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*Research Article*

# A New Refinement of Generalized Hölder's Inequality and Its Application

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We present a new refinement of generalized Hölder's inequality due to Vasić and Pečarić. Moreover, the obtained result is used to improve Beckenbach-type inequality due to Wang.

## 1. Introduction

If $a_k \geq 0$, $b_k \geq 0$ $(k = 1, 2, \ldots, n)$, $p > 1$, and $(1/p) + (1/q) = 1$, then

$$\sum_{k=1}^{n} a_k b_k \leq \left(\sum_{k=1}^{n} a_k^p\right)^{1/p} \left(\sum_{k=1}^{n} b_k^q\right)^{1/q}. \tag{1}$$

The sign of inequality is reversed for $p < 1$, $p \neq 0$ (for $p < 0$; we assume that $a_k, b_k > 0$). Inequality (1) and its reversed version are called Hölder's inequalities and are important in the study of inequalities and in the field of applied mathematics. The important inequalities have attracted interest of many mathematicians and have been improved as well as generalized in several different directions. For example, Barza et al. [1] presented matriceal versions of Hölder's inequality. Nikolova and Varošanec [2] obtained some new refinements of the classical Hölder's inequality by using a convex function. Tian and Hu [3] established a new reversed version of a generalized sharp Hölder's inequality. For more detailed expositions, the interested reader may consult [1–13] and the references therein. Among various generalizations of (1), Vasić and Pečarić in [14] presented the following interesting theorem.

**Theorem A.** *Let $A_{ij} \geq 0$ $(i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, m)$.*

(a) *If $\beta_j$ are positive numbers, such that $\sum_{j=1}^{m} (1/\beta_j) \geq 1$, then*

$$\sum_{i=1}^{n} \prod_{j=1}^{m} A_{ij} \leq \prod_{j=1}^{m} \left(\sum_{i=1}^{n} A_{ij}^{\beta_j}\right)^{1/\beta_j}. \tag{2}$$

(b) *If $\beta_1 > 0$, $\beta_j < 0$ $(j = 2, 3, \ldots, m)$ and if $\sum_{j=1}^{m} (1/\beta_j) \leq 1$, then*

$$\sum_{i=1}^{n} \prod_{j=1}^{m} A_{ij} \geq \prod_{j=1}^{m} \left(\sum_{i=1}^{n} A_{ij}^{\beta_j}\right)^{1/\beta_j}. \tag{3}$$

(c) *If $\beta_j < 0$ $(j = 1, 2, \ldots, m)$, then*

$$\sum_{i=1}^{n} \prod_{j=1}^{m} A_{ij} \geq \prod_{j=1}^{m} \left(\sum_{i=1}^{n} A_{ij}^{\beta_j}\right)^{1/\beta_j}. \tag{4}$$

The main objective of this paper is to build some new refinements of inequalities (2), (3), and (4). Moreover, the obtained results will be applied to improve Beckenbach-type inequality which is due to Wang [15].

## 2. A New Refinement of Generalized Hölder's Inequality

In this section, we first prove the following lemma, which plays a crucial role in proving our main results.

**Lemma 1.** *Let $X_{ij} > 0$ and let $1-\sum_{j=1}^{m} X_{ij}^{\beta_j} > 0$ $(i = 1, 2, \ldots, n,$ $j = 1, 2, \ldots, m)$.*

(a) *If $\beta_j > 0$ $(j = 1, 2, \ldots, m)$ and if $\sum_{j=1}^{m}(1/\beta_j) \geq 1$, then*

$$\prod_{j=1}^{m}\left(1-\sum_{i=1}^{n}X_{ij}^{\beta_j}\right)^{1/\beta_j}+\sum_{i=1}^{n}\prod_{j=1}^{m}X_{ij} \leq \left[1-\left(\sum_{i=1}^{n}X_{i2}^{\beta_2}-\sum_{i=1}^{n}X_{i1}^{\beta_1}\right)^{2}\right]^{1/\beta^*}, \tag{5}$$

*where $\beta^* = \max\{\beta_1, \beta_2\}$.*

(b) *If $\beta_1 > 0$, $\beta_j > 0$ $(j = 2, 3, \ldots, m)$ and if $\sum_{j=1}^{m}(1/\beta_j) \leq 1$, then*

$$\prod_{j=1}^{m}\left(1-\sum_{i=1}^{n}X_{ij}^{\beta_j}\right)^{1/\beta_j}+\sum_{i=1}^{n}\prod_{j=1}^{m}X_{ij} \geq \left[1-\left(\sum_{i=1}^{n}X_{i2}^{\beta_2}-\sum_{i=1}^{n}X_{i1}^{\beta_1}\right)^{2}\right]^{1/\beta_2}. \tag{6}$$

(c) *If $\beta_j < 0$ $(j = 1, 2, \ldots, m)$, then*

$$\prod_{j=1}^{m}\left(1-\sum_{i=1}^{n}X_{ij}^{\beta_j}\right)^{1/\beta_j}+\sum_{i=1}^{n}\prod_{j=1}^{m}X_{ij} \geq \left[1-\left(\sum_{i=1}^{n}X_{i2}^{\beta_2}-\sum_{i=1}^{n}X_{i1}^{\beta_1}\right)^{2}\right]^{1/\beta_2}. \tag{7}$$

*Proof.* (a) Without loss of generality, we assume that $\beta_1 \leq \beta_2$.

*Case 1* (when $0 < \beta_1 < \beta_2$). It implies that $1/\beta_2 > 0$ and $(1/\beta_1) - (1/\beta_2) > 0$. According to $(1/\beta_2) + (1/\beta_2) + ((1/\beta_1) - (1/\beta_2)) + (1/\beta_3) + \cdots + (1/\beta_m) \geq 1$, by using inequality (2), we have

$$\begin{aligned}
&\left[1-\left(\sum_{i=1}^{n}X_{i2}^{\beta_2}-\sum_{i=1}^{n}X_{i1}^{\beta_1}\right)^{2}\right]^{1/\beta_2}\\
&\quad=\left[\left(1-\sum_{i=1}^{n}X_{i1}^{\beta_1}\right)+\sum_{i=1}^{n}X_{i2}^{\beta_2}\right]^{1/\beta_2}\\
&\qquad\times\left[\left(1-\sum_{i=1}^{n}X_{i2}^{\beta_2}\right)+\sum_{i=1}^{n}X_{i1}^{\beta_1}\right]^{1/\beta_2}\\
&\qquad\times\left[\left(1-\sum_{i=1}^{n}X_{i1}^{\beta_1}\right)+\sum_{i=1}^{n}X_{i1}^{\beta_1}\right]^{(1/\beta_1)-(1/\beta_2)}\\
&\qquad\times\prod_{j=3}^{m}\left[\left(1-\sum_{i=1}^{n}X_{ij}^{\beta_j}\right)+\sum_{i=1}^{n}X_{ij}^{\beta_j}\right]^{1/\beta_j}\\
&\quad\geq\left(1-\sum_{i=1}^{n}X_{i1}^{\beta_1}\right)^{1/\beta_2}\left(1-\sum_{i=1}^{n}X_{i2}^{\beta_2}\right)^{1/\beta_2}\\
&\qquad\times\left(1-\sum_{i=1}^{n}X_{i1}^{\beta_1}\right)^{(1/\beta_1)-(1/\beta_2)}\times\prod_{j=3}^{m}\left(1-\sum_{i=1}^{n}X_{i1}^{\beta_1}\right)^{1/\beta_j}\\
&\qquad+\sum_{i=1}^{n}\left[\left(X_{i2}^{\beta_2}\right)^{1/\beta_2}\left(X_{i1}^{\beta_1}\right)^{1/\beta_2}\right.\\
&\qquad\left.\times\left(X_{i1}^{\beta_1}\right)^{(1/\beta_1)-(1/\beta_2)}\prod_{j=3}^{m}\left(X_{ij}^{\beta_j}\right)^{1/\beta_j}\right]\\
&\quad=\prod_{j=3}^{m}\left(1-\sum_{i=1}^{n}X_{ij}^{\beta_j}\right)^{1/\beta_j}+\sum_{i=1}^{n}\prod_{j=1}^{m}X_{ij},
\end{aligned} \tag{8}$$

which means that the desired inequality (5) holds for $0 < \beta_1 < \beta_2$.

*Case 2* (when $\beta_1 = \beta_2 > 0$). By applying inequality (2), we obtain

$$\begin{aligned}
&\left[1-\left(\sum_{i=1}^{n}X_{i2}^{\beta_2}-\sum_{i=1}^{n}X_{i1}^{\beta_1}\right)^{2}\right]^{1/\beta_2}\\
&\quad=\left[\left(1-\sum_{i=1}^{n}X_{i1}^{\beta_1}\right)+\sum_{i=1}^{n}X_{i2}^{\beta_2}\right]^{1/\beta_2}\\
&\qquad\times\left[\left(1-\sum_{i=1}^{n}X_{i2}^{\beta_2}\right)+\sum_{i=1}^{n}X_{i1}^{\beta_1}\right]^{1/\beta_2}\\
&\qquad\times\prod_{j=3}^{m}\left[\left(1-\sum_{i=1}^{n}X_{ij}^{\beta_j}\right)+\sum_{i=1}^{n}X_{ij}^{\beta_j}\right]^{1/\beta_j}\\
&\quad\geq\left(1-\sum_{i=1}^{n}X_{i1}^{\beta_1}\right)^{1/\beta_2}\left(1-\sum_{i=1}^{n}X_{i2}^{\beta_2}\right)^{1/\beta_2}\\
&\qquad\times\prod_{j=3}^{m}\left(1-\sum_{i=1}^{n}X_{ij}^{\beta_j}\right)^{1/\beta_j}\\
&\qquad+\sum_{i=1}^{n}\left[\left(X_{i2}^{\beta_2}\right)^{1/\beta_2}\left(X_{i1}^{\beta_1}\right)^{1/\beta_2}\times\prod_{j=3}^{m}\left(X_{ij}^{\beta_j}\right)^{1/\beta_j}\right]\\
&\quad=\prod_{j=1}^{m}\left(1-\sum_{i=1}^{n}X_{ij}^{\beta_j}\right)^{1/\beta_j}+\sum_{i=1}^{n}\prod_{j=1}^{m}X_{ij}.
\end{aligned} \tag{9}$$

That is, inequality (5) is true for $\beta_1 = \beta_2 > 0$.

(b) If $\beta_1 > 0$, $\beta_j < 0$ $(j = 2, 3, \ldots, m)$, then $(1/\beta_1) - (1/\beta_2) > 0$, $(1/\beta_j) < 0$ $(j = 2, 3, \ldots, m)$. By the same method as in Case 1, we obtain the desired inequality (6).

(c) The proof of inequality (7) is similar to the one of inequality (5), and we omit it.

The proof of Lemma 1 is completed. □

Next, we present new refinements of inequalities (2), (3), and (4).

**Theorem 2.** *Let $A_{ij} \geq 0$ $(i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, m)$, and let $s$ be any given natural number $(1 \leq s \leq n)$.*

(a) *If $\beta_j > 0$ $(j = 1, 2, \ldots, m)$ and if $\sum_{j=1}^{m}(1/\beta_j) \geq 1$, then*

$$\sum_{i=1}^{n}\prod_{j=1}^{m}A_{ij} \leq \left[\prod_{j=1}^{m}\left(\sum_{i=1}^{n}A_{ij}^{\beta_j}\right)^{1/\beta_j}\right] \times \left[1-\left(\frac{A_{s2}^{\beta_2}}{\sum_{i=1}^{n}A_{i2}^{\beta_2}}-\frac{A_{s1}^{\beta_1}}{\sum_{i=1}^{n}A_{i1}^{\beta_1}}\right)^2\right]^{1/\beta^*}, \tag{10}$$

*where $\beta^* = \max\{\beta_1, \beta_2\}$.*

(b) *If $\beta_1 > 0$, $\beta_j < 0$ $(j = 2, 3, \ldots, m)$ and if $\sum_{j=1}^{m}(1/\beta_j) \leq 1$, then*

$$\sum_{i=1}^{n}\prod_{j=1}^{m}A_{ij} \geq \left[\prod_{j=1}^{m}\left(\sum_{i=1}^{n}A_{ij}^{\beta_j}\right)^{1/\beta_j}\right] \times \left[1-\left(\frac{A_{s2}^{\beta_2}}{\sum_{i=1}^{n}A_{i2}^{\beta_2}}-\frac{A_{s1}^{\beta_1}}{\sum_{i=1}^{n}A_{i1}^{\beta_1}}\right)^2\right]^{1/\beta_2}. \tag{11}$$

(c) *If $\beta_j < 0 (j = 1, 2, \ldots, m)$, then*

$$\sum_{i=1}^{n}\prod_{j=1}^{m}A_{ij} \geq \left[\prod_{j=1}^{m}\left(\sum_{i=1}^{n}A_{ij}^{\beta_j}\right)^{1/\beta_j}\right] \times \left[1-\left(\frac{A_{s2}^{\beta_2}}{\sum_{i=1}^{n}A_{i2}^{\beta_2}}-\frac{A_{s1}^{\beta_1}}{\sum_{i=1}^{n}A_{i1}^{\beta_1}}\right)^2\right]^{1/\beta_2}. \tag{12}$$

*Proof.* Consider the following substitution:

$$X_{ij} = \frac{A_{ij}}{\left(\sum_{k=1}^{n}A_{kj}^{\beta_j}\right)^{1/\beta_j}} \quad (i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, m). \tag{13}$$

It is easy to see that, for any given natural number $s$ $(1 \leq s \leq n)$, the following inequalities hold:

$$X_{ij} > 0, \qquad 1 - \sum_{1\leq i\leq n, i\neq s} X_{ij}^{\beta_j} > 0. \tag{14}$$

Consequently, by using the substitution (13) and inequality (5), we have

$$\prod_{j=1}^{m}\left[1-\sum_{1\leq i\leq n, i\neq s}\left(\frac{A_{ij}^{\beta_j}}{\sum_{k=1}^{n}A_{kj}^{\beta_j}}\right)\right]^{1/\beta_j} + \sum_{1\leq i\leq n, i\neq s}\left(\prod_{j=1}^{m}\frac{A_{ij}}{\left(\sum_{k=1}^{n}A_{kj}^{\beta_j}\right)^{1/\beta_j}}\right)$$
$$\leq \left\{1-\left[\sum_{1\leq i\leq n, i\neq s}\left(\frac{A_{i2}^{\beta_2}}{\sum_{k=1}^{n}A_{k2}^{\beta_2}}\right) - \sum_{1\leq i\leq n, i\neq s}\left(\frac{A_{i1}^{\beta_1}}{\sum_{k=1}^{n}A_{k2}^{\beta_2}}\right)\right]^2\right\}^{1/\beta^*} \tag{15}$$

for $\beta_j > 0$ $(j = 1, 2, \ldots, m)$, $\sum_{j=1}^{m}(1/\beta_j) \geq 1$, and thus we have

$$\frac{\prod_{j=1}^{m}A_{sj}}{\prod_{j=1}^{m}\left(\sum_{k=1}^{n}A_{kj}^{\beta_j}\right)^{1/\beta_j}} + \frac{\sum_{1\leq i\leq n, i\neq s}\prod_{j=1}^{m}A_{ij}}{\prod_{j=1}^{m}\left(\sum_{k=1}^{n}A_{kj}^{\beta_j}\right)^{1/\beta_j}} \leq \left[1-\left(\frac{A_{s2}^{\beta_2}}{\sum_{k=1}^{n}A_{k2}^{\beta_2}}-\frac{A_{s1}^{\beta_1}}{\sum_{k=1}^{n}A_{k1}^{\beta_1}}\right)^2\right]^{1/\beta^*}, \tag{16}$$

that is,

$$\frac{\sum_{i=1}^{n}\left(\prod_{j=1}^{m}A_{ij}\right)}{\prod_{j=1}^{m}\left(\sum_{k=1}^{n}A_{kj}^{\beta_j}\right)^{1/\beta_j}} \leq \left[1-\left(\frac{A_{s2}^{\beta_2}}{\sum_{k=1}^{n}A_{k2}^{\beta_2}}-\frac{A_{s1}^{\beta_1}}{\sum_{k=1}^{n}A_{k1}^{\beta_1}}\right)^2\right]^{1/\beta^*}. \tag{17}$$

So, we have the desired inequality (10). The proof of inequalities (11) and (12) is similar to the one of inequality (10), and we omit it. The proof of Theorem 2 is completed. □

Putting $s = 1$ in (10), (11), and (12), respectively, we obtain the following corollary.

**Corollary 3.** *Let $A_{ij} \geq 0$ $(i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, m)$.*

(a) *If $\beta_j > 0$ $(j = 1, 2, \ldots, m)$ and if $\sum_{j=1}^{m}(1/\beta_j) \geq 1$, then*

$$\sum_{i=1}^{n}\prod_{j=1}^{m}A_{ij} \leq \left[\prod_{j=1}^{m}\left(\sum_{i=1}^{n}A_{ij}^{\beta_j}\right)^{1/\beta_j}\right] \times \left[1-\left(\frac{A_{12}^{\beta_2}}{\sum_{i=1}^{n}A_{i2}^{\beta_2}}-\frac{A_{11}^{\beta_1}}{\sum_{i=1}^{n}A_{i1}^{\beta_1}}\right)^2\right]^{1/\beta^*}, \tag{18}$$

*where $\beta^* = \max\{\beta_1, \beta_2\}$.*

(b) *If $\beta_1 > 0$, $\beta_j < 0$ $(j = 2, 3, \ldots, m)$ and if $\sum_{j=1}^{m}(1/\beta_j) \leq 1$, then*

$$\sum_{i=1}^{n}\prod_{j=1}^{m}A_{ij} \geq \left[\prod_{j=1}^{m}\left(\sum_{i=1}^{n}A_{ij}^{\beta_j}\right)^{1/\beta_j}\right] \times \left[1-\left(\frac{A_{12}^{\beta_2}}{\sum_{i=1}^{n}A_{i2}^{\beta_2}}-\frac{A_{11}^{\beta_1}}{\sum_{i=1}^{n}A_{i1}^{\beta_1}}\right)^2\right]^{1/\beta_2}. \tag{19}$$

(c) *If* $\beta_j < 0$ $(j = 1, 2, \ldots, m)$, *then*

$$\sum_{i=1}^{n}\prod_{j=1}^{m} A_{ij} \geq \left[\prod_{j=1}^{m}\left(\sum_{i=1}^{n} A_{ij}^{\beta_j}\right)^{1/\beta_j}\right] \times \left[1 - \left(\frac{A_{12}^{\beta_2}}{\sum_{i=1}^{n} A_{i2}^{\beta_2}} - \frac{A_{11}^{\beta_1}}{\sum_{i=1}^{n} A_{i1}^{\beta_1}}\right)^2\right]^{1/\beta_2}. \tag{20}$$

## 3. Application

In this section, we present a refinement of Beckenbach-type inequality by using Corollary 3. The classical Beckenbach inequality was proved by Beckenbach in [5]. Since Beckenbach discovered this inequality, it has been discussed by many researchers, who either improved it using various techniques or generalized it in many different ways. The interested reader may refer to [7, 16] and references therein. In 1983, Wang [15] established the following Beckenbach-type inequality.

**Theorem B.** *Let* $f(x)$ *and* $g(x)$ *be positive integrable functions defined on* $[0, T]$, *and let* $(1/p) + (1/q) = 1$. *If* $q \geq p > 1$, *then, for any positive numbers a, b, and c, the inequality*

$$\frac{\left(a + c\int_0^T h^p(x)\,dx\right)^{1/p}}{b + c\int_0^T h(x)\,g(x)\,dx} \leq \frac{\left(a + c\int_0^T f^p(x)\,dx\right)^{1/p}}{b + c\int_0^T f(x)\,g(x)\,dx} \tag{21}$$

*holds, where* $h(x) = ((ag(x))/b)^{q/p}$. *The sign of the inequality in* (21) *is reversed if* $0 < p < 1$.

**Theorem 4.** *Let* $f(x)$ *and* $g(x)$ *be positive integrable functions defined on* $[0, T]$, *and let* $(1/p) + (1/q) = 1$. *If* $q \geq p > 1$, *then, for any positive numbers a, b, and c, the inequality*

$$\begin{aligned}\frac{\left(a + c\int_0^T h^p(x)\,dx\right)^{1/p}}{b + c\int_0^T h(x)\,g(x)\,dx} &\leq \frac{\left(a + c\int_0^T f^p(x)\,dx\right)^{1/p}}{b + c\int_0^T f(x)\,g(x)\,dx} \\ &\times \left[1 - \left(\frac{a^{-(q/p)}b^q}{a^{-(q/p)}b^q + c\int_0^T g^q(x)\,dx} - \frac{a}{a + c\int_0^T f^p(x)\,dx}\right)^2\right]^{1/q}\end{aligned} \tag{22}$$

*holds, where* $h(x) = ((ag(x))/b)^{q/p}$. *The sign of the inequality in* (22) *is reversed if* $0 < p < 1$.

*Proof.* After some simple calculations, we have

$$\frac{\left(a + c\int_0^T h^p(x)\,dx\right)^{1/p}}{b + c\int_0^T h(x)\,g(x)\,dx} = \left(a^{-(q/p)}b^q + c\int_0^T g^q(x)\,dx\right)^{-1/q}. \tag{23}$$

On the other hand, putting $\beta_1 = p, \beta_2 = q, m = 2$ in (18), from the integral form of Hölder's inequality (1) and Corollary 3, we obtain

$$\begin{aligned}&b + c\int_0^T f(x)\,g(x)\,dx \\ &\leq b + c\left(\int_0^T f^p(x)\,dx\right)^{1/p}\left(\int_0^T g^q(x)\,dx\right)^{1/q} \\ &= a^{1/p}\left(ba^{-1/p}\right) + \left(c\int_0^T f^p(x)\,dx\right)^{1/p} \\ &\quad\times\left(c\int_0^T g^q(x)\,dx\right)^{1/q} \\ &\leq \left(a + c\int_0^T f^p(x)\,dx\right)^{1/p} \\ &\quad\times\left(a^{-(q/p)}b^q + c\int_0^T g^q(x)\,dx\right)^{1/q} \\ &\quad\times\left[1 - \left(\frac{a^{-(q/p)}b^q}{a^{-(q/p)}b^q + c\int_0^T g^q(x)\,dx} - \frac{a}{a + c\int_0^T f^p(x)\,dx}\right)^2\right]^{1/q},\end{aligned} \tag{24}$$

that is,

$$\begin{aligned}&\left(a^{-(q/p)}b^q + c\int_0^T g^q(x)\,dx\right)^{-1/q} \\ &\leq \frac{\left(a + c\int_0^T f^p(x)\,dx\right)^{1/p}}{b + c\int_0^T f(x)\,g(x)\,dx} \\ &\quad\times\left[1 - \left(\frac{a^{-(q/p)}b^q}{a^{-(q/p)}b^q + c\int_0^T g^q(x)\,dx} - \frac{a}{a + c\int_0^T f^p(x)\,dx}\right)^2\right]^{1/q}.\end{aligned} \tag{25}$$

Combining inequalities (23) and (25) yields inequality (22). In a similar way, we can prove that the reversed version of inequality (22) is true. Thus, the proof of Theorem 4 is complete. □

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