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Research Article

Space-Time Radar Waveforms: Circulating Codes

G. Babur, P. Aubry, and F. Le Chevalier

Microwave Sensing, Systems and Signals (MS3), Delft University of Technology, 2628 CD Delft, The Netherlands

Correspondence should be addressed to G. Babur; g.babur@tudelft.nl

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This paper describes a concept of the circulating codes covering the whole class of the space-time codes. The circulating codes do not narrow the radiated pattern of the antenna array, thus providing a wide angular coverage, possibly tunable. In turn, the beam-forming on transmit is achievable by means of the signal processing in one (or each) receiver channel. The modelling results demonstrate the efficiency of the circulating codes based on their multidimensional ambiguity functions.

1. Introduction

In many modern radar applications, pulse compression is a key element to achieve the required performance. The codes to be used must possess appropriate properties of auto- and cross-correlations. In turn, beam-forming is another well-known signal processing technique used in sensor arrays for directional signal transmission or reception [1].

Many engineers have noted that if one were able to form truly simultaneous transmit beams in independent directions, then a flexible radar system could be build that would be capable of true simultaneous multifunction operation [2]. Moreover, since digital array MIMO radars can be operated in either the MIMO or the more conventional transmit modes (as needed), a great deal of flexibility can be provided.

Until recently, radar transmitters have not enjoyed the same degree of software-driven agility as radar receivers [3]. Nowadays radar transmitters incorporate highly agile, software-driven waveform generators. In principle, the radar designer's choice of beam shape and waveform can be used to spread energy over space and time in many interesting and different ways [2].

The coloured transmission principles have been presented in [4]. They allow the transmission of the sounding signals in a wide angular angle, and, at the same time, to perform the beam-forming on transmit in one (or each) of the receiving channels. They allow for an effective monitoring of any number of beams formed on transmit by means of signal processing on receive. The technique, describing application of the digital beam-forming processing with a single receiver channel, can be found, for example, in [5]. At the same time, the amount of measurements stays the same as for one beam, with some increase of the computational cost in the receiver. So, the coloured waveform transmission offers vastly increased transmitter agility to identify ways to exploit agile transmitter capabilities in practice.

In the literature on MIMO radar, it is often assumed that the transmitted waveforms associated with each transmit antenna or subarray are orthogonal and have perfect waveform cross-correlation properties. In practice, this is hardly achievable, when the transmitted signals occupy the same time interval within the same bandwidth. When we consider the same waveform circulating through each antenna element with a very small relative time shift, as presented in this paper, it seems that achievement of good cross-correlation properties is simply impossible. However, the received results demonstrate the opposite. Consequently, the circulating codes for coherent MIMO deserve careful consideration and analysis.

Implicit in the waveform design is that the observed radar scene should be estimated accurately. The scattering response of the clutter and targets must be disentangled by the receiver. A commonly used measure for assessing waveform selection in radar systems is the ambiguity function, which will be used here for analyzing the properties of the proposed code. In conventional radar systems, Woodward's ambiguity function is used to characterize waveform resolution performance [6, 7]. For analysis of space-time radar waveforms, such as

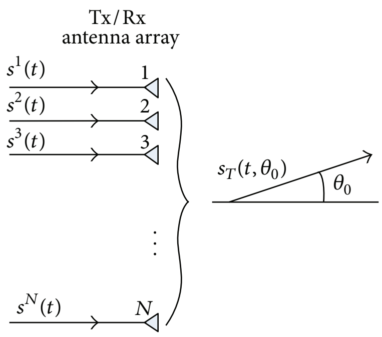


Figure 1: MIMO transmission.

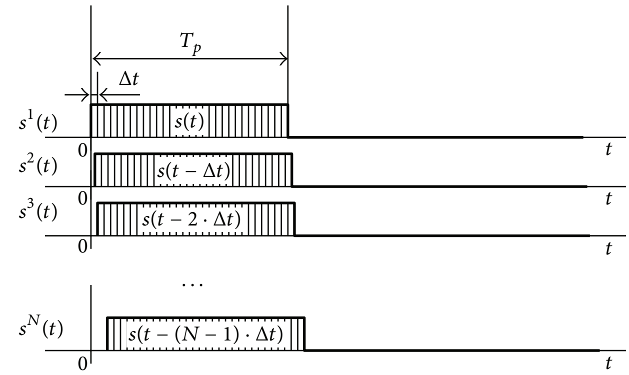


Figure 2: Circulating signals.

the circulating codes presented in this paper, the multidimensional ambiguity function should be employed [8, 9].

This paper describes a solution for the proposed waveforms based on the use of a novel and technically simple solution of transmitting only one waveform circulating from one antenna element to another (or from one subarray to another) with a very small relative time shift. In this way the radiated signals have exactly the same coding complex envelope, highly overlapping both in the time and frequency domains.

The paper is organized as follows. In Section 2 the concept of the circulating space-time codes, which applies to all possible waveforms having good autocorrelation properties, is introduced. Section 3 describes the multidimensional ambiguity function used for the space-time signals analysis. Section 4 gives the examples of the ambiguity functions for a number of circulating codes examples. The capabilities of the waveforms for the beam-forming on transmit and pulse compression are shown. Section 5 contains the conclusions.

2. Circulating Codes Concept

The transmitted waveforms are assumed to be encoded by the same waveform $s(t)$ circulating with a relative time shift through N MIMO transmitter channels (see Figure 1). The waveform $s^n(t)$ circulating through the nth channel can be written as

$$s^n(t) = a_n \cdot s(t-(n-1)\cdot\Delta t), \tag{1}$$

where the index n represents both the number of the transmitting channels and the number of the transmitted waveforms. The relative time shift Δt between the circulating signals is equal to 1-sample time, $\Delta t = 1/\Delta F$, where ΔF is the signal bandwidth. For waveforms with a large BT-product (compression ratio), the relative time shift is very small compared to the pulse duration T_p because, in fact, $\mathrm{BT} = T_p/\Delta t$. For the sake of simplicity, the amplitudes a_n are assumed to be the same and equal to one and will be omitted further.

The simplicity of implementation of the circulating codes should be emphasized. Because of the use of only one waveform, only one complex signal with a large BT product should be generated; and then it circulates in all the transmitter channels with a small relative offset.

Figure 2 shows the circulating signals $s^n(t)$, $n = 1\cdots N$. The total radiated signal $s_T(t)$ has the following properties. It has the same duration T as the signal $s(t)$. This is explained by the fact that the envelope of the sum signal has a trapezium shape with duration T at −3 dB level. The relative time delay of the signal $s_T(t)$ compared to $s(t)$ is equal to the half of the relative time shift between the first and the last circulating signals. This time delay is constant and it is equal to $(N-1)\cdot\Delta t/2$. It can be compensated at the signal generation stage in the MIMO radar transmitter by the equal shift of all the radiated signals or by compensation of this known time delay at the signal processing stage in the MIMO radar receiver.

The time offset used in the circulating signal can be looked at as a time-delay beam-steering (see, e.g., [10]). The time-delay steering of array antennas is used to steer the narrow main beam of the antenna array physically. When the circulating signals are used for sounding, the radiated beam is not steered, and the signals are radiated within the wide angle. The time shifts within the circulating codes are used not to steer the beam but to keep it as wide as possible. The beam-forming on transmit (narrowing of the main beam) is implemented by the signal processing means on receive.

Array theory tends to discuss things in phase rather than delay, so we can convert the delay experienced by the circulating signal into a phase shift, between adjacent antenna elements, at a given frequency:

$$\Delta\varphi = 2\pi\cdot f\cdot\Delta t = \frac{2\pi\cdot f}{\Delta F}, \tag{2}$$

where f is the radiated signal's frequency. Since the radiated signal bandwidth ΔF is not equal to zero, the phase shift is not fixed. The phase shift is a function of the frequency $\Delta\varphi(f)$ where the radiated signal's frequency belongs to the following interval: $f \in [(f_0 - \Delta F/2)\cdots(f_0+\Delta F/2)]$, where f_0 is the radiated signal's central frequency. Therefore, the range of the phase shift described by (2) can be written as

$$(\Delta\varphi_{\min}\cdots\Delta\varphi_{\max}) = ((\Delta\varphi_0-\pi)\cdots(\Delta\varphi_0+\pi)), \tag{3}$$

where $\Delta\varphi_0$ is the phase shift provided by the central radiated frequency. Equation (3) shows that the phase shift within the radiated signal's bandwidth changes by 2π, which characterizes the full angular coverage.

The uniformly spaced linear array can be steered by applying a phase shift such as $\Delta\varphi = 0$ at the angle of interest [11]:

$$\Delta\varphi_0 = 2\pi \cdot \frac{d}{c} \cdot f \cdot \sin\theta_0, \quad (4)$$

where d is the spacing between the antenna elements and c is the speed of light. However, for the circulating signals the phase shift varies by 2π along the array for any θ_0. It means that, regardless of the value of $\Delta\varphi_0$ or on the steering direction, the physically radiated circulating signals provide the full angular coverage, in practice, limited only by the radiated patterns of the antenna elements. So, in fact, the signals are radiated within a wide angular range, and the beam-forming on transmit (forming of multiple beams) is performed on receive by the signal processing means, which is explained further in the paper.

The group transmitted signal (see Figure 1) is angular dependent. For ease of exposition, let us make the usual narrowband assumption; this is not a requirement, but simplifies the discussion. So, in a given direction θ_0 the aggregate transmitted signal can be written as the sum of all transmitted signals, with appropriate phase shifts for this direction defined by the wavenumber $\vec{k}(\theta_0)$:

$$s_T(t,\theta_0) = \sum_{n=1}^{N} e^{j\vec{k}(\theta_0)\cdot\vec{x}(n)} \cdot s^n(t), \quad (5)$$

where $\bar{k}$ is a wave-vector, $\bar{x}(n)$ is the position vector of the nth radiating element, and $s^n(t)$ is a circulating signal described by (1).

The signal received by one receiver channel r is written as

$$s_R^r(t,\theta_0) = e^{-j\cdot\bar{k}(\theta_0)\cdot\bar{x}(r)} \cdot \dot{A} \cdot s_T(t-\tau_0,\theta_0) + e(t), \quad (6)$$

where $\dot{A}$ is the complex scattering coefficient of an observed target, τ_0 is a time delay defined by the travelling distance of the signal, and $e(t)$ is noise at time t. In this work, we assume that within one pulse the phase variance due to the Doppler shift is negligible and the Doppler shift becomes just a constant phase term. The multidimensional transmit ambiguity function presented in the next section is calculated for one pulse as a squared modulus of the matched filter output (see the end of Section 3). Since we take the modulus, we neglect the phase term defined by the Doppler shift imposed on the pulse. So, the use of the bank of Doppler filters together with the sounding circulating signals makes the observation of moving objects possible and, at the same time, provides a wide angular coverage with possibility of the beam-forming on transmit performed in radar receiver.

3. Transmit Ambiguity Function: Circulating Codes

As known, the matched filter is the solution for maximizing the output signal-to-noise ratio (SNR). So, the beam-forming on transmit is performed on the received signals in the rth channel as the filtering matched with the transmitted signal $s_T(t,\theta_0)$:

$$R_{\theta_0}^r(\tau,\theta') = \int s_T(t-\tau,\theta') \cdot (s_R^r(t,\theta_0))^* dt, \quad (7)$$

where θ' is a hypothesis about the arrival direction. The superscript "$*$" means complex conjugation. The extended form of the compressed signal is

$$R_{\theta_0}^r(\tau,\theta') = \dot{A}^* \cdot e^{+j\cdot\bar{k}(\theta_0)\cdot\bar{x}(r)} \cdot \int s_T(t-\tau,\theta') \cdot (s_T(t-\tau_0,\theta_0))^* dt. \quad (8)$$

Omitting the constant phase term characteristic for the rth receiving channel and changing the time variables $t' = (t-\tau_0)$, $\tau' = (\tau_0 - \tau)$, we receive

$$R_{\theta_0}(\tau',\theta') = \int s_T(t'+\tau',\theta') \cdot (s_T(t',\theta_0))^* dt', \quad (9)$$

which can be rewritten as

$$R_{\theta_0}(\tau',\theta') = \int \left(\sum_{n=1}^{N} e^{j\cdot\bar{k}(\theta')\cdot\bar{x}(n)} \cdot s(t'+\tau'-n\cdot\Delta t+\Delta t)\right) \cdot \left(\sum_{n'=1}^{N} e^{j\cdot\bar{k}(\theta_0)\cdot\bar{x}(n')} \cdot s(t'-n'\cdot\Delta t+\Delta t)\right)^* dt', \quad (10)$$

where, as we remember, n is the index for the transmitted signal and n' is the index for its replica.

$$R_{\theta_0}(\tau',\theta') = \sum_{n=1}^{N}\sum_{n'=1}^{N} e^{j\cdot(\bar{k}(\theta')\cdot\bar{x}(n)-\bar{k}(\theta_0)\cdot\bar{x}(n'))} \cdot \int s(t'+\tau'-(n-1)\cdot\Delta t) \cdot s^*(t'-(n'-1)\cdot\Delta t)\, dt'. \quad (11)$$

As was defined before, Δt is a 1-sample time shift between the replicas of $s(t)$ in the circulating code. At the same time, Δt is the width of the autocorrelation function $\mathrm{ACF}_S(\tau')$ of the signal $s(t')$ taking into account the time variables' change:

$$R_{\theta_0}(\tau',\theta') = \sum_{n=1}^{N}\sum_{n'=1}^{N} e^{j\cdot(\bar{k}(\theta')\cdot\bar{x}(n)-\bar{k}(\theta_0)\cdot\bar{x}(n'))} \cdot \mathrm{ACF}_S(\tau'-(n-1)\cdot\Delta t+(n'-1)\cdot\Delta t). \quad (12)$$

The normalized value of the autocorrelation function is equal to 1 within 1-sample time interval $\Delta t = 1/\Delta F$ around $\tau' = 0$. For the rest, the ACF is described by its side-lobe level (SLL). The SLL for signals having large BT

products and providing good autocorrelation properties can be approximated as 0. So, (12) can be written as

$$R_{\theta_0}\left(\tau',\theta'\right)=\sum_{n=1}^{N}\sum_{n'=1}^{N}e^{j\cdot(\bar{k}(\theta')\cdot\bar{x}(n)-\bar{k}(\theta_0)\cdot\bar{x}(n'))}\cdot 1\left(\tau'-(n-1)\cdot\Delta t+\left(n'-1\right)\cdot\Delta t\right). \quad (13)$$

The function $1(\tau')$ is equal to 1 within 1-sample time interval around $\tau'=0$ and zero for the rest. Consider

$$R_{\theta_0}\left(\tau',\theta'\right)=\sum_{n=1}^{N}\operatorname{rect}\left(\tau'-(n-1)\cdot\Delta t\right)\sum_{n'=1}^{N}e^{j\cdot(\bar{k}(\theta')\cdot\bar{x}(n)-\bar{k}(\theta_0)\cdot\bar{x}(n'))}, \quad (14)$$

where

$$\operatorname{rect}\left(\tau'\right)=\begin{cases}1 & \text{for } -(N-1)\cdot\Delta t\le\tau'\le 0,\\ 0 & \text{for elsewhere } \tau'.\end{cases} \quad (15)$$

We note that the function $\operatorname{rect}(\tau')$ for the circulating signals is N times wider than the achievable range resolution provided by one waveform, which is equal to 1 bit or Δt. This gives the proper width of its autocorrelation function (ACF) introduced below, and, therefore, the proper widening of the ambiguity function along ranges.

Equation (14) can be simplified as follows:

$$R_{\theta_0}\left(\tau',\theta'\right)=\begin{cases}\mathrm{ACF}_{\mathrm{rect}}\left(\tau'\right)\cdot\sum\limits_{n=1}^{N}\sum\limits_{n'=1}^{N}e^{j\cdot(\bar{k}(\theta')\cdot\bar{x}(n)-\bar{k}(\theta_0)\cdot\bar{x}(n'))} & \text{for } \left|\tau'\right|\le\dfrac{N\cdot\Delta t}{2},\\ 0 & \text{for elsewhere } \tau'.\end{cases} \quad (16)$$

Equation (16) is an adequate approximation in case of an ideal autocorrelation function of the signal $s(t)$ with one peak and zero side-lobe level (SLL).

The mathematical expression of the multiparameter signal $R_{\theta_0}(\tau',\theta')$ shows its dependency on the transmitted waveforms $s^n(t)$ (see (1)). The ambiguity function $|R_{\theta_0}(\tau',\theta')|^2$ is thus a 3-dimensional function (or 4-dimensional, if the amplitude is considered as a dimension as well), giving for each aiming direction θ_0 the delay-angle ambiguity. $R_{\theta_0}(\tau',\theta')$, being a 3-parameters function, cannot be visualized easily; a preferred way is to look at the angle-angle cut, for $\tau=0$, and at the range-angle ambiguity, for a specific θ_0:

(i) $|R_{\theta_0}(0,\sin\theta)|^2 = D(\theta,\theta_0)$, is the angular transmit diagram (at the exact range of the target, $\tau'=\tau_0$), as a function of the angular aiming position θ_0.

(ii) $|R_{\theta_0}((c\tau')/2,\theta')|^2$, where θ_0 is constant, is the range-angle ambiguity function, for boresight aiming direction—ideally, this range angle ambiguity function should also be analyzed for each possible aiming direction θ_0.

4. Modeling Results

Four very different complex waveforms have been chosen for operation in the circulating space-time mode as follows.

(1) Quadratic Alltop code, as a representative of the distinct complex-valued sequences constructed for the minimization of the magnitudes of autocorrelation side lobes and cross-correlations [12].

(2) M-sequence or a maximum length sequence, as a type of widely used pseudorandom binary sequences, which are inexpensive to implement in hardware or software. The autocorrelation function has a uniform side-lobe level.

(3) LFM signal, which is a very common radar waveform to realize pulse compression for its fairly ready generation and easy processing. However, without weighting, the autocorrelation function experiences a high side-lobe level.

(4) Nonlinear LFM signal (NLFM). Nonlinear frequency modulation can advantageously shape the "rectangular" energy spectrum of an LFM signals, such that the autocorrelation function exhibits substantially reduced side lobes. The NLFM signal with the tangent-based frequency variation law [13] has been chosen for modeling.

It is known that the number of orthogonal LFM signals within the same bandwidth and time interval is limited by a number of 2. In case an LFM signal is used according to the principle presented in this paper, all the signals circulating through the transmitting channels have the same shape, as can be seen in (1). The important conclusion for all waveforms, not necessarily having LFM modulation, is that the number of circulated signals (transmitters/antenna elements) is not really limited, and it can be definitely more than 2. What is really needed is the good autocorrelation properties of the circulating waveform to make the approximation (13) viable.

The pulse duration is equal to 100 μs. The carrier frequency is 10 GHz. The BT product of all the modelled signals was chosen relatively small, 255, for better visibility of the results. The antenna array is made of $N = 8$ elementary omnidirectional antennas spaced $\lambda/2$ from each other. For demonstrating the possibilities of the beam-forming on transmit for the analysed waveforms, only one element of the array was used on receive.

In Figure 3, four angle-angle cuts of the ambiguity function calculated for four considered types of circulating waveforms are shown. They are angle versus aiming (beam-formed on transmit) angle taken at the contingently zero range $\tau'=0$ ($\tau=\tau_0$). These cuts demonstrate the possibilities of the considered signals to implement beam-forming on transmit implemented in one (or each) of the receiving radar channels. Every single vertical slice for a definite possible angular direction θ_0 describes the array patterns formed on transmit by the signal processing means in the receiver, while physically the radiation pattern on transmit stays wide (omnidirectional for the presented modelling results). This is illustrated by eight vertical slices of the ambiguity function

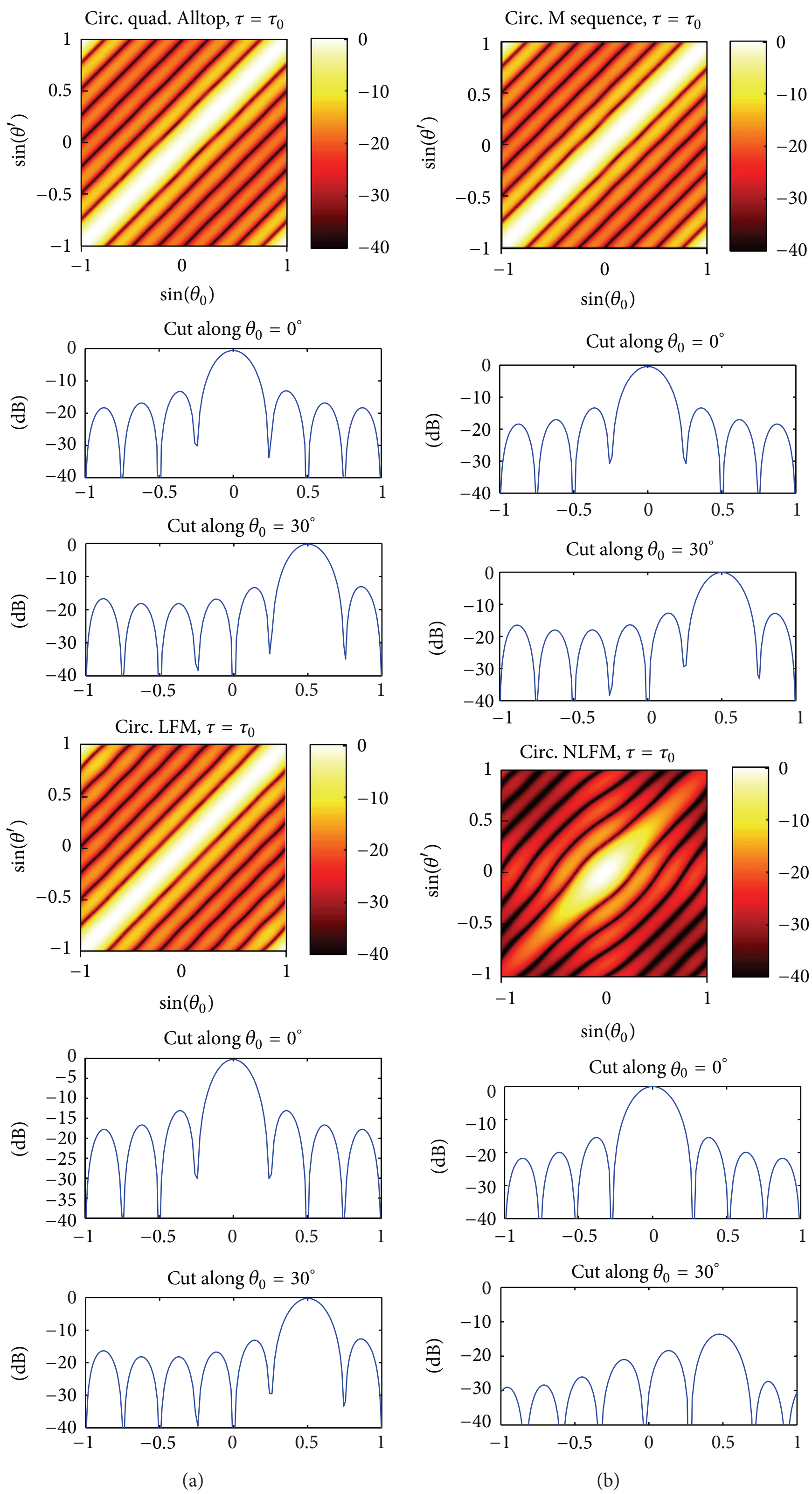
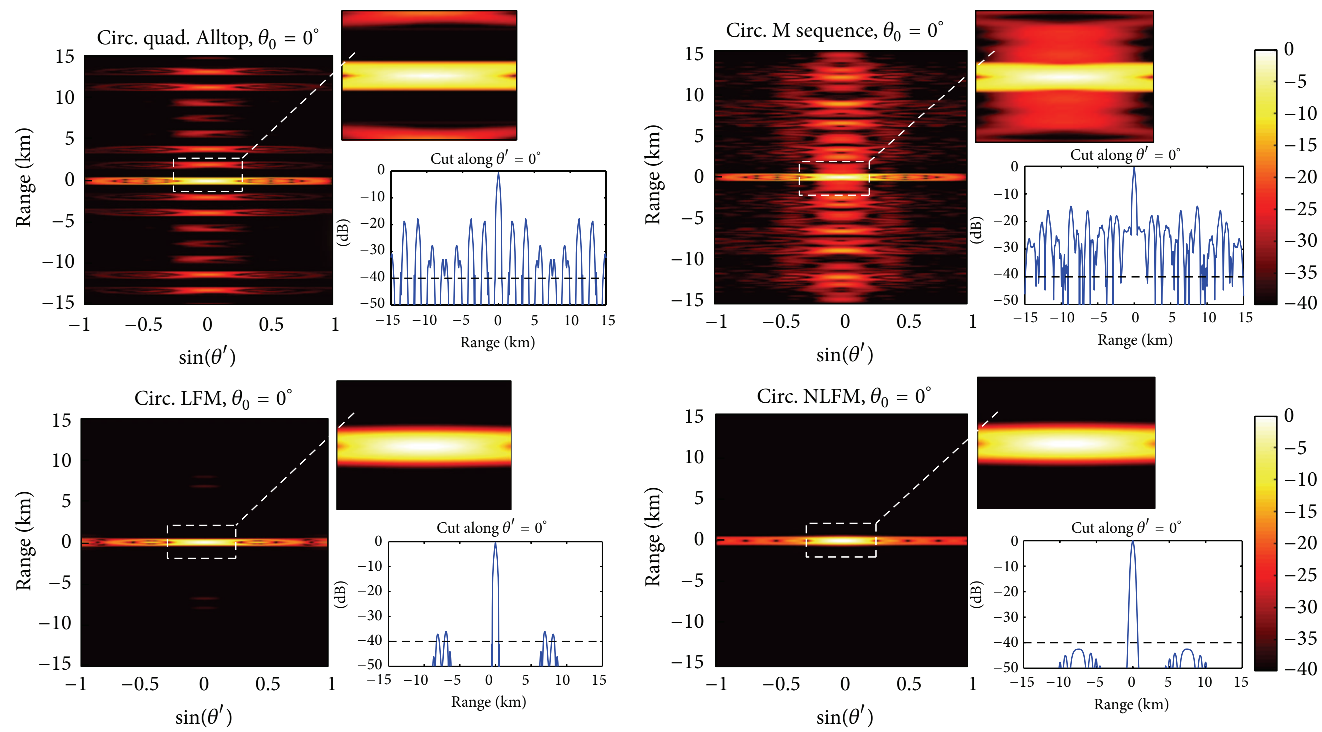


FIGURE 3: Angle-angle cuts of the ambiguity function $|R^0_{\theta_0}(\tau', \theta')|^2$ at $\tau' = 0$ in dB.



(a) (b)

Figure 4: Range-angle cuts of the ambiguity function $|R^0_{\theta_0}(\tau', \theta')|^2$ at $\theta_0 = 0°$ in dB.

cuts, namely, by eight antenna patterns, presented for the considered signals in two possible aiming angular directions, $\theta_0 = 0°, 30°$ in Figure 3. The angle-angle cuts demonstrate the uniform radiation for all the considered angular directions θ' for the circulating quadratic Alltop signal, M-sequence, and LFM-signal. There are no fluctuations along the diagonal, which means that the antenna array gain is constant with respect to the aiming angle. This is a clear demonstration of the fact that by circulating one waveform through the transmitting radar channels, the orthogonality of the radiated signals can be achieved.

As for the circulating NLFM signal, the beam-formed patterns have maximal amplitude, the main beam-width increases in the zero angular direction, and the gradual amplitude degrades for farther angles. The amplitude degradation is in order of 14 dB, which is certainly not negligible. The amplitude degradation is explained by the reduced equivalent bandwidth of the NLFM signal compared to the total occupied bandwidth and, therefore, by the inappropriate definition of the bandwidth dependent offset Δt. This problem can be solved by recalculation of the relative time shift Δt between the circulating signals $s^n(t)$ according to the reduced equivalent bandwidth of the NLFM signal. The reduced angular coverage for the circulating NLFM signal means also that the angular coverage of the antenna can be adapted by reducing the time shift between adjacent transmitters; this is also a valuable property of circulating codes, for use in different operational uses, requiring different angular coverages. In this work, for fair comparison, the relative time shifts are preserved equal.

We note that the results presented in Figure 3 indicate the inherent properties of the considered signals for beamforming because they have been obtained with the assumption of omnidirectional radiation patterns of the antenna array elements.

One of the important properties of transmitted circulating signals is the ability to detect weak (with small radar cross-sections) targets against strong targets or clutter. This property is normally illustrated by the range side lobes of the ambiguity function. Figures 4 and 5 demonstrate, as an example, this property for two aiming directions, while not changing the transmitted waveforms or array geometry.

Figure 4 presents four range-angle cuts of the ambiguity function. They are angle versus range taken at the aiming (beam-formed) angle $\theta_0 = 0°$. The zoomed figures display the main lobe of the ambiguity functions for four analysed signals. In turn, the vertical slices demonstrate the obtained main lobe and range side lobes for one chosen observed angular direction $\theta' = 0°$. In this way these vertical slices received for the space-time radar waveforms can be considered as an analogue of the traditional autocorrelation function obtained, however, for each observed angular direction.

The range side lobes are maximal for the observed direction which is equal to the aiming direction. The side-lobe level between the considered signals is maximal, about −15 dB, for the circulating M-sequence; and it is rather homogeneous

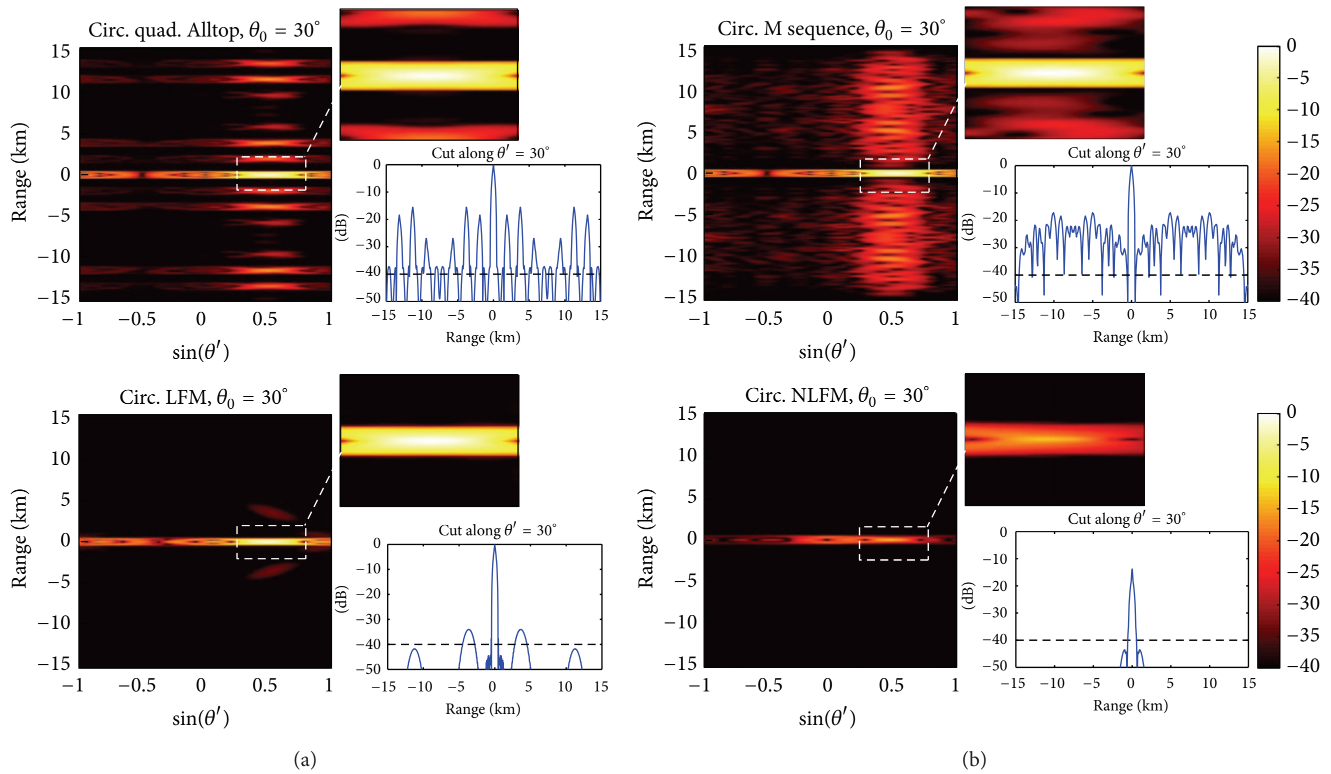


FIGURE 5: Range-angle cuts of the ambiguity function $|R^0_{\theta_0}(\tau',\theta')|^2$ at $\theta_0 = 30^\circ$ in dB.

in each chosen observation direction. Such side-lobe levels would certainly be detrimental in case of multiple targets situations. The minimal side-lobe level, about −42 dB, has been obtained for the circulating NLFM signal. Both, the circulating LFM and NLFM signals, are characterized by a very low side-lobe level for all the observed directions θ'. We remind that the BT product of all the signals has been chosen pretty small, 255. The main lobe is widened in range for the NLFM signal, which is visible in the corresponding zoomed figure in Figure 4. This is explained by the widened main beam of the antenna array for the circulating NLFM signal case in the aiming direction $\theta_0 = 0^\circ$; see the corresponding angle-angle cut of the ambiguity function in Figure 3.

Figure 5 shows another cut of the ambiguity function $|R^0_{\theta_0}(\tau',\theta')|^2$ obtained by means of the signal processing for the aiming angle $\theta_0 = 30^\circ$ in one of the receiving channels. Now the main lobe of the ambiguity function is shifted in the proper angular direction. The peak of the ambiguity function along the observed angular directions is located at the aiming angle. The zoomed figures display the main lobe of the ambiguity functions for four analysed signals, as in the previous figure. The vertical slices demonstrate the obtained main lobe and range side lobes for one observed angular direction $\theta' = 30^\circ$, which is equal to the aiming angle. The range side-lobes are maximal for the aiming directions for all the analysed waveforms. This time the range side-lobes are maximal for the circulating quadratic Alltop code, −15.6 dB, and lower for the M-sequence, −17 dB. The circulating LFM and NLFM signals are still providing very low side-lobe level in range for all the observed angles. In the aimed direction, the maximal values are −34 dB and −44 dB, respectively. It is quite impressive, taking into account the small BT-product of the waveforms. For the circulating NLFM signal, the normalized amplitude of the main lobe drops to −14 dB. This is caused by the amplitude degradation with the angle increase appropriate for the circulating NLFM signal (see the proper angle-angle function in Figure 3).

5. Conclusions

A concept of the circulating space-time code has been described. Basically the presented concept describes the whole class of space-time codes employing a selected type of a waveform having a large BT product. The circulating codes do not narrow the radiated pattern of the antenna array, thus providing a wide angular coverage, possibly tunable. In turn, the beam-forming on transmit is achievable by means of the signal processing in one (or each) receiver channel.

Four types of circulating codes have been analysed in the paper using their ambiguity functions. The circulating LFM and NLFM signals demonstrated a very clean range-angle ambiguity function, with some angular-dependent amplitude degradation of the latter.

This paper describes a novel and technically simple solution of transmitting only one waveform circulating with a very small offset from one antenna element to another

(or from one subarray to another). The signals can be uniformly radiated in free space, providing the beam-forming on transmit implemented in radar receiver.

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