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Research Article

Hyers-Ulam Stability of Differentiation Operator on Hilbert Spaces of Entire Functions

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We investigate the Hyers-Ulam stability of differentiation operator on Hilbert spaces of entire functions. We give a necessary and sufficient condition in order that the operator has the Hyers-Ulam stability and also show that the best constant of Hyers-Ulam stability exists.

1. Introduction

In 1940, the first stability problem concerning group homomorphisms was raised by Ulam [1]. Let G_1 be a group and let G_2 be a metric group with a metric $d(\cdot,\cdot)$. Given any $\epsilon > 0$, does there exist a $\delta > 0$ such that if a function $h : G_1 \to G_2$ satisfies the inequality $d(h(xy), h(x)h(y)) < \delta$ for all $x, y \in G_1$, then there exists a homomorphism $H : G_1 \to G_2$ with $d(h(x), H(x)) < \epsilon$ for all $x \in G_1$?

In the following years, Hyers affirmatively answered the question of Ulam for the case where G_1 and G_2 are Banach spaces (see [2]). Furthermore, the result of Hyers has been generalized by Rassias (see [3]).

Since then, the stability of many algebraic, differential, integral, operatorial, functional equations have been extensively investigated (see [4–17] and the references therein).

In this paper, we discuss the Hyers-Ulam stability of differentiation operator on Hilbert spaces of entire functions $E^2(\gamma)$ and give a necessary and sufficient condition in order that the operator has the Hyers-Ulam stability, and we show that the best constant of Hyers-Ulam stability exists.

2. Hilbert Spaces of Entire Functions

In this section, we describe the Hilbert spaces of entire functions in which the rest of our work is set and record their most basic properties. About the function spaces, we recommend the research papers [18, 19]. For the sake of coherency we recall a few basic definitions, notions, and theorems from [18], and we also give some typical examples; in particular Fock space in these examples is a very important tool for quantum stochastic calculus in the case of quantum probability (see [20–22]).

Let us call an entire function $\gamma(z) = \sum_{n=0}^{\infty} \gamma_n z^n$ a *comparison function* if $\gamma_n > 0$ for each n, and the sequence of ratios γ_{n+1}/γ_n decreases to zero as n increases to ∞. For each comparison function $\gamma(z)$, we define $E^2(\gamma)$ to be the Hilbert space of power series

$$f(z) = \sum_{n=0}^{\infty} \hat{f}(n) z^n \tag{1}$$

for which

$$\|f\|_{2,\gamma}^2 := \sum_{n=0}^{\infty} \gamma_n^{-2} \left|\hat{f}(n)\right|^2 < \infty. \tag{2}$$

It is easy to check that each element of $E^2(\gamma)$ is an entire function and that every sequence convergent in the norm of

the space is uniformly convergent on compact subsets of the plane. In this case, the inner product of $E^2(\gamma)$ is given by

$$\left\langle \sum_{n=0}^{\infty} a_n z^n, \sum_{n=0}^{\infty} b_n z^n \right\rangle := \sum_{n=0}^{\infty} \gamma_n^{-2} a_n \overline{b_n}, \tag{3}$$

and the functions

$$e_n(z) = \gamma_n z^n \quad (n = 0, 1, 2, \ldots) \tag{4}$$

form an orthonormal basis for $E^2(\gamma)$. We can see that the polynomials are dense in $E^2(\gamma)$.

Example 1. We consider the comparison function $\gamma_1(z) = \sum_{k=0}^{\infty}(1/\sqrt{k!})z^k$; that is, $\gamma_k = 1/\sqrt{k!}$; by a simple calculation, we can see that $\|z^k\| = \sqrt{k!}$, and then $E^2(\gamma_1)$ is the famous Fock space.

Example 2. If we put comparison function $\gamma_2(z) = \sum_{k=0}^{\infty}(1/k!)z^k$, that is, $\gamma_k = 1/k!$, we can see that $\|z^k\| = k!$ on $E^2(\gamma_2)$.

Throughout this paper, let $D : E^2(\gamma) \to E^2(\gamma)$ be the differentiation operator defined by

$$D(f)(z) = f'(z), \quad \left(f \in E^2(\gamma),\ z \in \mathbb{C}\right). \tag{5}$$

An important result about D is the following theorem.

Theorem 3 (see [18]). *The operator D is bounded on $E^2(\gamma)$ if and only if the sequence $\{w_n\}_{n=1}^{\infty}$ is bounded, where $w_n = n\gamma_n/\gamma_{n-1}$.*

By Theorem 3, we can obtain that the operator D is unbounded on Fock space $E^2(\gamma_1)$, and it is bounded on $E^2(\gamma_2)$.

Throughout this paper, we suppose that the sequence $\{w_n\}_{n=1}^{\infty}$ is bounded.

3. Hyers-Ulam Stability of Differentiation Operator

Let A, B be normed spaces and consider a mapping $T : A \to B$. The following definition can be found in [14].

Definition 4. We say that T *has the Hyers-Ulam stability property* (briefly, T is *HU-stable*) if there exists a constant $K > 0$ such that, for any $g \in T(A)$, $\epsilon > 0$, and $f \in A$ with $\|Tf - g\| \le \epsilon$, there exists an $f_0 \in A$ with $Tf_0 = g$ and $\|f - f_0\| \le K\epsilon$. The number K is called a *Hyers-Ulam stability constant* (briefly *HUS-constant*) and the infimum of all HUS constants of T is denoted by K_T; generally, K_T is not a HUS constant of T (see [9, 10]).

Theorem 5. *Let D be the differentiation operator on the Hilbert spaces of entire functions $E^2(\gamma)$. Then the following statements are equivalent:*

(a) *D has Hyers-Ulam stability on $E^2(\gamma)$;*

(b) *the sequence $\{1/w_n\}_{n=1}^{\infty}$ is bounded, where $w_n = n\gamma_n/\gamma_{n-1}$.*

Proof. (b)⇒(a). Suppose that the sequence $\{1/w_n\}_{n=1}^{\infty}$ is bounded, and let $R = \sup\{1/w_n : n \ge 1\}$. Since the polynomials are dense in $E^2(\gamma)$, we just need to show that D has Hyers-Ulam stability on the polynomials dense subspace. For each $\epsilon \ge 0$, we take any two polynomials f and g that satisfy $\|Df - g\| \le \epsilon$, f and g can be represented by the orthonormal basis. Then if $f(z) = \sum_{k=0}^{l}\langle f, e_k\rangle e_k$, $g(z) = \sum_{k=0}^{m}\langle g, e_k\rangle e_k$, we have

$$\begin{aligned} Df(z) &= \sum_{k=1}^{l} k\langle f, e_k\rangle \gamma_k z^{k-1} \\ &= \sum_{k=1}^{l} \frac{k\gamma_k}{\gamma_{k-1}} \langle f, e_k\rangle \gamma_{k-1} z^{k-1} \\ &= \sum_{k=1}^{l} \frac{k\gamma_k}{\gamma_{k-1}} \langle f, e_k\rangle e_{k-1} \\ &= \sum_{k=0}^{l-1} \frac{(k+1)\gamma_{k+1}}{\gamma_k} \langle f, e_{k+1}\rangle e_k \\ &= \sum_{k=0}^{l-1} w_{k+1} \langle f, e_{k+1}\rangle e_k. \end{aligned} \tag{6}$$

For any nonnegative integers m and l such that $m \ge l$, we can get

$$\begin{aligned} \|Df - g\|^2 &= \left\| \sum_{k=0}^{l-1} w_{k+1}\langle f, e_{k+1}\rangle e_k - \sum_{k=0}^{m}\langle g, e_k\rangle e_k \right\|^2 \\ &= \left\| \sum_{k=0}^{l-1} w_{k+1}\langle f, e_{k+1}\rangle e_k - \sum_{k=0}^{l-1}\langle g, e_k\rangle e_k \right. \\ &\qquad \left. - \sum_{k=l}^{m}\langle g, e_k\rangle e_k \right\|^2 \\ &= \left\| \sum_{k=0}^{l-1} \left[w_{k+1}\langle f, e_{k+1}\rangle - \langle g, e_k\rangle\right] e_k \right. \\ &\qquad \left. + \sum_{k=l}^{m} \left[-\langle g, e_k\rangle\right] e_k \right\|^2 \\ &= \sum_{k=0}^{l-1} \left|w_{k+1}\langle f, e_{k+1}\rangle - \langle g, e_k\rangle\right|^2 + \sum_{k=l}^{m} \left|\langle g, e_k\rangle\right|^2. \end{aligned} \tag{7}$$

Hence,

$$\sum_{k=0}^{l-1} \left|w_{k+1}\langle f, e_{k+1}\rangle - \langle g, e_k\rangle\right|^2 + \sum_{k=l}^{m} \left|\langle g, e_k\rangle\right|^2 \le \epsilon^2. \tag{8}$$

Let $f_0 \in E^2(\gamma)$ be the function defined by

$$f_0(z) = \langle f, e_0 \rangle e_0 + \sum_{k=1}^{m+1} \frac{\langle g, e_{k-1} \rangle \gamma_{k-1}}{k\gamma_k} e_k. \tag{9}$$

It is easy to check that $Df_0 = g$, also; from (8), we obtain

$$\begin{aligned}
\|f - f_0\|^2 &= \left\| \sum_{k=0}^{l} \langle f, e_k \rangle e_k - \left(\langle f, e_0 \rangle e_0 + \sum_{k=1}^{m+1} \frac{\langle g, e_{k-1} \rangle \gamma_{k-1}}{k\gamma_k} e_k \right) \right\|^2 \\
&= \left\| \sum_{k=1}^{l} \left[\langle f, e_k \rangle - \frac{\langle g, e_{k-1} \rangle \gamma_{k-1}}{k\gamma_k} \right] e_k + \sum_{k=l+1}^{m+1} \frac{\langle g, e_{k-1} \rangle \gamma_{k-1}}{k\gamma_k} e_k \right\|^2 \\
&= \sum_{k=1}^{l} \left| \langle f, e_k \rangle - \frac{\langle g, e_{k-1} \rangle \gamma_{k-1}}{k\gamma_k} \right|^2 + \sum_{k=l+1}^{m+1} \left| \frac{\langle g, e_{k-1} \rangle \gamma_{k-1}}{k\gamma_k} \right|^2 \\
&= \sum_{k=1}^{l} \left| \langle f, e_k \rangle - \langle g, e_{k-1} \rangle \cdot \frac{1}{w_k} \right|^2 + \sum_{k=l+1}^{m+1} \left| \langle g, e_{k-1} \rangle \cdot \frac{1}{w_k} \right|^2 \\
&= \sum_{k=1}^{l} \left| w_k \langle f, e_k \rangle - \langle g, e_{k-1} \rangle \right|^2 \left| \frac{1}{w_k} \right|^2 + \sum_{k=l+1}^{m+1} \left| \langle g, e_{k-1} \rangle \right|^2 \left| \frac{1}{w_k} \right|^2 \\
&\le \left\{ \sum_{k=1}^{l} \left| w_k \langle f, e_k \rangle - \langle g, e_{k-1} \rangle \right|^2 + \sum_{k=l+1}^{m+1} \left| \langle g, e_{k-1} \rangle \right|^2 \right\} \cdot R^2 \\
&\le R^2 \epsilon^2.
\end{aligned} \tag{10}$$

Next, assume that $m = l - 1$. It follows that

$$\begin{aligned}
\|Df - g\|^2 &= \left\| \sum_{k=0}^{l-1} w_{k+1} \langle f, e_{k+1} \rangle e_k - \sum_{k=0}^{l-1} \langle g, e_k \rangle e_k \right\|^2 \\
&= \left\| \sum_{k=0}^{l-1} \left[w_{k+1} \langle f, e_{k+1} \rangle - \langle g, e_k \rangle \right] e_k \right\|^2 \\
&= \sum_{k=0}^{l-1} \left| w_{k+1} \langle f, e_{k+1} \rangle - \langle g, e_k \rangle \right|^2.
\end{aligned} \tag{11}$$

Hence,

$$\sum_{k=0}^{l-1} \left| w_{k+1} \langle f, e_{k+1} \rangle - \langle g, e_k \rangle \right|^2 \le \epsilon^2. \tag{12}$$

Let

$$f_0(z) = \langle f, e_0 \rangle e_0 + \sum_{k=1}^{l} \frac{\langle g, e_{k-1} \rangle \gamma_{k-1}}{k\gamma_k} e_k. \tag{13}$$

Then $Df_0 = g$; by (12), we have

$$\begin{aligned}
\|f - f_0\|^2 &= \left\| \sum_{k=0}^{l} \langle f, e_k \rangle e_k - \left(\langle f, e_0 \rangle e_0 + \sum_{k=1}^{l} \frac{\langle g, e_{k-1} \rangle \gamma_{k-1}}{k\gamma_k} e_k \right) \right\|^2 \\
&= \left\| \sum_{k=1}^{l} \left[\langle f, e_k \rangle - \frac{\langle g, e_{k-1} \rangle \gamma_{k-1}}{k\gamma_k} \right] e_k \right\|^2 \\
&= \sum_{k=1}^{l} \left| \langle f, e_k \rangle - \frac{\langle g, e_{k-1} \rangle \gamma_{k-1}}{k\gamma_k} \right|^2 \\
&= \sum_{k=0}^{l-1} \left| \langle f, e_{k+1} \rangle - \frac{\langle g, e_k \rangle \gamma_k}{(k+1)\gamma_{k+1}} \right|^2 \\
&= \sum_{k=0}^{l-1} \left| \langle f, e_{k+1} \rangle - \langle g, e_k \rangle \frac{1}{w_{k+1}} \right|^2 \\
&= \sum_{k=0}^{l-1} \left| w_{k+1} \langle f, e_{k+1} \rangle - \langle g, e_k \rangle \right|^2 \left| \frac{1}{w_{k+1}} \right|^2 \\
&\le R^2 \epsilon^2.
\end{aligned} \tag{14}$$

Finally, if $m < l-1$, we get

$$\begin{aligned}
\|Df-g\|^2 &= \left\|\sum_{k=0}^{l-1} w_{k+1}\langle f, e_{k+1}\rangle e_k - \sum_{k=0}^{m}\langle g, e_k\rangle e_k\right\|^2 \\
&= \left\|\sum_{k=0}^{m} w_{k+1}\langle f, e_{k+1}\rangle e_k \right. \\
&\qquad \left. + \sum_{k=m+1}^{l-1} w_{k+1}\langle f, e_{k+1}\rangle e_k - \sum_{k=0}^{m}\langle g, e_k\rangle e_k\right\|^2 \\
&= \left\|\sum_{k=0}^{m}\left[w_{k+1}\langle f, e_{k+1}\rangle - \langle g, e_k\rangle\right] e_k \right. \\
&\qquad \left. + \sum_{k=m+1}^{l-1} w_{k+1}\langle f, e_{k+1}\rangle e_k\right\|^2 \\
&= \sum_{k=0}^{m}\left|w_{k+1}\langle f, e_{k+1}\rangle - \langle g, e_k\rangle\right|^2 \\
&\qquad + \sum_{k=m+1}^{l-1}\left|w_{k+1}\langle f, e_{k+1}\rangle\right|^2.
\end{aligned} \tag{15}$$

Hence,

$$\sum_{k=0}^{m}\left|w_{k+1}\langle f, e_{k+1}\rangle - \langle g, e_k\rangle\right|^2 + \sum_{k=m+1}^{l-1}\left|w_{k+1}\langle f, e_{k+1}\rangle\right|^2 \le \epsilon^2. \tag{16}$$

Thus, from (16), it follows that

$$\begin{aligned}
\|f-f_0\|^2 &= \left\|\sum_{k=0}^{l}\langle f, e_k\rangle e_k \right. \\
&\qquad \left. - \left(\langle f, e_0\rangle e_0 + \sum_{k=1}^{m+1}\frac{\langle g, e_{k-1}\rangle \gamma_{k-1}}{k\gamma_k} e_k\right)\right\|^2 \\
&= \left\|\sum_{k=1}^{m+1}\langle f, e_k\rangle e_k + \sum_{k=m+2}^{l}\langle f, e_k\rangle e_k \right. \\
&\qquad \left. - \sum_{k=1}^{m+1}\frac{\langle g, e_{k-1}\rangle \gamma_{k-1}}{k\gamma_k} e_k\right\|^2 \\
&= \left\|\sum_{k=1}^{m+1}\left[\langle f, e_k\rangle - \frac{\langle g, e_{k-1}\rangle \gamma_{k-1}}{k\gamma_k}\right] e_k \right. \\
&\qquad \left. + \sum_{k=m+2}^{l}\langle f, e_k\rangle e_k\right\|^2 \\
&= \sum_{k=1}^{m+1}\left|\langle f, e_k\rangle - \frac{\langle g, e_{k-1}\rangle \gamma_{k-1}}{k\gamma_k}\right|^2 \\
&\qquad + \sum_{k=m+2}^{l}\left|\langle f, e_k\rangle\right|^2 \\
&= \sum_{k=0}^{m}\left|\langle f, e_{k+1}\rangle - \frac{\langle g, e_k\rangle \gamma_k}{(k+1)\gamma_{k+1}}\right|^2 \\
&\qquad + \sum_{k=m+1}^{l-1}\left|\langle f, e_{k+1}\rangle\right|^2 \\
&= \sum_{k=0}^{m}\left|\langle f, e_{k+1}\rangle - \langle g, e_k\rangle \frac{1}{w_{k+1}}\right|^2 \\
&\qquad + \sum_{k=m+1}^{l-1}\left|\langle f, e_{k+1}\rangle\right|^2 \\
&= \sum_{k=0}^{m}\left|w_{k+1}\langle f, e_{k+1}\rangle - \langle g, e_k\rangle\right|^2 \cdot \left|\frac{1}{w_{k+1}}\right|^2 \\
&\qquad + \sum_{k=m+1}^{l-1}\left|w_{k+1}\langle f, e_{k+1}\rangle\right|^2 \cdot \left|\frac{1}{w_{k+1}}\right|^2 \\
&\le \left\{\sum_{k=0}^{m}\left|w_{k+1}\langle f, e_{k+1}\rangle - \langle g, e_k\rangle\right|^2 \right. \\
&\qquad \left. + \sum_{k=m+1}^{l-1}\left|w_{k+1}\langle f, e_{k+1}\rangle\right|^2\right\} \cdot R^2 \\
&\le R^2\epsilon^2.
\end{aligned} \tag{17}$$

Therefore, (a) holds.

(a)⇒(b). Suppose that D is stable with Hyers-Ulam stability constant K. For any nonnegative integer k, let $f(z) = (1/w_{k+1})e_{k+1}$. Then $\|Df(z)\| = 1$, so there exists f_0 such that $Df_0 = 0$ and $\|f(z) - f_0(z)\| \le K$. Hence,

$$\frac{1}{w_{k+1}} - \|f_0(z)\| = \|f(z)\| - \|f_0(z)\| \le \|f(z) - f_0(z)\| \le K, \tag{18}$$

and consequently $0 < 1/w_{k+1} \le K + \|f_0(z)\|$ for any nonnegative integer k. This completes the proof. □

Example 6. We consider the comparison function $\gamma_2(z) = \sum_{k=0}^{\infty}(1/k!)z^k$; that is, $\gamma_k = 1/k!$; by a simple calculation, we have $w_k = 1$ and $1/w_k = 1$ $(k = 1, 2, \ldots)$, and hence operator D has Hyers-Ulam stability on the $E^2(\gamma_2)$.

Example 7. If we put comparison function $\gamma_3(z) = \sum_{k=0}^{\infty}(1/(k!)^2)z^k$, that is, $\gamma_k = 1/(k!)^2$, we have $w_k = 1/k$ and

$1/w_k = k$ $(k = 1, 2, \ldots)$, $\{1/w_k\}_{k=1}^{\infty}$ is unbounded, and hence operator D is not Hyers-Ulam stable on the $E^2(\gamma_3)$.

Example 8. We consider the comparison function $\gamma_4(z) = 1 + 2z + \sum_{k=2}^{\infty}(1/k \cdot k!)z^k$; we get $\gamma_0 = 1$, $\gamma_1 = 2$, $\gamma_k = 1/k \cdot k!$ $(k = 2, 3, \ldots)$, and by a simple calculation, we have $w_1 = 2$, $w_2 = 1/4$, $w_k = (k-1)/k$ $(k = 3, 4, \ldots)$ and $1/w_1 = 1/2$, $1/w_2 = 4$, $1/w_k = 1 + (1/(k-1))$ $(k = 3, 4, \ldots)$, where $\{1/w_k\}_{k=1}^{\infty}$ is bounded; hence, operator D has Hyers-Ulam stability on the $E^2(\gamma_4)$.

Remark 9. From Theorem 5 and Examples 6–8, we can see that the Hyers-Ulam stability of differentiation operator D on Hilbert spaces of entire functions $E^2(\gamma)$ depends on the comparison functions $\gamma(z)$. It shows that the comparison functions affects the behaviors of the operators and the functions in the corresponding Hilbert space $E^2(\gamma)$.

Next, we will show that the best constant of Hyers-Ulam stability exists.

Theorem 10. *If differentiation operator D has the Hyers-Ulam stability on Hilbert spaces of entire functions $E^2(\gamma)$, then $K_D = \sup\{1/w_n : n \geq 1\}$ and K_D is a HUS constant of D.*

Proof. Suppose that D has Hyers-Ulam stability on $E^2(\gamma)$. By the proof of Theorem 5, we know that $\sup\{1/w_n : n \geq 1\}$ is a constant of the Hyers-Ulam stability of differentiation operator D. Next, we show that it is the infimum of all the Hyers-Ulam stability constants. Let $K < \infty$ be an arbitrary Hyers-Ulam stability constant for D; put $f(z) = (1/w_{k+1})e_{k+1}$, and for any nonnegative integer k, we can obtain $\|Df(z)\| = 1$, so there exists $f_0 = c_0$, such that $Df_0 = 0$ and $\|f(z) - c_0\| \leq K$, and hence

$$\frac{1}{w_{k+1}} \leq \left\| \frac{1}{w_{k+1}} e_{k+1} - c_0 \right\| = \|f(z) - c_0\| \leq K, \tag{19}$$

and so $\sup\{1/w_n : n \geq 1\} \leq K$. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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