Research Article

Two Chaotic Patterns of Dynamic Risk Definition for Solving Hazardous Materials Routing Problem

Abbas Mahmoudabadi

Department of Industrial Engineering, Mehrstan University, Kisom Road, Astaneh-ye Ashrafiyeh, Gilan, Iran

Correspondence should be addressed to Abbas Mahmoudabadi; mahmoudabadi@phd.pnu.ac.ir

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In the case of determining routes for hazardous material transportation, risk is considered as a main attribute. Transport risk, which is usually combined with other attributes such as cost or travel time, plays a significant role in determining paths for hazardous materials transportation. Since, risk is chaotically affected by road incidents, decision makers are dealing with selecting a method for defining chaotic risk factors in hazmat transportation. In this paper, transport risk has been defined as a chaotic variable using two different methods of generating chaotic patterns. In an experimental road network, which consists of eighty-nine nodes and one hundred and one two-way links, two different methods of generating chaotic variables have been used for applying the proposed procedure. In addition, results for different amounts of risk and cost have also been analyzed in case study. Results revealed that different cost and risk priorities change the frequencies of selected paths determined for hazmat transportation, but the route convergence of the route to chaos method is better than that of the logistic map equation.

1. Introduction

1.1. Hazmat Routing Problem. Hazardous materials (hazmat for short) which are classified into nine classes of industrial and natural substances [1] are involved in a large part of the economic activities, not only in industrialized [2] but also in developing countries [3]. Management of hazardous material is usually considered as an extremely complex issue involving a multitude of environmental, engineering, economic, social, and political concerns [4] due to their incident impacts. One of the most important issues in hazardous material management is to find the best path for transportation which is known as hazmat routing problem (HRP). HRP is usually a double-sided consideration problem in which local authorities are interested in minimizing public risk and carriers are dealing with minimizing transport cost [5]. In this case, a combination of risk and cost is usually used for developing mathematical models in order to find the best path for hazmat transportation. Mathematical modeling result in this matter is not necessarily the shortest path. Relevant studies show that different combinations of risk and cost lead researchers to use various methods for solving HRP [6, 7] and different

priorities for risk and cost or travel time change results on the above-mentioned problem [3].

1.2. Chaos Theory. Edvard Lorenz introduced the concept of chaos theory in 1963. He found the chaotic attractions in complex system of weather forecasting when he entered different values as starting points in computer program [8]. Chaos has also been studied within the engineering scientific and mathematical communities and was found to be useful in many disciplines such as high-performance circuits and devices, collapse prevention of power systems, and also information processing [9]. While chaos theory is used to analyze complex systems and transportation systems are complex entities [10], it may be found useful for applying in the field of transportation. In transportation systems, considering legal and social constraints helps researchers more accurately predict human actions and system evolution [11, 12].

Following the utilization of chaos theory in transportation and road safety, it has been approved that the behavior of road incident over the road network is chaotic [3, 13] and showed that the ratio of the fatal accidents in a specific part of road network to all accidents has a chaotic behavior which

could be defined as a chaotic variable. The normal form of logistic map equation is used to generate dynamic risk factors over a year and applied the concept of chaos theory to define dynamic risk factors [3, 13] in which the Lyapunov characteristic exponent was calculated for checking the presence of chaos [13, 14].

There are some equations used for generating chaotic variables in complex systems. Logistic map [9] and route to chaos [15] equations are well known. Logistic map equation is formulated by the following equation, where $R(t)$ is the chaotic variable at time t and K_l is chaos parameter mainly set to 4 [9]:

$$R(t) = K_l \times R(t-1) \times (1 - R(t-1)) \quad (1)$$

$\forall t \in \text{Defined Time Interval.}$

Route to chaos equation is formulated by the following equation, where $R(t)$ is chaotic variable at time t and K_r is also defined as chaos parameter mainly set to approximate interval 1.0624–1.0807 [15]:

$$R(t) = K_r \times R(t-1)^2 + 0.3 \times R(t-2) \quad (2)$$

$\forall t \in \text{Defined Time Interval.}$

Considering different durations is the most important difference between the above equations. Logistic map equation uses only the previous time while route to chaos equation is dependent on the previous and the preprevious terms of chaotic variable.

1.3. Vision. Following the above sections, it is concluded that transport risk is an important attribute in hazardous material transportation planning [16]. On the other hand, recent studies show that the risk regarding the road accidents can be defined as a chaotic variable in terms of probability and hazmat incident impacts [3]. Further research also shows that chaotic behavior of risk can be used for defining risk when a part of road network has been affected by natural disasters such as flood, heavy rain, or earthquake [17]. Risk factors in the previous studies are mainly formulated using logistic map equation, so utilizing the other structures of chaotic variables is an attractive topic in this field. Therefore, the aim of this research work is to apply both logistic map and route to chaos equations for defining chaotic risk variables in HRP. As a main criterion, path convergence according to the most frequent path is considered in order to check chaotic generation performances of the above equations.

2. Defining Procedure

Because of the existing chaotic behavior in daily road accidents [13], risk factors, associated with the edges (links) of network, are different over a year [3] and different paths will be outlined for hazmat transportation, accordingly. Therefore, the proposed procedure is capable of considering different chaotic variables over the network. In order to cover a wide range of variations, the proposed procedure

FIGURE 1: Overall view of the proposed procedure.

is designed as an iterative based method for solving HRP in which a wide range of risk and cost priorities are simultaneously used to find the best path over a year. The total combination of risk and cost priorities is defined as criterion in HRP objective function. Risk factors are updated utilizing logistic map and route to chaos equations over a specific number of iterations (such as 365 days over a year). Different priorities of risk and cost are also considered in the proposed procedure. Termination is decided by both iteration number and combinations of risk and cost. In other words, the proposed procedure is terminated when the number of iterations is met for all combinations of risk and cost. The most frequent selected path is eventually outlined to be considered as the best path for HRP in terms of their repetitions. Figure 1 shows an overall view of the proposed procedure including six steps as follows.

- (1) Set network parameters including nodes, edges (links), uniform risk, and cost for each edge.
- (2) Set model parameters including number of iterations, starting points, and priorities for risk and cost.
- (3) Update dynamic risk factors for all edges to the next iteration (day) using logistic map and route to chaos equations.
- (4) Run mathematical model to find the best path for different priorities of risk and cost and keep them according to dynamic risks outlined by both generation equations.

- (5) If stopping criteria (the number of iterations and different priorities of risk and cost) are met, stop and go to Step 6; otherwise go back to Step 3.
- (6) Choose the most frequent paths and compare the results.

3. Developing Mathematical Model

Assume that the road network is defined by graph $G(n, e)$ including predefined nodes (n) and edges (e). Nodes are categorized in to three main sets including origin, destination, and connecting nodes. Following the concept defined in Section 2, mathematical model is formulated to minimize the total combinations of risk and cost over the network. The uniform amounts of risk and cost should be used because of the existing different scales of variables in the proposed mathematical model, so the following equation is used to convert data into closed interval [0.05 0.95]:

$$X_{\text{new}} = \frac{(X_{\text{old}} - X_{\text{min}})}{(X_{\text{max}} - X_{\text{min}})} * 0.9 + 0.05, \quad (3)$$

where X_{new} is the uniformed amount of variable X_{old} , while X_{max} and X_{min} are the maximum and minimum of variable X , respectively [3].

In formulation process, $X_{ij}(t)$ is defined as a binary variable. If $X_{ij}(t)$ is assigned by 1, link (i, j) is located in the selected path; otherwise it is assigned by 0. There are two attributes of cost and risk for each link defined as C_{ij} and R_{ij} , respectively. As risk is updated in each iteration, it corresponds to t notated by $R_{ij}(t)$. P_c and P_r are also, respectively, defined to cost and risk priorities. Following the above notations, objective function which defines the total amount combinations of cost and risk is formulated as (4).

The best path is the most frequent path, in which

$$\text{Min } Z = \sum_G [P_r \times R_{ij}(t) + P_c \times C_{ij}] \times X_{ij}(t) \quad (4)$$

$$\forall (i, j) \in G, \quad t = 1, 2, \dots, \text{Number of iterations,}$$

where t is iteration number.

There are some constraints in developing mathematical model. The main constraint is to keep continuous paths over the network. The concept can be formulated as [18]

$$\sum_G X_{ij}(t) - \sum_G X_{ji}(t) = \begin{cases} 1 & \text{if } j \text{ is an origin node} \\ -1 & \text{if } j \text{ is a destination node} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

$$\forall j \in G, \quad t = 1, 2, \dots, \text{Number of iterations.}$$

The other equations are regarding chaotic behavior of risk associated with the network links. At the first round, one-dimensional logistic map equation is used for updating risk in analytical process formulated by (6). In the second round, route to chaos equation is also used for generating chaotic

variables using (7). Both parameters K_l and K_r are defined as chaotic variables for the following equations:

$$R_{ij}(t) = K_l \times R_{ij}(t-1) \times (1 - R_{ij}(t-1)) \quad (6)$$

$$\forall t = 1, 2, \dots, \text{Number of iterations,}$$

$$R_{ij}(t) = K_r \times R_{ij}(t-1)^2 + 0.3 \times R_{ij}(t-2) \quad (7)$$

$$\forall t = 1, 2, \dots, \text{Number of iterations.}$$

Eventually, in intercity transportation, two-way edges are usually available, so the following equation is inserted in mathematical model:

$$(i, j), (j, i) \in G. \quad (8)$$

To summarize the above notations and descriptions, mathematical model is formulated as follows:

$$\text{Min } Z = \sum_G [P_r \times R_{ij}(t) + P_c \times C_{ij}] \times X_{ij}(t)$$

$$\forall (i, j) \in G, \quad t = 1, 2, \dots, \text{Number of iterations,}$$

$$\text{S.T. } \sum_G X_{ij}(t) - \sum_G X_{ji}(t) = \begin{cases} 1 & \text{if } j \text{ is an origin node} \\ -1 & \text{if } j \text{ is a destination node} \\ 0 & \text{otherwise} \end{cases}$$

$$\forall j \in G, \quad t = 1, 2, \dots, \text{Number of iterations,}$$

$$R_{ij}(t) = K_l \times R_{ij}(t-1) \times (1 - R_{ij}(t-1))$$

$$\forall t = 1, 2, \dots, \text{Number of iterations,}$$

$$R_{ij}(t) = K_r \times R_{ij}(t-1)^2 + 0.3 \times R_{ij}(t-2)$$

$$\forall t = 1, 2, \dots, \text{Number of iterations,}$$

$$(i, j), (j, i) \in G, \quad X_{ij} \text{ Binary variable.} \quad (9)$$

4. Case Study and Experimental Data

Iranian north-west part including six provinces of West Azarbayegan, East Azarbayegan, Ardabil, Kordestan, Zanzan, and Hamadan has been selected as case study. Selected network includes 89 nodes and 101 two-way links as shown in Figure 2. Some nodes are border nodes which connect research area to other provinces. Vehicles will be assigned to transport hazardous materials subject to minimizing the combination of risk and cost in which different amounts of priorities are used over the network. More details on risk components and their assessment method are available in [3].

The other issue which is regarding the problem definition is to define origin-destination pairs. Two sets of origin-destination pairs have been used to check the proposed procedure and develop mathematical model. The first set includes origin node 43 (Orumieh) and destination node 71 (Khodabandeh) and the second consists of origin node 33 (Ardabil) and destination node 67 (Sanandaj).

TABLE 2: Best and secondary optimal paths and their lengths from Ardabil (node 33) to Sanandaj (node 67).

Chaos equation	(Cost, risk)*	Solution ranking	Ranked most probable path	Length (KM)	F**
Logistic map	(0, 1)	Best optimal	33 34 37 38 39 40 42 44 45 50 51 52 53 59 60 65 85 67	783	74
		Secondary optimal	33 34 37 38 39 40 42 44 45 50 51 52 53 58 61 64 85 67	657	63
	(0.3, 0.7)	Best optimal	33 34 37 38 39 40 42 44 45 50 51 52 53 58 61 64 85 67	657	116
		Secondary optimal	33 34 37 38 39 40 42 44 45 50 51 52 53 59 60 65 85 67	783	97
	(0.5, 0.5)	Best optimal	33 34 37 38 39 40 42 44 45 50 51 52 53 58 61 64 85 67	657	185
		Secondary optimal	33 34 37 38 39 40 42 44 45 50 51 52 53 59 60 65 85 67	783	93
	(0.7, 0.3)	Best optimal	33 34 37 38 39 40 42 44 45 50 51 52 53 58 61 64 85 67	657	318
		Secondary optimal	33 34 37 38 39 40 42 44 45 50 51 52 53 59 60 65 85 67	783	35
	(1, 0)	Best optimal	33 34 37 38 39 40 42 44 45 50 51 52 53 58 61 64 85 67	657	365
		Secondary optimal	No path		
	Best solution		(33 34 37 38 39 40 42 44 45 50 51 52 53 58 61 64 85 67)	644	
Route to chaos	(0, 1)	Best optimal	33 34 37 38 39 40 42 44 45 50 46 83 47 82 48 75 72 65 85 67	1023	99
		Secondary optimal	33 34 37 38 39 40 42 44 45 50 51 52 53 59 60 65 85 67	783	83
	(0.3, 0.7)	Best optimal	33 34 37 38 39 40 42 44 45 50 51 52 53 59 60 65 85 67	783	162
		Secondary optimal	33 34 37 38 39 40 42 44 45 50 46 83 47 82 48 75 72 65 85 67	1023	60
	(0.5, 0.5)	Best optimal	33 34 37 38 39 40 42 44 45 50 51 52 53 59 60 65 85 67	783	207
		Secondary optimal	33 34 37 38 39 40 42 44 45 50 46 83 47 82 48 75 72 65 85 67	1023	78
	(0.7, 0.3)	Best optimal	33 34 37 38 39 40 42 44 45 50 51 52 53 59 60 65 85 67	783	348
		Secondary optimal	33 34 37 38 39 40 42 44 45 50 46 83 47 82 48 75 72 65 85 67	1023	11
	(1, 0)	Best optimal	33 34 37 38 39 40 42 44 45 50 51 52 53 59 60 65 85 67	783	365
		Secondary optimal	No path		
	Best solution		(33 34 37 38 39 40 42 44 45 50 51 52 53 59 60 65 85 67)	783	

* (Cost, risk) priority set is ordered by left to right.

** Frequency is regarding 365 iterations.

TABLE 3: Frequency of the best optimal paths for different chaos equations.

Chaos equation	O-D Pairs	(Cost, risk) priorities				
		(0, 1)	(0.3, 0.7)	(0.5, 0.5)	(0.7, 0.3)	(1, 0)
Logistic map	43-71	262	288	302	319	365
	33-67	74	116	185	318	365
Route to chaos	43-71	301	304	305	336	365
	33-67	99	162	207	348	365
Logistic map	Average	168	202	243.5	318.5	365
Route to chaos		200	233	256	342	365

mathematical model. Although there are some methods for solving multiobjective programs (such as preemptive goal programming, weighted objective function), uniform amounts of variables have been used for developing mathematical model in this research work.

For more discussion, results which are tabulated in Tables 1 and 2 are summarized in Table 3 and graphically shown in Figure 3. The average frequency for each O-D pair is selected for discussion. As it can be observed, average frequency is raised when priority of risk is decreased. It goes up to 365 when risk has no effect to find hazmat transport path. The other finding is corresponding to the path convergence derived by the proposed procedure. Path convergence criterion for using route to chaos equation is much better than the one for logistic map equation. The main

FIGURE 3: Average path frequency using methods of generating chaotic patterns.

reason is that the chaotic variable in logistic map equation is just dependent on the previous time but it is dependent on the previous and preprevious times in route to chaos equation. Therefore, using route to chaos equation for considering dynamic risk factors in hazardous material transport planning is recommended to researchers who work in this field.

6. Conclusion

In this research work, an iterative procedure, proposed following the basic principle of hazmat routing problem and

the concept of chaos theory in dynamic risk definition, has been proposed and applied in hazmat transportation routing problem. Transport cost and risk have been considered as main attributes in HRP, so mathematical model has been developed to be capable of considering both risk and cost attributes. Risk is defined as a chaotic variable using two methods of generating chaotic variables including logistic map and route to chaos equations.

An experimental data set in a real road network, including eighty-nine nodes and one hundred and one two-way edges, has been used as a case study for analytical and evaluation process. The proposed procedure and mathematical model have been run in prespecified iteration number and origin-destination pairs. Results revealed that there are some differences between results when different equations for generating chaotic factors are used. It is also concluded that route to chaos equation is capable of determining the best path in better convergence rather than logistic map equation, because route to chaos equation uses two previous situations of chaotic variable but logistic map uses only the consecutive previous situation.

Further study may focus on the process of generating risk factors using different equations mainly utilized in the field of chaos theory. In a large network, since running time could be a concern, future research is also recommended to check the speed of procedure with more iterations and improve that for shorter running time.

Conflict of Interests

No funding or financial support from fellowship, scholarship, or any sponsors have been gained by the author. Therefore, the author declares that there is no conflict of interests regarding the publication of this research work.

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