Hindawi
Journal of Electrical and Computer Engineering
Volume 2017, Article ID 8653136, 13 pages
https://doi.org/10.1155/2017/8653136

Research Article

Self-Tuning Control Scheme Based on the Robustness σ-Modification Approach

Nabiha Touijer,[1] Samira Kamoun,[1] Najib Essounbouli,[2] and Abdelaziz Hamzaoui[2]

[1] *Laboratory of Sciences and Techniques of Automatic Control and Computer Engineering (Lab–STA), National School of Engineering of Sfax, University of Sfax, BP 1173, 3038 Sfax, Tunisia*
[2] *CReSTIC, IUT de Troyes, 9 Rue de Québec, BP 396, 10026 Troyes Cedex, France*

Correspondence should be addressed to Nabiha Touijer; nabiha_ettouijer@yahoo.fr

Received 24 July 2016; Revised 3 January 2017; Accepted 1 February 2017; Published 8 March 2017

Academic Editor: James Lam

This paper deals with the self-tuning control problem of linear systems described by autoregressive exogenous (ARX) mathematical models in the presence of unmodelled dynamics. An explicit scheme of control is described, which we use a recursive algorithm on the basis of the robustness σ-modification approach to estimate the parameters of the system, to solve the problem of regulation tracking of the system. This approach was designed with the assumptions that the norm of the vector of the parameters is well-known. A new quadratic criterion is proposed to develop a modified recursive least squares (M-RLS) algorithm with σ-modification. The stability condition of the proposed estimation scheme is proved using the concepts of the small gain theorem. The effectiveness and reliability of the proposed M-RLS algorithm are shown by an illustrative simulation example. The effectiveness of the described explicit self-tuning control scheme is demonstrated by simulation results of the cruise control system for a vehicle.

1. Introduction

Adaptive control has been known since 1950 by Caldwell [1]. Different types of adaptive controls were discussed and used to design adaptive laws of the proposed control schemes. Various studies have been focused on the development of adaptive control theory [2–4]. Stability theory was introduced. In this context, several studies have been developed [5–15].

Egardt [16] noted that the application of adaptive laws could easily be unstable in the presence of small perturbations. In the early 1980s, the robust adaptive control behavior has become much discussed [17, 18]. Several researches developed and studied the robust adaptive control [19–29]. In continuous time, Ioannou and Sun [10] developed the robust adaptive control (pole placement control and model reference control) for dynamic systems in presence of unmodelled dynamics. The different developed control scheme has been based on algorithms with different robustness approach (dead zone, normalization...) to estimate the parameters of the systems. In discrete time, different robust adaptive control schemes have been developed and applied to the class of linear systems described by a mathematical model ARX in the presence of unmodelled dynamics [30–32]. Different robust adaptive control of monovariable systems have been developed on the basis of the modified recursive least squares algorithm M-RLS with approach robustness dead zone [33–36]. The stability conditions of the different proposed estimation scheme have been demonstrated. A robust explicit scheme of self-tuning regulation using the modified filtering recursive algorithm with dead zone was applied to a temperature regulation system in the building [37]. The M-RLS algorithm was extended to a multivariable system, where the stability condition of estimation scheme has been shown and a robust self-tuning control has been developed [38]. The different parametric estimation algorithms were based on the knowledge of the bounds of the unmodelled dynamics.

This paper focuses on the regulation-tracking problem for the stochastic systems described by the ARX mathematical model, in the presence of unknown unmodelled dynamics in the parameters of the system. This problem consists of developing a control law (called the corrector) allowing the output of the system to follow a time-varying reference signal while reducing the effects of disturbances acting at different locations of the system to be controlled. An explicit scheme of self-tuning control has been designed with the assumptions that the norm of the vector of the parameters is known. A quadratic criterion is proposed to develop M-RLS algorithm with σ-modification that will be used in the estimation step of control scheme. The choice of parameter σ is given. The stability condition of the proposed parametric estimation scheme is proved using the small gain theorem [39] and based on the stability condition of the RLS algorithm [40].

The remainder of his paper is structured as follows. Section 2 describes the stochastic systems by ARX mathematical model in presence of unmodelled dynamics. Section 3, firstly, treats the RLS algorithm, and, secondly, a new quadratic criterion is proposed to develop M-RLS algorithm with σ-modification. Furthermore, the choice of σ is given. The stability condition of the developed parametric estimation scheme is shown on the basis of the concepts of the small gain theorem. Section 4 presents an explicit scheme of self-tuning control using the proposed recursive algorithm M-RLS with σ-modification to estimate the parameters of the system. Section 5 provides two simulation examples. Firstly, a simulation example is given to illustrate the reliability and the effectiveness of the proposed M-RLS algorithm with σ-modification which are compared to the RLS algorithm. And secondly the simulation results of the cruise control system for vehicles are given to show the performance of the explicit scheme of self-tuning control which is compared to the explicit scheme of self-tuning control based on the RLS algorithm. Finally, concluding remarks are given in Section 6.

2. System Description

This section describes a stochastic system by ARX mathematical model with unknown parameters and in the presence of unmodelled dynamics.

Let us consider a linear stochastic system, which can be described by the following discrete-time ARX mathematical model:

$$A\left(q^{-1}\right) y(k) = B\left(q^{-1}\right) u(k) + v(k), \tag{1}$$

where $u(k)$ and $y(k)$ represent, respectively, the input and the output of the system at the discrete-time k, $v(k) = e(k) + m(k)$ is the noise acting on the system, where $e(k)$ is an independent random variable with zero mean and constant variance and $m(k)$ is unknown unmodelled dynamics, and $A(q^{-1})$ and $B(q^{-1})$ are polynomials, which are defined, respectively, as

$$\begin{aligned} A\left(q^{-1}\right) &= 1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a}, \\ B\left(q^{-1}\right) &= b_1 q^{-1} + \cdots + b_{n_b} q^{-n_b}, \end{aligned} \tag{2}$$

where n_a and n_b are the orders of the polynomials $A(q^{-1})$ and $B(q^{-1})$, respectively.

We suppose that the orders n_a and n_b are known.

The output $y(k)$ of system (1) can be given by

$$\begin{aligned} y(k) = {} & -a_1 y(k-1) - \cdots - a_{n_a} y(k-n_a) + b_1 u(k-1) \\ & + \cdots + b_{n_b} u(k-n_b) + v(k). \end{aligned} \tag{3}$$

The mathematical model (3) can be written as follows:

$$y(k) = \theta^T \varphi(k) + v(k), \tag{4}$$

where θ^T and $\varphi^T(k)$ are the parameters vector and the observation vector, respectively, such that

$$\theta^T = \left[a_1 \cdots a_{n_a} \;\; b_1 \cdots b_{n_b}\right], \tag{5}$$

$$\begin{aligned} \varphi^T(k) = {} & \left[-y(k-1) - \cdots \right. \\ & \left. - y(k-n_a) \, u(k-1) \cdots u(k-n_b)\right]. \end{aligned} \tag{6}$$

3. Parametric Estimation Algorithm

This section concerns solving the parametric estimation problem for the considered stochastic system (1) on the basis of the two following assumptions.

Assumption 1. The parameters intervening in vector θ (5) are bounded; an upper bound M_0 of θ is known, such that

$$0 < \|\theta\| \le M_0. \tag{7}$$

Figure 1 represents the first area of θ.

Assumption 2. The parameters intervening in vector θ (5) are bounded; an upper bound and a lower bound, respectively, $M_{\max}$ and $M_{\min}$, are known, such that

$$0 < M_{\min} \le \|\theta\| \le M_{\max}. \tag{8}$$

Figure 2 represents the second area of θ with

$$\|\theta\| = \sqrt{\sum_{i=1}^{n_a} a_i^2 + \sum_{j=1}^{n_b} b_j^2}. \tag{9}$$

The aim of this section is the development of a robust recursive parametric estimation algorithm for uncertain dynamic system. Thus, we propose to use, in the parametric estimation algorithm RLS, a parameter of robustness which is known in the literature by σ-modification. The developed algorithm is called modified recursive least square (M-RLS) algorithm with σ-modification. However, before the formulation of this algorithm, we present, in the following subsection, the recursive least square algorithm RLS.

3.1. Recursive Least Square Algorithm RLS. To show the advantages of the proposed recursive parametric estimation algorithm RLS with σ-modification to be proposed later, the RLS algorithm is given to compare its performance to the performance of the proposed parametric estimation scheme, which is described in this subsection.

The recursive parametric estimation algorithm RLS is given by

$$\begin{aligned} \widehat{\theta}(k) &= \widehat{\theta}(k-1) + P(k)\varphi(k)\varepsilon(k), \\ P(k) &= P(k-1) - \frac{P(k-1)\varphi(k)\varphi^T(k)P(k-1)}{1 + \varphi^T(k)P(k-1)\varphi(k)}, \\ \varepsilon(k) &= y(k) - \varphi^T(k)\widehat{\theta}(k-1). \end{aligned} \tag{10}$$

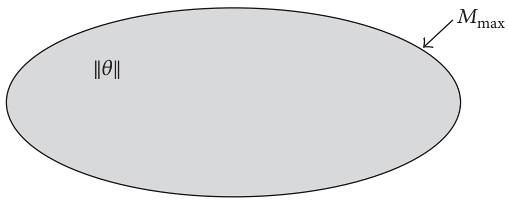


Figure 1: Representation of the first area.

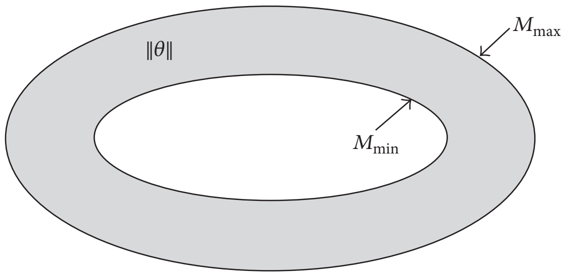


Figure 2: Representation of the second area.

Theorem 3 (see [40]). *Consider a linear system which can be described by input-output mathematical model (3) (without unmodelled dynamic). The estimation of the parameters intervening in the mathematical model can be made by using RLS algorithm (10). If the components of the vectors $\widehat{\theta}(0)$ and $\varphi(k)$ are finite, then the convergence of the RLS algorithm is ensured.*

Lemma 4 (see [40]). *Let us consider the RLS algorithm (10) to estimate the parameters intervening in (3). If the components of vectors $\widehat{\theta}(0)$ and $\varphi(k)$ are finite, and if again the adaptation gain $P(k)$ is decreasing, then the convergence of this algorithm is ensured.*

In the presence of unmodelled dynamics, the inconvenient of the RLS algorithm is that, at the computing of $\widehat{\theta}(k-1)$, the corresponding norm can exceed certain threshold. Then the effectiveness of this algorithm is not ensured.

The proposed key idea is based on the two following steps.

Step 1. If Assumption 1 (or Assumption 2) is verified, then $\widehat{\theta}(k)$ is given by the RLS algorithm (10).

Step 2. If Assumption 1 (or Assumption 2) is not verified, we propose to develop a parametric estimation algorithm such that

$$\widehat{\theta}(k) = f\left(\widehat{\theta}(k-1)\right) + P(k)\varphi(k)\varepsilon(k), \quad (11)$$

where $0 < \|f(\widehat{\theta}(k-1))\| \leq M_0$ or $M_{min} \leq \|f(\widehat{\theta}(k-1))\| \leq M_{max}$.

3.2. Modified Recursive Least Square Algorithm M-RLS with σ-Modification. In order to overcome the parametric estimation problem for the considered system, we will develop a modified algorithm M-RLS with σ-modification.

The following quadratic criterion is proposed to solve the parametric estimation problem for the considered system:

$$J(k) = \sum_{i=n_a+1}^{k} \frac{\left[y(i) - \widehat{\theta}^T(k)\varphi(i)\right]^2}{2} + \sum_{i=n_a+1}^{k} \widehat{\theta}^T(k)\zeta(i)\widehat{\theta}(i), \quad (12)$$

where $\zeta(i)$ is a symmetrical matrix, whose choice is to give certain robustness to the developed estimation scheme with respect to the unmodelled dynamics.

The optimal of the estimated vector of parameters $\widehat{\theta}(i)$, which is given by the minimization of the quadratic criterion $J(k)$, can be obtained by the calculation of the derived of this criterion, such that

$$\frac{\partial(J(k))}{\partial\left(\widehat{\theta}(i)\right)} = -\sum_{i=n_a+1}^{k} y(i)\varphi(i) + \sum_{i=n_a+1}^{k} \varphi(i)\varphi^T(i)\widehat{\theta}(k) + \sum_{i=n_a+1}^{k-1} \zeta(i)\widehat{\theta}(i). \quad (13)$$

In fact, the optimal of the vector of the estimated parameters $\widehat{\theta}(i)$ corresponds to the cancellation of (13). Thus, by cancelling expression (13) derivative of the $J(k)$ quadratic criterion considered, we can write the following expression:

$$P^{-1}(k)\widehat{\theta}(k) = \sum_{i=n_a+1}^{k} y(i)\varphi(i) - \sum_{i=n_a+1}^{k-1} \zeta(i)\widehat{\theta}(i), \quad (14)$$

such that

$$P^{-1}(k) = \sum_{i=n_a+1}^{k} \varphi(i)\varphi^T(i). \quad (15)$$

At the discrete-time $k+1$, (14) is written as follows:

$$\begin{aligned} P^{-1}(k+1)\widehat{\theta}(k+1) &= \sum_{i=n_a+1}^{k+1} y(i)\varphi(i) - \sum_{i=n_a+1}^{k} \zeta(i)\widehat{\theta}(i) \\ &= \sum_{i=n_a+1}^{k} y(i)\varphi(i) - \sum_{i=n_a+1}^{k-1} \zeta(i)\widehat{\theta}(i) + y(k+1)\varphi(k+1) - \zeta(k)\widehat{\theta}(k). \end{aligned} \quad (16)$$

Using (14) and (15) and adding and subtracting $\varphi(k+1)\varphi^T(k+1)\hat{\theta}(k)$ to the right member of (16), we obtain

$$\begin{aligned} P^{-1}(k+1)\hat{\theta}(k+1) = P^{-1}(k+1)\hat{\theta}(k) \\ + \varphi(k+1)\varepsilon(k+1) \\ - \zeta(k)\hat{\theta}(k) \end{aligned} \tag{17}$$

with

$$\varepsilon(k+1) = y(k+1) - \varphi^T(k+1)\hat{\theta}(k). \tag{18}$$

Multiplying (16) by $P(k+1)$, the estimated vector $\hat{\theta}(k+1)$ is given as follows:

$$\begin{aligned} \hat{\theta}(k+1) = \hat{\theta}(k) - \zeta(k)P(k+1)\hat{\theta}(k) \\ + P(k+1)\varphi(k+1)\varepsilon(k+1). \end{aligned} \tag{19}$$

Then, the deduced recursive parametric estimation algorithm RLS with σ-modification is defined by

$$\begin{aligned} \hat{\theta}(k) &= \hat{\theta}(k-1) - \zeta(k-1)P(k)\hat{\theta}(k-1) \\ &\quad + P(k)\varphi(k)\varepsilon(k), \\ P(k) &= P(k-1) - \frac{P(k-1)\varphi(k)\varphi^T(k)P(k-1)}{1+\varphi^T(k)P(k-1)\varphi(k)}, \\ \varepsilon(k) &= y(k) - \varphi^T(k)\hat{\theta}(k-1). \end{aligned} \tag{20}$$

The determination of the following function depends on the choice of parameter $\zeta(k-1)$:

$$f\left(\hat{\theta}(k-1)\right) = \hat{\theta}(k-1) - \zeta(k-1)P(k)\hat{\theta}(k-1). \tag{21}$$

Based on Assumption 1, parameter $\zeta(k-1)$ is defined as follows:

$$0 < \left\|\hat{\theta}(k-1) - \zeta(k-1)P(k)\hat{\theta}(k-1)\right\| < M_0. \tag{22}$$

We propose to write matrix $\zeta(k-1)$ as follows:

$$\zeta(k-1) = \sigma(k-1)P^{-1}(k). \tag{23}$$

The following conditions permit determining parameter $\sigma(k-1)$:

(1) If the next condition satisfies $0 < \|\hat{\theta}(k-1)\| \le M_0$, then we take $\sigma(k-1) = 0$.

(2) if the next condition satisfies $\|\hat{\theta}(k-1)\| > M_0$, then we must determine a value for the parameter $\sigma(k-1)$, while satisfying the following condition:

$$\sigma(k-1) < 1. \tag{24}$$

Using (23), (22) can be written as follows:

$$0 < (1-\sigma(k-1))\left\|\hat{\theta}(k-1)\right\| < M_0. \tag{25}$$

Using the second condition and dividing (25) by $\|\hat{\theta}(k-1)\|$, (25) can be written as follows:

$$0 < 1 - \frac{M_0}{\left\|\hat{\theta}(k-1)\right\|} < \sigma(k) < 1. \tag{26}$$

Then, there exists a finite scalar σ_0, such that

$$\sigma(k) = \sigma_0\left[1 - \frac{M_0}{\left\|\hat{\theta}(k-1)\right\|}\right], \tag{27}$$

if $\|\hat{\theta}(k-1)\| > M_0$, with $\sigma_0 > 1$.

Thus, the parameter $\sigma(k-1)$ is defined as follows:

$$\sigma(k-1) = \begin{cases} 0, & \text{if } \left\|\hat{\theta}(k-1)\right\| \le M_0 \\ \sigma_{\max}(k-1) = \sigma_0\left[1 - \dfrac{M_0}{\left\|\hat{\theta}(k-1)\right\|}\right], & \text{either, with } \sigma_0 > 1. \end{cases} \tag{28}$$

Based on Assumption 2, the parameter $\sigma(k-1)$ is defined as follows:

$$\begin{aligned} M_{\min} \le \left\|\hat{\theta}(k-1) - \sigma(k-1)P(k)\hat{\theta}(k-1)\right\| \\ \le M_{\max}. \end{aligned} \tag{29}$$

We must consider the conditions intervening in the three following situations, in order to determine the parameter $\sigma(k-1)$:

(1) If the following condition satisfies $\|\hat{\theta}(k-1)\| < M_{\min}$, then we must take

$$\sigma(k-1) = \sigma_{\min}(k-1). \tag{30}$$

(2) If the following condition satisfies $M_{\min} \le \|\hat{\theta}(k-1)\| \le M_{\max}$, then we must take

$$\sigma(k-1) = 0. \tag{31}$$

(3) If the following condition satisfies $\|\hat{\theta}(k-1)\| > M_{\max}$, then we must take $\sigma(k-1) = \sigma_{\max}(k-1)$, where $\sigma(k-1)$ is given by (27), with $M_0 = M_{\max}$.

By considering the first situation, we can define the parameter as $\sigma_{\min}(k-1)$ as follows:

$$\left\|\hat{\theta}(k-1) - \sigma_{\min}(k-1)\hat{\theta}(k-1)\right\| \ge M_{\min}. \tag{32}$$

Dividing (32) by $\|\hat{\theta}(k-1)\|$, we can write the following inequality:

$$\left[1 - \sigma_{\min}(k-1)\right] > \frac{M_{\min}}{\left\|\hat{\theta}(k-1)\right\|}. \tag{33}$$

In (33), adding $\sigma_{\min}(k-1)$ and subtracting $M_{\min}/\|\hat{\theta}(k-1)\|$, we obtain

$$\sigma_{\min}(k-1) < 1 - \frac{M_{\min}}{\left\|\hat{\theta}(k-1)\right\|} < 0. \tag{34}$$

Thus, we can affirm that there exists a finite scalar σ_1, such that

$$\sigma_{\min}(k) = \sigma_1 \left[1 - \frac{M_{\min}}{\left\|\widehat{\theta}(k-1)\right\|}\right], \tag{35}$$

where the following condition is supposed to satisfy $\|\widehat{\theta}(k-1)\| < M_{\min}$, with $\sigma_1 > 0$.

Thus, the parameter $\sigma(k-1)$ is defined as follows:

$$\sigma(k-1) = \begin{cases} \sigma_{\min}(k-1) = \sigma_1 \left(1 - \dfrac{M_{\min}}{\left\|\widehat{\theta}(k-1)\right\|}\right) & \text{if } \left\|\widehat{\theta}(k-1)\right\| < M_{\min} \\ 0, & \text{if } M_{\min} \le \left\|\widehat{\theta}(k-1)\right\| \le M_{\max} \\ \sigma_{\max}(k-1) = \sigma_0 \left(1 - \dfrac{M_{\max}}{\left\|\widehat{\theta}(k-1)\right\|}\right), & \text{either, with } \sigma_1, \sigma_0 > 1. \end{cases} \tag{36}$$

Consequently, the proposed recursive parametric estimation algorithm RLS with σ-modification is defined by

$$\begin{aligned} \widehat{\theta}(k) &= \widehat{\theta}(k-1) - \sigma(k-1)\widehat{\theta}(k-1) \\ &\quad + P(k)\varphi(k)\varepsilon(k), \\ P(k) &= P(k-1) - \frac{P(k-1)\varphi(k)\varphi^T(k)P(k-1)}{1 + \varphi^T(k)P(k-1)\varphi(k)}, \\ \varepsilon(k) &= y(k) - \varphi^T(k)\widehat{\theta}(k-1), \end{aligned} \tag{37}$$

where $\sigma(k-1)$ is defined by (28) (or (36)).

If $\sigma(k-1) = 0$, then the RLS algorithm has been used.

In the next, the convergence condition of the RLS algorithm is used to demonstrate the sufficient condition of stability of the proposed estimation scheme.

3.3. Stability Analysis of the Proposed Parametric Estimation Scheme. Based on the small gain theorem, the stability analysis of the proposed parametric estimation scheme is established.

Consider the closed-loop system Figure 3, where H_1 and H_2 are causal operators. The small gain theorem gives a sufficient condition for stability of the closed-loop system below, using the notion of the gain operator defined later.

Theorem 5 (small gain theorem [39]). *Consider the closed-loop system Figure 3, where the operators H_1 and H_2 are bounded. Let the gains of the systems H_1 and H_2 are γ_1 and γ_2, respectively. If $\gamma_1\gamma_2 < 1$, then the closed-loop system is input-output stable.*

The a posteriori prediction error $\varepsilon_\circ(k)$ is given by

$$\varepsilon_\circ(k) = y(k) - \varphi^T(k)\widehat{\theta}(k) = -\varphi^T(k)\widetilde{\theta}(k) \tag{38}$$

with

$$\widetilde{\theta}(k) = \widehat{\theta}(k) - \theta. \tag{39}$$

Subtracting θ of the first equation in (37) and based on (39), (39) can be given by

$$\begin{aligned} \widetilde{\theta}(k) &= \widetilde{\theta}(k-1) + P(k)\varphi(k)\varepsilon_\circ(k) \\ &\quad - \sigma(k-1)\widehat{\theta}(k-1). \end{aligned} \tag{40}$$

Using (40), the a posteriori prediction error $\varepsilon_\circ(k)$ is given by

$$\begin{aligned} \varepsilon_\circ(k) &= -\varphi^T(k)\left[P(k)\varphi(k)\varepsilon_\circ(k) + \widetilde{\theta}(k-1)\right. \\ &\quad \left. - \sigma(k-1)\widehat{\theta}(k-1)\right]. \end{aligned} \tag{41}$$

Let us consider parameter $w(k)$, which is defined as follows:

$$w(k) = -\varepsilon_\circ(k). \tag{42}$$

Using (41) and (42), the closed-loop system is shown in Figure 4.

Based on Lemma 4, we assume that the gain matrix $P(k)$ is decreasing and bounded and that the components of vector $\varphi(k)$ are finite. If $P(k)\varphi(k)$ and $\varphi(k)$ are bounded, then there exists $\gamma_1 \ge 0$, $\gamma_2 \ge 0$, β_1 and β_2, such that

$$\begin{aligned} \left\|P(k)\varphi(k)\varepsilon_\circ(k)\right\| &\le \gamma_1 \left\|\varepsilon_\circ(k)\right\| + \beta_1, \\ \left\|\varphi(k)\widetilde{\theta}(k)\right\| &\le \gamma_2 \left\|\widetilde{\theta}(k)\right\| + \beta_2. \end{aligned} \tag{43}$$

Based on the closed-loop system shown in Figure 4, $\varepsilon_\circ(k)$ and $\widetilde{\theta}(k)$ can be written, respectively, as follows:

$$\begin{aligned} \varepsilon_\circ(k) &= -\varphi^T(k)\widetilde{\theta}(k) + U_1 = -\varphi^T(k)\widetilde{\theta}(k), \\ \widetilde{\theta}(k) &= P(k)\varphi(k)\varepsilon_\circ(k) + U_2(k) \end{aligned} \tag{44}$$

with

$$\begin{aligned} U_1 &= 0, \\ U_2(k) &= \widetilde{\theta}(k-1) - \sigma(k-1)\widehat{\theta}(k-1). \end{aligned} \tag{45}$$

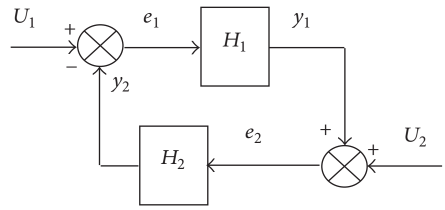


Figure 3: General closed-loop system.

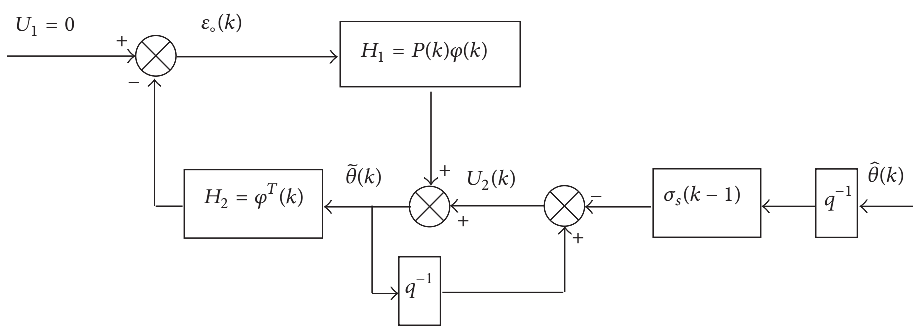


Figure 4: Closed-loop system of the parametric estimation scheme.

Based on (43) and using (44), we can write

$$\left\|\varepsilon_\circ(k)\right\| \le \gamma_2 \left\|\widetilde{\theta}(k)\right\| + \beta_2, \tag{46}$$

$$\left\|\widetilde{\theta}(k)\right\| \le \left\|U_2(k)\right\| + \gamma_1 \left\|\varepsilon_\circ(k)\right\| + \beta_1. \tag{47}$$

If

$$\gamma_1\gamma_2 < 1, \tag{48}$$

then

$$\left\|\varepsilon_\circ(k)\right\| \le \frac{1}{1-\gamma_1\gamma_2}\left[\gamma_2\left\|U_2(k)\right\| + \gamma_2\beta_1 + \beta_2\right], \tag{49}$$

$$\left\|\widetilde{\theta}(k)\right\| \le \frac{1}{1-\gamma_1\gamma_2}\left[\left\|U_2(k)\right\| + \gamma_1\beta_2 + \beta_1\right]. \tag{50}$$

So, if $U_2(k)$ is bounded, then the stability condition of the recursive parametric estimation scheme is ensured, such that

$$\begin{aligned} U_2(k) &= \widetilde{\theta}(k-1) - \sigma(k-1)\widehat{\theta}(k-1) \\ &= \widehat{\theta}(k-1) - \theta - \sigma(k-1)\widehat{\theta}(k-1) \\ &= \widehat{\theta}(k-1)\left[1-\sigma(k-1)\right] - \theta. \end{aligned} \tag{51}$$

The norm of $U_2(k)$ is given by

$$\begin{aligned} \left\|U_2(k)\right\| &= \left\|\widehat{\theta}(k-1)\left[1-\sigma(k-1)\right] - \theta\right\| \\ &\le \left\|\widehat{\theta}(k-1)\left[1-\sigma(k-1)\right]\right\| + \left\|\theta\right\|. \end{aligned} \tag{52}$$

Based on (7) (or (8)) and (25), $U_2(k)$ is bounded, such that

$$\left\|U_2(k)\right\| \le 2M_0. \tag{53}$$

Based on (47), (48) and (49) are given by, respectively,

$$\begin{aligned} \left\|\varepsilon_\circ(k)\right\| &\le \frac{1}{1-\gamma_1\gamma_2}\left[2\gamma_2 M_0 + \gamma_2\beta_1 + \beta_2\right], \\ \left\|\widetilde{\theta}(k)\right\| &\le \frac{1}{1-\gamma_1\gamma_2}\left[2M_0 + \gamma_1\beta_2 + \beta_1\right]. \end{aligned} \tag{54}$$

Theorem 6. *The closed-loop system of the proposed recursive parametric estimation scheme shown in Figure 4 is stable; if the operator $P(k)\varphi(k)$ and $\varphi(k)$ are bounded and have positive gain, respectively, γ_1 and γ_2 are defined, such that $\gamma_1\gamma_2 < 1$.*

4. Explicit Scheme of Self-Tuning Control

This section discusses the regulation-tracking problem for the considered system, where an explicit scheme of self-tuning control will be developed. The following quadratic criterion $J(k+d+1)$ is used to design the controller:

$$\begin{aligned} &J(k+d+1) \\ &= \left[S\left(q^{-1}\right)\left[y(k+d+1) - y_r(k+d+1)\right]\right]^2 + \left[Q\left(q^{-1}\right)u(k)\right]^2, \end{aligned} \tag{55}$$

where $y_r(k+d+1)$ represents the desired output signal, $u(k)$ is the control law, and $S(q^{-1})$ and $Q(q^{-1})$ are two polynomials, such that

$$S\left(q^{-1}\right) = 1 + s_1 q^{-1} + \cdots + s_{n_s} q^{-n_s},$$
$$Q\left(q^{-1}\right) = q_0 + q_1 q^{-1} + \cdots + q_{n_q} q^{-n_q}. \tag{56}$$

Note that the orders n_s and n_q of the polynomials $S(q^{-1})$ and $Q(q^{-1})$, respectively, are chosen by the designer.

The derivate of the criterion $J(k+d+1)$, which is described by (55), is given by

$$\begin{aligned}\frac{\partial J(k+d+1)}{\partial(u(k))} &= 2b_1 S\left(q^{-1}\right)\left[y(k+d+1) - y_r(k+d+1)\right] \\ &\quad + 2q_0 Q\left(q^{-1}\right) u(k)\end{aligned} \tag{57}$$

with

$$\begin{aligned}S\left(q^{-1}\right) y(k+d+1) = qB\left(q^{-1}\right) F\left(q^{-1},k\right) u(k) \\ + G\left(q^{-1},k\right) y(k) \\ + F\left(q^{-1},k\right) v(k+d+1),\end{aligned} \tag{58}$$

where $F(q^{-1},k)$ and $G(q^{-1},k)$ are solutions of the following polynomial equation:

$$S\left(q^{-1}\right) = A\left(q^{-1}\right) F\left(q^{-1},k\right) + q^{-d-1} G\left(q^{-1},k\right). \tag{59}$$

The polynomials $F(q^{-1},k)$ and $G(q^{-1},k)$ are given by

$$\begin{aligned}F\left(q^{-1},k\right) &= 1 + F_1(k) q^{-1} + \cdots + F_d(k) q^{-d}, \\ G\left(q^{-1},k\right) &= G_0(k) + G_1(k) q^{-1} + \cdots \\ &\quad + G_{n_a-1}(k) q^{1-n_a}.\end{aligned} \tag{60}$$

Thus, the optimal control law $u(k)$ can be written by

$$\begin{aligned}u(k) = \frac{1}{Z\left(q^{-1},k\right)}\Bigg[-G\left(q^{-1},k\right) y(k) \\ + \frac{1}{b_1} S\left(q^{-1}\right) y_r(k+d+1)\Bigg],\end{aligned} \tag{61}$$

where the polynomials $H(q^{-1},k)$ and $Z(q^{-1},k)$ are given by, respectively,

$$\begin{aligned}Z\left(q^{-1},k\right) &= H\left(q^{-1},k\right) + \frac{1}{b_1} q_0 Q\left(q^{-1}\right), \\ H\left(q^{-1},k\right) &= qB\left(q^{-1}\right) F\left(q^{-1},k\right).\end{aligned} \tag{62}$$

4.1. Explicit Scheme of Self-Tuning Control. The recursive algorithm of the explicit robust self-tuning control scheme is formulated by the following steps.

Step 1. Estimate the parameters intervening in the ARX mathematical model (1) using the M-RLS algorithm with σ-modification (37).

Step 2. Calculate the parameters intervening in the polynomials $F(q^{-1},k)$ and $G(q^{-1},k)$ by solving the polynomial equation defined as follows:

$$S\left(q^{-1}\right) = \widehat{A}\left(q^{-1}\right) F\left(q^{-1},k\right) + q^{-d-1} G\left(q^{-1},k\right). \tag{63}$$

Step 3. Calculate the control law $u(k)$ given by the following equation:

$$\begin{aligned}u(k) = \frac{1}{z_1(k)}\Bigg[-\sum_{r=2}^{n_b+d-1} z_r(k) u(k-r+1) \\ - \sum_{t=0}^{n_a-1} g_t(k) y(k-t) + \frac{1}{\widehat{b}_1(k)} y_r(k+d+1-j) \\ + \sum_{j=1}^{n_s} \frac{s_t}{\widehat{b}_1(k)} y_r(k+d+1-j)\Bigg].\end{aligned} \tag{64}$$

Note that if $\widehat{b}_1(k) = 0$, then we take $h_1(k) = 0.01$.

5. Simulation Results

5.1. Simulation Example 1. Let us consider that the dynamic system can be described by the following mathematical model ARX:

$$\begin{aligned}y(k) &= -a_1(k) y(k-1) - a_2(k) y(k-2) \\ &\quad + b_1(k) u(k-2) + b_2(k) u(k-3) + e(k),\end{aligned} \tag{65}$$

where $y(k)$ and $u(k)$ are the output and the input of the second-order system with time delay being one and $e(k)$ is white noise acting on the system.

The output of the system can be given as follows:

$$y(k) = \theta^T(k) \varphi(k) \tag{66}$$

with

$$\theta^T(k) = \left[a_1(k) \quad a_2(k) \quad b_1(k) \quad b_2(k)\right], \tag{67}$$

$$\begin{aligned}&\varphi^T(k) \\ &= \left[-y(k-1) \quad -y(k-2) \quad u(k-2) \quad u(k-3)\right],\end{aligned} \tag{68}$$

where $\theta(k)$ and $\varphi(k)$ represent, respectively, the vector of the parameters and the vector of the observations.

The bounds of unmodelled dynamic presented in the system are unknown, but the norm of the vector of the parameters is given by the following inequality:

$$M_{\min} \leq \left\|\theta^*\right\| \leq M_{\max} \tag{69}$$

with $M_{\min} = 0.85$ and $M_{\max} = 1.05$.

In simulation, the nominal values of the uncertain parameters of system are defined by the following.

$a_1(k) = 0.79 + 0.1\sin(0.1k)$, $a_2(k) = 0.2 + 0.1\cos(0.1k)$, $b_1(k) = 0.4 + 0.1\sin(0.1k)$, and $b_2(k) = 0.3 + 0.1\cos(0.1k)$. The evolution curve of the norm of the vector of the nominal values of parameters is given Figure 5.

The objective of this simulation example is the demonstration of the performance of the robust recursive algorithm for parameter estimation M-RLS with σ-modification (37). A comparative study between the recursive algorithm RLS (10) and the proposed recursive algorithm (37) is treated. The more robust algorithm is the algorithm which can estimate the parameters such that the norm of the vector of the estimated parameters is inside the desired area.

The input signal is a square signal with amplitude that equals two and a period that equal 100, $\{e(k)\}$ is a sequence of random variables with zero mean and variance $\sigma^2 = 0.2$, and the gain matrix $P(0) = 1000I$.

We use the recursive algorithm RLS (10) to estimate the parameters involved in (67). Figure 6 shows the evolution curve of the variance of the prediction error $\sigma_\varepsilon^2(k)$ and Figure 7 shows the evolution curve of the norm of the vector of the estimated parameters $\|\widehat{\theta}(k)\|$.

We use the proposed recursive algorithm M-RLS with σ-modification (37) to estimate the parameters involved in (67). Figure 8 represents the evolution curve of the variance of the prediction error $\sigma_\varepsilon^2(k)$ and Figure 9 represents the evolution curve of the norm of the vector of the estimated parameters $\|\widehat{\theta}(k)\|$.

Based on the simulation results, we conclude that the proposed recursive algorithm M-RLS with σ-modification (37) is more robust than the recursive algorithm defined by (10).

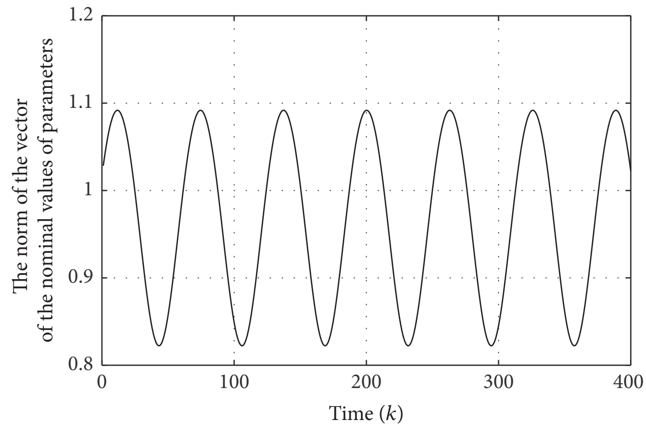


FIGURE 5: Evolution curve of the norm of the vector of the nominal values of parameters $\|\theta(k)\|$.

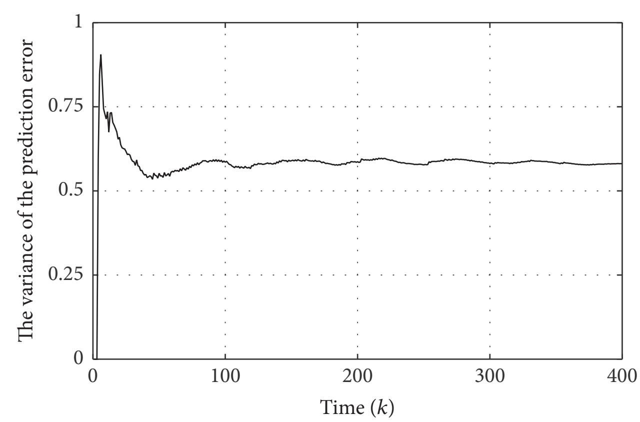


FIGURE 6: Evolution curve of the variance of the prediction error $\sigma_\varepsilon^2(k)$.

5.2. Simulation Example 2: The Vehicle. We treat here an example of numerical simulation which is related to the control of a vehicle of laboratory, by using the described algorithm of the explicit scheme of the self-tuning control. Figure 10 represents the scheme of this vehicle, as considered by Sam Fadali [41], in which U is the input force, V is the velocity of this vehicle, and b is the coefficient of viscous friction.

Sam Fadali [41] determined the following transfer function $G(s)$ in open loop, such that describes the dynamic behavior of the vehicle:

$$G(s) = \frac{K}{s+3}. \tag{70}$$

The discrete transfer function $G(q^{-1})$ relating to (70) can be defined as follows, such that the used sampling period is $T_e = 0.02$ sec:

$$G(q^{-1}) = \frac{b_1(k)\,q^{-1}}{1 - 0.9418q^{-1}}. \tag{71}$$

For the system to be stable, the closed-loop poles or the roots of the following characteristic equation

$$1 + G(q^{-1}) = 0 \tag{72}$$

must lie within the unit circle.

To ensure this condition of the stability of the system in closed loop, the parameter $b_1(k)$ must be defined as follows:

$$-0.054 < b_1(k) < 1.941. \tag{73}$$

This system can be described by the following mathematical model ARX:

$$y(k) = -a_1 y(k-1) + b_1(k)\,u(k-1) + v(k), \tag{74}$$

where $y(k)$ represents the velocity of vehicle, $u(k)$ represents the input force, a_1 and $b_1(k)$ are unknown parameters, and $v(k)$ is noise which can be given by the following equation:

$$v(k) = e(k) + m(k), \tag{75}$$

in which the element $m(k)$ designates the unmodelled dynamics related to the parameter $b_1(k)$.

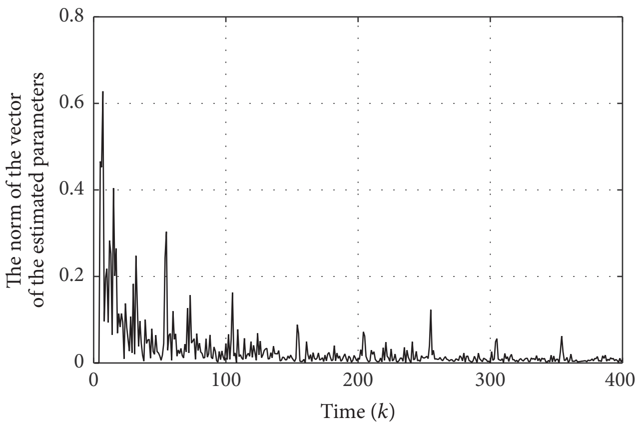


Figure 7: Evolution curve of the norm of the vector of the estimated parameters $\|\hat{\theta}(k)\|$.

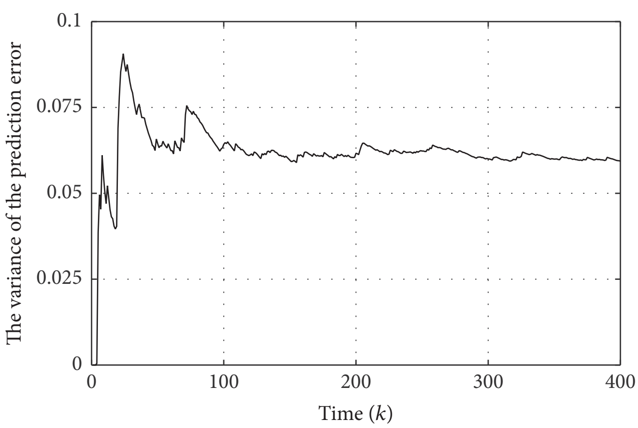


Figure 8: Evolution curve of the variance of the prediction error $\sigma_{\varepsilon}^2(k)$.

The output of the vehicle can be defined as follows:

$$y(k) = \theta^T(k)\varphi(k) + v(k), \tag{76}$$

where the vectors of the parameters $\theta(k)$ and of the observation $\varphi(k)$ are given by the following expression, respectively:

$$\theta^T(k) = [a_1 \quad b_1(k)], \tag{77}$$

$$\varphi(k) = [-y(k-1) \quad u(k-1)]. \tag{78}$$

Thus, the data of the explicit scheme of the proposed robust self-tuning control are as follows:

(1) The different values of the parameters involved in (77) are chosen such that $a_1 = -0.9418$, $b_1(k) = 2 + 0.1\sin(0.2k)$, $m(k) = 0.05\sin(0.2k)u(k-1)$.

(2) The sequence of noise $\{e(k)\}$ is composed of independent random variables with zero mean and constant variance $\sigma^2 = 0.04$.

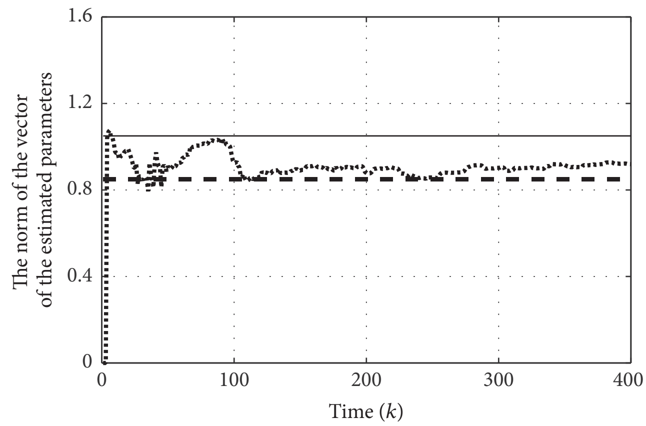


Figure 9: Evolution curve of the norm of the vector of the estimated parameters $\|\hat{\theta}(k)\|$.

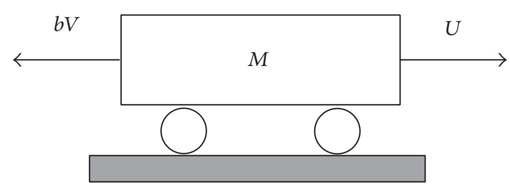


Figure 10: Schematic representation of the cruise control system for a vehicle.

(3) We will take $\hat{\theta}(0) = 0$ and $P(0) = 100I$ (where I is an identity matrix).

(4) The application of the recursive algorithm M-RLS with σ-modification is based on the knowledge of the region Σ where $\theta \in \Sigma$, such that

$$\Sigma = \{\theta \in \mathfrak{R}^n,\ 0 < \|\theta(k)\| < M_0 = 0.9433\} \cup \{a_1 \in \mathfrak{R}^n,\ b_1(k) > 0,\ M_{\min} = 0.9433 < \|\theta(k)\| < M_{\max} = 2.1628\}. \tag{79}$$

(5) $s_1 = 0.51$, $q_0 = 2.6$, and $q_1 = -2.1$.

(6) The evolution curve of the reference velocity $y_r(k)$ is shown in Figure 11.

The tracking error is defined by

$$v(k) = y_r(k) - y(k). \tag{80}$$

Using the same initial conditions, we will compare the numerical simulation results of the explicit scheme of self-tuning based on the recursive algorithm RLS (10) (control scheme (1)) and of the robust explicit scheme of self-tuning control based on the robust recursive algorithm M-RLS with σ-modification (37) (control scheme (2)). Control laws are applied to the example of the vehicle in the presence of unmodelled dynamics.

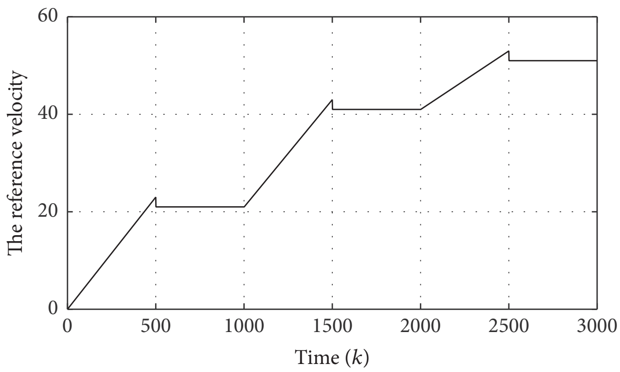


FIGURE 11: Evolution curve of the reference velocity $y_r(k)$.

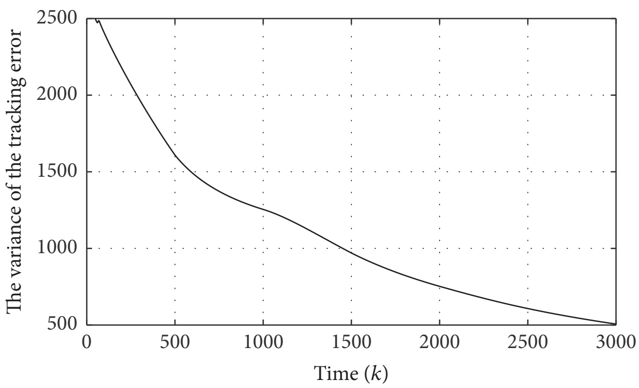


FIGURE 12: Evolution curve of the variance of the tracking error $\sigma_v^2(k)$ (control scheme (1)).

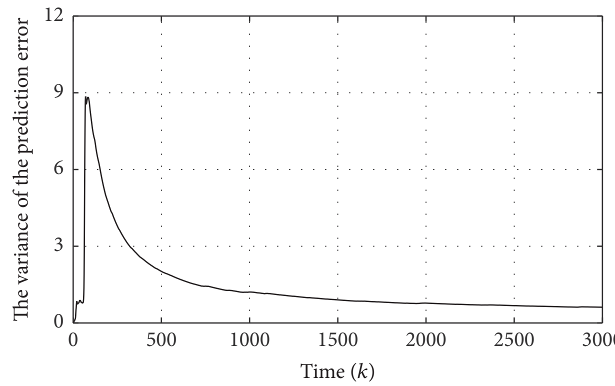


FIGURE 13: Evolution curve of the variance of the prediction error $\sigma_\varepsilon^2(k)$ (control scheme (1)).

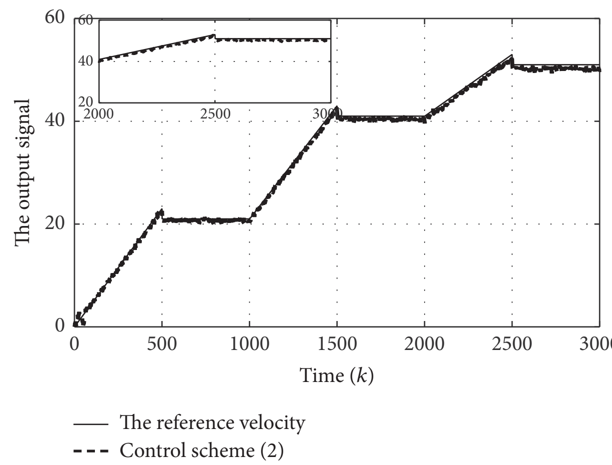


FIGURE 14: Evolution curve of the output signal $y(k)$ and of the reference velocity $y_r(k)$.

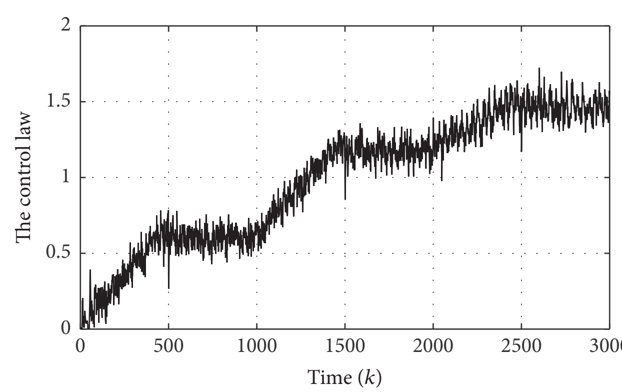


FIGURE 15: Evolution curve of the control law $u(k)$.

Figure 12 show the evolution curve of the variance of the tracking error and Figure 13 show the evolution curve of the variance of the prediction error in the control scheme (1) based on the recursive algorithm RLS (10) to estimate the parameters involved in (79) with considering (77).

In control scheme (2), Figure 14 shows the evolution curve of the velocity $y(k)$, Figure 15 shows the evolution curve of the input force $u(k)$ (or the control law), Figure 16 shows the evolution curve of the estimated parameter $\widehat{a}_1(k)$, Figure 17 shows the evolution curve of the estimated parameter $\widehat{b}_1(k)$, Figure 18 shows the evolution curve of the norm of the vector of the estimated parameters $\|\widehat{\theta}(k)\|$, Figure 19 shows the evolution curve of the variance of the tracking error $\sigma_v^2(k)$, and Figure 20 shows the evolution curve of the variance of the prediction error $\sigma_\varepsilon^2(k)$.

The different illustrated simulation results in Figures 11–20 show the performance of the developed robust explicit self-tuning control scheme on the basis of the proposed M-RLS algorithm with the robustness σ-modification approach. This control scheme is robust, in the presence of unknown unmodelled dynamics, and allows the output to follow the desired velocity while reducing the effects of disturbances acting at different locations in the system. In addition, the estimated parameters are within the desired region.

6. Conclusion

In this paper, we have proposed the M-RLS algorithm with the robustness σ-modification approach. This approach was designed assuming that the bound of desired system parameters norm is known. The stability condition of the parametric estimation scheme was established using the concepts of

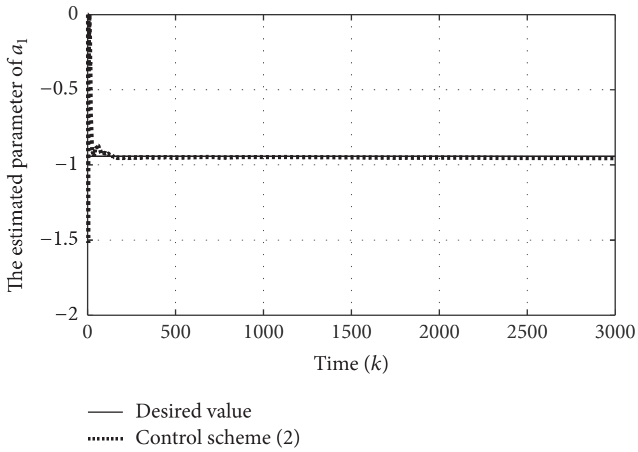


Figure 16: Evolution curve of the estimated parameter $\widehat{a}_1(k)$.

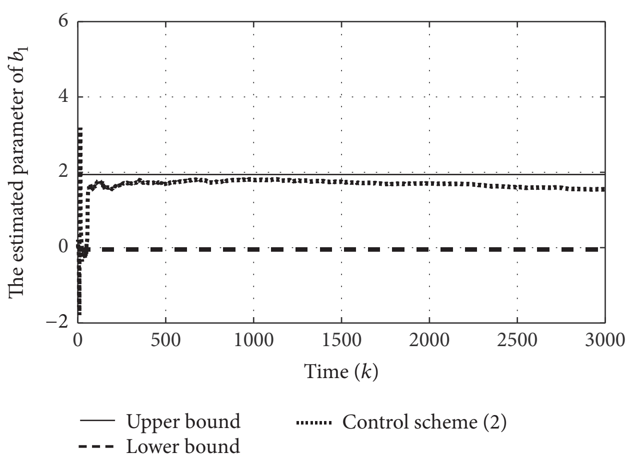


Figure 17: Evolution curve of the estimated parameter $\widehat{b}_1(k)$.

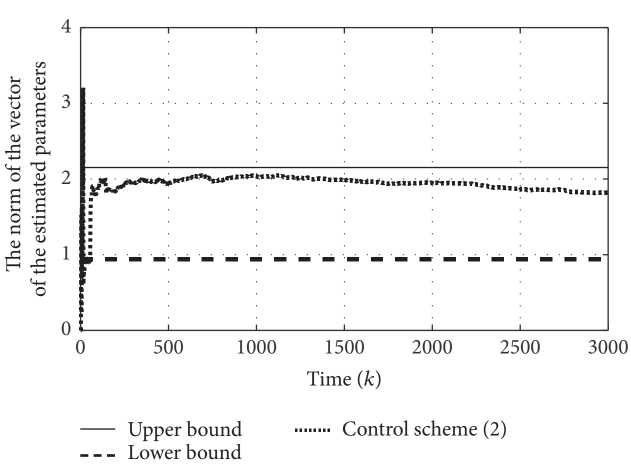


Figure 18: Evolution curve of the norm of the vector of the estimated parameters $\|\widehat{\theta}(k)\|$.

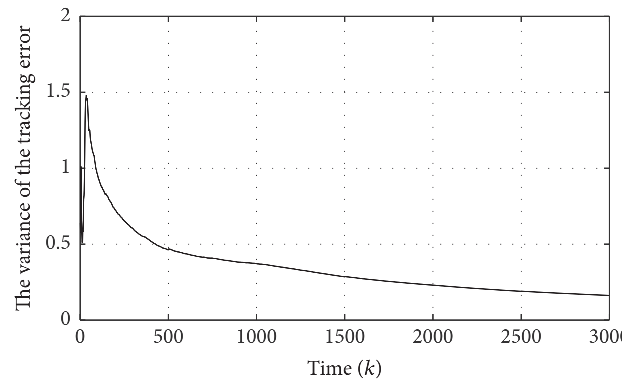


Figure 19: Evolution curve of the variance of the tracking error $\sigma_v^2(k)$.

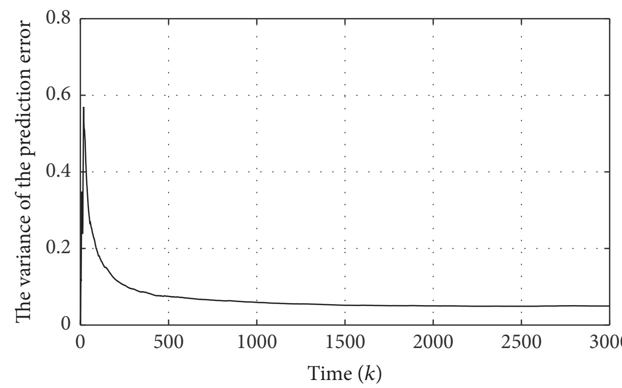


Figure 20: Evolution curve of the variance of the prediction error $\sigma_\varepsilon^2(k)$.

the small gain theorem. A numerical simulation example has shown the effectiveness and the performance of M-RLS algorithm with σ-modification.

An explicit scheme of self-tuning control was developed to solve the regulation-tracking problem for the linear systems in the presence of unknown unmodelled dynamics. This control scheme was based on the proposed M-RLS algorithm with σ-modification approach. The robustness of the proposed control scheme for the stochastic system, in the presence of unknown unmodelled dynamics, is shown using the simulation results of the cruise control system for the vehicle.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References

[1] W. I. Caldwell, "Control system with automatic response adjustment," American Patent, 2,517,081. Filed 25, 1950.

[2] K. J. Åström, "Theory and applications of adaptive control—a survey," *Automatica*, vol. 19, no. 5, pp. 471–486, 1983.

[3] K. J. Åström and B. Wittenmark, *Adaptive Control*, Addison-Wesley, New York, NY, USA, 1989.

[4] R. Isermann, K. H. Lachmann, and D. Matko, *Adaptive Control Systems*, Prentice-Hall, New York, NY, USA, 1992.

[5] T. R. Fortescue, L. S. Kershenbaum, and B. E. Ydstie, "Implementation of self-tuning regulators with variable forgetting factors," *Automatica*, vol. 17, no. 6, pp. 831–835, 1981.

[6] K. J. Åström and B. Wittenmark, "On self tuning regulators," *Automatica*, vol. 9, no. 2, pp. 185–199, 1973.

[7] S. Boyd and S. S. Sastry, "Necessary and sufficient conditions for parameter convergence in adaptive control," *Automatica*, vol. 22, no. 6, pp. 629–639, 1986.

[8] D. W. Clarke and P. J. Gawthrop, "SELF-tuning control," *Proceedings of the IEE*, vol. 126, no. 6, pp. 633–640, 1979.

[9] Z. Li, "Discrete-time adaptive control for time-varying systems subject to unknown fast time-varying deterministic disturbances," *IEE Proceedings. D. Control Theory and Applications*, vol. 135, no. 6, pp. 445–450, 1988.

[10] P. A. Ioannou and J. Sun, *Robust Adaptive Control*, Prentice-Hall, 1996.

[11] Z. Li and R. J. Evans, "Generalised minimum variance control of linear time-varying systems," *IEE Proceedings: Control Theory and Applications*, vol. 149, no. 1, pp. 111–116, 2002.

[12] R. Marino and P. Tomei, "Adaptive control of linear time-varying systems," *Automatica*, vol. 39, no. 4, pp. 651–659, 2003.

[13] A. Patete, K. Furuta, and M. Tomizuka, "Self-tuning control based on generalized minimum variance criterion," in *Proceedings of the 9th IFAC Workshop Adaptation and Learning in Control and Signal Processing (ALCOSP '07)*, pp. 411–416, August 2007.

[14] A. Patete, K. Furuta, and M. Tomizuka, "Stability of self-tuning control based on Lyapunov function," *International Journal of Adaptive Control and Signal Processing*, vol. 22, no. 8, pp. 795–810, 2008.

[15] A. Patete, K. Furuta, and M. Tomizuka, "Self-tuning control based on generalized minimum variance criterion for auto-regressive models," *Automatica*, vol. 44, no. 8, pp. 1970–1975, 2008.

[16] B. Egardt, *Stability of adaptive controllers*, vol. 20 of *Lecture Notes in Control and Information Sciences*, Springer, Berlin, Germany, 1979.

[17] P. A. Ioannou and P. V. Kokotovic, *Adaptive Systems with Reduced Models*, vol. 47 of *Lecture Notes in Control and Information Sciences*, Springer, Berlin, Germany, 1983.

[18] C. E. Rohrs, L. Valavani, M. Athans, and G. Stein, "Robustness of continuous-time adaptive control algorithms in the presence of unmodeled dynamics," *IEEE Transactions on Automatic Control*, vol. 30, no. 9, pp. 881–889, 1985.

[19] P. A. Ioannou and A. Datta, "Robust adaptive control: design, analysis and robustness bounds," in *Grainger Lectures: Foundations of Adaptive Control*, Springer, New York, NY, USA, 1991.

[20] M. Kamoun, "Design of robust adaptive regulators for large-scale systems," *International Journal of Systems Science*, vol. 26, no. 1, pp. 47–63, 1995.

[21] V. J. Leite and P. L. Peres, "Robust control through piecewise Lyapunov functions for discrete time-varying uncertain systems," *International Journal of Control*, vol. 77, no. 3, pp. 230–238, 2004.

[22] C. W. Lim, Y. J. Park, and S. J. Moon, "Robust saturation controller for linear time-invariant system with structured real parameter uncertainties," *Journal of Sound and Vibration*, vol. 294, no. 1-2, pp. 1–14, 2006.

[23] K. El Rifai and K. Youcef-Toumi, "Switched systems," in *Robust Adaptive Control of Switched Systems*, chapter 2, pp. 35–50, Prentice Hall, 2009.

[24] S. Kamoun, "Design of optimal self-tuning regulators for large-scale stochastic systems," *International Journal of Information and Systems Science*, vol. 6, no. 2, pp. 201–219, 2010.

[25] M. Hedayati, N. Mariun, H. Hizam, and S. Bahari, "Design of robust controller for STATCOM applied to large induction motor using normalized coprime factorization approach," *Arabian Journal for Science and Engineering*, vol. 38, no. 10, pp. 2713–2723, 2013.

[26] M. B. Kadri, "Disturbance rejection in nonlinear uncertain systems using feedforward control," *Arabian Journal for Science and Engineering*, vol. 38, no. 9, pp. 2439–2450, 2013.

[27] Y. Chen, G. Mei, G. Ma, S. Lin, and J. Gao, "Robust adaptive inverse dynamics control for uncertain robot manipulator," *International Journal of Innovative Computing, Information and Control*, vol. 10, no. 2, pp. 575–587, 2014.

[28] C. Wang, "Adaptive tracking control of uncertain MIMO switched nonlinear systems," *International Journal of Innovative Computing, Information and Control*, vol. 10, no. 3, pp. 1149–1159, 2014.

[29] H. Sheng, W. Huang, T. Zhang, and X. Huang, "Robust adaptive fuzzy control of compressor surge using backstepping," *Arabian Journal for Science and Engineering*, vol. 39, no. 12, pp. 9301–9308, 2014.

[30] G. Feng and T. Fu, "Robust adaptive rejection of unknown deterministic disturbances," *Computers and Electrical Engineering*, vol. 21, no. 1, pp. 1–12, 1995.

[31] A. Jerbi and E. W. Kamen, "Robust adaptive control of discrete-time systems arbitrary rate of variation," *Journal of Mathematical Systems Estimation and Control*, vol. 6, no. 4, pp. 1–31, 1996.

[32] M. Makoudi and L. Radouane, "A robust model reference adaptive control for non-minimum phase systems with unknown or time-varying delay," *Automatica*, vol. 36, no. 7, pp. 1057–1065, 2000.

[33] X. Y. Gu and C. Shao, "Robust adaptive control of time-varying linear plants using polynomial approximation," *IEE Proceedings D: Control Theory and Applications*, vol. 140, no. 2, pp. 111–118, 1993.

[34] X. Wei, L. Del Re, and J. Tan, "Robust adaptive control of quasi-LPV systems," in *Proceedings of the IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM '05)*, pp. 1617–1622, July 2005.

[35] S. Alonso-Quesada and M. De la Sen, "A discrete robust adaptive control to stabilize LTI plants by using multirate sampling," in *Proceedings of the IEEE International Conference on Control and Automation (ICCA '09)*, pp. 240–245, IEEE, Christchurch, New Zealand, December 2009.

[36] Y. Fu and T. Chai, "Robust self-tuning PID-like control with a filter for a class of discrete time systems," in *Proceedings of the 50th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC '11)*, pp. 6783–6787, December 2011.

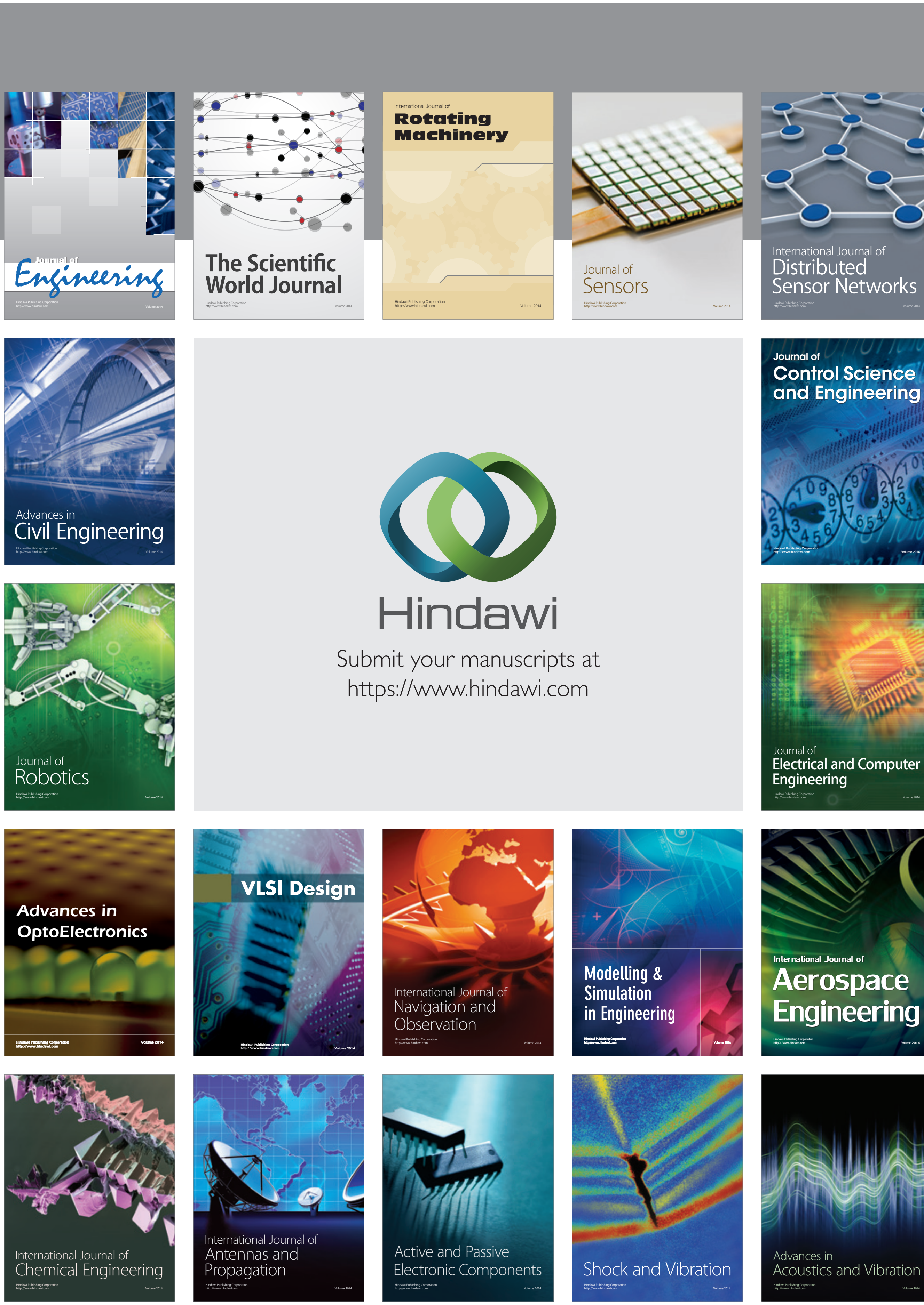
[37] N. Touijer and S. Kamoun, "Design of robust self-tuning control schemes for stochastic systems described by input-output mathematical models," *International Journal of Innovative Computing, Information and Control*, vol. 11, no. 3, pp. 1101–1112, 2015.

[38] N. Touijer and S. Kamoun, "Development of robust self-tuning control for MIMO linear systems with dead-zone approach," *International Journal of Circuits, Systems and Signal Processing*, vol. 8, pp. 101–108, 2014.

[39] G. Zames, "On the input-output stability of time-varying nonlinear feedback systems. Part I: conditions using concepts of loop gain, conicity, and positivity," *IEEE Transactions on Automatic Control*, vol. 11, pp. 228–238, 1966.

[40] M. Kamoun, *Modeling, identification and decentralized adaptive control of discrete-time large-scale systems [Thesis in Electrical Engineering—Automatic Control]*, University of Tunis 2, Tunis, Tunisia, 1994 (French).

[41] M. Sam Fadali, *Digital Control Engineering: Analysis and Design*, Prentice Hall, Upper Saddle River, NJ, USA, 2009.



Journal of Engineering
The Scientific World Journal
International Journal of Rotating Machinery
Journal of Sensors
International Journal of Distributed Sensor Networks
Advances in Civil Engineering
Hindawi
Submit your manuscripts at
https://www.hindawi.com
Journal of Control Science and Engineering
Journal of Robotics
Journal of Electrical and Computer Engineering
Advances in OptoElectronics
VLSI Design
International Journal of Navigation and Observation
Modelling & Simulation in Engineering
International Journal of Aerospace Engineering
International Journal of Chemical Engineering
International Journal of Antennas and Propagation
Active and Passive Electronic Components
Shock and Vibration
Advances in Acoustics and Vibration