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Research Article

State Fusion of Decentralized Optimal Unbiased FIR Filters

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The paper presents a decentralized fusion strategy based on the optimal unbiased finite impulse response (OUFIR) filter for discrete systems with correlated process and measurement noise. We extend OUFIR filter to apply in the model with control inputs. Taking it as local filters, cross covariance between any two is calculated; then it is expressed to the fast iterative form. Finally based on cross covariance, optimal weights are utilized to fuse local estimates and the overall outcome is obtained. The numerical examples show that the proposed filter exhibits better robustness against temporary modeling uncertainties than the fusion Kalman filter used commonly.

1. Introduction

State estimation plays an important role in many applications such as control, moving target tracking, and timekeeping and clock synchronization [1]. The existing filtering methods can be classified by two types: the infinite impulse response (IIR) filter and the finite impulse response (FIR) filter [2]. Specifically, the former uses all the historical measurements, and a special case is the Kalman filter (KF) [3], while the latter utilizes limited memory over the most recent time interval [4]. It is due to the difference of structure that FIR-type filters exhibit some useful engineering features such as bounded input/bounded output (BIBO) stability, round off errors [5], and better robustness against temporary uncertainties [6].

With higher requirements of flexibility and accuracy, information fusion filtering theory for multisensor systems has been studied and widely applied [7, 8]. For example, [9, 10] proposed the weighted fusion estimator according to the maximum likelihood and weight least square respectively with local filtering error cross covariance zero or not. In [11], an optimal decentralized fusion weighted by matrix was proposed to fuse local KF in the linear unbiased minimum variance (LUMV) sense. Furthermore, the situation of correlated multiplicative and additive noise was analyzed in [8], and the decentralized KF fusion problem for cross-correlated measurement noises was investigated in [12] which also discussed the cases with feedback from the fusion center to local sensors. Reference [13] focused on the decentralized fusion estimation problem for networked systems with random delays and packet losses, while [14] aimed at missing measurements and correlated noises. Reference [15] applied decentralized fusion to the maneuvering target tracking with multirate sampling and uncertainties in wireless sensor networks.

The fusion filtering methods nowadays take the KF as local filters mostly, so that they inherit the properties of KF, optimal but not robust under some uncertainties. Meanwhile there are few results considering decentralized FIR-type filter to deal with multisensor systems. Basically optimal FIR (OFIR) filter was obtained in [16] by minimum mean square error sense. Besides unbiased FIR (UFIR) filter was derived for real-time models and realized iteratively in [17], which can ignore the noise statistics but does not guarantee optimality. In between, [18] represents two forms of optimal unbiased FIR (OUFIR) filter, minimum variance unbiased FIR and embedded unbiasedness (EU) constraint on OFIR, and proved the identity of the two further. Contrary to the KF, the OUFIR filter is more robust and performs lower sensitivity to the initial condition [19]. In addition, there are a few new study and improved solutions on FIR filtering [20–22] developed nowadays. Some practical applications were reported [23, 24] as well.

In this correspondence, we firstly develop OUFIR filter to an extension to handle the system with or without the control inputs universally. Then considering it as local filters, cross covariance between any two is determined by batch and iterative form under the LUMV sense. Finally an optimal decentralized state fusion filter is proposed, which shows better immunity against temporary model uncertainties than traditional KF. The rest of this paper is organized as follows. In Section 2, we describe the preliminaries of system model. Local OUFIR filters are developed in Section 3. Main results about fusion and cross covariance are given in Section 4. The simulation and conclusions are provided in Sections 5 and 6, respectively.

2. System Model and Preliminaries

Consider the following linear discrete time-invariant system with multiple sensors:

$$x_n = Ax_{n-1} + Eu_n + Bw_n, \tag{1}$$

$$y_n^{[i]} = C^{[i]}x_n + v_n^{[i]}, \tag{2}$$

where $i = 1, 2, \cdots, r$ denotes the index of sensor, n is the discrete time index, $x_n \in \mathbb{R}^k$ and $y_n^{[i]} \in \mathbb{R}^{m_i}$ denote the state and measurement vectors, respectively, $u_n \in \mathbb{R}^p$ is the known control signal, and the system matrices A, E, B, and $C^{[i]}$ are known with appropriate dimensions. In this paper, the process noise $w_n \in \mathbb{R}^q$ and measurement noise $v_n^{[i]} \in \mathbb{R}^{m_i}$ are assumed to be correlated white noises with zero mean and

$$E\left\{\begin{bmatrix} w_n \\ v_n^{[i]} \end{bmatrix}\begin{bmatrix} w_m^T & \left(v_m^{[i]}\right)^T \end{bmatrix}\right\} = \begin{bmatrix} Q & S_i \\ S_i^T & R_i \end{bmatrix}\delta_{nm}, \tag{3}$$

$$E\left\{v_n^{[i]}\left(v_m^{[j]}\right)^T\right\} = R_{ij}\delta_{nm}, \quad i \neq j \tag{4}$$

where δ_{nm} is the Kronecker delta function, i.e., if $n = m$, $\delta_{nm} = 1$, otherwise, $\delta_{nm} = 0$. For notational convenience, we assign $R_{ii} = R_i$, $Q_{ii} = Q_i$ when $i = j$. That is, the process noise w_n is correlated with each sensor noise $v_n^{[i]}$ which is also correlated with each other.

Firstly the original state-space model (1) and (2) needs to be transformed to be a form with uncorrelated noise. In this way, we add the right-hand side of (1) with a zero term $y_n^{[i]} - C^{[i]}x_n - v_n^{[i]}$ as

$$\begin{aligned} x_n &= Ax_{n-1} + Eu_n + Bw_n + J_i\left[y_n^{[i]} - C^{[i]}x_n - v_n^{[i]}\right] \\ &= Ax_{n-1} + Eu_n + Bw_n \\ &\quad + J_i\left[y_n^{[i]} - C^{[i]}\left(Ax_{n-1} + Eu_n + Bw_n\right) - v_n^{[i]}\right] \\ &= \overline{A}^{[i]}x_{n-1} + \overline{u}_n^{[i]} + \overline{w}_n^{[i]}, \end{aligned} \tag{5}$$

where $\overline{A}^{[i]} \triangleq (I - J_iC^{[i]})A$, $\overline{u}_n^{[i]} \triangleq (I - J_iC^{[i]})Eu_n + J_iy_n^{[i]}$, $\overline{w}_n^{[i]} \triangleq (I - J_iC^{[i]})Bw_n - J_iv_n^{[i]}$, and J_i is a coefficient matrix. To remove the relevance between $\overline{w}_n^{[i]}$ and $v_n^{[i]}$, the expectation term $\mathbb{E}\{\overline{w}_n^{[i]}(v_n^{[i]})^T\}$ should be zero by definition. That is,

$$0 = \mathbb{E}\left\{\overline{w}_n^{[i]}\left(v_m^{[i]}\right)^T\right\} = \left[\left(I - J_iC^{[i]}\right)BS_i - J_iR_i\right]\delta_{nm}, \tag{6}$$

which further gives us $J_i = BS_i(C^{[i]}BS_i + R_i)^{-1}$. It is not difficult to find that $\mathbb{E}\{\overline{w}_n^i\} = \mathbb{E}\{(I - J_iC^{[i]})Bw_n - J_iv_n^{[i]}\} = 0$, which means that the new-defined noise term $\overline{w}_n^{[i]}$ is zero mean also. Accordingly, the cross state noise variance can be computed as

$$\begin{aligned} Q_{ij} &= \mathbb{E}\left\{\overline{w}_n^{[i]}\left(\overline{w}_m^{[j]}\right)^T\right\} \\ &= \mathbb{E}\left\{\left[\left(I - J_iC^{[i]}\right)Bw_n - J_iv_n^{[i]}\right][\cdots]^T\right\} \\ &= \left\{\left(I - J_iC^{[i]}\right)BQ\left[\left(I - J_iC^{[i]}\right)B\right]^T - \left(I - J_iC^{[i]}\right)\right. \\ &\quad \left.\times BS_jJ_j^T - J_i^TS_i^T\left[\left(I - J_jC^{[j]}\right)B\right]^T + J_iR_{ij}J_j^T\right\}\delta_{nm}. \end{aligned} \tag{7}$$

As can be seen, with an auxiliary matrix J_i, there is no correlation between $\overline{w}_m^{[i]}$ and $v_n^{[i]}$, and we consider the state transition equation (5) and measurement equation (2) subsequently.

By defining an estimation horizon as $[m, n]$, where $m \triangleq n - N + 1$ is the initial time step and N is the horizon length, models (5) and (2) within the estimation horizon can be extended as [16] similarly,

$$X_{n,m} = A_{n,m}^{[i]}x_m + E_{n,m}^{[i]}U_{n,m}^{[i]} + B_{n,m}^{[i]}W_{n,m}^{[i]}, \tag{8}$$

$$Y_{n,m}^{[i]} = C_{n,m}^{[i]}x_m + G_{n,m}^{[i]}U_{n,m}^{[i]} + F_{n,m}^{[i]}W_{n,m}^{[i]} + V_{n,m}^{[i]}, \tag{9}$$

where the initial state x_m is assumed to be known, and the extended vectors are specified as

$$\begin{aligned} X_{n,m} &= \begin{bmatrix} x_n^T & x_{n-1}^T & \cdots & x_m^T \end{bmatrix}_{kN\times 1}^T, \\ U_{n,m}^{[i]} &= \begin{bmatrix} \left(\overline{u}_n^{[i]}\right)^T & \left(\overline{u}_{n-1}^{[i]}\right)^T & \cdots & \left(\overline{u}_m^{[i]}\right)^T \end{bmatrix}_{pN\times 1}^T, \\ Y_{n,m}^{[i]} &= \begin{bmatrix} \left(y_n^{[i]}\right)^T & \left(y_{n-1}^{[i]}\right)^T & \cdots & \left(y_m^{[i]}\right)^T \end{bmatrix}_{m_iN\times 1}^T, \\ W_{n,m}^{[i]} &= \begin{bmatrix} \left(\overline{w}_n^{[i]}\right)^T & \left(\overline{w}_{n-1}^{[i]}\right)^T & \cdots & \left(\overline{w}_m^{[i]}\right)^T \end{bmatrix}_{qN\times 1}^T, \\ V_{n,m}^{[i]} &= \begin{bmatrix} \left(v_n^{[i]}\right)^T & \left(v_{n-1}^{[i]}\right)^T & \cdots & \left(v_m^{[i]}\right)^T \end{bmatrix}_{m_iN\times 1}^T. \end{aligned} \tag{10}$$

The involved extended matrices $A_{n,m}^{[i]} \in \mathbb{R}^{kN\times k}$, $E_{n,m}^{[i]} \in \mathbb{R}^{kN\times kN}$, $B_{n,m}^{[i]} \in \mathbb{R}^{kN\times kN}$, $C_{n,m}^{[i]} \in \mathbb{R}^{kN\times k}$, $G_{n,m}^{[i]} \in \mathbb{R}^{m_iN\times kN}$, and $F_{n,m}^{[i]} \in \mathbb{R}^{m_iN\times kN}$ are all N-dependent, which have the forms of

$$A_{n,m}^{[i]} = \left[\left(\overline{A}^{[i]^T}\right)^{N-1} \quad \left(\overline{A}^{[i]^T}\right)^{N-2} \quad \cdots \quad \overline{A}^{[i]^T} \quad I\right]^T, \tag{11}$$

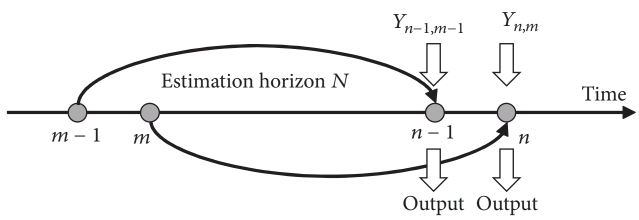


Figure 1: Operation time diagrams of the FIR structures.

$$B_{n,m}^{[i]} = \begin{bmatrix} I & \overline{A}^{[i]} & \cdots & \left(\overline{A}^{[i]}\right)^{N-2} & \left(\overline{A}^{[i]}\right)^{N-1} \\ 0 & I & \cdots & \left(\overline{A}^{[i]}\right)^{N-3} & \left(\overline{A}^{[i]}\right)^{N-2} \\ 0 & 0 & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & \overline{A}^{[i]} \\ 0 & 0 & \cdots & 0 & I \end{bmatrix}, \quad (12)$$

$$C_{n,m}^{[i]} = \overline{C}_{n,m}^{[i]} A_{n,m}^{[i]}, \quad (13)$$

$$G_{n,m}^{[i]} = \overline{C}_{n,m}^{[i]} E_{n,m}^{[i]}, \quad (14)$$

$$F_{n,m}^{[i]} = \overline{C}_{n,m}^{[i]} B_{n,m}^{[i]}, \quad (15)$$

where we assign

$$\overline{C}_{n,m}^{[i]} = \operatorname{diag}\underbrace{\left(C^{[i]} \quad C^{[i]} \quad \cdots \quad C^{[i]}\right)}_{N}, \quad (16)$$

and $E_{n,m}^{[i]} = B_{n,m}^{[i]}$.

Once the extended state-space models (8) and (9) are available, the problem considered in this paper can be formulated as follows. Given the state-space model (1) and (2), we would like to design local OUFIR filter for each sensor.

3. Local Optimal Unbiased FIR Filter

A demonstration of the FIR filter structure is provided in Figure 1. From the figure, FIR estimators explicitly employ N most recent measurements unlike KF. It is in this way that some nice properties like better robustness are achieved. Based on it, for i^{th} subsystem represented as (8) and (9), we design a local OUFIR filter in this section.

3.1. Batch Form. The linear FIR filter for each sensor can be expressed as the following batch form [25] generally:

$$\hat{x}_n^{[i]} = H_n^{[i]} Y_{n,m}^{[i]} + L_n^{[i]} U_{n,m}^{[i]} \quad (17)$$

for gain matrices $H_n^{[i]}$ and $L_n^{[i]}$. It is noted that the filter defined as (17) handles all the measurements and input values collected within the estimation horizon at one time.

Introduce unbiasedness constraint (or deadbeat constraint) [26]:

$$\mathbb{E}\left\{\hat{x}_n^{[i]}\right\} = \mathbb{E}\left\{x_n\right\} \quad (18)$$

in the estimation (17) where x_n can be specified as

$$x_n = \left(\overline{A}^{[i]}\right)^{n-m} x_m + \overline{E}_{n,m}^{[i]} U_{n,m}^{[i]} + \overline{B}_{n,m}^{[i]} W_{n,m}^{[i]}, \quad (19)$$

and $\overline{E}_{n,m}^{[i]}$, $\overline{B}_{n,m}^{[i]}$ are the first vector rows of $E_{n,m}^{[i]}$, $B_{n,m}^{[i]}$ respectively. By substituting (19) and (17) into (18), replacing the term $Y_{n,m}^{[i]}$ with (9), and providing the averaging, one arrives at two following constraints

$$H_n^{[i]} C_{n,m}^{[i]} = \left(\overline{A}^{[i]}\right)^{n-m}, \quad (20)$$

$$L_n^{[i]} = \overline{E}_{n,m}^{[i]} - H_n^{[i]} G_{n,m}^{[i]}. \quad (21)$$

Substituting (21) into (17) yields

$$\hat{x}_n^{[i]} = H_n^{[i]} \left(Y_{n,m}^{[i]} - G_{n,m}^{[i]} U_{n,m}^{[i]}\right) + \overline{E}_{n,m}^{[i]} U_{n,m}^{[i]}. \quad (22)$$

Provided $\hat{x}_n^{[i]}$, the instantaneous filtering error can be defined as $e_n^{[i]} = x_n - \hat{x}_n^{[i]}$, then the OUFIR filter is derived by solving the following optimization problem

$$H_n^{[i]} = \underset{H_n^{[i]}}{\arg\min}\, \mathbb{E}\left\{\left[e_n^{[i]} \left(e_n^{[i]}\right)^T\right]\right\}, \quad (23)$$

where $H_n^{[i]}$ should satisfy the constrain (20).

By replacing x_n with (19) and $\hat{x}_n^{[i]}$ with (22), and substituting the unbiasedness constrain (20), filtering error is obtained

$$\begin{aligned} e_n^{[i]} &= \left[\left(\overline{A}^{[i]}\right)^{n-m} - H_n^{[i]} C_{n,m}^{[i]}\right] x_m \\ &\quad + \left(\overline{B}_{n,m}^{[i]} - H_n^{[i]} F_{n,m}^{[i]}\right) W_{n,m}^{[i]} - H_n^{[i]} V_{n,m}^{[i]} \\ &= \left(\overline{B}_{n,m}^{[i]} - H_n^{[i]} F_{n,m}^{[i]}\right) W_{n,m}^{[i]} - H_n^{[i]} V_{n,m}^{[i]}. \end{aligned} \quad (24)$$

Employing the trace operation and providing the averaging, the optimization problem (23) can be rewritten as

$$\begin{aligned} H_n^{[i]} &= \underset{H_n^{[i]}}{\arg\min}\, \operatorname{tr}\left[\left(\overline{B}_{n,m}^{[i]} - H_n^{[i]} F_{n,m}^{[i]}\right)\right. \\ &\quad \left. \cdot Q_{n,m}^{[ii]} \left(\overline{B}_{n,m}^{[i]} - H_n^{[i]} F_{n,m}^{[i]}\right)^T - H_n^{[i]} R_{n,m}^{[ii]} \left(H_n^{[i]}\right)^T\right]. \end{aligned} \quad (25)$$

It is not difficult to get

$$\begin{aligned} Q_{n,m}^{[ij]} &= \operatorname{diag}\underbrace{\left(Q_{ij} \quad Q_{ij} \quad \cdots \quad Q_{ij}\right)}_{N}, \\ R_{n,m}^{[ij]} &= \operatorname{diag}\underbrace{\left(R_{ij} \quad R_{ij} \quad \cdots \quad R_{ij}\right)}_{N}, \end{aligned} \quad (26)$$

where Q_{ij} is given in (7), and R_{ij} is defined in (4). Referring to [19], we can obtain the following gain:

$$\begin{aligned} H_n^{[i]} &= \left(\overline{A}^{[i]}\right)^{n-m} \left[\left(C_{n,m}^{[i]}\right)^T Z_{w+v,n}^{-1} C_{n,m}^{[i]}\right]^{-1} \left(C_{n,m}^{[i]}\right)^T \\ &\quad \times Z_{w+v,n}^{-1} + \overline{B}_{n,m}^{[i]} Q_{n,m} \left(F_{n,m}^{[i]}\right)^T Z_{w+v,n}^{-1} \left\{I - C_{n,m}^{[i]}\right. \\ &\quad \left. \times \left[\left(C_{n,m}^{[i]}\right)^T Z_{w+v,n}^{-1} C_{n,m}^{[i]}\right]^{-1} \left(C_{n,m}^{[i]}\right)^T Z_{w+v,n}^{-1}\right\}, \end{aligned} \quad (27)$$

where $Z_{w+v,n} = Z_{w,n} + R_{n,m}$ and

$$Z_{w,n} = F^{[i]}_{n,m} Q^{[ii]}_{n,m} \left(F^{[i]}_{n,m}\right)^T. \tag{28}$$

At this point, the local OUFIR filter with batch form is specified by (22) with filter gain $H^{[i]}_n$ provided as (26). As mentioned, this structure is not suitable for the subsequent fusion step, as N measurements are operated at one time and there is no error covariance available to quantify the estimation accuracy. To address these issues, an equivalent iterative computational formula is given below.

3.2. Equivalent Iterative Computation. Introducing $\hat{x}^{[i]}_n = \hat{x}^{[i]}_{1,n} + \hat{x}^{[i]}_{2,n}$, (22) can be divided into two parts shown as follows:

$$\hat{x}^{[i]}_{1,n} = \overline{E}^{[i]}_{n,m} U^{[i]}_{n,m}, \tag{29}$$

$$\hat{x}^{[i]}_{2,n} = H^{[i]}_n \left(Y^{[i]}_{n,m} - G^{[i]}_{n,m} U^{[i]}_{n,m}\right), \tag{30}$$

and consider each components separately.

By former transformations, for (29), recursive algorithm is represented easily as

$$\begin{aligned}\hat{x}^{[i]}_{1,n} &= \overline{u}^{[i]}_n + \overline{A}^{[i]} \overline{u}^{[i]}_{n-1} + \left(\overline{A}^{[i]}\right)^2 \overline{u}^{[i]}_{n-2} + \cdots \\ &\quad + \left(\overline{A}^{[i]}\right)^{N-1} \overline{u}^{[i]}_m = \overline{u}^{[i]}_n + \overline{A}^{[i]} \overline{E}^{[i]}_{n-1,m} U^{[i]}_{n-1,m} \\ &= \overline{u}^{[i]}_n + \overline{A}^{[i]} \hat{x}^{[i]}_{1,n-1}\end{aligned} \tag{31}$$

Assigning $\tilde{Y}^{[i]}_{n,m} = Y^{[i]}_{n,m} - G^{[i]}_{n,m} U^{[i]}_{n,m}$, this part becomes $\hat{x}^{[i]}_{2,n} = H^{[i]}_n \tilde{Y}^{[i]}_{n,m}$. For this estimate, we notice a iterative form proposed in [19], which is represented similarly

$$\begin{aligned}\hat{x}^{[i]}_{2,n} &= A^{[i]} \hat{x}^{[i]}_{2,n-1} \\ &\quad + \left(\widetilde{H}^{[i]}_n + \overline{H}^{[i]}_n\right) \left(\tilde{y}^{[i]}_n - C^{[i]} \times A^{[i]} \hat{x}^{[i]}_{2,n-1}\right)\end{aligned} \tag{32}$$

where $\widetilde{H}^{[i]}_n$ and $\overline{H}^{[i]}_n$ are gain matrices. Besides, $\tilde{y}^{[i]}_n$ is the first vector rows of $\tilde{Y}^{[i]}_{n,m}$, substituting (31), which is given by

$$\tilde{y}^{[i]}_n = y^{[i]}_n - C^{[i]} \overline{E}^{[i]}_{n,m} U^{[i]}_{n,m} \tag{33}$$

$$= y^{[i]}_n - C^{[i]} \left(u^{[i]}_n + \overline{A}^{[i]} \hat{x}^{[i]}_{1,n-1}\right) \tag{34}$$

Then putting (33) into (32), iterative form is

$$\begin{aligned}\hat{x}^{[i]}_{2,n} &= \overline{A}^{[i]} \hat{x}^{[i]}_{2,n-1} + \left(\widetilde{H}^{[i]}_n + \overline{H}^{[i]}_n\right) \\ &\quad \cdot \left[y^{[i]}_n - C^{[i]} \left(u^{[i]}_n + \overline{A}^{[i]} \hat{x}^{[i]}_{1,n-1}\right) - C^{[i]} \overline{A}^{[i]} \hat{x}^{[i]}_{2,n-1}\right].\end{aligned} \tag{35}$$

Table 1: Computation of the initial state.

1: $m = n - N + 1$, $s = m + k$
2: $M_{s-2} = \overline{B}^{[i]}_{s-2,m} Q_{s-2,m} (F^{[i]}_{s-2,m})^T$
3: $M_{s-1} = \overline{B}^{[i]}_{s-1,m} Q_{s-1,m} (F^{[i]}_{s-1,m})^T$
4: $U_{s-1} = \overline{B}^{[i]}_{s-1,m} Q_{s-1,m} (\overline{B}^{[i]}_{s-1,m})^T$
5: $P_{s-1} = U_{s-1} - \overline{A}^{[i]} M_{s-2} Z^{-1}_{w+v,s-2} (M_{s-2})^T (\overline{A}^{[i]})^T$
6: $N_{s-1} = [(C^{[i]}_{s-1,m})^T Z^{-1}_{w+v,s-1} C^{[i]}_{s-1,m}]^{-1}$
7: $G_{s-1} = A^{[i]}_{s-1,m} - \overline{A}^{[i]} M_{s-2} Z^{-1}_{w+v,s-2} C^{[i]}_{s-1,m}$
8: $\hat{x}^{[i]}_{s-1} = [(\overline{A}^{[i]})^{s-m-1} N_{s-1} (C^{[i]}_{s-1,m})^T + M_{s-1} - M_{s-1}$ $\times Z^{-1}_{w+v,s-1} C^{[i]}_{s-1,m} N_{s-1} (C^{[i]}_{s-1,m})^T] Z^{-1}_{w+v,s-1}$ $\times (Y^{[i]}_{s-1,m} - G^{[i]}_{s-1,m} U^{[i]}_{s-1,m}) + \overline{E}^{[i]}_{s-1,m} U^{[i]}_{s-1,m}$

Combining (31) and (35), we arrive at

$$\begin{aligned}\hat{x}^{[i]}_n &= \hat{x}^{[i]}_{1,n} + \hat{x}^{[i]}_{2,n} = u^{[i]}_n + \overline{A}^{[i]} \hat{x}^{[i]}_{1,n-1} + \overline{A}^{[i]} \hat{x}^{[i]}_{2,n-1} \\ &\quad + \left(\widetilde{H}^{[i]}_n + \overline{H}^{[i]}_n\right) \\ &\quad \cdot \left[y^{[i]}_n - C^{[i]} \times \left(u^{[i]}_n + \overline{A}^{[i]} \hat{x}^{[i]}_{1,n-1}\right) - C^{[i]} \overline{A}^{[i]} \hat{x}^{[i]}_{2,n-1}\right] \\ &= \hat{x}^{[i]*}_n + \left(\widetilde{H}^{[i]}_n + \overline{H}^{[i]}_n\right) \left(y^{[i]}_n - C^{[i]} \hat{x}^{[i]*}_n\right).\end{aligned} \tag{36}$$

which suggests easily that the prior estimate is $\hat{x}^{[i]*}_n = \overline{A}^{[i]} \hat{x}^{[i]}_{n-1} + u^{[i]}_n$. Referring to [27], we finally come up with a general iterative OUFIR filtering algorithm similarly whose pseudocode is summarized as Algorithm 1 in which l is an auxiliary variable replaced with n. Unlike the OFIR or KF, it is because of unbiasedness condition that the filter does not require initial information. Instead that can be defined at $s = m + k$ using a short batch form given in Table 1; k is the number of system states. In the Algorithm 1, variable l ranges from s to n. The true filtering result corresponds to $l = n$.

Once the local estimates $\hat{x}^{[i]}_n$ are corrected, $i = 1, 2, \ldots, r$, an optimal fusion strategy based on cross covariance is employed to get the overall estimates, which is introduced below.

4. Decentralized Fusion Filter

In this section, we will investigate the decentralized fusion filters and first deduce the corresponding filtering error cross covariance between any two local subsystems with FIR structure. Due to inefficient computation, then we find its fast iterative form. Based on these, a multisensor decentralized fusion OUFIR filtering algorithm is finally presented.

4.1. Cross Covariance Matrices and Iterative Computation. For the i^{th} sensor subsystem, filtering error $e^{[i]}_n$ is written as (24) where filtering gain $H^{[i]}_n$ is specified as (27). So the cross covariance matrix between the i^{th} and the j^{th} subsystem can be showed as

$$\begin{aligned}P^{[ij]}_n &= E\left\{e^{[i]}_n \left(e^{[j]}_n\right)^T\right\} \\ &= E\left\{\left[\left(\overline{B}^{[i]}_{n,m} - H^{[i]}_n F^{[i]}_{n,m}\right) W^{[i]}_{n,m} - H^{[i]}_n V^{[i]}_{n,m}\right] [\cdots]^T\right\}\end{aligned}$$

for $n = N-1, N, \ldots$ **do**
  $m = n - N + 1,\ s = m + k$
  Table 1: *Computation of the initial state*
  for $l = s : n$ **do**
    $P_l^{[i]} = \overline{A}^{[i]} P_{l-1}^{[i]} (\overline{A}^{[i]})^T + Q_{ii} - \overline{A}^{[i]} P_{l-1}^{[i]} \times (C^{[i]})^T \Upsilon_{l-1}^{[i]} C^{[i]} P_{l-1}^{[i]} (\overline{A}^{[i]})^T$
    $G_l^{[i]} = \overline{A}^{[i]} [I - P_{l-1}^{[i]} (C^{[i]})^T \Upsilon_{l-1}^{[i]} C^{[i]}] G_{l-1}^{[i]}$
    $\Upsilon_l^{[i]} = [C^{[i]} P_l^{[i]} (C^{[i]})^T + R_{ij}]^{-1}$
    $N_l^{[i]} = [N_{l-1}^{[i]} + (G_l^{[i]})^T (C^{[i]})^T \Upsilon_l^{[i]} C^{[i]} G_l^{[i]}]^{-1}$
    $\widetilde{H}_l^{[i]} = P_l^{[i]} (C^{[i]})^T \Upsilon_l^{[i]}$
    $\overline{H}_l^{[i]} = (I - \widetilde{H}_l^{[i]} C^{[i]}) G_l^{[i]} N_l^{[i]} (G_l^{[i]})^T (C^{[i]})^T \Upsilon_l^{[i]}$
    $\widehat{x}_l^{[i]*} = \overline{A}^{[i]} \widehat{x}_{l-1}^{[i]} + u_l^{[i]}$
    $\widehat{x}_l^{[i]} = \widehat{x}_l^{[i]*} + (\widetilde{H}_l^{[i]} + \overline{H}_l^{[i]})(y_l^{[i]} - C^{[i]} \widehat{x}_l^{[i]*})$
  end for
  $\widehat{x}_n^{[i]} = \widehat{x}_l^{[i]}$
end for

Algorithm 1: General iterative OUFIR filtering algorithm.

$$= \left(\overline{B}_{n,m}^{[i]} - H_n^{[i]} F_{n,m}^{[i]}\right) Q_{n,m}^{[ij]} \left(\overline{B}_{n,m}^{[j]} - H_n^{[j]} F_{n,m}^{[j]}\right)^T + H_n^{[i]} R_{n,m}^{[ij]} \left(H_n^{[j]}\right)^T \tag{37}$$

where $Q_{n,m}^{[ij]}, R_{n,m}^{[ij]}$ are defined in (26).

The batch cross covariance given by (37) is exact but takes a large amount of computation. To find a recursive formula, one may combine $\widehat{x}_l^{[i]}$ given in Algorithm 1 and (5) with respect to an auxiliary variable l, so $e_l^{[i]}$ can be rewritten as

$$e_l^{[i]} = \overline{A}^{[i]} x_{l-1} + \overline{u}_l^{[i]} + \overline{w}_l^{[i]} - \widehat{x}_l^{[i]*} - \left(\widetilde{H}_l^{[i]} + \overline{H}_l^{[i]}\right) \times \left(y_l^{[i]} - C^{[i]} \widehat{x}_l^{[i]*}\right). \tag{38}$$

Substitute x_l given by (5) and (2) into (38), then transform it into

$$\begin{aligned} e_l^{[i]} &= \overline{A}^{[i]} x_{l-1} + \overline{u}_l^{[i]} + \overline{w}_l^{[i]} - \overline{A}^{[i]} \widehat{x}_{l-1}^{[i]} - u_l^{[i]} - \left(\widetilde{H}_l^{[i]} + \overline{H}_l^{[i]}\right) \left(C^{[i]} \overline{A}^{[i]} x_{l-1} + C^{[i]} \overline{u}_l^{[i]} + C^{[i]} \overline{w}_l^{[i]} + v_l^{[i]} - C^{[i]} \overline{A}^{[i]} \widehat{x}_{l-1}^{[i]} - C^{[i]} u_l^{[i]}\right) \\ &= \left[\overline{A}^{[i]} - \left(\widetilde{H}_l^{[i]} + \overline{H}_l^{[i]}\right) C^{[i]} \overline{A}^{[i]}\right] e_{l-1}^{[i]} - \left[I - \left(\widetilde{H}_l^{[i]} + \overline{H}_l^{[i]}\right) C^{[i]}\right] \overline{w}_l^{[i]} - \left(\widetilde{H}_l^{[i]} + \overline{H}_l^{[i]}\right) v_l^{[i]}. \end{aligned} \tag{39}$$

So $P_l^{[ij]}$ can now be computed iteratively over the recursive form of

$$\begin{aligned} P_l^{[ij]} &= E\left\{\left(\left[\overline{A}^{[i]} - \left(\widetilde{H}_l^{[i]} + \overline{H}_l^{[i]}\right) C^{[i]} \overline{A}^{[i]}\right] e_{l-1}^{[i]} - \left[I - \left(\widetilde{H}_l^{[i]} + \overline{H}_l^{[i]}\right) C^{[i]}\right] \overline{w}_l^{[i]} - \left(\widetilde{H}_l^{[i]} + \overline{H}_l^{[i]}\right) v_l^{[i]}\right) (\cdots)^T\right\} \\ &= \left[\overline{A}^{[i]} - \left(\widetilde{H}_l^{[i]} + \overline{H}_l^{[i]}\right) C^{[i]} \overline{A}^{[i]}\right] P_{l-1}^{[ij]} (\cdots)^T + \left[I - \left(\widetilde{H}_l^{[i]} + \overline{H}_l^{[i]}\right) \cdot C^{[i]}\right] Q_{ij} (\cdots)^T + \left(\widetilde{H}_l^{[i]} + \overline{H}_l^{[i]}\right) R_{ij} (\cdots)^T \end{aligned} \tag{40}$$

where l ranges from s to n, and the output is taken when $l = n$ similarly. The initial value $P_{s-1}^{[ij]}$ is given in a short batch form by (37) as

$$\begin{aligned} P_{s-1}^{[ij]} &= E\left\{e_{s-1}^{[i]} \left(e_{s-1}^{[i]}\right)^T\right\} = \left(\overline{B}_{s-1,m}^{[i]} - H_{s-1}^{[i]} F_{s-1,m}^{[i]}\right) \cdot Q_{n,m}^{[ij]} \left(\overline{B}_{s-1,m}^{[j]} - H_{s-1}^{[j]} F_{s-1,m}^{[j]}\right)^T \\ &\quad + H_{s-1}^{[j]} R_{n,m}^{[ij]} \left(H_{s-1}^{[j]}\right)^T. \end{aligned} \tag{41}$$

Clearly the pseudocode is shown in Algorithm 2.

4.2. Decentralized OUFIR Filtering Algorithm. Based on the local filters $\widehat{x}_n^{[i]}$ in Algorithm 1 and the filtering error cross covariance matrices between any two local filters in Algorithm 2, we can get the decentralized fusion filter weighted by matrices in the LUMV sense [7]. Introduce the synthetically unbiased estimator

$$\widehat{x}_n^o = \alpha_1 \widehat{x}_n^{[1]} + \alpha_2 \widehat{x}_n^{[2]} + \cdots + \alpha_r \widehat{x}_n^{[r]}, \tag{42}$$

where α_i are arbitrary matrices. From the unbiasedness assumption, we have $E\{\widehat{x}_n\} = E\{x_n\}$ and $E\{\widehat{x}_n^{[i]}\} = E\{x_n\}$. Taking the expectation in (42) yields

$$\alpha_1 + \alpha_2 + \cdots + \alpha_r = \overline{I} \tag{43}$$

for $n = N-1, N, \ldots$ **do**
  $m = n - N + 1,\ s = m + k$
  for $i = 1 : r$ **do**
    for $j = 1 : r$ **do**
      $P^{[ij]}_{s-1} = (\overline{B}^{[i]}_{s-1,m} - H^{[i]}_{s-1}F^{[i]}_{s-1,m})Q^{[ij]}_{s-1,m}(\overline{B}^{[j]}_{s-1,m} - H^{[j]}_{s-1}F^{[j]}_{s-1,m})^T + H^{[j]}_{s-1}R^{[ij]}_{s-1,m}(H^{[j]}_{s-1})^T$
      for $l = s : n$ **do**
        $P^{[ij]}_{l} = [\overline{A}^{[i]} - (\widetilde{H}^{[i]}_l + \overline{H}^{[i]}_l)C^{[i]}\overline{A}^{[i]}]P^{[ij]}_{l-1}(\cdots)^T + [I - (\widetilde{H}^{[i]}_l + \overline{H}^{[i]}_l)C^{[i]}]Q_{ij}(\cdots)^T + (\widetilde{H}^{[i]}_l + \overline{H}^{[i]}_l)R_{ij}(\cdots)^T$
      end for
      $P^{[ij]}_n = P^{[ij]}_l$
    end for
  end for
end for

Algorithm 2: Iterative form for $P^{[ij]}_n$.

$J_i = BS_i(C^{[i]}BS_i + R_i)^{-1}$
$\overline{A}^{[i]} = (I - J_iC^{[i]})A$
$\overline{u}^{[i]}_n = (I - J_iC^{[i]})Eu_n + J_iy^{[i]}_n$
$\overline{w}^{[i]}_n = (I - J_iC^{[i]})Bw_n - J_iv^{[i]}_n$
Algorithm 1: *General Iterative OUFIR Filtering Algorithm*
Algorithm 2: *Iterative Form for* $P^{[ij]}_n$
$\Sigma = [P^{[ij]}_n],\ i = 1, 2 \cdots r$
$\overline{\Lambda} = (\overline{I}^T\Sigma^{-1}\overline{I})^{-1}\overline{I}^T\Sigma^{-1}$
$\widehat{x}^o_n = \overline{\alpha}_1\widehat{x}^{[1]}_n + \overline{\alpha}_2\widehat{x}^{[2]}_n + \cdots + \overline{\alpha}_r\widehat{x}^{[r]}_n$

Algorithm 3: Optimal decentralized fusion OUFIR filter.

where $\overline{I} = \begin{bmatrix} I_k & I_k & \cdots & I_k \end{bmatrix}^T_{kr\times k}$ and I_k denotes $k \times k$ identity matrix. Then fusion estimation error is $e_n = x_n - \widehat{x}^o_n = \sum_{i=1}^{r}\alpha_i e^{[i]}_n$. Let $\Lambda = \begin{bmatrix}\alpha_1 & \alpha_2 & \cdots & \alpha_r\end{bmatrix}_{kr\times k}$; the error variance matrix of the fusion estimator is

$$P = E\left\{e_ne_n^T\right\} = \Lambda\Sigma\Lambda^T \quad (44)$$

where $\Sigma \triangleq [P^{[ij]}_n]_{i,j=1,2,\cdots,r}$ is an $kr \times kr$ symmetric positive definite matrix where $P^{[ij]}_n \triangleq \{(x_n - \widehat{x}^{[i]}_n)(x_n - \widehat{x}^{[j]}_n)^T\}$ is from (40).

To find the optimal weights Λ under the constrain (43), applying the Lagrange multiplier approach, the target function is defined as

$$J = \operatorname{tr}\left(\Lambda\Sigma\Lambda^T\right) + 2\Gamma\left(\Lambda\overline{I} - I_k\right) \quad (45)$$

where $\Gamma = (\lambda_{ij})_{k\times k}$. Set $\partial J/\partial\Lambda = 0$ and note that $\Sigma^T = \Sigma$; we have

$$2\Lambda\Sigma + 2\Gamma\overline{I}^T = 0 \quad (46)$$

Combining (43) and (46) yields the matrix equation as

$$\begin{bmatrix}\Lambda & \Gamma\end{bmatrix} = \begin{bmatrix}0 & I_k\end{bmatrix}\begin{bmatrix}\Sigma & \overline{I} \\ \overline{I}^T & 0\end{bmatrix}^{-1}. \quad (47)$$

Noted Σ is a symmetric positive definite matrix and using the formula of the inverse matrix [28] we have

$$\Lambda = \left(\overline{I}^T\Sigma^{-1}\overline{I}\right)^{-1}\overline{I}^T\Sigma^{-1} \quad (48)$$

Pseudocode is summarized by Algorithm 3, and the corresponding structure is shown in Figure 2. As can be seen, each sensor subsystem estimates the state independently at time n. Next fusion layer determines optimal matrix weights. As a basic requirement, the proposed decentralized fusion filter will have better accuracy than any local one. Different from fusion KF, the proposed method does not relay on the initial information about $\widehat{x}_0, P_0$, and $P^{[ij]}_0$, and it performs better robustness.

5. Simulation

In this section, we consider two basic models of moving target tracking system and 1 degree of freedom (1-DOF) torsion system. Fusion OUFIR filter is applied to above models under various conditions such as temporary modeling uncertainties and unknown noise statistics. Compared to fusion Kalman filter (conveniently FOUF and FKF denote as fusion OUFIR filter and fusion KF filter respectively next), it demonstrates better performance.

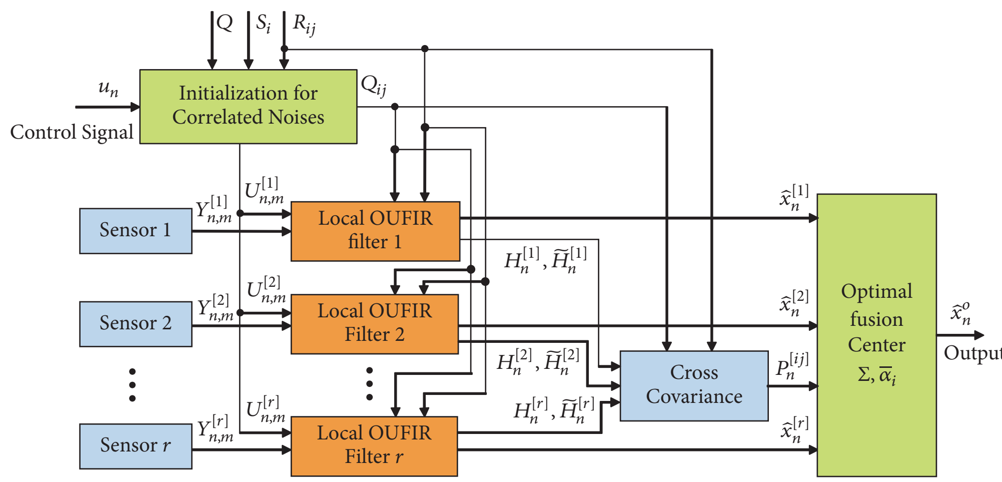


FIGURE 2: The structure of decentralized OUFIR filter.

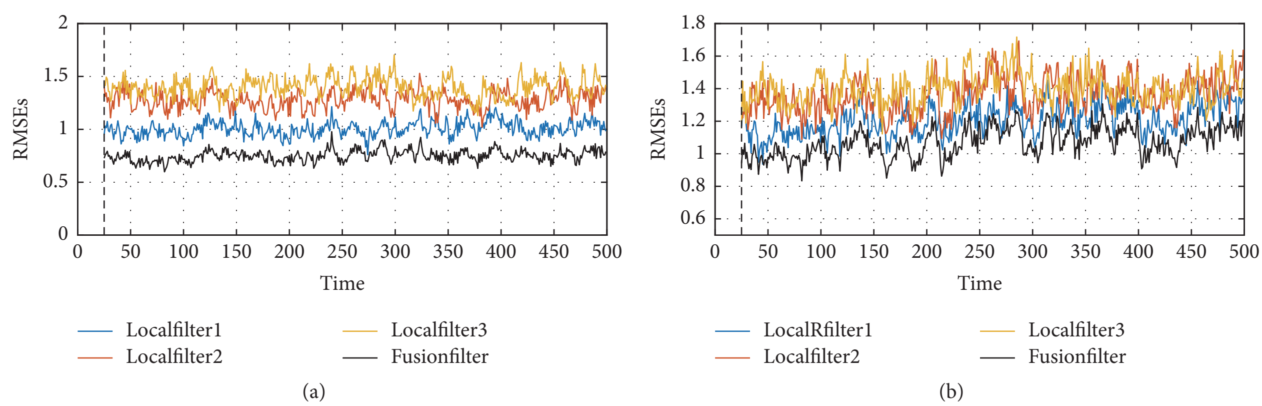


FIGURE 3: RMSEs produced by local and fusion filters: (a) the first state and (b) the second state.

5.1. Moving Target Tracking System. With three sensors, a simplified model can be specified by (1) and (2) with $E = [1 \ \ 1]^T$, $B = [0.1 \ \ 1]^T$, and

$$A = \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix} \tag{49}$$

where $i = 1, 2, 3$, $\tau = 0.1$, control signal $u_n = 0.1$, and $C_1 = C_2 = C_3 = [1 \ \ 0]$. And the measurement noise $v_n^{[i]}$ is represented as $v_n^{[i]} = b_i w_n + \xi_n^{[i]}$, which are correlated with the white process noise $\sigma_w^2 = 0.1$. The correlation coefficients are $b_1 = 0.5$, $b_2 = 0.8$, and $b_3 = 0.4$. $\xi_n^{[i]}$ are white Gaussian noise with zero mean and variances $R_{\xi^{[1]}} = 5$, $R_{\xi^{[2]}} = 8$, and $R_{\xi^{[3]}} = 10$, respectively, and are independent of w_n. Easily we can get R_{ij} and S_i as

$$R_{ij} = b_i Q b_j^T + R_{\xi^{[i]}} \delta_{ij} \tag{50}$$

$$S_i = E\left\{ w_n \left(v_n^{[i]} \right)^T \right\} = Q b_i^T \tag{51}$$

5.1.1. Estimation Accuracy with Correct Noise Variances. In this section, we assume that all the modeling parameters are known accurately and show that FOUF outperforms any local filter. The process was simulated over 500 points, and the estimation horizon length used is $N = 20$. The average root mean square error (RMSE) based on 100 Monte-Carlo (MC) runs is shown in Figure 3. The result proves that the proposed approach has better accuracy exactly.

5.1.2. Sensitivity to Incorrect Noise Variance. The knowledge of noise is typically not known exactly in practice, which does not guarantee optimality. To figure out the effect of noise variances, we give an idea about the FKF and FOUF immunity to noise statistics' errors in the worst case. Introducing error coefficient β, designed covariances are substituted here with $Q^* = \beta Q$.

$\sqrt{\mathrm{tr}(J_n)}$ of FKF and FOUF over 100 MC runs for $\beta = 0.01, 0.1, 1, 10, 100$ are shown in Figure 4 and also listed in Table 2. As can be seen, errors in the FKF grow rapidly with β and even become unacceptable, while the proposed FOUF has much higher immunity against noise error when $\beta \leq 1$. Otherwise, performance of FOUF exhibits closely to FKFs.

TABLE 2: Performance of FOUF and FKF with respect to different β.

β	**0.01**	**0.1**	**1**	**10**	**100**
FKF	4.42	1.86	1.24	1.28	2.53
FOUF	1.58	1.47	1.28	1.29	2.58

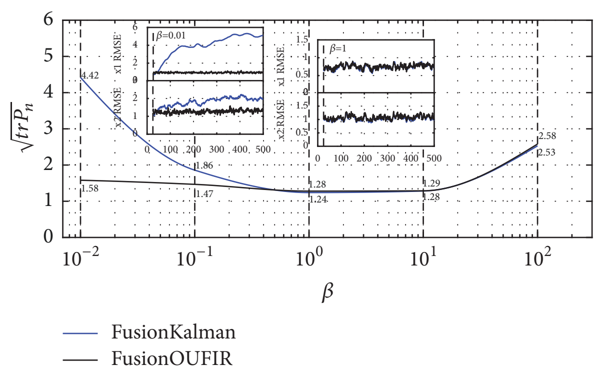


FIGURE 4: Average $\sqrt{\text{tr}(J_n)}$ against system noises uncertainties $\beta = 0.01, 0.1, 1, 10, 100$ between FKF and FOUF.

5.1.3. Immunity to Errors in the Noise Statistics. To investigate the effect of temporary inaccurate statistics, the state noise variance is set to vary as follows: $\sigma_w^2 = 0.5$ when $100 \le n \le 200$ and $\sigma_w^2 = 0.1$ otherwise. And invariable value $\sigma_w^2 = 0.1$ is used in all the algorithms. The average RMSEs based on 100 MC runs are provides by Figure 5. One can see that FOUF has much higher immunity against errors of noise statistic than FKF. Besides it is noted that the first state is more obvious about robustness performance.

5.1.4. Robustness against Model Uncertainties. To identify the robustness of each tracker against temporary model uncertainties, we set

$$A = \begin{bmatrix} 1 & \tau + d \\ 0 & 1 \end{bmatrix} \tag{52}$$

where $d = 0.1$ when $100 \le n \le 300$ and $d = 0$ otherwise. The process is simulated over 500 subsequent points with correct noise statistics. Comparing the FKF as a benchmark, Figures 6(a) and 6(b) illustrate the typical response under the model mismatch introduced. From figures, one infers that the FOUF's peak value is smaller and recovers to normal faster than the FKF's, particularly with the first state.

5.2. 1-DOF Torsion System. We consider 1-DOF torsion system [29] as the object whose schematic and parameters are provided by Figure 7 and Table 3. Referring from [29] of the equations of motion, the continuous-time model can be described by

$$\frac{\partial x}{\partial t} = Ax + Bu, \tag{53}$$

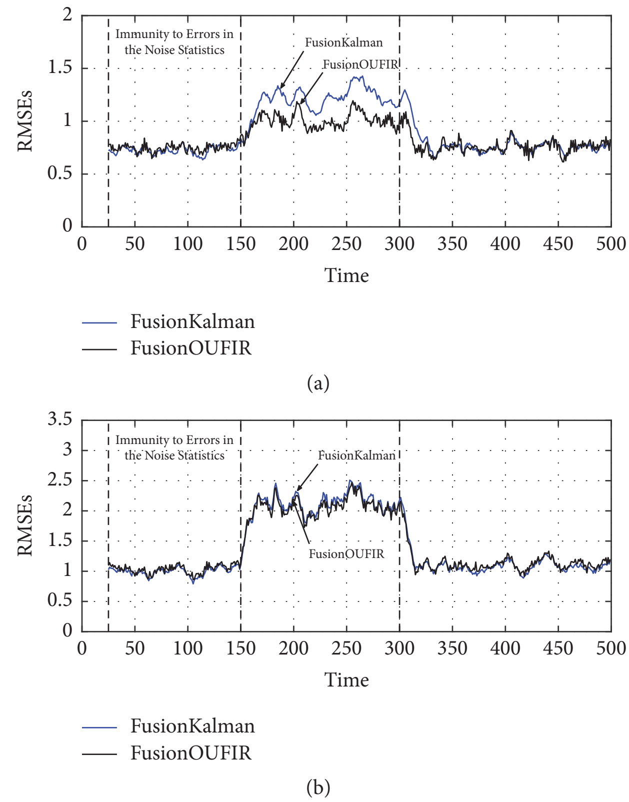


FIGURE 5: RMSEs produced by FKF and FUF with imprecisely defined noise statistics: (a) the first state and (b) the second state.

where $x = \begin{bmatrix} \theta_1 & \theta_2 & \dot{\theta}_1 & \dot{\theta}_2 \end{bmatrix}^T$ and

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{K_s}{J_1} & \dfrac{K_s}{J_1} & -\dfrac{B_1}{J_1} & 0 \\ \dfrac{K_s}{J_2} & -\dfrac{K_s}{J_2} & 0 & -\dfrac{B_2}{J_2} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{J_1} \\ 0 \end{bmatrix}. \tag{54}$$

Discretize this process with sampling time $T_s = 0.01s$, and the discrete state-space model is achieved as

$$x_{n+1} = Gx_n + Fu_n + w_n, \tag{55}$$

where w_n is introduced to describe the stochastic properties of states, u_n is input signal, $G = e^{AT_s}$, and $F = \int_0^{T_s} e^{A\tau} d\tau B$. The angular positions of two loads are measured using three groups of sensors, and the observation equation is given by $y_n^{[i]} = C^{[i]} x_n + v_n^{[i]}$, where $i = 1, 2, 3$,

$$C^{[1]} = C^{[2]} = C^{[3]} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \tag{56}$$

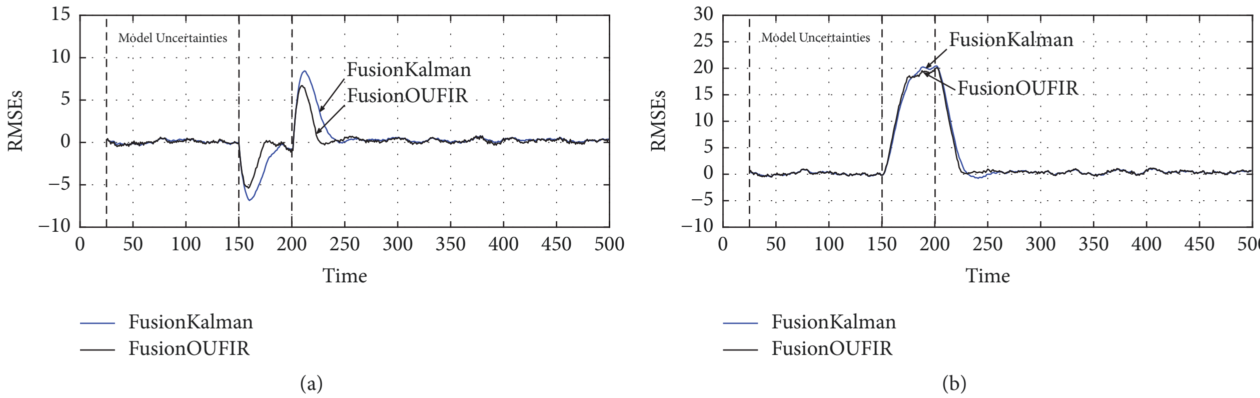


Figure 6: Estimation errors by FKF and FOUF filter against temporary model uncertainty in a gap of $100 \leq n \leq 150$: (a) the first state and (b) the second state.

Table 3: Physical parameters of 1-DOF torsion system, where EMI is equal to equivalent moment of inertia, and EVD is equivalent viscous damping.

Constant	Value	Unit	Description
K_s	1.0	N.m/rad	Stiffness of flexible coupling
J_1	2.2e-4	Kg.m^2	EMI at the rigid load shaft
J_2	5.45e-4	Kg.m^2	EMI at the torsion load shaft
B_1	0.015	N.m.s/rad^2	EVD of rigid load
B_2	0.015	N.m.s/rad	EVD of torsion load
θ_1	–	rad	angular positions of rigid load
θ_1	–	rad	angular positions of torsion load

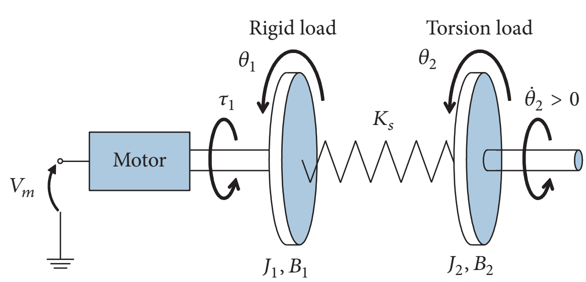


Figure 7: Schematic of 1-DOF Torsion system.

The correlation between state and measurement noises is described as $v_n^{[i]} = b^{[i]} w_n + \xi_n^{[i]}$, where the coefficients are

$$b^{[1]} = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \end{bmatrix},$$
$$b^{[2]} = \begin{bmatrix} 0.8 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \end{bmatrix}, \qquad (57)$$
$$b^{[3]} = \begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 \end{bmatrix}.$$

The measurement noises variances are assigned as $\sigma^2_{\xi_1^{[1]}} = \sigma^2_{\xi_2^{[1]}} = 5$, $\sigma^2_{\xi_1^{[2]}} = \sigma^2_{\xi_2^{[2]}} = 8$, and $\sigma^2_{\xi_1^{[3]}} = \sigma^2_{\xi_2^{[3]}} = 3$.

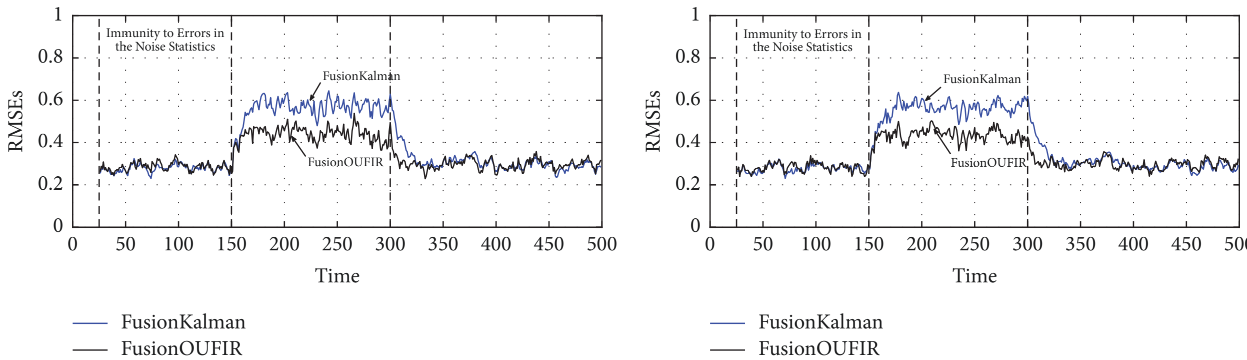
5.2.1. Imprecisely Defined Noise Statistics. With inaccurateness, the state noise variance is set to vary as follows: $\sigma^2_{w_j} = 0.05$ when $150 \leq n \leq 300$ and $\sigma^2_{w_j} = 0.01$ otherwise. Meanwhile invariable value $\sigma^2_{w_j} = 0.01$, $j = 1, 2, 3, 4$, is used in the filter. The average RMSEs based on 100 MC runs are provided by Figure 8. It suggests again that FOUF performs closely to the FKF under the correct noise, but it has better robustness against noise errors than FKF.

5.2.2. Temporary Model Uncertainty. Similarly, we assume that B_2 changes unpredictably as $B_2 = 0.5$ when $150 \leq n \leq 300$ and $B_2 = 0.015$ otherwise, which makes A vary simultaneously. Average RMSEs produced by FKF and FOUF over 100 Monte-Carlo simulations are shown Figure 9.

Clearly the result obtained in this case coincides with that in the previous model. At this point, one can conclude that proposed FOUF is less influenced by noise and more robust against temporary model mismatch.

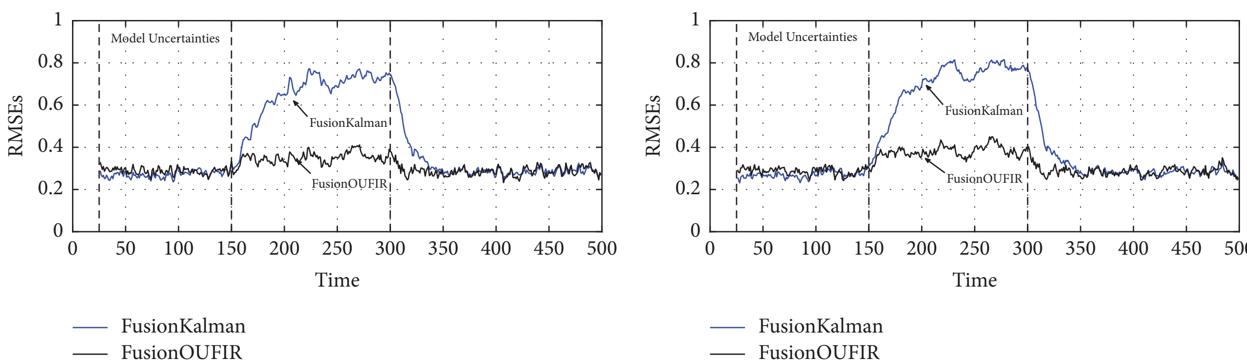
6. Conclusion

Based on optimal fusion weighted by matrices in the LUMV sense, this paper presents a decentralized fusion OUFIR filter for discrete-invariant control system. Fundamentally the FOUF has higher accuracy than each local filter. The simulation results show that the proposed algorithm maintains the advantages of OUFIR filter and suggests lowlier



(a) (b)

Figure 8: RMSEs produced by FKF and FOUF with imprecisely defined noise statistics: (a) the first state and (b) the second state.



(a) (b)

Figure 9: RMSEs produced by FKF and FOUF with temporary model uncertainty: (a) the first state and (b) the second state.

sensitive to noise statistics and more robust against modeling uncertainties.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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