Research Article

Strong Convergence Theorems for Variational Inequalities and Split Equality Problem

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Let H_1, H_2 , and H_3 be real Hilbert spaces, let $C \subseteq H_1, Q \subseteq H_2$ be two nonempty closed convex sets, and let $A : H_1 \rightarrow H_3, B : H_2 \rightarrow H_3$ be two bounded linear operators. The split equality problem (SEP) is to find $x \in C, y \in Q$ such that $Ax = By$. Let $H = H_1 \times H_2$; consider $f : H \rightarrow H$ a contraction with coefficient $0 < \alpha < 1$, a strongly positive linear bounded operator $T : H \rightarrow H$ with coefficient $\bar{\gamma} > 0$, and $M : H \rightarrow H$ is a β -inverse strongly monotone mapping. Let $0 < \gamma < \bar{\gamma}/\alpha, S = C \times Q$ and $G : H \rightarrow H_3$ be defined by restricting to H_1 is A and restricting to H_2 is $-B$, that is, G has the matrix form $G = [A, -B]$. It is proved that the sequence $\{w_n\} = \{(x_n, y_n)\} \subseteq H$ generated by the iterative method $w_{n+1} = P_S[\alpha_n \gamma f(w_n) + (I - \alpha_n T)P_S(I - \gamma_n G^* G)P_S(w_n - \lambda_n M w_n)]$ converges strongly to $\bar{w}$ which solves the SEP and the following variational inequality: $\langle (T - \lambda f)\bar{w}, w - \bar{w} \rangle \geq 0$ and $\langle M\bar{w}, w - \bar{w} \rangle \geq 0$ for all $w \in S$. Moreover, if we take $M = G^* G : H \rightarrow H, \gamma_n = 0$, then M is a β -inverse strongly monotone mapping, and the sequence $\{w_n\}$ generated by the iterative method $w_{n+1} = \alpha_n \gamma f(w_n) + (I - \alpha_n T)P_S(w_n - \lambda_n G^* G w_n)$ converges strongly to $\bar{w}$ which solves the SEP and the following variational inequality: $\langle (T - \lambda f)\bar{w}, w - \bar{w} \rangle \geq 0$ for all $w \in S$.

1. Introduction and Preliminaries

Throughout this paper, let H be a real Hilbert space and S a nonempty closed convex subset of H , and let I denote the identity operator on Hilbert space H , $\text{Fix}(T)$ the set of the fixed points of an operator T . An operator T on a Hilbert space H is nonexpansive if, for each x and y in H , $\|Tx - Ty\| \leq \|x - y\|$. T is said to be strongly positive, if there exists $\bar{\gamma} > 0$ such that $\langle Tx, x \rangle \geq \bar{\gamma}\|x\|^2$ for all $x \in H$. T is said to be inverse strongly monotone if there exists $\beta > 0$ such that $\langle x - y, Tx - Ty \rangle \geq \beta\|Tx - Ty\|^2$ for all $x, y \in H$. A bounded linear operator T on a Hilbert space H is positive operator if $T^* = T$ and $\lambda \geq 0$ for all λ in the spectrum of T or equivalent to $\langle Tx, x \rangle \geq 0$ for all $x \in H$.

Let P_S denote the projection from H onto a nonempty closed convex subset S of H ; that is,

$$P_S(w) = \min_{x \in S} \|x - w\|. \quad (1)$$

It is well known that $P_S(w)$ is characterized by the inequality

$$\langle w - P_S(w), x - P_S(w) \rangle \leq 0, \quad \forall x \in S, \quad (2)$$

and P_S is nonexpansive.

Let C and Q be nonempty closed convex subsets of real Hilbert spaces H_1 and H_2 , respectively, and let $A : H_1 \rightarrow H_2$ be a bounded linear operator. The split feasibility problem (SFP) is to find a point x satisfying the property

$$x \in C, \quad Ax \in Q. \quad (3)$$

The SFP was first introduced by Censor and Elfving [1], which attracts many authors' attention due to its application in signal processing [1]. Various algorithms have been invented to solve it (see [2–7]).

Recently, Moudafi [8] proposes a new split equality problem (SEP): let H_1, H_2 , and H_3 be real Hilbert spaces, let $C \subseteq H_1, Q \subseteq H_2$ be two nonempty closed convex sets, and

let $A : H_1 \rightarrow H_3, B : H_2 \rightarrow H_3$ be two bounded linear operators. Find $x \in C, y \in Q$ satisfying

$$Ax = By. \quad (4)$$

When $B = I$, SEP reduces to the well-known SFP. In the paper [8], Moudafi gives the alternating CQ-algorithm and relaxed alternating CQ-algorithm iterative algorithm to solve the split equality problem. We must point out that the above algorithms converge weakly to a solution of the SEP.

We use $\Gamma = \{(x, y) \in H_1 \times H_2, Ax = By, x \in C, y \in Q\}$ to denote the solution set of SEP, and letting $S = C \times Q$ in $H = H_1 \times H_2$ define $G : H \rightarrow H_3$ by $G = [A, -B]$, then $G^*G : H \rightarrow H$ has the matrix form

$$G^*G = \begin{bmatrix} A^*A & -A^*B \\ -B^*A & B^*B \end{bmatrix}. \quad (5)$$

The original problem can now be reformulated as finding $w = (x, y) \in S$ with $Gw = 0$ or, more generally, minimizing the function $\|Gw\|$ over $w \in S$. Therefore solving SEP (4) is equivalent to solving the following minimization problem:

$$\min_{w \in S} f(w) = \frac{1}{2} \|Gw\|^2. \quad (6)$$

In the paper [9], we use the well-known Tychonov regularization to get some algorithms converge strongly to the minimum-norm solution of the SEP. Moreover, in the paper [9], we obtain that, for $0 < \gamma < 2/\|G\|^2$, $\text{Fix}(P_S(I - \gamma G^*G)) = \text{Fix}(P_S) \cap \text{Fix}(I - \gamma G^*G) = \Gamma$. That is to say, the set of the solution of SEP is equal to the fixed points set of the nonexpansive mapping $P_S(I - \gamma G^*G)$.

Recall that the variational inequality problem is to find $p \in H$ such that $\langle Mp, x - p \rangle \geq 0$, for all $x \in S$, where S is closed, convex, and nonempty, and M is a mapping on H . The set of solutions of the variational inequality is denoted by $\text{VI}(S, M)$. In the paper [9], we also get that $w \in \text{Fix}(P_S(I - \gamma G^*G))$ if and only if w is a solution of the variational inequality $\langle G^*Gw, v - w \rangle \geq 0$, for all $v \in S$.

In this paper, we consider a contraction f on $H = H_1 \times H_2$ with coefficient $0 < \alpha < 1$, a strongly positive linear bounded operator T with coefficient $\bar{\gamma} > 0$, and M is a β -inverse strongly monotone mapping. Let $0 < \gamma < \bar{\gamma}/\alpha$, $S = C \times Q$. We prove that the sequence $\{w_n\} = \{(x_n, y_n)\} \subseteq H$ generated by the iterative method $w_{n+1} = P_S[\alpha_n \gamma f(w_n) + (I - \alpha_n T)P_S(I - \gamma_n G^*G)P_S(w_n - \lambda_n M w_n)]$ converges strongly to $\bar{w}$ which solves the SEP and the following variational inequality: $\langle (A - \lambda f)\bar{w}, w - \bar{w} \rangle \geq 0$ and $\langle M\bar{w}, w - \bar{w} \rangle \geq 0$ for all $w \in S$ under some mild conditions for α_n, γ_n , and λ_n .

We now collect some elementary facts which will be used in the proofs of our main results.

Lemma 1. Assuming T is a positive operator, then T is $1/\|T\|$ -inverse strongly monotone.

Proof. Since T is a positive operator on H , by the functional calculus of positive operator (for more details see [10, Chapter 4]), there exists positive operator T_1 such that $T = T_1 T_1$ and $\|T_1\| = \|T\|^{1/2}$.

Hence, for any $x, y \in H$, $\|Tx - Ty\|^2 = \|T_1 T_1(x - y)\|^2 \leq \|T_1\|^2 \|T_1(x - y)\|^2 = \|T\| \langle T_1(x - y), T_1(x - y) \rangle = \|T\| \langle x - y, T_1 T_1(x - y) \rangle = \|T\| \langle x - y, Tx - Ty \rangle$. It follows that $\langle x - y, Tx - Ty \rangle \geq (1/\|T\|) \|Tx - Ty\|^2$. That is to say, T is $1/\|T\|$ -inverse strongly monotone. $\square$

Lemma 2 (see [11, 12]). Let X be a Banach space, C a closed convex subset of X , and $T : C \rightarrow C$ a nonexpansive mapping with $\text{Fix}(T) \neq \emptyset$. If $\{x_n\}$ is a sequence in C weakly converging to x and if $\{(I - T)x_n\}$ converges strongly to y , then $(I - T)x = y$.

Lemma 3 (see [13]). Let $\{s_n\}$ be a sequence of nonnegative real numbers, $\{\alpha_n\}$ a sequence of real numbers in $[0, 1]$ with $\sum_{n=1}^{\infty} \alpha_n = \infty$, $\{u_n\}$ a sequence of nonnegative real numbers with $\sum_{n=1}^{\infty} u_n < \infty$, and $\{t_n\}$ a sequence of real numbers with $\limsup_{n \rightarrow \infty} t_n \leq 0$. Suppose that

$$s_{n+1} = (1 - \alpha_n) s_n + \alpha_n t_n + u_n, \quad \forall n \in \mathbb{N}. \quad (7)$$

Then $\lim_{n \rightarrow \infty} s_n = 0$.

Lemma 4 (see [14]). Assuming T is a strongly positive linear bounded operator on a Hilbert space H with coefficient $\bar{\gamma} > 0$, then $\|I - \rho T\| \leq 1 - \rho \bar{\gamma}$ for $0 < \rho \leq \|T\|^{-1}$.

Lemma 5 (see [13]). Let S be a nonempty and closed subset of a Banach space X , and let $\{R_n\}$ be a family of mappings of S into itself which satisfies the condition: for each bounded subset E of S ,

$$\sum_{n=1}^{\infty} \sup \{\|R_{n+1}w - R_n w\|, w \in E\} < \infty. \quad (8)$$

Then, for each $w \in S$, $\{R_n w\}$ converges strongly to a point in S . Moreover, R is defined by

$$Rw = \lim_{n \rightarrow \infty} R_n w, \quad w \in S. \quad (9)$$

Then, for each bounded subset E of S ,

$$\limsup_{n \rightarrow \infty} \{\|Rw - R_n w\|, w \in E\} = 0, \quad (10)$$

and $\text{Fix}(R) = \cap \text{Fix}(R_n)$, if $\cap \text{Fix}(R_n) \neq \emptyset$.

2. Main Results

Theorem 6. Let Γ denote the solutions set of the SEP, $S = C \times Q$, f a contraction on $H = H_1 \times H_2$ with coefficient $0 < \alpha < 1$, a strongly positive linear bounded operator T with coefficient $\bar{\gamma} > 0$, and M a β -inverse strongly monotone mapping. Assume that $0 < \gamma < \bar{\gamma}/\alpha$, $\Gamma \cap \text{VI}(S, M) \neq \emptyset$, and $\{w_n\} = \{(x_n, y_n)\} \subseteq H$ is the sequence generated by the following algorithm: $w_0 \in H$,

$$w_{n+1} = P_S [\alpha_n \gamma f(w_n) + (I - \alpha_n T) P_S (I - \gamma_n G^* G) P_S (w_n - \lambda_n M w_n)] \quad (11)$$

for all $n = 0, 1, 2, \dots$, where $\alpha_n \in (0, 1)$, $\lambda_n \in (0, 2\beta)$, and $\gamma_n \in (0, 2/\|G\|^2)$. If α_n , λ_n , and γ_n are chosen such that $\lambda_n \in [a, b]$ for some a, b with $0 < a < b < 2\beta$ and

- (i) $\lim_n \alpha_n = 0$,
- (ii) $\sum_{n=0}^{\infty} \alpha_n = \infty$,
- (iii) $\sum_{n=0}^{\infty} |\alpha_{n+1} - \alpha_n| < \infty$,
- (v) $\sum_{n=0}^{\infty} |\lambda_{n+1} - \lambda_n| < \infty$,
- (iv) $\sum_{n=0}^{\infty} |\gamma_{n+1} - \gamma_n| < \infty$,

then the sequence $\{w_n\}$ converges strongly to $\tilde{w}$ which solves the SEP and the following variational inequality: $\langle (T - \lambda f)\tilde{w}, w - \tilde{w} \rangle \geq 0$ and $\langle M\tilde{w}, w - \tilde{w} \rangle \geq 0$ for all $w \in S$.

Proof. First, we show that the sequence $\{w_n\}$ is bounded. Consider the mapping $I - \lambda_n M$. Since M is a β -inverse strongly monotone mapping, we have that, for all $w, v \in S$,

$$\begin{aligned} & \|(I - \lambda_n M)w - (I - \lambda_n M)v\|^2 \\ &= \|(w - v) - \lambda_n(Mw - Mv)\|^2 \\ &= \|w - v\|^2 - 2\lambda_n \langle w - v, Mw - Mv \rangle \\ & \quad + \lambda_n^2 \|Mw - Mv\|^2 \\ &\leq \|w - v\|^2 + \lambda_n (\lambda_n - 2\beta) \|Mw - Mv\|^2. \end{aligned} \quad (12)$$

Since $0 < \lambda_n < 2\beta$, we deduce that $\|(I - \lambda_n M)w - (I - \lambda_n M)v\|^2 \leq \|w - v\|^2$. That is to say, $I - \lambda_n M$ is nonexpansive.

Letting $v_n = P_S(w_n - \lambda_n Mw_n)$, $R_n = P_S(I - \gamma_n G^* G)$, choosing $u \in \Gamma \cap \text{VI}(S, M)$, then $u = P_S(I - \gamma G^* G)u$ and $u = P_S(I - \lambda_n M)u$. P_S is nonexpansive, which implies that

$$\begin{aligned} \|v_n - u\| &= \|P_S(w_n - \lambda_n Mw_n) - P_S(u - \lambda_n Mu)\| \\ &\leq \|(w_n - \lambda_n Mw_n) - (u - \lambda_n Mu)\| \\ &= \|(I - \lambda_n M)w_n - (I - \lambda_n M)u\| \\ &\leq \|w_n - u\|, \end{aligned}$$

$$\begin{aligned} \|w_{n+1} - u\| &= \|P_S[\alpha_n \gamma f(w_n) + (I - \alpha_n T)R_n v_n] - u\| \\ &\leq \|\alpha_n \gamma f(w_n) + (I - \alpha_n T)R_n v_n - u\| \\ &= \|\alpha_n (\gamma f(w_n) - Tu) + (I - \alpha_n T)(R_n v_n - u)\|. \end{aligned} \quad (13)$$

Since T is strongly positive linear bounded operator, R_n is nonexpansive, and f is contraction, we have

$$\begin{aligned} \|w_{n+1} - u\| &\leq \alpha_n \|\gamma f(w_n) - Tu\| + (1 - \alpha_n \bar{\gamma}) \|R_n v_n - u\| \\ &\leq \alpha_n \|\gamma f(w_n) - \gamma f(u)\| + \alpha_n \|\gamma f(u) - Tu\| \\ & \quad + (1 - \alpha_n \bar{\gamma}) \|R_n v_n - u\| \\ &\leq \alpha \gamma \alpha_n \|w_n - u\| + \alpha_n \|\gamma f(u) - Tu\| \\ & \quad + (1 - \alpha_n \bar{\gamma}) \|R_n v_n - u\| \end{aligned}$$

$$\begin{aligned} &= \alpha \gamma \alpha_n \|w_n - u\| + \alpha_n \|\gamma f(u) - Tu\| \\ & \quad + (1 - \alpha_n \bar{\gamma}) \|R_n v_n - R_n u\| \\ &\leq \alpha \gamma \alpha_n \|w_n - u\| + \alpha_n \|\gamma f(u) - Tu\| \\ & \quad + (1 - \alpha_n \bar{\gamma}) \|v_n - u\| \\ &\leq \alpha \gamma \alpha_n \|w_n - u\| + \alpha_n \|\gamma f(u) - Tu\| \\ & \quad + (1 - \alpha_n \bar{\gamma}) \|w_n - u\| \\ &= (1 - \alpha(\bar{\gamma} - \gamma \alpha_n)) \|w_n - u\| \\ & \quad + \alpha_n \|\gamma f(u) - Tu\| \\ &\leq \max \left\{ \|w_n - u\|, \frac{1}{\bar{\gamma} - \gamma \alpha_n} \|\gamma f(u) - Tu\| \right\}. \end{aligned} \quad (14)$$

It follows from induction that

$$\|w_n - u\| \leq \max \left\{ \|w_0 - u\|, \frac{1}{\bar{\gamma} - \gamma \alpha_n} \|\gamma f(u) - Tu\| \right\}, \quad (15)$$

$n > 0$.

Hence, $\{w_n\}$ is bounded, so are $\{v_n\}$, $\{R_n v_n\}$, $\{Mw_n\}$, and $\{f(w_n)\}$.

Next, we show that $\|w_{n+1} - w_n\| \rightarrow 0$ and $\|v_n - R_n v_n\| \rightarrow 0$. In fact, in view that P_S and $I - \lambda_n M$ are nonexpansive, we can get that

$$\begin{aligned} \|v_{n+1} - v_n\| &= \|P_S(w_{n+1} - \lambda_{n+1} Mw_{n+1}) - P_S(w_n - \lambda_n Mw_n)\| \\ &\leq \|(w_{n+1} - \lambda_{n+1} Mw_{n+1}) - (w_n - \lambda_n Mw_n)\| \\ &\leq \|(w_{n+1} - \lambda_{n+1} Mw_{n+1}) - (w_n - \lambda_{n+1} Mw_n)\| \\ & \quad + |\lambda_{n+1} - \lambda_n| \|Mw_n\| \\ &\leq \|w_{n+1} - w_n\| + |\lambda_{n+1} - \lambda_n| \|Mw_n\|. \end{aligned} \quad (16)$$

Hence,

$$\begin{aligned} \|w_{n+1} - w_n\| &= \|P_S[\alpha_n \gamma f(w_n) + (I - \alpha_n T)R_n v_n] \\ & \quad - P_S[\alpha_{n-1} \gamma f(w_{n-1}) + (I - \alpha_{n-1} T)R_{n-1} v_{n-1}]\| \\ &\leq \|[\alpha_n \gamma f(w_n) + (I - \alpha_n T)R_n v_n] \\ & \quad - [\alpha_{n-1} \gamma f(w_{n-1}) + (I - \alpha_{n-1} T)R_{n-1} v_{n-1}]\| \\ &= \|\alpha_n \gamma (f(w_n) - f(w_{n-1})) + \gamma (\alpha_n - \alpha_{n-1}) \\ & \quad \times f(w_{n-1}) + (I - \alpha_n T)(R_n v_n - R_{n-1} v_{n-1}) \\ & \quad + (\alpha_n - \alpha_{n-1}) T R_{n-1} v_{n-1}\| \\ &\leq \alpha_n \alpha \gamma \|w_n - w_{n-1}\| + \gamma |\alpha_n - \alpha_{n-1}| \|f(w_{n-1})\| \\ & \quad + (1 - \alpha_n \bar{\gamma}) \|R_n v_n - R_{n-1} v_{n-1}\| \\ & \quad + |\alpha_n - \alpha_{n-1}| \|T R_{n-1} v_{n-1}\| \end{aligned}$$

$$\begin{aligned}
& \leq \alpha_n \alpha \gamma \|w_n - w_{n-1}\| + \gamma |\alpha_n - \alpha_{n-1}| \|f(w_{n-1})\| \\
& \quad + (1 - \alpha_n \bar{\gamma}) \|R_n v_n - R_n v_{n-1}\| \\
& \quad + (1 - \alpha_n \bar{\gamma}) \|R_n v_{n-1} - R_{n-1} v_{n-1}\| \\
& \quad + |\alpha_n - \alpha_{n-1}| \|TR_{n-1} v_{n-1}\| \\
& \leq \alpha_n \alpha \gamma \|w_n - w_{n-1}\| + \gamma |\alpha_n - \alpha_{n-1}| \|f(w_{n-1})\| \\
& \quad + (1 - \alpha_n \bar{\gamma}) \|v_n - v_{n-1}\| + (1 - \alpha_n \bar{\gamma}) \\
& \quad \times \|R_n v_{n-1} - R_{n-1} v_{n-1}\| \\
& \quad + |\alpha_n - \alpha_{n-1}| \|TR_{n-1} v_{n-1}\| \\
& \leq \alpha_n \alpha \gamma \|w_n - w_{n-1}\| + \gamma |\alpha_n - \alpha_{n-1}| \|f(w_{n-1})\| \\
& \quad + (1 - \alpha_n \bar{\gamma}) \|w_n - w_{n-1}\| \\
& \quad + (1 - \alpha_n \bar{\gamma}) |\lambda_n - \lambda_{n-1}| \\
& \quad \times \|Mw_{n-1}\| + (1 - \alpha_n \bar{\gamma}) \|R_n v_{n-1} - R_{n-1} v_{n-1}\| \\
& \quad + |\alpha_n - \alpha_{n-1}| \|TR_{n-1} v_{n-1}\| \\
& = (1 - \alpha_n (\bar{\gamma} - \alpha \gamma)) \|w_n - w_{n-1}\| \\
& \quad + \gamma |\alpha_n - \alpha_{n-1}| \|f(w_{n-1})\| + (1 - \alpha_n \bar{\gamma}) \\
& \quad \times |\lambda_n - \lambda_{n-1}| \|Mw_{n-1}\| + (1 - \alpha_n \bar{\gamma}) \\
& \quad \times \|R_n v_{n-1} - R_{n-1} v_{n-1}\| \\
& \quad + |\alpha_n - \alpha_{n-1}| \|TR_{n-1} v_{n-1}\|. \tag{17}
\end{aligned}$$

Based on

$$\begin{aligned}
& \|R_n v_{n-1} - R_{n-1} v_{n-1}\| \\
& = \|P_S (I - \gamma_n G^* G) v_{n-1} - P_S (I - \gamma_{n-1} G^* G) v_{n-1}\| \\
& \leq \|(I - \gamma_n G^* G) v_{n-1} - (I - \gamma_{n-1} G^* G) v_{n-1}\| \\
& \leq |\gamma_n - \gamma_{n-1}| \|G\|^2 \|v_{n-1}\|, \tag{18}
\end{aligned}$$

we deduce that

$$\begin{aligned}
\|w_{n+1} - w_n\| & \leq (1 - \alpha_n (\bar{\gamma} - \alpha \gamma)) \|w_n - w_{n-1}\| \\
& \quad + \gamma |\alpha_n - \alpha_{n-1}| \|f(w_{n-1})\| \\
& \quad + (1 - \alpha_n \bar{\gamma}) |\lambda_n - \lambda_{n-1}| \|Mw_{n-1}\| \\
& \quad + (1 - \alpha_n \bar{\gamma}) |\gamma_n - \gamma_{n-1}| \|G\|^2 \|v_{n-1}\| \\
& \quad + |\alpha_n - \alpha_{n-1}| \|TR_{n-1} v_{n-1}\|. \tag{19}
\end{aligned}$$

By Lemma 3, we have

$$\|w_{n+1} - w_n\| \longrightarrow 0. \tag{20}$$

On the other hand, since $u = P_S(u - \lambda_n M u)$ and $u = R_n u$ we have

$$\begin{aligned}
\|w_{n+1} - u\|^2 & = \|P_S [\alpha_n \gamma f(w_n) + (I - \alpha_n T) R_n v_n] - P_S(u)\|^2 \\
& \leq \|\alpha_n \gamma f(w_n) + (I - \alpha_n T) R_n v_n - u\|^2 \\
& = \|\alpha_n (\gamma f(w_n) - T u) + (I - \alpha_n T) (R_n v_n - u)\|^2 \\
& \leq (\|\alpha_n (\gamma f(w_n) - T u)\| \\
& \quad + \|(I - \alpha_n T) (R_n v_n - u)\|)^2 \\
& \leq \alpha_n \|\gamma f(w_n) - T u\|^2 + \|(I - \alpha_n \bar{\gamma}) R_n v_n - u\|^2 \\
& \quad + 2\alpha_n (I - \alpha_n \bar{\gamma}) \|\gamma f(w_n) - T u\| \|R_n v_n - u\| \\
& \leq \alpha_n \|\gamma f(w_n) - T u\|^2 + (I - \alpha_n \bar{\gamma}) \|v_n - u\|^2 \\
& \quad + 2\alpha_n (I - \alpha_n \bar{\gamma}) \|\gamma f(w_n) - T u\| \|v_n - u\| \\
& \leq \alpha_n \|\gamma f(w_n) - T u\|^2 + (I - \alpha_n \bar{\gamma}) \\
& \quad \times \|(I - \lambda_n M) w_n - (I - \lambda_n M) u\|^2 \\
& \quad + 2\alpha_n (I - \alpha_n \bar{\gamma}) \|\gamma f(w_n) - T u\| \|v_n - u\| \\
& \leq \alpha_n \|\gamma f(w_n) - T u\|^2 + (I - \alpha_n \bar{\gamma}) \\
& \quad \times (\|w_n - u\|^2 - 2\lambda_n \langle w_n - u, Mw_n - Mu \rangle \\
& \quad + \lambda_n^2 \|Mw_n - Mu\|^2) + 2\alpha_n (I - \alpha_n \bar{\gamma}) \\
& \quad \times \|\gamma f(w_n) - T u\| \|v_n - u\| \\
& \leq \alpha_n \|\gamma f(w_n) - T u\|^2 \\
& \quad + (I - \alpha_n \bar{\gamma}) (\|w_n - u\|^2 - 2\lambda_n \beta \|Mw_n - Mu\|^2 \\
& \quad + \lambda_n^2 \|Mw_n - Mu\|) \\
& \quad + 2\alpha_n (I - \alpha_n \bar{\gamma}) \|\gamma f(w_n) - T u\| \|v_n - u\| \\
& \leq \alpha_n \|\gamma f(w_n) - T u\|^2 + (I - \alpha_n \bar{\gamma}) \\
& \quad \times (\|w_n - u\|^2 + \lambda_n (\lambda_n - 2\beta) \|Mw_n - Mu\|^2) \\
& \quad + 2\alpha_n (I - \alpha_n \bar{\gamma}) \|\gamma f(w_n) - T u\| \|v_n - u\| \\
& \leq \alpha_n \|\gamma f(w_n) - T u\|^2 + \|w_n - u\|^2 \\
& \quad + (I - \alpha_n \bar{\gamma}) b (b - 2\beta) \|Mw_n - Mu\|^2 \\
& \quad + 2\alpha_n (I - \alpha_n \bar{\gamma}) \|\gamma f(w_n) - T u\| \|v_n - u\|. \tag{21}
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
& (I - \alpha_n \bar{\gamma}) b (2\beta - b) \|Mw_n - Mu\|^2 \\
& \leq \alpha_n \|\gamma f(w_n) - T u\|^2 + \|w_n - u\|^2 - \|w_{n+1} - u\|^2 \\
& \quad + 2\alpha_n (I - \alpha_n \bar{\gamma}) \|\gamma f(w_n) - T u\| \|v_n - u\|
\end{aligned}$$

$$\begin{aligned}
&= \alpha_n \|\gamma f(w_n) - Tu\|^2 + 2\alpha_n (I - \alpha_n \bar{\gamma}) \\
&\quad \times \|\gamma f(w_n) - Tu\| \|v_n - u\| + (\|w_n - u\| - \|w_{n+1} - u\|) \\
&\quad \times (\|w_n - u\| - \|w_{n+1} - u\|) \\
&\leq \alpha_n \|\gamma f(w_n) - Tu\|^2 \\
&\quad + 2\alpha_n (I - \alpha_n \bar{\gamma}) \|\gamma f(w_n) - Tu\| \|v_n - u\| \\
&\quad + \|w_n - w_{n+1}\| (\|w_n - u\| - \|w_{n+1} - u\|). \tag{22}
\end{aligned}$$

Since $\alpha_n \rightarrow 0$, $\|w_n - w_{n+1}\| \rightarrow 0$, we can get that

$$\|Mw_n - Mu\| \rightarrow 0. \tag{23}$$

Furthermore, by the property of the projection P_S , we have

$$\begin{aligned}
\|v_n - u\| &= \|P_S(I - \lambda_n M)w_n - P_S(I - \lambda_n M)u\| \\
&\leq \langle (w_n - \lambda_n Mw_n) - (u - \lambda_n Mu), v_n - u \rangle \\
&= \frac{1}{2} (\|(w_n - \lambda_n Mw_n) - (u - \lambda_n Mu)\|^2 + \|v_n - u\|^2 \\
&\quad - \|(w_n - \lambda_n Mw_n) - (u - \lambda_n Mu) - (v_n - u)\|^2) \\
&\leq \frac{1}{2} (\|w_n - u\|^2 + \|v_n - u\|^2 \\
&\quad - \|(w_n - v_n) - \lambda_n (Mw_n - Mu)\|^2) \\
&\leq \|w_n - u\|^2 - \|w_n - v_n\|^2 \\
&\quad + 2\lambda_n \langle w_n - v_n, Mw_n - Mu \rangle - \lambda_n^2 \|Mw_n - Mu\|^2. \tag{24}
\end{aligned}$$

Then, we have

$$\begin{aligned}
\|w_{n+1} - u\|^2 &= \|P_S[\alpha_n \gamma f(w_n) + (I - \alpha_n T)R_n v_n] - P_S(u)\|^2 \\
&\leq \alpha_n \|\gamma f(w_n) - Tu\|^2 + (1 - \alpha_n \bar{\gamma}) \|v_n - u\|^2 \\
&\quad + 2\alpha_n (I - \alpha_n \bar{\gamma}) \|\gamma f(w_n) - Tu\| \|v_n - u\| \\
&\leq \alpha_n \|\gamma f(w_n) - Tu\|^2 + (1 - \alpha_n \bar{\gamma}) \|w_n - u\|^2 \\
&\quad - (1 - \alpha_n \bar{\gamma}) \|w_n - v_n\|^2 + 2(1 - \alpha_n \bar{\gamma}) \\
&\quad \times \lambda_n \langle w_n - v_n, Mw_n - Mu \rangle - (1 - \alpha_n \bar{\gamma}) \\
&\quad \times \lambda_n^2 \|Mw_n - Mu\|^2 + 2\alpha_n (I - \alpha_n \bar{\gamma}) \\
&\quad \times \|\gamma f(w_n) - Tu\| \|v_n - u\|, \tag{25}
\end{aligned}$$

which yields that

$$\begin{aligned}
&(1 - \alpha_n \bar{\gamma}) \|w_n - v_n\|^2 \\
&\leq \alpha_n \|\gamma f(w_n) - Tu\|^2 \\
&\quad + (\|w_n - u\| + \|w_{n+1} - u\|) \|w_n - w_{n+1}\| \\
&\quad + 2(1 - \alpha_n \bar{\gamma}) \lambda_n \langle w_n - v_n, Mw_n - Mu \rangle \\
&\quad - (1 - \alpha_n \bar{\gamma}) \lambda_n^2 \|Mw_n - Mu\|^2 \\
&\quad + 2\alpha_n (I - \alpha_n \bar{\gamma}) \|\gamma f(w_n) - Tu\| \|v_n - u\|. \tag{26}
\end{aligned}$$

Since $\alpha_n \rightarrow 0$, $\|w_n - w_{n+1}\| \rightarrow 0$, and $\|Mw_n - Mu\| \rightarrow 0$, we can get that

$$\|w_n - v_n\| \rightarrow 0. \tag{27}$$

Next, since $\alpha_n \rightarrow 0$ we can get that

$$\begin{aligned}
&\|w_{n+1} - R_n v_n\| \\
&= \|P_S[\alpha_n \gamma f(w_n) + (I - \alpha_n T)R_n v_n] - P_S(R_n v_n)\| \\
&\leq \|\alpha_n \gamma f(w_n) + (I - \alpha_n T)R_n v_n - R_n v_n\| \\
&= \alpha_n \|\gamma f(w_n) - TR_n v_n\| \rightarrow 0. \tag{28}
\end{aligned}$$

Hence,

$$\begin{aligned}
&\|w_n - R_n v_n\| \leq \|w_n - w_{n+1}\| + \|w_{n+1} - R_n v_n\| \rightarrow 0, \\
&\|w_n - R_n w_n\| \leq \|w_n - R_n v_n\| + \|R_n v_n - R_n w_n\| \\
&\leq \|w_n - R_n v_n\| + \|v_n - w_n\| \rightarrow 0, \\
&\|v_n - R_n v_n\| \leq \|v_n - w_n\| + \|w_n - R_n v_n\| \rightarrow 0. \tag{29}
\end{aligned}$$

Let the mapping R be defined by

$$Rw = \lim_{n \rightarrow \infty} R_n w, \quad \forall w \in H. \tag{30}$$

By Lemma 5 we can get that $\text{Fix}(R) = \cap \text{Fix}(R_n)$ and

$$\begin{aligned}
&\|Rw_n - w_n\| \leq \|Rw_n - Rv_n\| + \|Rv_n - R_n v_n\| \\
&\quad + \|R_n v_n - w_n\| \\
&\leq \|w_n - v_n\| \\
&\quad + \sup \{\|R_n z - Rz\|, z \in \{v_n\}\} \\
&\quad + \|R_n v_n - w_n\| \\
&\rightarrow 0. \tag{31}
\end{aligned}$$

Note that it is easy to check that $P_{\Gamma \cap \text{VI}(S, M)}(\gamma f + I - T)$ is a contraction; by Banach contraction mapping principle, $P_{\Gamma \cap \text{VI}(S, M)}(\gamma f + I - T)$ has a unique fixed point $p \in H$. By the property of the projection $P_{\Gamma \cap \text{VI}(S, M)}$, we can get that

$$\langle (\gamma f - T)p, w - p \rangle \leq 0, \quad \forall w \in \Gamma \cap \text{VI}(S, M). \tag{32}$$

Choose a subsequence v_{nk} of v_n such that

$$\begin{aligned} \limsup_{n \rightarrow \infty} \langle (\gamma f - T)p, R_n v_n - p \rangle \\ = \lim_{n \rightarrow \infty} \langle (\gamma f - T)p, R_{nk} v_{nk} - p \rangle. \end{aligned} \quad (33)$$

Since $\{v_n\}$ is bounded, there exists a subsequence $\{v_{nk_i}\}$ of $\{v_{nk}\}$ which converges weakly q . Without loss of generality, we may assume that v_{nk} converges weakly q . Note that, by $\|v_n - R_n v_n\| \rightarrow 0$, we can get $R_{nk} v_{nk}$ converges weakly q . Furthermore, $\|w_n - R w_n\| \rightarrow 0$, $\|v_n - w_n\| \rightarrow 0$; by Lemmas 2 and 5, we obtain that $q \in \text{Fix}(R) = \cap \text{Fix}(R_n)$.

Next we will prove that $q \in \text{VI}(S, M)$.

Choose $v \in S$, and let $w_v \in \{w \in H, \langle v - u, w \rangle \geq 0, \forall u \in S\}$. Since $v_n \in S$, we can get $\langle v - v_n, w_v \rangle \geq 0$. On the other hand, by the property of the projection P_S and $v_n = P_S(w_n - \lambda_n M w_n)$, we have

$$\begin{aligned} \langle v - v_n, v_n - (w_n - \lambda_n M w_n) \rangle &\geq 0, \\ \left\langle v - v_n, \frac{v_n - w_n}{\lambda_n} + M w_n \right\rangle &\geq 0. \end{aligned} \quad (34)$$

It follows that

$$\begin{aligned} \langle v - v_{nk}, M v + w_v \rangle &\geq \langle v - v_{nk}, M v \rangle \\ &\geq \langle v - v_{nk}, M v \rangle \\ &\quad - \left\langle v - v_{nk}, \frac{v_{nk} - w_{nk}}{\lambda_{nk}} + M w_{nk} \right\rangle \\ &= \left\langle v - v_{nk}, M v - M w_{nk} - \frac{v_{nk} - w_{nk}}{\lambda_{nk}} \right\rangle \\ &= \langle v - v_{nk}, M v - M v_{nk} \rangle \\ &\quad + \langle v - v_{nk}, M v_{nk} - M w_{nk} \rangle \\ &\quad - \left\langle v - v_{nk}, \frac{v_{nk} - w_{nk}}{\lambda_{nk}} \right\rangle \\ &\geq \langle v - v_{nk}, M v_{nk} - M w_{nk} \rangle \\ &\quad - \left\langle v - v_{nk}, \frac{v_{nk} - w_{nk}}{\lambda_{nk}} \right\rangle. \end{aligned} \quad (35)$$

Since $\|M v_n - M w_n\| \leq \|v_n - w_n\| \rightarrow 0$ and v_{nk} converges weakly q , we can get that $\langle v - q, M v + w_v \rangle \geq 0$.

Note that M is a β -inverse strongly monotone mapping, and if $w_v \in \{w \in H, \langle v - u, w \rangle \geq 0, \forall u \in S\}$, then $\alpha w_v \in \{w \in H, \langle v - u, w \rangle \geq 0, \forall u \in S\}$ for all $\alpha > 0$. Hence, we can choose $w_v \in \{w \in H, \langle v - u, w \rangle \geq 0, \forall u \in S\}$ such that $\langle v - q, w_v \rangle \geq \langle v - q, M v - M q \rangle$. By

$$\langle v - q, M v + w_v \rangle \geq 0, \quad (36)$$

we can get that

$$\langle v - q, M v - M q \rangle + \langle v - q, M q - w_v \rangle \geq 0. \quad (37)$$

Hence

$$\langle v - q, M q \rangle \geq \langle v - q, w_v \rangle - \langle v - q, M v - M q \rangle \geq 0. \quad (38)$$

It follows that $q \in \text{VI}(S, M)$. We obtain that $q \in P_{\Gamma \cap \text{VI}(S, M)}$ and $\langle (\gamma f - T)p, q - p \rangle \leq 0$, for all $w \in \Gamma \cap \text{VI}(S, M)$.

Finally, we prove that $\lim_{n \rightarrow \infty} \|w_n - q\| = 0$. Since $\|v_n - u\| \leq \|w_n - u\|$, we have

$$\begin{aligned} \|w_{n+1} - p\|^2 &= \|P_S [\alpha_n \gamma f(w_n) + (I - \alpha_n T) R_n v_n] - P_S(p)\|^2 \\ &\leq \|\alpha_n (\gamma f(w_n) - T p) + (I - \alpha_n T) (R_n v_n - p)\|^2 \\ &\leq \|(I - \alpha_n T) (R_n v_n - p)\|^2 + \alpha_n^2 \|\gamma f(w_n) - T q\|^2 \\ &\quad + 2\alpha_n \langle (I - \alpha_n T) (R_n v_n - p), \gamma f(w_n) - T q \rangle \\ &\leq (1 - \alpha_n \bar{\gamma})^2 \|v_n - p\|^2 + \alpha_n^2 \|\gamma f(w_n) - T q\|^2 \\ &\quad + 2\alpha_n \langle R_n v_n - p, \gamma f(w_n) - T p \rangle \\ &\quad - 2\alpha_n^2 \langle T (R_n v_n - p), \gamma f(w_n) - T p \rangle \\ &\leq (1 - \alpha_n \bar{\gamma})^2 \|v_n - p\|^2 + \alpha_n^2 \|\gamma f(w_n) - T q\|^2 \\ &\quad + 2\alpha_n \langle R_n v_n - p, \gamma f(w_n) - \gamma f(p) \rangle \\ &\quad + 2\alpha_n \langle R_n v_n - p, \gamma f(p) - T p \rangle \\ &\quad - 2\alpha_n^2 \langle T (R_n v_n - p), \gamma f(w_n) - T p \rangle \\ &\leq (1 - \alpha_n \bar{\gamma})^2 \|v_n - p\|^2 + \alpha_n^2 \|\gamma f(w_n) - T q\|^2 \\ &\quad + 2\alpha_n \|R_n v_n - p\| \|\gamma f(w_n) - \gamma f(p)\| \\ &\quad + 2\alpha_n \langle R_n v_n - p, \gamma f(p) - T p \rangle \\ &\quad - 2\alpha_n^2 \langle T (R_n v_n - p), \gamma f(w_n) - T p \rangle \\ &\leq (1 - \alpha_n \bar{\gamma})^2 \|v_n - p\|^2 + \alpha_n^2 \|\gamma f(w_n) - T q\|^2 \\ &\quad + 2\gamma \alpha \alpha_n \|v_n - p\| \|w_n - p\| \\ &\quad + 2\alpha_n \langle R_n v_n - p, \gamma f(p) - T p \rangle \\ &\quad - 2\alpha_n^2 \langle T (R_n v_n - p), \gamma f(w_n) - T p \rangle \\ &\leq (1 - \alpha_n \bar{\gamma})^2 \|v_n - p\|^2 + \alpha_n^2 \|\gamma f(w_n) - T q\|^2 \\ &\quad + 2\gamma \alpha \alpha_n \|w_n - p\|^2 \\ &\quad + 2\alpha_n \langle R_n v_n - p, \gamma f(p) - T p \rangle \\ &\quad - 2\alpha_n^2 \langle T (R_n v_n - p), \gamma f(w_n) - T p \rangle \\ &\leq [(1 - \alpha_n \bar{\gamma})^2 + 2\gamma \alpha \alpha_n] \|w_n - p\|^2 \\ &\quad + \alpha_n [2 \langle R_n v_n - p, \gamma f(p) - T p \rangle \\ &\quad + \alpha_n \|\gamma f(w_n) - T q\|^2 \\ &\quad + 2\alpha_n \|T (R_n v_n - p)\| \|\gamma f(w_n) - T p\|] \end{aligned}$$

$$\begin{aligned}
&= [1 - 2(\bar{\gamma} - \gamma\alpha)\alpha_n] \|w_n - p\|^2 \\
&\quad + \alpha_n [2\langle R_n v_n - p, \gamma f(p) - Tp \rangle \\
&\quad + \alpha_n \|\gamma f(w_n) - Tq\|^2 + 2\alpha_n \|T(R_n v_n - p)\| \\
&\quad \times \|\gamma f(w_n) - Tp\| + \alpha_n \bar{\gamma}^2 \|w_n - p\|^2].
\end{aligned} \tag{39}$$

Since $\{w_n\}$, $\{f(w_n)\}$, $\{R_n v_n\}$, and $\limsup_{n \rightarrow \infty} \langle (\gamma f - T)p, R_n v_n - p \rangle \leq 0$, using Lemma 3, we can conclude that

$$\lim_{n \rightarrow \infty} \|w_n - q\| = 0, \tag{40}$$

which completes the proof. $\square$

By Lemma 1, we know that G^*G is $1/\|G\|^2$ -inverse strongly monotone. Hence, letting $M = G^*G$, we can get the following Corollary.

Corollary 7. Let Γ denote the solutions set of the SEP, $S = C \times Q$, f a contraction on $H = H_1 \times H_2$ with coefficient $0 < \alpha < 1$, a strongly positive linear bounded operator T with coefficient $\bar{\gamma} > 0$. Assume that $0 < \gamma < \bar{\gamma}/\alpha$, $\Gamma \neq \emptyset$, and $\{w_n\} = \{(x_n, y_n)\} \subseteq H$ is the sequence generated by the following algorithm: $w_0 \in H$,

$$\begin{aligned}
w_{n+1} &= P_S [\alpha_n \gamma f(w_n) + (I - \alpha_n T) P_S \\
&\quad \times (I - \gamma_n G^* G) P_S (w_n - \lambda_n G^* G w_n)]
\end{aligned} \tag{41}$$

for all $n = 0, 1, 2, \dots$, where $\alpha_n \in (0, 1)$, $\lambda_n, \gamma_n \in (0, 2/\|G\|^2)$. If α_n, λ_n , and γ_n are chosen so that $\lambda_n \in [a, b]$ for some a, b with $0 < a < b < 2/\|G\|^2$ and

- (i) $\lim_n \alpha_n = 0$,
- (ii) $\sum_{n=0}^{\infty} \alpha_n = \infty$,
- (iii) $\sum_{n=0}^{\infty} |\alpha_{n+1} - \alpha_n| < \infty$,
- (v) $\sum_{n=0}^{\infty} |\lambda_{n+1} - \lambda_n| < \infty$,
- (iv) $\sum_{n=0}^{\infty} |\gamma_{n+1} - \gamma_n| < \infty$,

then the sequence $\{w_n\}$ converges strongly to $\bar{w}$ which solves the SEP and the following variational inequality: $\langle (T - \lambda f)\bar{w}, w - \bar{w} \rangle \geq 0$ for all $w \in S$.

Furthermore, if $M = G^*G$ and $\gamma_n = 0$, changed slightly for Theorem 6, we get the following result.

Corollary 8. Let Γ denote the solutions set of the SEP, $S = C \times Q$, f a contraction on $H = H_1 \times H_2$ with coefficient $0 < \alpha < 1$, a strongly positive linear bounded operator T with coefficient $\bar{\gamma} > 0$. Assume that $0 < \gamma < \bar{\gamma}/\alpha$, $\Gamma \neq \emptyset$, and $\{w_n\} = \{(x_n, y_n)\} \subseteq H$ is the sequence generated by the following algorithm: $w_0 \in H$,

$$w_{n+1} = \alpha_n \gamma f(w_n) + (I - \alpha_n T) P_S (w_n - \lambda_n G^* G w_n) \tag{42}$$

for all $n = 0, 1, 2, \dots$, where $\alpha_n \in (0, 1)$, $\lambda_n \in (0, 2/\|G\|^2)$. If α_n, λ_n are chosen so that $\lambda_n \in [a, b]$ for some a, b with $0 < a < b < 2/\|G\|^2$ and

- (i) $\lim_n \alpha_n = 0$,
- (ii) $\sum_{n=0}^{\infty} \alpha_n = \infty$,
- (iii) $\sum_{n=0}^{\infty} |\alpha_{n+1} - \alpha_n| < \infty$,
- (v) $\sum_{n=0}^{\infty} |\lambda_{n+1} - \lambda_n| < \infty$,

then the sequence $\{w_n\}$ converges strongly to $\bar{w}$ which solves the SEP and the following variational inequality: $\langle (T - \lambda f)\bar{w}, w - \bar{w} \rangle \geq 0$ for all $w \in S$.

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