Research Letter

Design Two-Dimensional IIR Filters Based on Clonal Selection Algorithm with Singular Value Decomposition

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In this letter, clonal selection algorithm (CSA) with singular value decomposition (SVD) method is investigated for the realization of two-dimensional (2D) infinite-impulse response (IIR) filters with arbitrary magnitude responses. The CSA is applied to optimize the sampled frequencies of transition band of digital filters, then producing a planar response matrix of a 2D IIR digital filter. By using the SVD, 2D magnitude specifications can be decomposed into a pair of 1D filters, and thus the problem of designing a 2D digital filter can be reduced to the one of designing a pair of 1D digital filters or even only one 1D digital filter. The stimulation results show the proposed method has the better performance of the minimum attenuation between the passband and stopband.

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1. INTRODUCTION

In the past few years, there has been an ever-increasing interest in the area of algorithms that are inspired by biological behaviors and their applications. One of them, clonal selection algorithm (CSA), which was developed by De Castro and Von Zuben [1–4], is presented to design 2D infinite-impulse response (IIR) filters in this letter. CSA is used to explain how an immune response is mounted when a nonself antigenic pattern is recognized by a B-cell, and used to describe the immune response process which is caused by the recognition of antibody to the antigen. In the past two decades, many techniques have been proposed to design 2D digital filters and the technique that received a considerable attention is based on SVD [5–8]. The SVD is applied to the so-called planar response matrix, which is obtained by truncating the impulse response array. It is used to express a specified 2D-sampled amplitude response into a sum of products. The outcome will be a class of 2D digital filters. We used the SVD property to design 2D IIR filters.

2. CLONAL SELECTION ALGORITHM

The clone selection and mutation process are similar to natural selection process. The antibodies are soluble the forms of the B-cell receptors that are released from the B-cell surface to cope with the invading nonself antigen. When the affinity between antibody and antigen meets some criteria, the B-cell begins clonal proliferation and secretes new antibody. During reproduction, the B-cell progenies (clones) undergo a hypermutation process with antigenic receptors presenting higher affinities with the selective antigen. In the process of clonal proliferation, the progeny of B-cell goes through the mutation process that makes the B-cell combine with antigen by high affinity.

In addition to differentiating into antibody producing cells, the B-cell with high antigen affinity converts to long-life memory cells. Figure 1 illustrates the clonal selection, expansion (proliferation), and affinity maturation process.

From the aforementioned description of the basic mechanism, we can use immunological evolution for engineering applications, such as pattern recognition, machine learning, and multimodal/multiobjective function optimization. The flow chart of the clonal selection algorithm is shown in Figure 2, and the step-by-step procedure is presented as follows (Table 1) [1–4].

(1) Create the first iteration antibodies $A$ randomly (initial condition).

(2) Selecting the $n$ best individuals of the population antibodies $A$ is based on an affinity measure. Put these into a subpopulation ($A_n$).

(3) Reproduce (clone) the elements of ($A_n$), giving rise to a temporary population of clones ($C$). The clone size of
an individual in \((A_n)\) is an increasing function of its affinity with the antigens.

(4) Apply a mutation scheme for each individual in \(C\), where the mutation rate is proportional to the affinity of the antibody with the antigens. As a result, a mutated antibody population \(C^*\) is generated.

(5) Reselect the improved individuals from \(C^*\) to compose the memory set \(M\). Some members of \(A\) can be replaced by other improved members of \(C^*\).

(6) Replace \(A_p\) antibodies by novel ones (diversity introduction). The lower affinity cells have higher probabilities of being replaced.

(7) Repeat steps (2) to (6) until a certain criterion is met, such as completing the iterations set before.

### Table 1: Optimal problem versus clonal selection algorithm.

<table>
<thead>
<tr>
<th>Optimal Problem</th>
<th>CSA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem to be solved</td>
<td>Antigen</td>
</tr>
<tr>
<td>Candidate solution sets</td>
<td>Antibody</td>
</tr>
<tr>
<td>Identification of the problem</td>
<td>Recognition of antigen</td>
</tr>
<tr>
<td>Recalling a past successful solution</td>
<td>Production of antibody from memory cells</td>
</tr>
<tr>
<td>Searching process</td>
<td>Reproduce, Clone</td>
</tr>
<tr>
<td>Find the optimal solution</td>
<td>Mutation</td>
</tr>
</tbody>
</table>

**Figure 3:** Realization of quadrantally symmetric 2D filter.

### 3. SINGULAR VALUE DECOMPOSITION

The approach yields a class of multistage separable planar filters, and it will be shown in Figure 3.

Let matrix \(B = \{b_{pq}\}\) be the desired sampled amplitude response of a 2D filter with quadrant symmetry, in the first quadrant

\[
b_{pq} = |H(e^{j\pi p}, e^{j\pi q})|, \quad \text{for } 1 \leq p, q \leq M,
\]

(1)

where \(M\) means the dimensions of frequency vectors \(\mu_p\) and \(\mu_q\), while \(\mu_p\) and \(\mu_q\) are the normalized frequencies as follows:

\[
\mu_p = \frac{p - 1}{M - 1}, \quad \mu_q = \frac{q - 1}{M - 1}.
\]

(2)

The singular value decomposition of matrix \(B\) can be expressed as

\[
B = \sum_{i=1}^{r} \sigma_i u_i v_i',
\]

(3)

where “\(\cdot'\)" means the transpose of a matrix or a vector. \(\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r\) are the singular values of matrix \(B\), \(u_i\) is the \(i\)th eigenvector of \(BB'\) associated with the \(i\)th eigenvalue \(\sigma_i^2\), \(v_i\) is the \(i\)th eigenvector of \(B'B\) associated with the \(i\)th eigenvalue \(\sigma_i^2\) and they are both orthonormal column vectors, \(r\) is the rank of \(B\), and \(v_i'\) denotes the transpose of \(v_i\).

If we let \(\alpha_i = \sigma_i^{1/2} u_i\) and \(\beta_i = \sigma_i^{1/2} v_i\), then (3) can be rewritten as follows:

\[
B = \sum_{i=1}^{r} \alpha_i \beta_i',
\]

(4)
where \( \{ \alpha_i, 1 \leq i \leq r \} \) and \( \{ \beta_j, 1 \leq i \leq r \} \) are the sets of orthogonal \( M \)-dimensional column vectors. These vectors can be considered as the sampled frequency responses of 1D subfilters. Figure 3 shows the process of the structure.

If \( B \) is a symmetric matrix, then (4) becomes as follows:

\[
B = \sum_{i=1}^{r} s_i \alpha_i \alpha_i',
\]

where \( s_1 = 1 \) and \( s_i = \pm 1 \) for \( 2 \leq i \leq r \). Therefore, each section requires only one 1D subfilter to be designed. To decompose a 2D magnitude samples of a IIR digital filter, one can use shift up of the values of each decomposed vector with the absolute value of the most negative elements in the magnitude specifications of each subfilter.

4. CSA DESIGN APPROACH

Due to SVD, we just need to design 1D IIR filter to compose 2D IIR filter. For this reason, an easy-to-use, numerically reliable software package, called MATLAB, has been used to perform SVD. The coefficients of 1D IIR digital filters are designed by modified Yule-Walker method of autoregressive moving average models [9]. The modified Yule-Walker method reduces the complexity of the design of the IIR filters effectively. We can get the coefficients of IIR filter through the given orders of IIR filter, the frequency sampled points, and the magnitude of sampled points. Noting that the first sampled point value and the final sampled point value must be zero and one, respectively.

We take an antibody series that stand for the frequencies between \( 0 \sim \pi \) to be encoded to binary code for computing convenience. In this letter, the antibody is encoded with 36 bits binary string \( d_1, d_2, \ldots, d_{36} \) in which the first 12 bits, the second 12 bits, and the third 12 bits are denoted as \( x_1, x_2, \) and \( x_3, \) respectively, as follows:

\[
x_1 = \sum_{i=1}^{12} d_i 2^{-i}, \quad x_2 = \sum_{i=1}^{12} d_{i+12} 2^{-i}, \quad x_3 = \sum_{i=1}^{12} d_{i+24} 2^{-i}.
\]

From (6) above, two ranges, \( \omega_p < x_1 < x_2 < x_3 < \omega_s \) in the transition band of filters are easily found, where \( \omega_1 \) is the stopband frequency and \( \omega_p \) is the passband frequency.

5. EXPERIMENT

Design a 2D lowpass IIR filter specified by

\[
A_L = | H(\omega_1, \omega_2) | = \begin{cases} 
1, & 0 \leq \omega_s \leq \omega_p, \\
\omega_p - \omega_s, & \omega_p < \omega_s < \omega_x, \\
\omega_x - \omega_p, & 0 < \omega_s < \omega_x, \\
0, & \omega_x \leq \omega_x \leq \pi,
\end{cases}
\]

where \( A_L \) is the planar magnitude response matrix of a 2D lowpass IIR filter,

\[
\omega_s = \sqrt{\frac{1}{2} (\omega_1^2 - \omega_2^2) + \frac{1}{4} (\omega_1^2 + \omega_2^2)},
\]

and \( \omega_p = 0.4\pi, \omega_s = 0.6\pi \).

We would like to normalize frequency \( \omega_1 \) and \( \omega_2 \) for \( 0 < \omega_1, \omega_2 < 1 \) as

\[
F = \text{linspace}(0, \pi, 21)/\pi.
\]

The linspace is the command in the well-known program language of MATLAB. Its work is to divide the frequencies between \( 0 \sim \pi \) into equal ranges. So \( F \) means the vector of frequencies which distribute to zero and one. In the transition band \( 0.4\pi \sim 0.6\pi \), there are three sampled points. The transition sampled points are represented as \( 0.4 < x_3 < x_2 < x_1 < 0.6 \). After sampling transition band points, we obtain that

\[
\omega_1 = \omega_2 = [ F(1 : 9), x_3, x_2, x_1, F(13 : 21)]
\]

for

\[
F(1 : 9) = [0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4], \\
F(13 : 21) = [0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 1].
\]

Figures 4(a), 4(b) show the magnitude response of lowpass IIR filters with order 4 and order 6, respectively. Figures 5(a), 5(b) show the tendency of attenuation of stopband of the lowpass IIR filter with CSA with order 4 and order 6 by CSA.

Genetic algorithm (GA) [10], Chebyshev sampling method (Chebyshev) [11], and frequency sampling method (FSM) [12] have been taken to compare with CSA for the minimum attenuation as in the stopband. The results are shown in Table 2.

**Remark 1.** In this example, the transition areas have been chose between 0.4–0.6 \pi. There are three sample points at the transition band by average sampling, as 0.45 \pi, 0.5 \pi, and 0.55 \pi. Therefore, there will be three values at the optimization of the sampling points. This is our rule of choosing the number of sampling points.

**Remark 2.** Why we need to use 12 bits? Because the method of variable xi is binary system to decimal system, the more bit size the more exactly value. Of course you can try to add the number of bits, but this would also increase difficulty of the system and operating time of the program. The range of decimal system is \( 0 \sim 0.99998 \) by using 12 bits.

**Table 2:** The stimulation results and comparison.

<table>
<thead>
<tr>
<th></th>
<th>Order</th>
<th>FSM</th>
<th>GA</th>
<th>Chebyshev</th>
<th>CSA</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_L ) (dB)</td>
<td>4</td>
<td>-22.1862</td>
<td>-21.6623</td>
<td>-22.5932</td>
<td>-35.7297</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>-26.2988</td>
<td>-29.9445</td>
<td>-28.1792</td>
<td>-38.0128</td>
</tr>
</tbody>
</table>
6. CONCLUSIONS

CSA, based on specificity and memory characteristics of biological immune system, has been applied to solve the problem of the design of the 2D IIR digital filters. By CSA and SVD, we improve the transition band frequencies to construct the whole magnitude of planar response matrix to design 2D lowpass IIR filter. Comparing with other well-known methods (FSM, GA, Chebyshev), the proposed method has the better performance of the minimum attenuation between the passband and stopband.

REFERENCES


Figure 4: Magnitude response of 2D lowpass IIR. (a) 4-order, (b) 6-order.

Figure 5: The tendency of $A_s$ of 2D lowpass IIR. (a) Order 4, (b) Order 6.


