), \\ V \in V_1 &= \{(x, y, z) \mid x > 0\}, \end{aligned}$$

FIGURE 1: The eigenspaces corresponding to P_1 and P_2 .

$$\begin{aligned} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} &= \begin{pmatrix} -a & a & 0 \\ 1 & -1 & \sqrt{c(b-1)} \\ -\sqrt{c(b-1)} & -\sqrt{c(b-1)} & -c \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \\ &- F(x, y, z) = J_2 \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - F(x, y, z) \right), \\ V \in V_2 &= \{(x, y, z) \mid x < 0\}, \end{aligned} \quad (9)$$

where $F(x, y, z) = [f_1(x, y, z), f_2(x, y, z), f_3(x, y, z)]^T$ is the switching controller whose concrete form is determined by the conditions of forming heteroclinic loops for system (9).

Let the equilibrium points of system (9) be $P_1(x_1, y_1, z_1) \in V_1$ and $P_2(x_2, y_2, z_2) \in V_2$. At the equilibria P_1 and P_2 , the space straight lines, $E^S(P_1)$ and $E^S(P_2)$, and the space planes, $E^U(P_1)$ and $E^U(P_2)$, are shown in Figure 1.

From (7), the space straight line equations of one-dimensional stable manifold and space plane equation at equilibrium point $P_1(x_1, y_1, z_1) \in V_1$ are obtained as

$$\begin{aligned} E^S(P_1): \quad \frac{x - x_1}{l_1} &= \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}, \\ E^U(P_1): \quad A_1(x - x_1) + B_1(y - y_1) + C_1(z - z_1) &= 0. \end{aligned} \quad (10)$$

Similarly, from (8), we can get the space straight line equations of one-dimensional stable manifold and space plan equation at equilibrium point $P_2(x_2, y_2, z_2) \in V_2$:

$$\begin{aligned} E^S(P_2): \quad \frac{x-x_2}{l_2} &= \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}, \\ E^U(P_2): \quad & \\ A_2(x-x_2) + B_2(y-y_2) + C_2(z-z_2) &= 0. \end{aligned} \quad (11)$$

Let M_1 and M_2 (L_1 and L_2) be the intersection points (lines) of $E^S(P_1)$, $E^S(P_2)$, ($E^U(P_1)$ and $E^U(P_2)$) with the switching plane $S = \{(x, y, z) \mid x = 0\}$; that is,

$$\begin{aligned} M_1 &= E^S(P_1) \cap S: \left(0, y_1 - \frac{m_1}{l_1}x_1, z_1 - \frac{n_1}{l_1}x_1\right), \\ M_2 &= E^S(P_2) \cap S: \left(0, y_2 - \frac{m_2}{l_2}x_2, z_2 - \frac{n_2}{l_2}x_2\right), \\ L_1 &= E^U(P_1) \cap S: \\ &\begin{cases} A_1(x-x_1) + B_1(y-y_1) + C_1(z-z_1) = 0 \\ x = 0, \end{cases} \quad (12) \\ L_2 &= E^U(P_2) \cap S: \\ &\begin{cases} A_2(x-x_2) + B_2(y-y_2) + C_2(z-z_2) = 0 \\ x = 0. \end{cases} \end{aligned}$$

If M_1 lies on L_2 , there exists a heteroclinic orbit $H_1 = E^U(P_2) \cup M_1 \cup E^S(P_1)$ from P_2 to P_1 . Similarly, if M_2 lies on L_1 , then there exists a heteroclinic orbit $H_2 = E^U(P_1) \cup M_2 \cup E^S(P_2)$ from P_1 to P_2 . If the above two conditions are satisfied simultaneously, there must exist a heteroclinic loop, which consists of heteroclinic orbits H_1 and H_2 and connects P_1 with P_2 . According to the heteroclinic Shil'nikov theorem, system (9) exhibits the Smale horseshoe chaos.

In the following parts, the conditions that the coordinates of the equilibrium point must meet will be analyzed when the heteroclinic loop of system (9) exists. From the invariance of transformation $(x, y, z) \rightarrow (-x, -y, z)$, one can assume that the coordinates of the equilibria $P_1 = (x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ of system (9) satisfy the following formula:

$$\begin{aligned} x_1 &= -x_2 = x_0, \\ y_1 &= -y_2 = y_0, \\ z_1 &= z_2 = z_0. \end{aligned} \quad (13)$$

From (12) and (13), one can get the necessary conditions of $M_1 \in L_2$ and $M_2 \in L_1$ as

$$\frac{A_2 l_1 - B_2 m_1 - C_2 n_1}{B_2 l_1} = \frac{A_1 l_2 - B_1 m_2 - C_1 n_2}{B_1 l_2}. \quad (14)$$

When the parameters A_i, B_i, C_i, l_i, m_i , and n_i ($i = 1, 2$) satisfy (14), from (12) and (13), y_0 can be obtained:

$$\begin{aligned} y_0 &= \frac{(A_2 l_1 - B_2 m_1 - C_2 n_1) x_0}{2 B_2 l_1} \\ &= -\frac{(A_1 l_2 - B_1 m_2 - C_1 n_2) x_0}{2 B_1 l_2}. \end{aligned} \quad (15)$$

Equation (15) indicates that y_0 depends on the value of x_0 but is not associated with z_0 that can take any value. Here, let $x_1 = -x_2 = x_0 = 1$ and $z_1 = z_2 = z_0 = 0$; we can get $y_0 = 0.9870$. And when a heteroclinic loop of system (9) exists, the coordinates of the equilibria $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$ of system (9) are

$$\begin{aligned} P_1 &= (x_1, y_1, z_1) = (x_0, y_0, z_0) = (1, 0.9870, 0), \\ P_2 &= (x_2, y_2, z_2) = (-x_0, -y_0, z_0) = (-1, -0.9870, 0). \end{aligned} \quad (16)$$

To make the coordinates of the equilibria of system (9) satisfy (16), the controller of system (9) should be selected as following form:

$$F(x, y, z) = \begin{pmatrix} f_1(x, y, z) \\ f_2(x, y, z) \\ f_3(x, y, z) \end{pmatrix} = \begin{pmatrix} x_0 \operatorname{sgn}(x) \\ y_0 \operatorname{sgn}(x) \\ z_0 \operatorname{sgn}(z) \end{pmatrix}, \quad (17)$$

where $x_0 = 1$, $y_0 = 0.9870$, and $z_0 = 0$.

According to the above analysis and by introducing switching function $T = T(x, y, z) = \operatorname{sgn}(x)$ to (9), it is expressed that

$$\begin{aligned} &\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} \\ &= \begin{pmatrix} -a & a & 0 \\ 1 & -1 & -T\sqrt{c(b-1)} \\ T\sqrt{c(b-1)} & T\sqrt{c(b-1)} & -c \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \\ &\quad - F(x, y, z). \end{aligned} \quad (18)$$

Equation (18) is the expected two-piecewise linear chaotic system, which has a double-wing chaotic attractor, shown in Figure 2.

Based on (15), we can construct a multipiecewise linear chaotic system via alternate translational transform of the two fundamental linear systems and heteroclinic loop construction of adjacent equilibria for the multipiecewise linear system.

FIGURE 2: Two-wing chaotic attractor of system (18). (a) x - z plane projection. (b) y - z plane projection.

The mathematical expression for the multilinear chaotic systems is described as

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} -a & a & 0 \\ 1 & -1 & -T\sqrt{c(b-1)} \\ T\sqrt{c(b-1)} & T\sqrt{c(b-1)} & -c \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} - F(x, y, z), \quad (19)$$

where

$$F(x, y, z) = \begin{pmatrix} f_1(x, y, z) \\ f_2(x, y, z) \\ f_3(x, y, z) \end{pmatrix} = \begin{pmatrix} x_0 \operatorname{sgn}(x) + \sum_{m=1}^M x_0 [\operatorname{sgn}(x + 2mx_0) + \operatorname{sgn}(x - 2mx_0)] \\ y_0 \operatorname{sgn}(y) + \sum_{m=1}^M y_0 [\operatorname{sgn}(y + 2my_0) + \operatorname{sgn}(y - 2my_0)] \\ z_0 \operatorname{sgn}(z) + \sum_{n=1}^N z_0 [\operatorname{sgn}(z + 2nz_0) + \operatorname{sgn}(z - 2nz_0)] \end{pmatrix}, \quad (20)$$

$$T = T(x, y, z)$$

$$= \operatorname{sgn}(x) + \sum_{m=1}^M -1^m [\operatorname{sgn}(x + 2mx_0) + \operatorname{sgn}(x - 2mx_0)].$$

When $z_0 = 0$ and $f_3(x, y, z) = 0$, system (19) can generate a chaotic attractor with $(2M + 2)$ -wings, while when $z_0 \neq 0$, a chaotic attractor with $(2M + 2) \times (2N + 2)$ -grid wings can be obtained. Figure 3(a) shows a 6-wing chaotic attractor with $x_0 = 1$, $y_0 = 0.9870$, $z_0 = 0$, and $M = 2$. When $x_0 = 1$,

$y_0 = 0.9870$, $z_0 = 1.6$, $M = 2$, and $N = 0$, a 6×2 -grid wing chaotic attractor is found, which is shown in Figure 3(b).

5. Basic Dynamics of Two-Piecewise Linear Chaotic System

In this section, some basic dynamics of system (18) will be analyzed, including the dissipation, the Lyapunov exponents, and the bifurcation diagram.

5.1. Dissipation of System. It is well known that a chaotic system is a dissipative system. According to (17) and (18), we can get the dissipation character of the two-piecewise linear system from the following equation:

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -a - 1 - c = -\frac{38}{3} < 0. \quad (21)$$

5.2. Lyapunov Exponent Spectrum and Bifurcation Diagram. Lyapunov exponent is the most direct evidence to determine a chaotic system. Figure 4 shows the Lyapunov exponent spectrum versus parameter a in system (18), from which we can see one of the Lyapunov exponents is always positive when the parameter a changes in a large range, indicating that system (18) is chaotic. Larger range of chaotic parameter can provide larger space of key which can increase the difficulty of unmasking signals and improve the security of communications. Therefore, system (18) has significance if it is applied to secure communications as pseudorandom signal source.

Figure 5 shows the bifurcation diagram versus parameter a of system (18), from which we can see that system (18) is chaotic apart from a few tiny periodic windows. From Figure 5, we also can see the road of system (18) to chaos. System (18) evolves into chaotic state not through period-doubling bifurcation but directly from stable state or from periodic motion, while the system parameters vary.

FIGURE 3: The x - z plane projections of multiwing and grid wing chaotic attractors of system (19). (a) Six-wing chaotic attractor. (b) 6×2 -grid wing chaotic attractor.

FIGURE 4: The diagram of LE versus parameter a of system (18).

6. Circuit Implementation of the Piecewise Linear Chaotic System

According to (19)-(20), a circuit of realizing system (19) is shown in Figure 6, which can generate 6-wing chaotic attractor and 6×2 -grid wing chaotic attractor. The modular circuits for implementing functions $\pm(x - f_1)$, $\pm(y - f_2)$, $(z - f_3)$, and T are designed and shown in Figure 7, which consist of adders, inverters, multipliers, and integrators. In the modular circuits, the model of operational amplifiers is TL082 and that of multipliers is AD633.

According to Figure 6, the circuit equation of system (19) can be obtained:

$$\frac{dx}{dt} = \frac{1}{RC} \left[-\frac{R}{R_1} (x - f_1) + \frac{R}{R_2} (y - f_2) \right],$$

$$\frac{dy}{dt} = \frac{1}{RC} \left[\frac{R}{R_3} (x - f_1) - \frac{R}{R_4} (y - f_2) - T \frac{R}{10R_5} (z - f_3) \right],$$

$$\frac{dz}{dt} = \frac{1}{RC} \left[T \frac{R}{10R_6} (x - f_1) + T \frac{R}{10R_7} (y - f_2) - \frac{R}{R_8} (z - f_3) \right].$$

(22)

Let $t = 10RCt'$; one has

$$\frac{dx}{10RCdt'} = \frac{1}{RC} \left[-\frac{R}{R_1} (x - f_1) + \frac{R}{R_2} (y - f_2) \right],$$

$$\frac{dy}{10RCdt'} = \frac{1}{RC} \left[\frac{R}{R_3} (x - f_1) - \frac{R}{R_4} (y - f_2) \right.$$

$$\left. - T \frac{R}{10R_5} (z - f_3) \right],$$

(23)

$$\frac{dz}{10RCdt'} = \frac{1}{RC} \left[T \frac{R}{10R_6} (x - f_1) + T \frac{R}{10R_7} (y - f_2) \right.$$

$$\left. - \frac{R}{R_8} (z - f_3) \right].$$

Let again $t' = t$; (23) can be changed as follows:

$$\frac{dx}{dt} = 10 \left[-\frac{R}{R_1} (x - f_1) + \frac{R}{R_2} (y - f_2) \right],$$

$$\frac{dy}{dt} = 10 \left[\frac{R}{R_3} (x - f_1) - \frac{R}{R_4} (y - f_2) \right.$$

$$\left. - T \frac{R}{10R_5} (z - f_3) \right],$$

FIGURE 5: The bifurcation diagram versus parameter a of system (18). (a) a varies from 2 to 10. (b) a varies from 3.24 to 4.

FIGURE 6: Module-based circuit diagram for realizing system (19).

$$\frac{dz}{dt} = 10 \left[T \frac{R}{10R_6} (x - f_1) + T \frac{R}{10R_7} (y - f_2) - \frac{R}{R_8} (z - f_3) \right]. \quad (24)$$

$$\frac{R}{R_5} = \frac{R}{R_6} = \frac{R}{R_7} = \sqrt{c(b-1)},$$

$$\frac{10R}{R_8} = c.$$

(25)

Comparing (24) with (19), one has

$$\begin{aligned} \frac{10R}{R_1} &= \frac{10R}{R_2} = a, \\ \frac{10R}{R_3} &= \frac{10R}{R_4} = 1, \end{aligned}$$

When $a = 10$, $b = 55$, $c = 5/3$, and $R = 10 \text{ k}\Omega$, the resistance values in Figure 6 can be obtained:

$$\begin{aligned} R_1 &= R_2 = 10 \text{ k}\Omega, \\ R_3 &= R_4 = 100 \text{ k}\Omega, \\ R_5 &= R_6 = R_7 = 1.054 \text{ k}\Omega, \\ R_8 &= 60 \text{ k}\Omega. \end{aligned} \quad (26)$$

FIGURE 7: Module-based circuit diagram for realizing functions $\pm(x - f_1)$, $\pm(y - f_2)$, $\pm(z - f_3)$, and T .

FIGURE 8: Experimental observation of multiwing and grid wing chaotic attractors of system (19). (a) Six-wing chaotic attractor. (b) 6×2 -grid wing chaotic attractor.

When the capacitance $C = 10$ nF, the 6-wing chaotic attractors and 6×2 -grid wing chaotic attractors obtained observed from oscilloscope are shown in Figure 8.

7. Conclusion

Based on the heteroclinic Shil'nikov theorem, a method for constructing a kind of multipiecewise linear chaotic system has been proposed via switching control in this paper. By using this method, a two-piecewise linear and a multipiecewise linear chaotic systems are constructed.

It is worth pointing that this method of constructing chaotic system has general significance. It does not only suit Lorenz system in this paper but also can be applied to the three-dimensional autonomous system which has saddle focus equilibrium points $P_{1,2}(\pm x, \pm y, z)$ and satisfies $\gamma_1 \gamma_2 > 0$ or $\sigma_1 \sigma_2 > 0$ and $|\gamma_i| > |\sigma_i| > 0$ ($i = 1, 2$). In addition, because the circuit structure of implementing the multipiecewise linear chaotic system is simple and easy to implement, this chaos anticontrol method has more potential applications than other chaotic anticontrol methods of generating multiwings and multigrid wing chaotic attractors in engineering applications.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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