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Research Article

Linear Processing Design of Amplify-and-Forward Relays for Maximizing the System Throughput

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In this paper, firstly, we study the linear processing of amplify-and-forward (AF) relays for the multiple relays multiple users scenario. We regard all relays as one special "relay", and then the subcarrier pairing, relay selection and channel assignment can be seen as a linear processing of the special "relay". Under fixed power allocation, the linear processing of AF relays can be regarded as a permutation matrix. Employing the partitioned matrix, we propose an optimal linear processing design for AF relays to find the optimal permutation matrix based on the sorting of the received SNR over the subcarriers from BS to relays and from relays to users, respectively. Then, we prove the optimality of the proposed linear processing scheme. Through the proposed linear processing scheme, we can obtain the optimal subcarrier paring, relay selection and channel assignment under given power allocation in polynomial time. Finally, we propose an iterative algorithm based on the proposed linear processing scheme and Lagrange dual domain method to jointly optimize the joint optimization problem involving the subcarrier paring, relay selection, channel assignment and power allocation. Simulation results illustrate that the proposed algorithm can achieve a perfect performance.

1. Introduction

Relay-assisted cooperative communication and orthogonal frequency-division multiplexing (OFDM) are core technologies for the next generation mobile communication system which attract tremendous attentions. Relay-assist cooperative communication can effectively extend communication coverage, and reduce the consumption of transmit power. It can also improve overall system throughput due to its exploiting spatial diversity and combating channel fading [1, 2]. OFDM divides a wideband channel into some orthogonal narrow band subcarriers. Then, the fast data flow can be transformed to a set of slow data flows. Furthermore, inner-symbol interference can be eliminated significantly. OFDM can also increase the flexibility for coding and modulation. Therefore, relay-assisted cooperative communication and OFDM are adopted by a lot of communication standards, such as 3GPP and IEEE 802.16e.

There are two common categories of relay strategies: amplify-and-forward (AF) and decode-and forward (DF). The DF relay can decode and re-encode the received signals, and then retransmit these signals to destination node. Different from DF relay, the AF relay amplifies the received signals linearly and forwards these signals to destination node without decoding. The AF strategy is the most practical relay approach as the result of its low complexity transceiver design [3]. Moreover, AF relays are more transparent to adaptive modulation techniques than DF relays.

Due to the independent fading on each subcarrier, the incoming and outgoing subcarrier should be matched carefully to maximize the overall system throughput. This problem is also called subcarrier paring. Recently, subcarrier paring has gained a lot of attentions, such as [4–6]. Reference [4] proposes an ordered subcarrier pairing (OSP) method, in which it is proved that subcarrier paring according to the channel state is optimality for equal power allocation. The authors in [5] further prove that the OSP is still optimality for optimal power allocation. Reference [6] considers a linear processing design at the relay for amplified-and-forward (AF) relaying communication system without direct path and with direct path under fixed power gain. Different from [4, 5], [6] dose not only consider the cooperative mode without direct path, but also study the the cooperative mode with direct path. In [6], the authors separate the processing structure into two components which are the power amplification matrix and the unitary linear processing matrix, respectively. They show the optimal unitary processing matrix is of permutation structure.

The joint resource allocation for cooperative communication has been also attracted tremendous attentions. For the scenario with multiple relays multiple users, the joint optimization problem involves subcarrier pairing, relay selection, channel assignment and power allocation. Due to the combinational nature of the subcarrier pairing, relay selection and channel assignment, the joint optimization problem is difficult to solve when the number of subcarriers, relays and users are large. Therefore, most previous works only consider a subset of these issues.

In this paper, firstly, we exploit the linear processing scheme for multiple relays multiple users network to maximize the system throughput. We regard all relays as one special "relay", and the subcarrier pairing, channel assignment and relay selection can be seen as the linear processing of the special "relay" under fixed power allocation. It can be shown that the optimal linear processing matrix of the special "relay" is a promotion of the permutation matrix. To maximize the system throughput under fixed power allocation, the optimal linear processing matrix should be found out. To this end, we propose a linear processing scheme for the AF relays under fixed power allocation which can find out the optimal permutation matrix, and then we prove the optimality of this linear processing design. It is interesting that, through the proposed linear processing scheme, we can obtain the optimal subcarrier paring, relay selection and channel assignment with fixed power allocation in accordance with the ordered SNR over the incoming and outgoing channels in polynomial time. To the best of our knowledge, this paper is the first work to investigate the linear processing of the AF relays for the multiple relays multiple users network. Finally, based on the proposed linear process scheme and Lagrange dual domain method, we propose an iterative algorithm to solve the joint optimization problem involving subcarrier pairing, relay selection, channel assignment as well as power allocation. Simulation results illustrate that the proposed iterative algorithm is effective to find out an *asymptotically* optimal solution in polynomial time.

The rest of the paper is organized as follows. In Section 2, the previous related works are reviewed. We describe the system model in Section 3. The optimal linear processing under fixed power gain is introduced in Section 4. In Section 5, we extend the linear process scheme to the scenario with multiple relays multiple users. In Section 6, we describe the iterative algorithm based on the proposed linear process scheme and Lagrange dual domain method to solve the joint resource allocation problem. In Section 7, simulation results are provided to evaluate the performance of the proposed algorithm. Finally, we conclude this paper in Section 8.

2. Related Work

The resource allocation for the scenario with single BS, multiple relays and multiple users is a joint optimization problem which involves subcarrier paring, relay selection, channel assignment and power allocation. This joint optimization problem can be formulated as a mixed-integer programming due to the combinatorial nature of the subcarrier paring, relay selection and channel assignment. It becomes more and more intractable with the increase of the number of subcarriers, relays and users because the problem of subcarrier pairing, relay selection and channel assignment is NP hard [7]. To decrease the computational complexity, most previous works consider a subset of the joint optimization problems.

References [8–11] consider the scenario with one BS, one relay and one user. Reference [8] jointly optimizes the subcarrier pairing and power allocation, and a mixed-integer programming problem is formulated. Then, the authors in [8] transform the mixed integer programming problem into a convex optimization through continuous relaxation. [9] proposes a hybrid scheme in which the full-duplex and half-duplex relaying modes can be switched opportunistically to obtain a tradeoff between spectral efficiency and self-interference. For DF relay strategy, if the ratio of the time allocation in the first transmission phase over the whole period is not large enough, the DF relay can not decode the received signals accurately. It is necessarily to optimize the the ratio of the time allocation in the first transmission phase over the whole period. Therefore, reference [10] optimizes not only the power allocation but also time allocation for DF relay strategy to minimize the outage probability. The perfect channel state information (CSI) at the source is impractical, thus it is reasonable to study the resource allocation problem with limited feedback [11].

The resource allocation for the scenario with one BS, one relay and multiple users is studied by reference [7]. The authors in [7] propose an optimal algorithm for the joint resource allocation including subcarrier pairing, channel assignment and power allocation through transforming the original problem into some simple linear programming problems. In [12], a combination of transmit-receive weights and Tomlinson-Harashima precoding is used to cancel the interference, and then a low-complexity power allocation is proposed to achieve data rate fairness among different users.

The cooperative communication scenario with one BS, multiple relays and multiple users is studied by [13, 14], and the direct transmission mode is also considered at the same time. The authors in [13] transform the combinational optimization problem of subcarrier paring, channel assignment, relay selection and transmission mode selection into a minimum cost network flow problem, and apply the linear optimal distribution algorithm to solve the minimum cost network flow problem. Then, the Lagrange dual domain method is used to solve the power allocation. Different from [13], reference [14] only considers the relay selection and subcarrier assignment for multiuser cooperative network to decrease the computational complexity. In the first, [14] simplifies the original problem into a new problem through decreasing the number of variables which is easily to handle. Then, branch-and-cut is introduced to optimize the new problem.

There are also a lot of works on the resource allocation for the multiple-input and multiple-output (MIMO) cooperative communication networks, such as [3, 15–21].

Referencee [15] considers the problem of minimizing the total power consumption in a two-hop single-relay MIMO network with QoS requirements. In [15], a nonconvex power allocation problem is approximated with a convex problem, and then the convex problem is computed in closed-form through a multistep procedure. [16] studies the problem of resource allocation in relay-enhanced bidirectional MIMO-OFDM networks to minimize the total power consumption. The authors in [16] propose a green resource allocation scheme to jointly optimize the subchannel assignment, power allocation and phase duration assignment, in which both the separate-downlink (DL)-and-uplink (UL) and mixed-DL-and-UL relaying assignments and the linear block diagonalization (LBD) technique are adopted. Reference [17] study the optimal power allocation structure for the multiple relays network. The authors in [17] show that the power allocation at BS and each relay follow a matching structure. The cooperative communication with virtual MIMO has also attracted a lot of attentions, such as [18]. Reference [18] studies a wireless network where multiple users cooperate with each other. The authors in [18] propose a new auction-based power allocation framework with multiple auctioneers and multiple bidders to maximize the weighted sum-rates of the users.

Recently, energy efficient (EE) resource allocation for MIMO cooperative communication has become attractive, such as [22–27]. Reference [22] considers the energy efficient joint source-relay power allocation problem for MIMO AF relaying system. In [22], the objective of optimization is the number of the bits per second per hertz per Joule with the guarantee of the minimum spectral efficiency, and the objective function and the constraint are not convex. Firstly, [22] transforms the original problem into a pseudo-convex problem by employing the high signal-to-noise ratio (SNR) approximation. Then, the authors in [22] propose a relaxation method according to the Jensen inequality to solve the pseudo-convex problem. [23] studies the energy-spectral efficiency (EE-SE) trade-off of the uplink of a multi-user cellular virtual MIMO system. Finally, a heuristic resource allocation algorithm is proposed to optimize the EE-SE tradeoff in [23]. The energy-efficient resource allocation for OFDMA cellular networks with user cooperation is studied by [24]. In [24], a mixed-integer nonlinear programming problem is formulated, and then an optimal algorithm is proposed to solve the problem.

3. System Model

In this paper, for matrix M, we let $\mathrm{Tr}\{\mathrm{M}\}$, $\mathrm{M}^{\dagger}$ and $|\mathrm{M}|$ denote the trace, conjugate transpose and determinant of M, respectively. I denote the identity matrix. $\mathrm{diag}\{\mathrm{M}_1, \dots, \mathrm{M}_n\}$ represents the block diagonal matrix with $\mathrm{M}_1, \dots, \mathrm{M}_n$ on its main diagonal. We assume that the perfect CSI is obtained by the technology of channel estimation, and then the accurate channel gain is known.

In this paper, we intend to exploit the optimal linear processing of relays in OFDMA AF-based relaying network consisting multiple relays multiple users, similar as Figure 1. For easy to exposition, we first study the linear processing scheme for the scenario with multiple relays and single user in this section. Then, we will show that our proposed linear processing scheme can be readily extended to the multiple relays multiple users scenario in the next section. In this work, we assume there are N subcarriers and M AF relays. The half-duplex relay is considered, that is, each relay can not transmit and receive the signals in the same subcarrier at the same time. The transmission duration is divided into two equal time slot. Let $\mathrm{H}_{sr}^{i} \in \mathbb{C}^{N\times N}$ and $\mathrm{H}_{rd}^{i} \in \mathbb{C}^{N\times N}$ denote the channel gain matrices between BS and relay i and between relay i and the user, respectively.

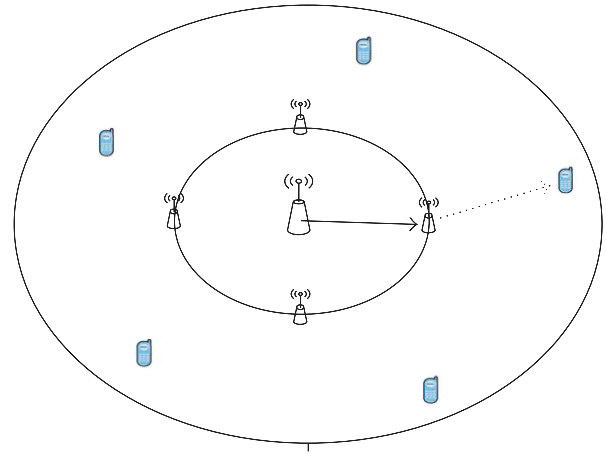

Figure 1: Illustration of dual-hop multi-relay multi-user AF relaying.

The received signals at the relay i in the first time slot are given by $\mathrm{y}_r^{i} = \mathrm{H}_{sr}^{i}\mathrm{D}_{sr}^{i}\mathrm{s} + \mathrm{n}_r^{i}$, $i = 1, 2, \dots, M$, where $\mathbf{s} \in \mathbb{C}^{N\times 1}$ is the transmission symbol vector and $\mathrm{D}_{sr}^{i} = \mathrm{diag}\{\sqrt{p_{b,i,1}}, \dots, \sqrt{p_{b,i,N}}\} \in \mathbb{C}^{N\times N}$ is the transmission power coefficient matrix from BS to relay i, and $\mathrm{n}_r^{i} \in \mathbb{C}^{N\times 1}$ represents the AWGN at relay i, with $\sim\mathcal{CN}(0, \sigma_r^2\mathrm{I})$. Next, we let $\mathrm{H}_{sr} = \mathrm{diag}\{\mathrm{H}_{sr}^{1}, \mathrm{H}_{sr}^{2}, \dots, \mathrm{H}_{sr}^{M}\} \in \mathbb{C}^{MN\times MN}$ denote the combined relay channel matrix between BS and all relays. Similarly, we let $\mathrm{D}_{sr} = \mathrm{diag}\{\mathrm{D}_{sr}^{1}, \mathrm{D}_{sr}^{2}, \dots, \mathrm{D}_{sr}^{M}\} \in \mathbb{C}^{MN\times MN}$, $\mathrm{s} = \underbrace{[s^{\dagger}, \dots, s^{\dagger}]^{\dagger}}_{M} \in \mathbb{C}^{MN\times 1}$ and $\mathrm{n}_r = [\mathrm{n}_r^{[1]\dagger}, \dots, \mathrm{n}_r^{[M]\dagger}]^{\dagger} \in \mathbb{C}^{MN\times 1}$. Thus, the combined received signal of all relays can be given as $\mathrm{y}_r = \mathrm{H}_{sr}\mathrm{D}_{sr}\mathrm{s} + \mathrm{n}_r$.

In the second time slot, the AF relays process the received signals linearly and then retransmit these signals to the user. Since the relays cannot share their received signals with each other, we let $\mathrm{W} = \mathrm{diag}\{\mathrm{W}_1, \dots, \mathrm{W}_M\} \in \mathbb{C}^{MN\times MN}$ denote the linear processing matrix of all relays. The processed signals at relay i can be represented as $\mathrm{x}_r^{[i]} = \mathrm{W}_i\mathrm{y}_r^{[i]}$. Then, the overall relay signals can be given as $\mathrm{x}_r = \mathrm{W}\mathrm{y}_r = \mathrm{W}(\mathrm{H}_{sr}\mathrm{D}_{sr}s + \mathrm{n}_r)$, where $\mathrm{x}_r = [\mathrm{x}_r^{[1]\dagger}, \dots, \mathrm{x}_r^{[M]\dagger}]^{\dagger}$.

Also, we let $\mathrm{H}_{rd} = \mathrm{diag}\{\mathrm{H}_{rd}^{1}, \dots, \mathrm{H}_{rd}^{M}\} \in \mathbb{C}^{MN\times MN}$ represent the combined channel gain matrix between all relays and the user. Then, the received signals at user can be written as:

$$y_d = H_{rd}x_r + n_d = H_{rd}W\left(H_{sr}D_{sr}s + n_r\right) + n_d, \quad (1)$$

where $n_d \in \mathbb{C}^{MN\times 1}$ is the AWGN, with $n_d \sim \mathcal{CN}(0, \sigma_d^2 I)$ at the user in the second time slot.

In the second time slot, the relays combine the received signals linearly and then retransmit the amplified version of the processed signal to the user. Therefore, the processing matrix W can be represented as $W = D_r U$, where $U \in \mathbb{C}^{MN\times MN}$ is a special permutation matrix in which there are only N rows and N columns with nonzero entry, which determines how to combine the incoming signals linearly and $D_r = \mathrm{diag}\{D_r^1, \ldots, D_r^M\} \in \mathbb{C}^{MN\times MN}$ is the power amplification matrix of all relays, where $D_r^i = \mathrm{diag}\{d_{i,1}, \ldots, d_{i,N}\}$ and $d_{i,n}$ means the power amplification factor for the nth subcarrier at relay i. In this section, we intend to investigate the linear processing matrix U with fixed D_s and D_r. That is, our goal is to investigate what form of the linear processing scheme leads to the optimal relay selection and subcarrier pairing. Since we are interesting in the permutation matrix U, the received signals at the user can be rewritten as

$$y_{dr} = \widetilde{H}(U)s + \tilde{n}(U), \quad (2)$$

where $\widetilde{H}(U) \triangleq H_{rd}D_r U H_{sr}D_s$ and $\tilde{n}(U) \triangleq H_{rd}D_r U n_r + n_d$ are the functions of matrix U, and they can be seen as the equivalent channel matrix and noise term.

In fact, there are M relays and N subcarriers so that there are MN incoming subchannels and MN outgoing subchannels. To avoid interference, we can only select N incoming subchannels and N outgoing subchannels (each subchannel can only be assigned to one relay node at each time slot). Therefore, U is a non-full rank permutation matrix in which there only N rows and N columns with nonzero entry.

4. Overall System Throughput

As aforementioned, the end-to-end achievable rate can be given by

$$R(U) = \frac{1}{2}\log\left|I + R_n^{-1}\widetilde{H}(U)\widetilde{H}^{\dagger}(U)\right|, \quad (3)$$

where $R_n \triangleq E[\tilde{n}(U)\tilde{n}^{\dagger}(U)]$ represents the covariance matrix of the equivalent noise term. The factor 1/2 in (3) accounts for the two time slots required to complete a transmission. Due to the maximum transmission power limit at BS and each relay i, we have the following peak power constraints:

$$E\left\|D_s s\right\| \le P_s, \quad (4)$$

$$\mathrm{Tr}\left\{W_i\left(H_{sr}^{[i]}D_s^{[i]2}H_{sr}^{[i]\dagger} + \sigma_r^2 I\right)W_i^{\dagger}\right\} \le P_i, \quad i = 1, \ldots, M, \quad (5)$$

where P_s and P_i are the maximum transmission power of BS and relay i, respectively.

In this section, our objective is to find the optimal U^* to maximize the system achievable $R(U)$, which can be formulated as the following optimization problem:

$$\begin{aligned}\max_{U}\quad & R(U) = \frac{1}{2}\log\left|I + R_n^{-1}\widetilde{H}(U)\widetilde{H}^{\dagger}(U)\right| \\ \text{s.t.}\quad & (4), (5).\end{aligned} \quad (6)$$

For the scenario with multiple relays and single user, relay selection and subcarrier pairing can be essentially represented as a kind of U. In this case, $U \in \mathbb{C}^{MN\times MN}$ is a non-full rank permutation matrix in which there are only N rows and N columns with nonzero entries.

Next, we give our method to find the optimal matrix U^*. In accordance with $|I + AB| = |I + BA|$, (6) can be rewritten as:

$$\max_{U}\quad R(U) = \frac{1}{2}\log\left|I + \widetilde{H}^{\dagger}(U)R_n^{-1}\widetilde{H}(U)\right|. \quad (7)$$

Substituting $\widetilde{H}(U) \triangleq H_{rd}D_r U H_{sr}D_s$ and $\tilde{n}(U) \triangleq H_{rd}D_r U n_r + n_d$ into (7), we have

$$\begin{aligned}R(U) &= \frac{1}{2}\log\left|I + \left(R_n^{-1/2}\widetilde{H}(U)\right)^{\dagger}R_n^{-1/2}\widetilde{H}(U)\right| = \frac{1}{2} \\ &\cdot \log\left|I + \left(R_n^{-1/2}H_{rd}D_r U H_{sr}D_{sr}\right)^{\dagger}\right. \\ &\left.\cdot R_n^{-1/2}H_{rd}D_r U H_{sr}D_{sr}\right|.\end{aligned} \quad (8)$$

Let $F = R_n^{-1/2}H_{rd}D_r$ and $T = H_{sr}D_{sr}$, and then (8) can be rewritten as

$$\max_{U}\quad \frac{1}{2}\log\left|I + (FUT)^{\dagger}FUT\right|. \quad (9)$$

The jth diagonal entries of F and T are given by:

$$\begin{aligned}f_j &= \frac{h_{rd}^j d_r^j}{\sqrt{\sigma_d^2 + \sigma_r^2\left|h_{rd}^j d_r^j\right|^2}}, \\ t_j &= h_{sr}^j d_s^j.\end{aligned} \quad (10)$$

Let $\mathrm{SNR}_{sr,j} = |h_{sr}^j|^2|d_s^j|^2/\sigma_r^2$ and $\mathrm{SNR}_{rd,j} = |h_{rd}^j|^2|d_r^j|^2/\sigma_d^2$ be the received SNR from BS to relay and from relay to user respectively, and then there are

$$\begin{aligned}f_j^2 &= \frac{\mathrm{SNR}_{rd,j}}{1 + \sigma_r^2\mathrm{SNR}_{rd,j}}, \\ t_j^2 &= \sigma_r^2\mathrm{SNR}_{sr,j}.\end{aligned} \quad (11)$$

It is clear that $\{f_j\}$ and $\{t_j\}$ have the same sorting order with $\{\mathrm{SNR}_{rd,j}\}$ and $\{\mathrm{SNR}_{sr,j}\}$. Therefore, f_j and t_j are equivalent to $\mathrm{SNR}_{rd,j}$ and $\mathrm{SNR}_{sr,j}$, respectively.

To maximize (9), in the first, we give the following Lemma.

Lemma 1. *Let* $\mathrm{P} = \operatorname{diag}\{p_1, \ldots, p_N\}$ *and* $\mathrm{Q} = \operatorname{diag}\{q_1, \ldots, q_N\}$ *be two diagonal matrices. We let* $\{|p_{(i)}|\}$ *and* $\{|q_{(i)}|\}$ *be the ordered sequences of* $\{|p_i|\}$ *and* $\{|q_i|\}$ *in descending order, respectively. The optimal* U *to maximize (9) is as follows:*

$$\left|\mathrm{I} + (\mathrm{PU}^*\mathrm{Q})^{\dagger}(\mathrm{PUQ})\right| = \prod_{i}^{N}\left(1 + |p_i|\,|q_i|\right). \tag{12}$$

That is, U^* *is the permutation matrix that matches* p_i *and* q_i.

Proof. Please refer to Appendix □

For the scenario with one relay and one user, U is a full rank permutation matrix since there is only subcarrier pairing that need to be optimized. According to Lemma 1, we can readily obtained the optimal subcarrier pairing with fixed power allocation, which is that the incoming subcarrier with the best SNR is paired with the outgoing subcarrier with the best SNR, and the incoming subcarrier with the worst SNR is paired with the outgoing subcarrier with the worst SNR, and so on. However, when the number of relays is more than one, the best incoming subcarrier and the best outgoing subcarrier may do not belong to the same relay which makes these two subcarrier can not be matched. Therefore, the case with multiple relays differs significantly from the case with single relay. To exploit the optimal linear processing scheme for the scenario with multiple relays, in the following, we first give two constraints to avoid interference and make the linear processing scheme be in line with the reality, and then we will give a Theorem to exploit the optimal linear processing for multiple relays network.

Constraint 2. Only N orthogonal subchannels can be selected at each hop to avoid co-channel interference.

Constraint 3. Two subcarriers which are matched to one pair should belong to the same relay node.

Indeed, Constraint 2 means that each subcarrier can be only assigned to one relay at each transmission slot to avoid interference.

Next, we give a Theorem as follows:

Theorem 4. *Let* $\mathrm{F} = \operatorname{diag}\{f_1, f_2, \ldots, f_{MN}\}$ *and* $\mathrm{T} = \operatorname{diag}\{t_1, t_2, \ldots, t_{MN}\}$. *Let* $\mathrm{G} = \{g_1, g_2, \ldots, g_{(MN)^2}\}$, *where* $g_i = f_k q_l, k, l \in \{1, 2, \ldots, MN\}$. *It is clear that the cardinality of* G *is* $(MN)^2$ *since there are* $(MN)^2$ *combinations of* f_k, $k = 1, 2, \ldots, MN$ *and* $t_l, l = 1, 2, \ldots, MN$. *Let* $\{|g_i|\}$ *be a ordered sequence in descending order. Under Constraints 2 and 3, the optimal* U^* *to maximize (9) is as follows:*

$$\left|\mathrm{I} + (\mathrm{FU}^*\mathrm{T})^{\dagger}\,\mathrm{FU}^*\mathrm{T}\right| = \prod_{1}^{N}\left(1 + |g_i|^2\right). \tag{13}$$

Proof. We assume the optimal incoming and outgoing channels obtained from Theorem 4 are $\{f_{(1)}, f_{(2)}, \ldots, f_{(N)}\}$ and $\{t_{(1)}, t_{(2)}, \ldots, t_{(N)}\}$ respectively, which correspond to a set of $\{g_{(1)}, g_{(2)}, \ldots, g_{(N)}\}$, where $g_{(1)} = f_{(1)}t_{(1)}$, $g_{(2)} = f_{(2)}t_{(2)}$, and so on. Without loss of generality, we assume $\{g_{(i)}\}$ are ordered sequence with descending order. Then, the corresponding permutation matrix $\mathbf{U}^*$ satisfies:

$$\left|\mathrm{I} + (\mathrm{F}\mathbf{U}^*\mathrm{T})^{\dagger}\,\mathrm{F}\mathbf{U}^*\mathrm{T}\right| = \prod_{1}^{N}\left(1 + |g_{(i)}|^2\right). \tag{14}$$

For any other permutation matrix $\widetilde{\mathrm{U}}$ which also satisfies Constraints 2 and 3, we assume the corresponding subcarrier pair set is $\{\widetilde{g}_{(1)}, \widetilde{g}_{(2)}, \ldots, \widetilde{g}_{(N)}\}$. From Lemma 1, there is

$$\left|\mathrm{I} + \left(\mathrm{F}\widetilde{\mathrm{U}}\mathrm{T}\right)^{\dagger}\mathrm{F}\widetilde{\mathrm{U}}\mathrm{T}\right| = \prod_{1}^{N}\left(1 + |\widetilde{g}_{(i)}|^2\right). \tag{15}$$

According to the selection criterion of Theorem 4, there is at least one $\widetilde{g}_{(i)} \le g_{(i)}$. So, there is

$$\prod_{1}^{N}\left(1 + |\widetilde{g}_{(i)}|^2\right) \le \prod_{1}^{N}\left(1 + |g_{(i)}|^2\right). \tag{16}$$

Therefore, the permutation matrix U^* is optimal for (9). □

Theorem 4 gives an optimal linear processing scheme of relays corresponding to one optimal relay selection and subcarrier pairing scheme for the scenario with multiple relays. In fact, as aforementioned, when there is only one relay, the linear processing matrix "U" is a full rank permutation matrix which only corresponds to the subcarrier pairing. In accordance with Lemma 1, the optimal pairing strategy is based on the sorted received SNRs. However, for the scenario with multiple relays considered in this paper, the processing matrix "U" is a non-full rank matrix since each subcarrier can only be allocated to one relay node to avoid interference. Theorem 4 shows that the optimal relay selection and subcarrier pairing strategy is based on the sorted products of the incoming and outgoing subchannel's SNRs.

In fact, there are $M \times N$ "possible" incoming channels between BS and all relays and $M \times N$ "possible" outgoing channels between all relays and the user, and we can only select N incoming and N outgoing channels. There are $(MN)^2$ combinations of incoming and outgoing channels. Each combination corresponds to one product of one incoming and one outgoing subchannel's SNR. Theorem 4 means the optimal scheme is that, firstly, these combinations are ordered according to their corresponding products. Then, N best combinations satisfying Constraints 2 and 3 would be selected from the ordered $(MN)^2$ combinations.

For example, if the number of subcarriers, relays and users are 2, 2 and 1 respectively, there are 4 "possible" incoming channels: $\{f_{11}^1, f_{12}^1, f_{21}^1, f_{22}^1\}$, and 4 "possible" outgoing channels: $\{q_{11}^1, q_{12}^1, q_{21}^1, q_{22}^1\}$. There are 16 combinations of the incoming and outgoing channels. Assume $f_{11}^1 = 5$, $f_{12}^1 = 4$, $f_{21}^1 = 2$, $f_{22}^1 = 1$, $q_{11}^1 = 2$, $q_{12}^1 = 3$, $q_{21}^1 = 4$, $q_{22}^1 = 1$. Then, $\{|g_i|\} = \{f_{11}^1 q_{21}^1 = 20$, $f_{12}^1 q_{21}^1 = 16$, $f_{11}^1 q_{12}^1 = 15$, $f_{12}^1 q_{12}^1 = 12$, $f_{11}^1 q_{11}^1 = 10$, $f_{12}^1 q_{11}^1 = 8$, $f_{21}^1 q_{21}^1 = 8$, $f_{21}^1 q_{12}^1 = 6$, $f_{11}^1 q_{22}^1 = 5$, $f_{12}^1 q_{22}^1 = 4$, $f_{21}^1 q_{11}^1 = 4$, $f_{22}^1 q_{21}^1 = 4$, $f_{22}^1 q_{12}^1 = 3$,

$f_{21}^1 q_{22}^1 = 2$, $f_{22}^1 q_{11}^1 = 2$, $f_{22}^1 q_{22}^1 = 1\}$. Therefore, $\{f_{11}^1, f_{22}^1\}$ and $\{q_{12}^1, q_{21}^1\}$ are selected because another channels do not satisfy Constraints 2 and 3. Thus, the optimal transmission paths are $(2,1,1,1)$ and $(1,2,1,1)$, which mean that the subcarrier pair $(2,1)$ is assigned to user 1 with the help of relay 1 and the subcarrier pair $(1,2)$ is assigned to user 1 with the help of relay 1, respectively. The corresponding permutation U is the form as follows:

$$\mathrm{U} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{17}$$

Note that it is possible that there exists tie, for instance, there are two combinations of subcarriers with the same product and both these two combinations satisfy Constraints 2 and 3. For this case, arbitrary tie-breaking will be adopted which does not change the optimality of the algorithm.

5. The Scenario with Multiple Relays Multiple Users

In this section, we aim to extend the linear processing scheme proposed in the previous section to the scenario with multiple relays multiple users. We assume there are one BS, M relays, K users and N subcarriers. To make the computation feasible, in the first time slot, we denote the channel matrix between BS and all relays, the transmit power coefficient matrix and the transmission symbol vector as $\mathrm{H}_{sr} = \mathrm{diag}\{\underbrace{\mathrm{H}_{sr}, \mathrm{H}_{sr}, \ldots, \mathrm{H}_{sr}}_{K}\} \in \mathbb{C}^{MNK\times MNK}$ $\mathrm{D}_{sr} = \mathrm{diag}\{\underbrace{\mathrm{D}_{sr}, \mathrm{D}_{sr}, \ldots, \mathrm{D}_{sr}}_{K}\} \in \mathbb{C}^{MNK\times MNK}$ and $\mathrm{s} = [\underbrace{s^\dagger, \ldots, s^\dagger}_{K}]^\dagger \in \mathbb{C}^{MNK\times 1}$, respectively. Then, the combined received signals of all relays can be given by $y_r = \mathrm{H}_{br}\mathrm{D}_{br}s + \mathrm{n}_r$, where $n_r \in \mathbb{C}^{MNK\times 1}$. Similarly, the processing matrix of all relays is extended to $\mathrm{W} = \mathrm{diag}\{\underbrace{\mathrm{W}, \ldots, \mathrm{W}}_{K}\} \in \mathbb{C}^{MNK\times MNK}$, and the linear processing matrix $\mathrm{U} = \mathrm{diag}\{\underbrace{\mathrm{U}, \ldots, \mathrm{U}}_{K}\} \in \mathbb{C}^{MNK\times MNK}$. Note that U is also a special permutation matrix, in which there are only N columns and N rows with nonzero entries.

We let $\mathrm{H}_{rd}^k = \mathrm{diag}\{\mathrm{H}_{rd}^{1,k}, \mathrm{H}_{rd}^{2,k}, \ldots, \mathrm{H}_{rd}^{M,k}\}$ denote the combined channel gain matrix from all relays to user k, where $\mathrm{H}_{rd}^{i,k} \in \mathbb{C}^{N\times N}$ is the channel gain from relay i to user k, and let $\mathrm{D}_{rd}^k = \mathrm{diag}\{\mathrm{D}_{rd}^{1,k}, \mathrm{D}_{rd}^{2,k}, \ldots, \mathrm{D}_{rd}^{M,k}\}$ denote the power amplification matrix from all relays to user k, where $\mathrm{D}_{rd}^{[i,k]} = \mathrm{diag}\{d_{i,n,k}, \ldots, d_{i,N,k}\} \in \mathbb{C}^{N\times N}$ denote the power amplification matrix from relay i to user k. $d_{i,n,k}$ denotes the power amplification factor from relay i to user k over subcarrier n.

Next, let $\mathrm{H}_{rd} = \mathrm{diag}\{\mathrm{H}_{rd}^1, \mathrm{H}_{rd}^2, \ldots, \mathrm{H}_{rd}^K\} \in \mathbb{C}^{MNK\times MNK}$ and $\mathrm{D}_{rd} = \mathrm{diag}\{\mathrm{D}_{rd}^1, \mathrm{D}_{rd}^2, \ldots, \mathrm{D}_{rd}^K\} \in \mathbb{C}^{MNK\times MNK}$ denote the combined channel gain matrix from all relays to all users and the combined power amplification matrix from all relays to all users, respectively. The received signals of all users can be given by: $\mathrm{y}_d = \mathrm{H}_{rd}\mathrm{D}_{rd}\mathrm{s} + \mathrm{n}_d$, where $\mathrm{n}_d = [\mathrm{n}_d^{[1]\dagger}, \ldots, \mathrm{n}_d^{[K]\dagger}] \in \mathbb{C}^{MNK\times 1}$ represents the AWGN at users. Finally, the received signals at all users can be expressed as:

$$\begin{aligned} \mathrm{y}_d &= \mathrm{H}_{rd}\mathrm{W}\left(\mathrm{H}_{sr}\mathrm{D}_{sr}s + \mathrm{n}_r\right) + \mathrm{n}_d \\ &= \mathrm{H}_{rd}\mathrm{D}_{rd}\mathrm{U}\left(\mathrm{H}_{sr}\mathrm{D}_{sr}s + \mathrm{n}_r\right) + \mathrm{n}_d. \end{aligned} \tag{18}$$

The achievable rate in this scenario with multiple relays multiple users can be given by

$$R(\mathrm{U}) = \frac{1}{2}\log\left|\mathrm{I} + \mathrm{R}_n^{-1}\widetilde{\mathrm{H}}(\mathrm{U})\widetilde{\mathrm{H}}^\dagger(\mathrm{U})\right|, \tag{19}$$

where $\mathrm{H}(\mathrm{U})$ and R_n have the similar form as that in previous section.

So far, it is easy to prove that the Theorem 4 can be extended to the scenario with multiple relays multiple users. Thus, for the multiple relays multiple users network, when the power allocation is fixed, we can obtain the optimal permutation U which corresponds to the optimal subcarrier pairing, relay selection and channel assignment through the linear processing scheme proposed in this paper.

The computational complexity of the proposed linear processing scheme for the scenario with multiple relays multiple users is given by $\mathcal{O}((MN)^2K\log(MN)^2K + (MN)^2K)$. In general, M and K are in the order of $\mathcal{O}(\sqrt{N})$. Therefore, the overall complexity of the proposed linear process scheme is $\mathcal{O}((7/2)N^3\sqrt{N}\log N + N^{7/2})$. Note that the computational complexity of the proposed algorithm is less than most of the algorithms which solve the similar problems, such as network flow method.

Remark 5. Reference [20] studies the resource allocation of MIMO-OFDM relaying system and achieves excellent performance. However, it is worth nothing that [20] considers the power allocation and subcarrier assignment of relay system, and studies resource allocation problem from scalar optimization. In this work, we consider the linear processing scheme of AF relay system, which is a signal processing problem. We formulate the problem as a matrix optimization problem, and solve it using the method of matrix analysis.

6. Joint Optimization with Power Allocation

The relay selection and subcarrier pairing under fixed power allocation have been optimized in the previous section. In this section, we intend to tackle the joint resource allocation involving relay selection, subcarrier pairing, channel assignment as well as power allocation using the Lagrange dual domain method and the proposed linear processing scheme.

Firstly, we give a set of binary assignment variables corresponding to the permutation matrix U:

u_{mnik}: indicates whether subcarrier pairing (m,n) is assigned to relay i to help user k. Note that u_{mnik} is dependent on the permutation matrix U. In the aforementioned section, we have optimized the binary assignment variables u_{mnik} which indicates the subcarrier pairing, relay selection and channel-user assignment under fixed power allocation through optimizing the linear processing of the relays. In this section, we take into account to optimize the power allocation.

Since both F and T are diagonal matrices, we can readily convert (19) into scalar form as follows:

$$R = \sum_{k=1}^{K}\sum_{i=1}^{M}\sum_{m=1}^{N}\sum_{n=1}^{N} u_{mnik} \log\left(\frac{\left|h_{i,n,k}d_{i,n,k}h_{b,r,n}\sqrt{p_{b,i,n}}\right|^2}{\sigma_d^2 + \sigma_r^2\left|h_{i,n,k}d_{i,n,k}\right|^2}\right). \tag{20}$$

In accordance with the definition of D_r, $d_{i,n,k}$ can be represented as

$$d_{i,n,k}^2 = \frac{p_{i,n,k}}{\sigma_i^2 + h_{b,i,n}^2 p_{b,i,n}}, \tag{21}$$

where $p_{i,n,k}$ is the transmit power of relay i to user k over subcarrier n in the second time slot. Let $g_{b,i,n} = |h_{i,n,k}|^2/\sigma_r^2$, and $g_{i,n,k} = |h_{i,n,k}|^2/\sigma_d^2$. Substituting $d_{i,n,k}^2 = p_{i,n,k}/(\sigma_r^2 + h_{b,r,n}^2 p_{b,i,n})$, $g_{b,i,n} = |h_{i,n,k}|^2/\sigma_r^2$ and $g_{i,n,k} = |h_{i,n,k}|^2/\sigma_d^2$ into (20), and then (20) can be rewritten as:

$$R = \sum_{k=1}^{K}\sum_{i=1}^{M}\sum_{m=1}^{N}\sum_{n=1}^{N} u_{mnik} \log\left(1 + \frac{g_{b,i,m}p_{b,i,m}g_{i,n,k}p_{i,n,k}}{1 + g_{b,i,m}p_{b,r,m} + g_{i,n,k}p_{i,n,k}}\right). \tag{22}$$

Our objective is to jointly optimize the subcarrier pairing, relay selection, channel assignment and power allocation to maximize the overall system throughput. Let $\mathbf{U} \triangleq [u_{mnik}]_{N\times N\times M\times K}$, $\mathbf{P} \triangleq \{[p_{b,i,m}]_{M\times N}, [p_{i,n,k}]_{M\times N\times K}\}$ and

$$R_{mnik} = \log\left(1 + \frac{g_{b,i,m}p_{b,r,m}g_{i,n,k}p_{i,n,k}}{1 + g_{b,i,m}p_{b,i,m} + g_{i,n,k}p_{i,n,k}}\right). \tag{23}$$

Then, the joint optimization problem can be formulated as:

$$\mathbf{P1}\ \max_{\mathbf{U},\mathbf{P}} \quad \sum_{k=1}^{K}\sum_{i=1}^{M}\sum_{m=1}^{N}\sum_{n=1}^{N} u_{mnik}R_{mnik} \tag{24}$$

$$\text{s.t.} \quad \sum_{k\in K}\sum_{i\in M}\sum_{n\in N} u_{mnik} = 1 \quad m = 1,2,\ldots,N, \tag{25}$$

$$\sum_{k\in K}\sum_{i\in M}\sum_{m\in N} u_{mnik} = 1 \quad n = 1,2,\ldots,N, \tag{26}$$

$$\sum_{i\in M}\sum_{m\in N} p_{b,i,m} \le P_b, \tag{27}$$

$$\sum_{k\in K}\sum_{n\in N} p_{i,k,n} \le P_i, \quad i = 1,2,\ldots,M, \tag{28}$$

$$\mathbf{P} \ge 0, \tag{29}$$

where constraint (25) and (26) mean each subcarrier can be assigned at most one node (user or relay) at each time slot to avoid interference. (27) and (28) mean the peak power constraints of BS and relays which are converted from (4) and (5), respectively.

Problem **P1** is a mixed-integer nonlinear programming problem. Furthermore, **P1** is nonconvex, which is difficult to tackle in general. However, it has been shown by [13] that the duality gap is close to zero as the number of subcarriers goes to infinity. Therefore, the Lagrange dual domain method can be used to solve problem **P1**.

It is worth nothing that the logarithm function in (23) is not concave. To make it concave, we remove the "1" in the denominator. Then, the original logarithm function becomes:

$$R_{mnik} \approx \log\left(1 + \frac{g_{b,i,m}p_{b,i,m}g_{i,n,k}p_{i,n,k}}{g_{b,i,m}p_{b,i,m} + g_{i,n,k}p_{i,n,k}}\right). \tag{30}$$

It is clear that (30) is an upper bound of (23). The authors in [24] have shown that this upper bound is tight at moderate high SNR.

Next, we solve the problem **P1** using Lagrange dual domain method. The Lagrangian function of **P1** is:

$$\begin{aligned} L(\mathbf{U},\mathbf{P},\boldsymbol{\lambda}) &= \sum_{k=1}^{K}\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{i=1}^{M} u_{mnik}R_{mnik} \\ &\quad + \lambda_b\left(P_b - \sum_{r\in M}\sum_{m\in N} p_{b,i,m}\right) \\ &\quad + \sum_{i\in M}\lambda_i\left(P_i - \sum_{k\in K}\sum_{n\in N} p_{i,k,n}\right), \end{aligned} \tag{31}$$

where λ_b and λ_i, $i = 1,2,\ldots,M$ are the Lagrangian multipliers associated with the constraint (27) and (28).

The dual function of **P1** can be formulated as:

$$g(\boldsymbol{\lambda}) = \max_{\mathbf{U},\mathbf{P}} \quad L(\mathbf{U},\mathbf{P},\boldsymbol{\lambda}). \tag{32}$$

6.1. Optimizing **U** *and* **P** *with Given* $\boldsymbol{\lambda}$. In this subsection, we optimize **U** and **P** with given $\boldsymbol{\lambda}$ to compute the dual function.

In the first, we optimize **P** with fixed **U**. The optimal **P** can be readily obtained using KKT conditions as follows:

$$\begin{aligned} p_{b,i,m} &= \frac{1}{1+\sqrt{g_{b,i,m}/g_{i,k,n}}}\left[\frac{1}{\lambda_b} - \frac{\left(\sqrt{g_{b,i,m}}+\sqrt{g_{i,k,n}}\right)^2}{g_{b,i,m}g_{i,k,n}}\right]^+, \\ p_{i,k,n} &= \frac{1}{1+\sqrt{g_{r,i,n}/g_{b,i,m}}}\left[\frac{1}{\lambda_i} - \frac{\left(\sqrt{g_{b,i,m}}+\sqrt{g_{i,k,n}}\right)^2}{g_{b,i,m}g_{i,k,n}}\right]^+, \end{aligned} \tag{33}$$

where $[a]^+ = \max\{a, 0\}$.

In the previous section, we can obtain the optimal $\mathbf{U}^*$ with fixed power allocation at the BS and the power amplification at the relays using the proposed linear processing scheme. Substituting $\mathbf{U}^*$ into (24), we can obtaine the corresponding optimal **P**. Then, an iterative algorithm can be designed. The details will be shown in the following subsection.

6.2. Optimizing the Dual Function. In the aforementioned subsection, the dual function of the problem **P1** has been

Initialize the Lagrange multiplier $\boldsymbol{\lambda}^{(0)}$ and power allocation $\mathbf{P}^{(0)}$
repeat
set $t \leftarrow t+1$
For given $\boldsymbol{\lambda}^{(t)}$ and $\mathbf{P}^{(t)}$, obtain the optimal $\mathbf{U}^{(t)}(\boldsymbol{\lambda})$ using the proposed linear process scheme. Then, according to the $\mathbf{U}^{(t)}(\boldsymbol{\lambda})$, obtain $\mathbf{P}^{(\mathbf{t}+1)}(\boldsymbol{\lambda})$ using the Lagrange dual domain method
Update $\boldsymbol{\lambda}$ through $\boldsymbol{\lambda}^{t+1} = [\boldsymbol{\lambda}^t - \Delta\boldsymbol{\lambda}\nu^t]^+$, where ν^t is the step size at the tth iteration.
until The convergence of $\min_t g(\boldsymbol{\lambda}^t)$

Algorithm 1

computed. Now, we minimize the dual problem using subgradient method:

$$\begin{aligned} \min \quad & g(\boldsymbol{\lambda}) \\ \text{s.t.} \quad & \boldsymbol{\lambda} \geq 0. \end{aligned} \tag{34}$$

Any subgradient-based method can be adopted to minimize $g(\boldsymbol{\lambda})$. Then, a subgradient at the point $\boldsymbol{\lambda}$ can be given by

$$\begin{aligned} \Delta\lambda_b &= p_b - \sum_{r\in M}\sum_{m\in N} p_{b,r,m}, \\ \Delta\lambda_i &= p_i - \sum_{k\in K}\sum_{n\in N} p_{i,k,n}, \quad \forall i \in M. \end{aligned} \tag{35}$$

Now, we propose an iterative algorithm to optimize the variables $\mathbf{U}$ and $\mathbf{P}$. Firstly, Lagrangian multiplier $\boldsymbol{\lambda}$ and power allocation $\mathbf{P}$ are initialized, and then $\mathbf{U}(\boldsymbol{\lambda})$ can be obtained by the proposed linear processing scheme. In accordance with $\mathbf{U}(\boldsymbol{\lambda})$, we can obtain corresponding power allocation $\mathbf{P}(\boldsymbol{\lambda})$. If $\mathbf{P}(\boldsymbol{\lambda})$ satisfies constraints (27) and (28), the solution $\mathbf{U}(\boldsymbol{\lambda})$ and $\mathbf{P}(\boldsymbol{\lambda})$ are global optimal solution. otherwise, iterative algorithm would continue to refine the solution. The details of the iterative algorithm is given in Algorithm 1.

7. Simulation Results

In this section, we evaluate the performance of the proposed algorithm using simulations. We assume that the BS is located in the center of the cell, and all users are placed randomly, and the relays are uniformly distributed on the centered circle. Cost231-Hata model is adopted as the path loss model, and the small scale fading is modeled as Rayleigh fading. The simulation parameters are summarized in Table 1.

Table 1: Simulation Parameters.

simulation parameter	value
Cell radius	1 Km
Inner boundary radius	0.6 Km
Subcarrier bandwidth	15 KHz
Number of relays	4
Noise power spectral density	−174 dBm/Hz
Path loss coefficient	4
Peak power of BS	30 dBm
Peak power of relay	25 dBm

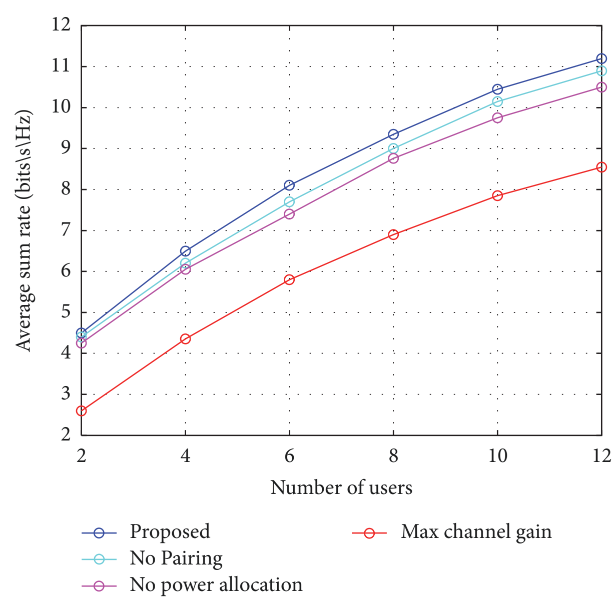

Figure 2: Operator's profits versus different number of users.

7.1. Comparing with Benchmarks. In this subsection, we compare the performance of the proposed iterative algorithm with that of some common benchmarks as follows:

7.1.1. No Pairing. In this scheme, the subcarrier pairing is not considered, that is, the subcarrier in the first hop is the same with that in the second hop. Thus, we should only optimize the channel assignment, relay selection and power allocation.

7.1.2. No PA. In this scheme, we assume each subcarrier is allocated uniform power. Therefore, the original problem becomes a combinatorial problem since the original problem only consists of the subcarrier pairing and relay selection.

7.1.3. Max Channel Gain. In this scheme, there is no subcarrier pairing and power allocation. That is, the subcarrier in the first hop and in the second hop are the same. The channel is assigned according to the channel gain over the second hop.

Figure 2 shows the system throughput versus the number of users. The number of subcarriers and relays are set to $N = 16$, $M = 4$, respectively. The number of users changes from 2 to 12. It can be seen from Figure 2 that the system

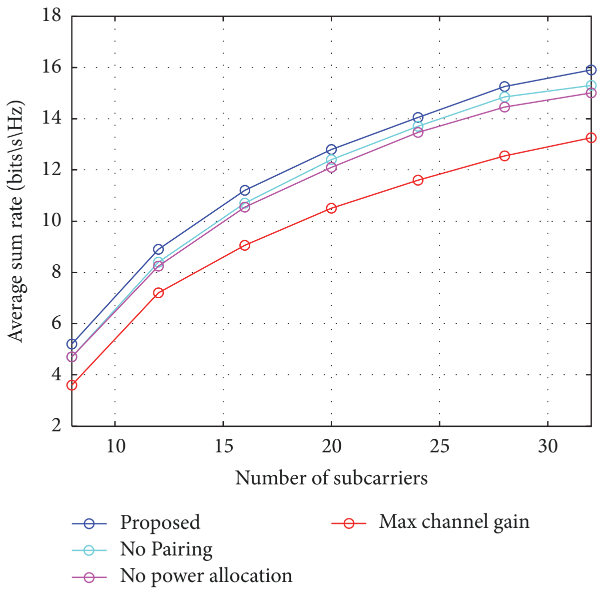

FIGURE 3: Operator's profits versus different number of users.

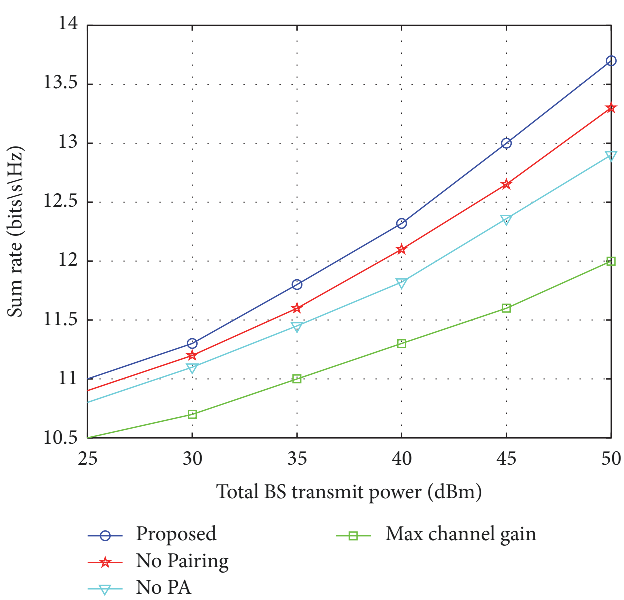

FIGURE 4: Operator's profits versus different number of users.

throughput increases with the increase of the number of users since the subcarriers have higher possibility to be allocated to the users who have better channel gain. Meanwhile, the proposed algorithm achieve 15 percent system throughput improvement than the *No Pairing* scheme.

Figure 3 illustrates the system throughput versus the number of subcarriers. It can be shown that the system throughput increases with the increase of the number of users. The number of subcarriers changes from 8 to 32. This result can be easily explained as that the system throughput is proportional to the system bandwidth.

Figure 4 depicts the system throughput versus the total transmit power. The number of subcarriers, relays and users

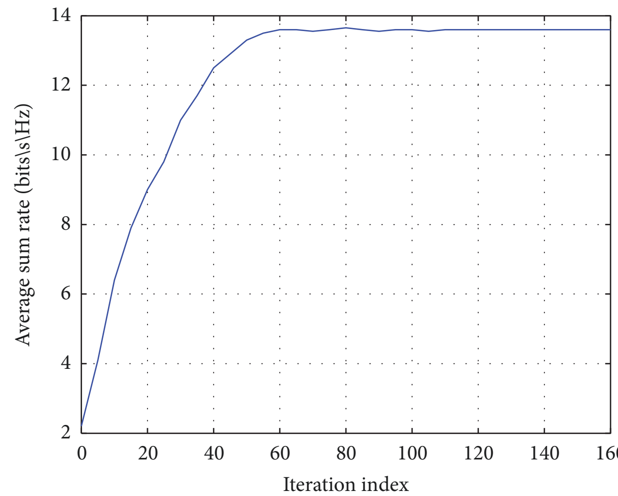

FIGURE 5: Operator's profits versus different number of users.

are set to $N = 16$, $M = 2$ and $K = 4$, respectively. Figure 3 shows that the proposed algorithm achieves more system throughput than these benchmark schemes. The result illustrates that it is necessary to jointly optimize the subcarrier pairing, relays selection and power allocation, and the proposed algorithm is effective.

7.2. Convergence Performance and System Capacity. In this subsection, we evaluate the convergence performance of the proposed algorithm and the system capacity versus the number of subcarriers.

Figure 5 shows the system throughput of the proposed algorithm versus the iteration indices. From Figure 5, it can be seen that, since the proposed linear processing scheme is polynomial time algorithm, the proposed algorithm have satisfactory convergence performance, and it converges within 50 iterations.

Figure 6 depicts the system throughput of the proposed algorithm versus the number of subcarriers with different total transmit power. The result shows that the system throughput can be improved significantly with the increase of the system bandwidth and SNR.

8. Conclusion

This paper investigates the joint resource allocation problem in multiple relays multiple users MIMO AF relaying network. The joint resource allocation problem involves subcarrier pairing, relay selection, channel assignment and power allocation which is difficult to tackle due to the combinatorial nature of the subcarrier pairing, relay selection and channel assignment. The subcarrier pairing, relay selection and channel assignment can be regarded as a linear processing of the relays which can be represented by a permutation matrix. We exploit the linear processing scheme for the scenario with multiple relays multiple users, and then propose an optimal linear processing design with fixed power gain to find the optimal permutation matrix which corresponds to

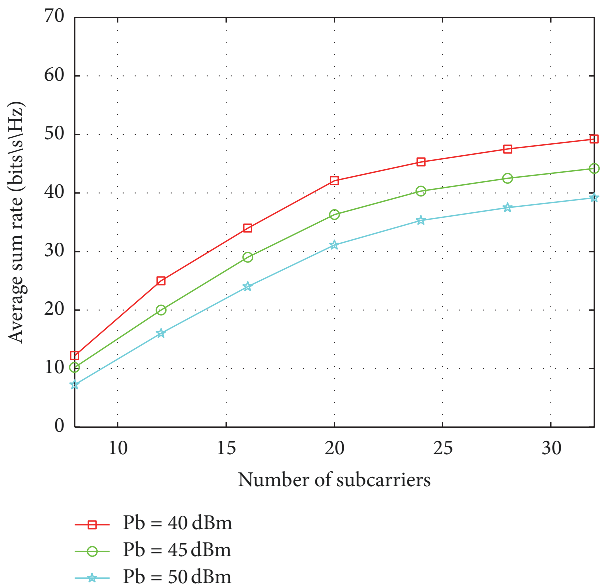

FIGURE 6: Operator's profits versus different number of users.

the optimal subcarrier pairing, relay selection and channel assignment. Through this linear processing design, we can obtain the optimal subcarrier pairing, relay selection and channel assignment with fixed power allocation. Then, the Lagrange dual domain method is adopted to solve the power allocation. Finally, an iterative algorithm is proposed to solve the combinatorial optimization problem and power allocation alternately.

The massive MIMO technology is one of the key topics of 5G wireless networks [28]. An issue with this technique is to eliminate the pilot population. In the future works, we will pay more attention to the resource allocation and pilot population elimination for the relay-enhanced massive MIMO system.

Appendix

Proof. In accordance with the property of the determinant $|AB| = |A||B|$, the objective function (9) can be rewritten as:

$$\begin{aligned}&\frac{1}{2}\log\left|I + (PUQ)^{\dagger}PUQ\right| \\ &\quad = \left|Q^{\dagger}\right|\left|\left(Q^{\dagger}Q\right)^{-1} + U^{\dagger}P^{\dagger}UP\right||Q| \\ &\quad = \left|Q^{\dagger}Q\right|\left|\left(Q^{\dagger}Q\right)^{-1} + U^{\dagger}P^{\dagger}UP\right|.\end{aligned} \tag{A.1}$$

Since $Q^{\dagger}Q$ is not a function of U, we only need to consider the second determinant:

$$\max_{U} \quad \left|\left(Q^{\dagger}Q\right)^{-1} + U^{\dagger}P^{\dagger}UP\right|. \tag{A.2}$$

By the property of determinant

$$|A + B| \leq \prod_{n=1}^{N}\left(\lambda_n(A) + \lambda_{N+1-n}(B)\right), \tag{A.3}$$

where $\lambda_n(A)$ and $\lambda_n(B)$ are the eigenvalues of A and B, respectively, sorted in ascending order, the equality is reached when A and B are both diagonal with the diagonal entries being inverse-order matched. Then, we have

$$\left|\left(Q^{\dagger}Q\right)^{-1} + U^{\dagger}P^{\dagger}UP\right| \leq \prod_{n=1}^{N}\left(\frac{1}{\left|q_{(n)}\right|^{2}} + \left|p_{N+1-n}\right|^{2}\right). \tag{A.4}$$

The equality is reached when U^{*} is the permutation matrix such that the entries of the ordered sequences $\{|p_{(i)}|\}$ and $\{|q_{(i)}|\}$ are one-to-one matched. □

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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