Research Letter

Improved Parameter Estimation for First-Order Markov Process

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This correspondence presents a linear transformation, which is used to estimate correlation coefficient of first-order Markov process. It outperforms zero-forcing (ZF), minimum mean-squared error (MMSE), and whitened least-squares (WTLSs) estimators by controlling output noise variance at the cost of increased computational complexity.

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1. Introduction

Let us consider a linear multi-input multioutput model with an unknown $N \times 1$ parameter vector $\vec{x}(n) = [a_1 a_2 \dots a_N]^T$, such that

$$\vec{y} = H\vec{x} + \vec{v}, \quad (1)$$

where, $\vec{y}(n) = [h(n)h(n-1) \dots h(n-M+1)]^T$ is $M \times 1$ output vector,

$$H(n) = \begin{bmatrix} h(n-1) & h(n-2) & \dots & h(n-N) \\ h(n-2) & h(n-3) & \dots & h(n-N-1) \\ h(n-M) & h(n-M-1) & \dots & h(n-N-M+1) \end{bmatrix} \quad (2)$$

is $M \times N$ complex matrix, and $\vec{v}(n) = [v(n)v(n-1) \dots v(n-M+1)]^T$ is white Gaussian process noise $M \times 1$ vector with zero-mean and covariance matrix $\sigma_v^2 I_M$, where I_M is $M \times M$ identity matrix. The estimated unknown parameter vector may be defined as $\hat{x} = T\vec{y}$, where “ T ” is linear transformation involving pseudoinverse. The application of ZF linear transformation $T_{ZF} = (H^H H)^{-1} H^H$ to $\vec{y}$ results in nonwhite noise with covariance matrix $C_{ZF} = \sigma_v^2 (H^H H)^{-1}$. On the other hand, MMSE linear transformation $T_{MMSE} = (H^H H + \sigma_v^2 I_N)^{-1} H^H$ alleviates output noise variance by finding the optimum balance between data detection and noise reduction [1]. However, the modification of least squares estimation is based on the concept of MMSE whitening;

that is, WTLS performs well at low to moderate signal-to-noise ratios by using linear transformation $T_{WTLS} = B(H^H H)^{-1/2} H^H$ [2], where $B = \text{diag}[\beta_1, \beta_2, \dots, \beta_N]$ with $\beta = \beta_1 = \beta_2 = \dots = \beta_N = \text{Tr}\{(H^H H)^{1/2}\} / \text{Tr}\{H^H H\}$. It follows that

$$\hat{x}_{WTLS} = B(H^H H)^{1/2} \vec{x} + B(H^H H)^{-1/2} H^H \vec{v}. \quad (3)$$

Substitution of the unique QR-decomposition $H = QD\bar{M}$ in (3) leads to

$$\hat{x}_{WTLS} = BD\bar{M}\vec{x} + BQ^H \vec{v} \quad (4)$$

where, $Q = [q_1, q_2, \dots, q_N]$ is an $M \times N$ matrix with orthonormal columns, D is an $N \times N$ real diagonal matrix whose diagonal elements are positive, and $\bar{M}$ is an $N \times N$ upper triangular matrix with ones on the diagonal (on contrary to [3]). Incorporation of $BD = I_N$ in (4) yields $\hat{x}_{WTLS} = \bar{M}\vec{x} + D^{-1}Q^H \vec{v}$, where $D^{-1}Q^H$ is referred to as noise whitening-matched filter [4].

2. AR(1) Parameter Estimation

In the presented exposition, the posited linear transformation is $T_{MZF} = \bar{M}(H^H H)^{-1} H^H$. Consequently,

$$\hat{x}_{MZF} = T_{MZF} \vec{y} = \bar{M}\vec{x} + \bar{M}(H^H H)^{-1} H^H \vec{v} \quad (5)$$

with noise covariance matrix $C_{MZF} = \sigma_v^2 \bar{M}(H^H H)^{-1} \bar{M}^H$. This transformation also performs noise whitening. It is

FIGURE 1: MSIE versus process noise variance (σ_v^2 dB).

apparent that output noise variance is controlled and reduced, since $0 \leq m_{i,j} < 1$ for $i \neq j$ (i.e., i th row and j th column element of matrix $\bar{M}$). For AR(1) correlation coefficient (a_1) estimation, the unknown parameter vector $\bar{x}$ in (1) and (5) is replaced by $\bar{x} = [a_1 \Delta a_2 \Delta a_3 \cdots \Delta a_N]^T$ with leakage coefficients $\Delta a_j \rightarrow 0$. Thus, the estimated parametric value is

$$\hat{x}_{\text{MZF},1} = \hat{a}_1 = a_1 + \lim_{\Delta a_j \rightarrow 0} \sum_{j=2}^N m_{1,j} \Delta a_j + v_{\text{MZF},1} \quad (6)$$

with $0 \leq m_{1,j} < 1$ for $j \neq 1$. For parameter values $a_1 = 0.95$ (true correlation coefficients) and $\Delta a_2 = \Delta a_3 = \Delta a_4 \approx 0.0001$ (assumed), the simulation results depicted in Figure 1 demonstrate that the proposed technique outperforms other aforementioned linear transformations. However under similar conditions, the value of β is found to be high in case of WTLS transform, which in turn increases the output noise variance.

3. Conclusions

The presented linear transformation based on a typical QR-decomposition (i.e., $QD\bar{M}$) reduces output noise, which is utilized for the efficient estimation of correlation coefficient in first-order Markov process.

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