Research Article

SPDIEs and BSDEs Driven by Lévy Processes and Countable Brownian Motions

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The paper is devoted to solving a new class of backward stochastic differential equations driven by Lévy process and countable Brownian motions. We prove the existence and uniqueness of the solutions to the backward stochastic differential equations by constructing Cauchy sequence and fixed point theorem. Moreover, we give a probabilistic solution of stochastic partial differential integral equations by means of the solution of backward stochastic differential equations. Finally, we give an example to illustrate.

1. Introduction

The backward stochastic differential equations (BSDEs for short), in the nonlinear cases, were firstly introduced by Pardoux and Peng [1] in order to give a probabilistic interpretation for the solution of semilinear parabolic partial differential equations. In the past decades, the equations have been extensively considered because of the applications in mathematic finance [2, 3], stochastic games [4–6], and partial differential equations (PDEs for short) [7–10].

As the applications developed, different settings of BSDEs have been introduced. Pardoux and Peng [11] proposed a new class of BSDEs driven by two Brownian motions, which are called backward doubly stochastic differential equations (BDSDEs for short), in order to give a probabilistic interpretation for the solution of quasi-linear stochastic partial differential equations (SPDEs for short). Since then, many authors discussed various settings of BDSDEs, for example, Bally and Matoussi [12], Matoussi and Scheutzwow [13], Zhang and Zhao [14, 15], and the references therein.

In 2000, Nualart and Schoutens [16] gave a martingale representation of Lévy process. Furthermore, they [17] discussed the BSDEs driven by Lévy process and the application in finance. Following it, many authors were devoted to the BSDEs driven by Lévy process. Bahlali et al. [18] generalized the results [17] to the BSDEs driven by Teugels martingales associated with Lévy process and a Brownian motion. Also,

they gave the application in partial differential integral equations (PDIEs for short). Ren et al. [19] introduced a class of BDSDEs driven by Teugels martingales associated with Lévy process and two Brownian motions. They obtained the existence and uniqueness of solution and gave the probabilistic interpretation for solutions of stochastic partial differential integral equations (SPDIEs for short). Later, Hu and Ren [20] discussed BDSDEs driven by Teugels martingales associated with Lévy process and an adapted continuous increasing process. Recently, Duan et al. [21] made further discussion of reflected backward stochastic differential equations driven by countable Brownian motions under Lipschitz conditions. Owo [22] studied the equations with continuous coefficients.

To the best of our knowledge, there are no works on the BSDEs driven by Teugels martingales associated with Lévy processes and countably many Brownian motions. Thus, we will make the first attempt to study such problem in this paper.

The structure of this paper is organized as follows. In Section 2, we present some basic notions and assumptions. Section 3 is devoted to the existence and uniqueness of solutions for BSDEs driven by Teugels martingales associated with Lévy processes and countably many Brownian motions by means of martingale representation theorem, fixed point theorem, and constructing Cauchy sequence. In Section 4, we discuss the connection between the BSDEs and SPDIEs.

2. Notations

Let $T > 0$ be a fixed terminal time. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{0 \leq t \leq T}, P)$ be a complete probability space, let $\{\beta_j(t), t \in [0, T], j = 1, 2, \dots\}$ and $\{L_t, t \in [0, T]\}$ be mutually independent processes, where $\{\beta_j(t)\}$ is a sequence of $\mathbb{R}$ -valued standard Brownian motion and mutually independent, and $\{L_t\}$ is $\mathbb{R}$ -valued Lévy process corresponding to a standard Lévy measure ν such that $\int_{\mathbb{R}} (1 \wedge y)\nu(dy) < \infty$.

Let $\mathcal{N}$ denote the totality of P -null sets of $\mathcal{F}$. For each $t \in [0, T]$, we define

$$\mathcal{F}_t = \left(\bigvee_{j=1}^{\infty} \mathcal{F}_{t,T}^{\beta_j} \right) \vee \mathcal{F}_t^L, \quad (1)$$

where for any process η_t , $\mathcal{F}_{s,t}^{\eta} = \sigma\{\eta_r - \eta_s, s \leq r \leq t\} \vee \mathcal{N}$, $\mathcal{F}_t^{\eta} = \mathcal{F}_{0,t}^{\eta}$.

Let us introduce some spaces which will be carried out in the following parts.

- (i) $L^2(\Omega, \mathcal{F}_T, P)$ denotes the set of all $\mathcal{F}_T$ -measurable random variables ξ such that $E|\xi|^2 < \infty$.
- (ii) $\mathcal{H}^2$ denotes the space of $\mathbb{R}$ -valued, square integrable, and $\mathcal{F}_t$ -progressively measurable processes $\{\varphi_t : t \in [0, T]\}$ such that

$$|\varphi|^2 = E \int_0^T |\varphi_t|^2 dt < \infty. \quad (2)$$

And we denote by $\mathcal{P}^2$ the subspace of $\mathcal{H}^2$ formed by the predictable processes.

- (iii) $\mathcal{S}^2$ denotes the set of $\mathbb{R}$ -valued, $\mathcal{F}_t$ -measurable processes $\{\varphi_t : t \in [0, T]\}$ such that

$$E \left(\sup_{0 \leq t \leq T} |\varphi_t|^2 \right) < \infty. \quad (3)$$

Let l^2 be the space of $\mathbb{R}$ -valued sequences $\{x_n\}_{n=1}^{\infty}$ such that $\sum_{n=1}^{\infty} x_n^2 < \infty$. $\mathcal{H}^2(l^2)$ and $\mathcal{P}^2(l^2)$ denote the corresponding space of l^2 -valued processes endowed with the norm

$$\|\varphi\|^2 = \sum_{i=1}^{\infty} E \int_0^T |\varphi_t^{(i)}|^2 dt. \quad (4)$$

Now, we give the definition of the Teugels martingales denoted by $\{H_t^{(i)}\}$, associated with the Lévy processes $\{L_t, t \in [0, T]\}$, which is given by

$$H_t^{(i)} = c_{i,i} Y_t^{(i)} + c_{i,i-1} Y_t^{(i-1)} + \dots + c_{i,1} Y_t^{(1)}, \quad (5)$$

where $Y_t^{(i)} = L_t^{(i)} - E[L_t^{(i)}] = L_t^{(i)} - tE[L_1^{(i)}]$ for all $i \geq 1$ and $L_t^{(i)}$ are power-jump processes. That is, $L_t^{(1)} = L_t$ and $L_t^{(i)} = \sum_{0 \leq s \leq t} (\Delta L_s)^i$ for $i \geq 2$, where $\Delta L_t = L_t - L_{t-}$ and $L_{t-} = \lim_{s \uparrow t} L_s$. The coefficients $c_{i,k}$ correspond to the orthonormalization of the polynomials $1, x, x^2, \dots$ with respect to the measure $\mu(dx) = x^2 \nu(dx) + \sigma^2 \delta_0(dx)$:

$$q_{i-1} = c_{i,i} x^{i-1} + c_{i,i-1} x^{i-2} + \dots + c_{i,1}. \quad (6)$$

We set

$$p_i(x) = x q_{i-1}(x) = c_{i,i} x^i + c_{i,i-1} x^{i-1} + \dots + c_{i,1} x. \quad (7)$$

For more details on Teugels martingales associated with the Lévy process $\{L_t, t \in [0, T]\}$, we can refer to [16, 17].

In this paper, we will discuss the following backward stochastic differential equations driven by Lévy process and countably many Brownian motions:

$$\begin{aligned} Y_t &= \xi + \int_t^T f(s, Y_{s-}, Z_s) ds \\ &+ \sum_{j=1}^{\infty} \int_t^T g_j(s, Y_{s-}, Z_s) d\beta_j(s) - \sum_{i=1}^{\infty} \int_t^T Z_s^{(i)} dH_s^{(i)}, \end{aligned} \quad (8)$$

$t \in [0, T],$

where the integral with respect to $\beta_j(s)$ is the classical backward Itô integral and the integral with respect to $H_t^{(i)}$ is standard forward Itô integral.

With the above preparation, we introduce the definition of solution of (8).

Definition 1. A pair of processes $(Y_t, Z_t)_{t \in [0, T]} \in \mathcal{S}^2 \times \mathcal{P}^2(l^2)$ is a solution to (8), if it satisfies (8).

In order to get the solution of (8), we propose the following assumptions:

(H1) $\xi \in L^2(\Omega, \mathcal{F}_T, P)$.

(H2) The functions $f : [0, T] \times \Omega \times \mathbb{R} \times l^2 \rightarrow \mathbb{R}$ and $g : [0, T] \times \Omega \times \mathbb{R} \times l^2 \rightarrow \mathbb{R}$ are progressively measurable such that

$$\begin{aligned} E \int_0^T |f(t, 0, 0)|^2 dt &< \infty, \\ \sum_{j=1}^{\infty} E \int_0^T |g_j(t, 0, 0)|^2 dt &< \infty. \end{aligned} \quad (9)$$

(H3) There exist some nonnegative constants C, C_j, α_j with $\sum_{j=1}^{\infty} C_j < \infty$ and $\alpha = \sum_{j=1}^{\infty} \alpha_j < 1$ such that, for any $t \in [0, T], y_1, y_2 \in \mathbb{R}, z_1, z_2 \in l^2$,

$$\begin{aligned} &|f(t, y_1, z_1) - f(t, y_2, z_2)|^2 \\ &\leq C(|y_1 - y_2|^2 + |z_1 - z_2|^2), \\ &|g_j(t, y_1, z_1) - g_j(t, y_2, z_2)|^2 \\ &\leq C_j |y_1 - y_2|^2 + \alpha_j |z_1 - z_2|^2. \end{aligned} \quad (10)$$

Our conclusions depend on the extensive Itô formula in [19].

Lemma 2. Let $\alpha \in \mathcal{S}^2, \beta, \gamma, \eta$ and $\zeta^{(i)} \in \mathcal{H}^2$ such that

$$\begin{aligned} \alpha_t &= \alpha_0 + \int_0^t \beta_s ds + \int_0^t \gamma_s dB_s + \int_0^t \eta_s dW_s \\ &\quad + \sum_{i=1}^{\infty} \int_0^t \zeta_s^{(i)} dH_s^{(i)}, \quad 0 \leq t \leq T. \end{aligned} \quad (11)$$

Then

$$\begin{aligned} |\alpha_t|^2 &= |\alpha_0|^2 + 2 \int_0^t \alpha_s \beta_s ds + 2 \int_0^t \alpha_s \gamma_s dB_s \\ &\quad + 2 \int_0^t \alpha_s \eta_s dW_s + 2 \sum_{i=1}^{\infty} \int_0^t \alpha_s \zeta_s^{(i)} dH_s^{(i)} \\ &\quad - \int_0^t |\gamma_s|^2 ds + \int_0^t |\eta_s|^2 ds \\ &\quad + \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \int_0^t \zeta_s^{(i)} \zeta_s^{(j)} d[H^{(i)}, H^{(j)}]_s. \end{aligned} \quad (12)$$

Noting that $\langle H^{(i)}, H^{(j)} \rangle_t = \delta_{ij}t$, we have

$$\begin{aligned} E|\alpha_t|^2 &= |\alpha_0|^2 + 2E \int_0^t \alpha_s \beta_s ds - E \int_0^t |\gamma_s|^2 ds \\ &\quad + E \int_0^t |\eta_s|^2 ds + \sum_{i=1}^{\infty} E \int_0^t (\zeta_s^{(i)})^2 ds. \end{aligned} \quad (13)$$

3. Existence and Uniqueness

In this section, we begin with establishing the existence and uniqueness of (8) in the case that f and g do not depend on Y and Z with finite noise; that is,

$$\begin{aligned} Y_t &= \xi + \int_t^T f(s) ds + \sum_{j=1}^n \int_t^T g_j(s) d\beta_j(s) \\ &\quad - \sum_{i=1}^{\infty} \int_t^T Z_s^{(i)} dH_s^{(i)}, \quad t \in [0, T]. \end{aligned} \quad (14)$$

Theorem 3. Assume that (H1)–(H3) hold. Then, there exists a unique solution $(Y_t, Z_t) \in \mathcal{S}^2 \times \mathcal{P}^2(l^2)$ satisfying (14).

Proof. For $E \int_0^T |f(s)|^2 ds < \infty$, $\sum_{j=1}^{\infty} \int_0^T |g_j(s)|^2 ds < \infty$, we set the filtration $\{\mathcal{G}_t : t \in [0, T]\}$ as follows:

$$\mathcal{G}_t = \mathcal{F}_t^L \vee \left(\bigvee_{j=1}^n \mathcal{F}_t^{\beta_j} \right) \quad (15)$$

and the $\mathcal{G}_t$ -square integrable martingale is as follows:

$$\begin{aligned} M_t &= E \left[\xi + \int_0^T f(s) ds + \sum_{j=1}^n \int_0^T g_j(s) d\beta_j(s) \mid \mathcal{G}_t \right]. \end{aligned} \quad (16)$$

By the predictable representation property, there exists $Z \in \mathcal{P}^2(l^2)$ such that

$$M_t = M_0 + \sum_{i=1}^{\infty} \int_0^t Z_s^{(i)} dH_s^{(i)}. \quad (17)$$

So, we have

$$M_T = M_t + \sum_{i=1}^{\infty} \int_t^T Z_s^{(i)} dH_s^{(i)}. \quad (18)$$

Let

$$\begin{aligned} Y_t &= M_t - \int_0^t f(s) ds - \sum_{j=1}^n \int_0^t g_j(s) d\beta_j(s) \\ &= M_T - \int_0^t f(s) ds - \sum_{j=1}^n \int_0^t g_j(s) d\beta_j(s) \\ &\quad - \sum_{i=1}^{\infty} \int_t^T Z_s^{(i)} dH_s^{(i)}. \end{aligned} \quad (19)$$

Hence, we have

$$\begin{aligned} Y_t &= \xi + \int_t^T f(s) ds + \sum_{j=1}^n \int_t^T g_j(s) d\beta_j(s) \\ &\quad - \sum_{i=1}^{\infty} \int_t^T Z_s^{(i)} dH_s^{(i)}. \end{aligned} \quad (20)$$

From the above equality, we can deduce the existence of solution of (14). The proof of uniqueness is a procedure similar to that in [11]; we omit it. $\square$

With the preparation of above, we consider the following BSDEs with finite noise:

$$\begin{aligned} Y_t &= \xi + \int_t^T f(s, Y_{s-}, Z_s) ds \\ &\quad + \sum_{j=1}^n \int_t^T g_j(s, Y_{s-}, Z_s) d\beta_j(s) \\ &\quad - \sum_{i=1}^{\infty} \int_t^T Z_s^{(i)} dH_s^{(i)}, \quad t \in [0, T]. \end{aligned} \quad (21)$$

Theorem 4. Assume that (H1)–(H3) hold. Then, there exists a unique solution $(Y_t, Z_t) \in \mathcal{S}^2 \times \mathcal{P}^2(l^2)$ satisfying (21).

Proof. From Theorem 3, for each $(\bar{Y}_t, \bar{Z}_t)$, there exists (Y_t, Z_t) satisfying

$$\begin{aligned} Y_t &= \xi + \int_t^T f(s, \bar{Y}_{s-}, \bar{Z}_s) ds \\ &\quad + \sum_{j=1}^n \int_t^T g_j(s, \bar{Y}_{s-}, \bar{Z}_s) d\beta_j(s) \\ &\quad - \sum_{i=1}^{\infty} \int_t^T Z_s^{(i)} dH_s^{(i)}, \quad t \in [0, T]. \end{aligned} \quad (22)$$

Following it, we define a map Φ from $\mathcal{S}^2 \times \mathcal{P}^2(l^2)$ to itself; that is, $\Phi(\bar{Y}_t, \bar{Z}_t) = (Y_t, Z_t)$. In the following parts, we will show that Φ is a strict contraction with the norm

$$\|(Y, Z)\|_\beta^2 = E \int_0^T e^{\beta s} [|Y_{s-}|^2 + \|Z_s\|^2] ds, \quad (23)$$

for suitable constant $\beta > 0$. In addition, $\mathcal{S}^2 \times \mathcal{P}^2(l^2)$ is a Banach space.

Set $(Y, Z) = \Phi(\bar{Y}, \bar{Z})$, $(Y', Z') = \Phi(\bar{Y}', \bar{Z}')$, where (Y, Z) and (Y', Z') are the solutions of (22) associated with $(\xi, f(t, \bar{Y}, \bar{Z}), g_j(t, \bar{Y}, \bar{Z}))$ and $(\xi, f(t, \bar{Y}', \bar{Z}'), g_j(t, \bar{Y}', \bar{Z}'))$, respectively. Let $(\bar{Y}_t, \bar{Z}_t) = (Y_t - Y'_t, Z_t - Z'_t)$. Applying Itô formula to $e^{\beta t} |Y_t - Y'_t|^2$, we have

$$\begin{aligned} e^{\beta t} (Y_t - Y'_t)^2 &= 2 \int_t^T e^{\beta s} (Y_{s-} - Y'_{s-}) \left[f(s, \bar{Y}_{s-}, \bar{Z}_s) \right. \\ &\quad \left. - f(s, \bar{Y}'_{s-}, \bar{Z}'_s) \right] ds + 2 \sum_{j=1}^n \int_t^T e^{\beta s} (Y_{s-} - Y'_{s-}) \\ &\quad \cdot \left[g_j(s, \bar{Y}_{s-}, \bar{Z}_s) - g_j(s, \bar{Y}'_{s-}, \bar{Z}'_s) \right] d\beta_j(s) \\ &\quad - 2 \sum_{i=1}^\infty \int_t^T e^{\beta s} (Y_{s-} - Y'_{s-}) (Z_s^{(i)} - Z_s'^{(i)}) dH_s^{(i)} \\ &\quad - \beta \int_t^T e^{\beta s} (Y_{s-} - Y'_{s-})^2 ds \\ &\quad + \sum_{j=1}^n \int_t^T e^{\beta s} \left| g_j(s, \bar{Y}_{s-}, \bar{Z}_s) - g_j(s, \bar{Y}'_{s-}, \bar{Z}'_s) \right|^2 ds \\ &\quad - \sum_{i=1}^\infty \sum_{k=1}^\infty \int_t^T e^{\beta s} (Z_s^{(i)} - Z_s'^{(i)}) \\ &\quad \cdot (Z_s^{(k)} - Z_s'^{(k)}) d[H^{(i)}, H^{(k)}]_s. \end{aligned} \quad (24)$$

Taking mathematical expectation on both sides, we obtain

$$\begin{aligned} E e^{\beta t} (Y_t - Y'_t)^2 &+ \sum_{i=1}^\infty E \int_t^T e^{\beta s} (Z_s^{(i)} - Z_s'^{(i)})^2 ds \\ &+ \beta E \int_t^T e^{\beta s} (Y_{s-} - Y'_{s-})^2 ds = 2E \int_t^T e^{\beta s} (Y_{s-} \\ &\quad - Y'_{s-}) \left[f(s, \bar{Y}_{s-}, \bar{Z}_s) - f(s, \bar{Y}'_{s-}, \bar{Z}'_s) \right] ds \\ &\quad + \sum_{j=1}^n E \int_t^T e^{\beta s} \left| g_j(s, \bar{Y}_{s-}, \bar{Z}_s) - g_j(s, \bar{Y}'_{s-}, \bar{Z}'_s) \right|^2 ds. \end{aligned} \quad (25)$$

With the conditions of (H1)–(H3), it follows that

$$\begin{aligned} E e^{\beta t} (Y_t - Y'_t)^2 &+ \sum_{i=1}^\infty E \int_t^T e^{\beta s} |Z_s^{(i)} - Z_s'^{(i)}|^2 ds \\ &+ \beta E \int_t^T e^{\beta s} (Y_{s-} - Y'_{s-})^2 ds \leq \frac{2C}{1 - \sum_{j=1}^\infty \alpha_j} \\ &\cdot E \int_t^T e^{\beta s} |Y_{s-} - Y'_{s-}|^2 ds + \frac{1 - \sum_{j=1}^\infty \alpha_j}{2} \\ &\cdot E \int_t^T e^{\beta s} \left(|\bar{Y}_{s-} - \bar{Y}'_{s-}|^2 + \|\bar{Z}_s - \bar{Z}'_s\|^2 \right) ds \\ &+ \left(\sum_{j=1}^n C_j \right) E \int_t^T e^{\beta s} |\bar{Y}_{s-} - \bar{Y}'_{s-}|^2 ds + \left(\sum_{j=1}^n \alpha_j \right) \\ &\cdot E \int_t^T e^{\beta s} \|\bar{Z}_s - \bar{Z}'_s\|^2 ds \leq \frac{2C}{1 - \alpha} \\ &\cdot E \int_t^T e^{\beta s} |Y_{s-} - Y'_{s-}|^2 ds + \left(\sum_{j=1}^\infty C_j + \frac{1 - \alpha}{2} \right) \\ &\cdot E \int_t^T e^{\beta s} |\bar{Y}_{s-} - \bar{Y}'_{s-}|^2 ds + \frac{1 + \alpha}{2} \\ &\cdot E \int_t^T e^{\beta s} \|\bar{Z}_s - \bar{Z}'_s\|^2 ds. \end{aligned} \quad (26)$$

Furthermore, we have

$$\begin{aligned} E e^{\beta t} (Y_t - Y'_t)^2 &+ E \int_t^T e^{\beta s} \|Z_s - Z'_s\|^2 ds \\ &+ \left(\beta - \frac{2C}{1 - \alpha} \right) E \int_t^T e^{\beta s} (Y_{s-} - Y'_{s-})^2 ds \\ &\leq \left(\sum_{j=1}^\infty C_j + \frac{1 - \alpha}{2} \right) E \int_t^T e^{\beta s} |\bar{Y}_{s-} - \bar{Y}'_{s-}|^2 ds \\ &\quad + \frac{1 + \alpha}{2} E \int_t^T e^{\beta s} \|\bar{Z}_s - \bar{Z}'_s\|^2 ds. \end{aligned} \quad (27)$$

Let $\gamma = 2C/(1 - \alpha)$, $\bar{C} = 2(\sum_{j=1}^\infty C_j + (1 - \alpha)/2)/(1 + \alpha)$, and $\beta = \gamma + \bar{C}$, and we have

$$\begin{aligned} E e^{\beta t} |Y_t - Y'_t|^2 &+ E \int_t^T e^{\beta s} \|Z_s - Z'_s\|^2 ds \\ &+ \bar{C} E \int_t^T e^{\beta s} (Y_{s-} - Y'_{s-})^2 ds \\ &\leq \frac{1 + \alpha}{2} E \int_t^T e^{\beta s} \left(\bar{C} |\bar{Y}_{s-} - \bar{Y}'_{s-}|^2 + \|\bar{Z}_s - \bar{Z}'_s\|^2 \right) ds. \end{aligned} \quad (28)$$

Moreover,

$$\begin{aligned} & E \int_t^T e^{\beta s} \bar{C} |Y_{s-} - Y'_{s-}|^2 ds + E \int_t^T e^{\beta s} \|Z_s - Z'_s\|^2 ds \\ & \leq \frac{1+\alpha}{2} E \int_t^T e^{\beta s} \left(\bar{C} |\bar{Y}_{s-} - \bar{Y}'_{s-}|^2 + \|\bar{Z}_s - \bar{Z}'_s\|^2 \right) ds. \end{aligned} \quad (29)$$

That is,

$$\|(Y_t, Z_t)\|_\beta^2 \leq \frac{1+\alpha}{2} \|(\bar{Y}_t, \bar{Z}_t)\|_\beta^2. \quad (30)$$

It follows that Φ is a strict contraction with the norm $\|\cdot\|_\beta$. Then, from B-D-G inequality, Φ has a unique fixed point $(Y_t, Z_t) \in \mathcal{S}^2 \times \mathcal{P}^2(l^2)$, which is the unique solution of (21). $\square$

Theorem 5. Under the conditions (H1)–(H3), there exists a unique solution $(Y_t, Z_t) \in \mathcal{S}^2 \times \mathcal{P}^2(l^2)$ satisfying (8).

Proof (existence). From Theorem 4, for each n , there exists a unique solution of (21) under the conditions (H1)–(H3) denoted by (Y_t^n, Z_t^n) :

$$\begin{aligned} Y_t^n &= \xi + \int_t^T f(s, Y_{s-}^n, Z_s^n) ds \\ &+ \sum_{j=1}^n \int_t^T g_j(s, Y_{s-}^n, Z_s^n) d\beta_j(s) \\ &- \sum_{i=1}^\infty \int_t^T Z_s^{n(i)} dH_s^{(i)}, \quad t \in [0, T]. \end{aligned} \quad (31)$$

In the following part, we claim that (Y_t^n, Z_t^n) is Cauchy sequence in $\mathcal{S}^2 \times \mathcal{P}^2(l^2)$. Applying Itô formula to $|Y_t^n - Y_t^m|^2$, without loss of generality, we let $n < m$; then

$$\begin{aligned} & |Y_t^n - Y_t^m|^2 + \int_t^T \|Z_s^n - Z_s^m\|^2 ds = 2 \int_t^T (Y_{s-}^n - Y_{s-}^m) \\ & \cdot (f(s, Y_{s-}^n, Z_s^n) - f(s, Y_{s-}^m, Z_s^m)) ds \\ & + 2 \sum_{j=n+1}^m \int_t^T |g_j(s, Y_{s-}^n, Z_s^n) - g_j(s, Y_{s-}^m, Z_s^m)|^2 ds \\ & + 2 \sum_{j=n+1}^m \int_t^T (Y_{s-}^n - Y_{s-}^m) (g_j(s, Y_{s-}^n, Z_s^n) \\ & - g_j(s, Y_{s-}^m, Z_s^m)) d\beta_j(s) \\ & - 2 \sum_{i=1}^\infty \int_t^T (Y_{s-}^n - Y_{s-}^m) (Z_s^{n(i)} - Z_s^{m(i)}) dH_s^{(i)} \\ & - \int_t^T \sum_{i=1}^\infty \sum_{j=1}^\infty (Z_s^{n(i)} - Z_s^{m(i)}) \\ & \cdot (Z_s^{m(j)} - Z_s^{n(j)}) d[H^{(i)}, H^{(j)}]_s. \end{aligned} \quad (32)$$

Taking mathematical expectation on both sides, we have

$$\begin{aligned} & E |Y_t^n - Y_t^m|^2 + E \int_t^T \|Z_s^n - Z_s^m\|^2 ds \\ & = 2E \int_t^T (Y_{s-}^n - Y_{s-}^m) \\ & \cdot (f(s, Y_{s-}^n, Z_s^n) - f(s, Y_{s-}^m, Z_s^m)) ds \\ & + 2 \sum_{j=n+1}^m E \int_t^T |g_j(s, Y_{s-}^n, Z_s^n) - g_j(s, Y_{s-}^m, Z_s^m)|^2 ds \\ & \leq \frac{2C}{1-\alpha} E \int_t^T |Y_{s-}^n - Y_{s-}^m|^2 ds + \frac{1-\alpha}{2} \\ & \cdot E \int_t^T |Y_{s-}^n - Y_{s-}^m|^2 ds + \frac{1+\alpha}{2} \\ & \cdot E \int_t^T \|Z_s^n - Z_s^m\|^2 ds + \left(\sum_{j=n+1}^m \alpha_j \right) \\ & \cdot E \int_t^T \|Z_s^n - Z_s^m\|^2 ds + \left(\sum_{j=n+1}^m C_j \right) \\ & \cdot E \int_t^T |Y_{s-}^n - Y_{s-}^m|^2 ds \leq \left(\frac{2C}{1-\alpha} + \frac{1-\alpha}{2} \right. \\ & \left. + \sum_{j=1}^m C_j \right) E \int_t^T |Y_{s-}^n - Y_{s-}^m|^2 ds + \frac{1+\alpha}{2} \\ & \cdot E \int_t^T \|Z_s^n - Z_s^m\|^2 ds. \end{aligned} \quad (33)$$

Therefore, we obtain

$$\begin{aligned} & E |Y_t^n - Y_t^m|^2 + \frac{1-\alpha}{2} E \int_t^T \|Z_s^n - Z_s^m\|^2 ds \\ & \leq \left(\frac{2C}{1-\alpha} + \frac{1-\alpha}{2} + \sum_{j=1}^m C_j \right) E \int_t^T |Y_{s-}^n - Y_{s-}^m|^2 ds. \end{aligned} \quad (34)$$

By the Gronwall inequality and B-D-G inequality, we have

$$E \left[\sup_{0 \leq t \leq T} \int_t^T |Y_{s-}^n - Y_{s-}^m|^2 ds \right] \rightarrow 0. \quad (35)$$

Denote its limit by (Y_t, Z_t) ; from the continuity of f and g and Lebesgue dominated convergence theorem, we can imply that it is the solution of (8).

Uniqueness. We set

$$\begin{aligned} \Psi_M(x) &= x^2 1_{\{-M \leq x \leq M\}} + M(2x - M) 1_{\{x > M\}} \\ &- M(2x + M) 1_{\{x < -M\}}. \end{aligned} \quad (36)$$

If we define $\Psi'_M(x)/x = 2$, when $x = 0$, then, $0 \leq \Psi'_M(x)/x \leq 2$. Let (Y_t^i, Z_t^i) ($i = 1, 2$) be two solutions of (8); we apply Itô

formula to $e^{\beta t} \Psi_M(\bar{Y}_t)$, where $(\bar{Y}_t, \bar{Z}_t) = (Y_t^1 - Y_t^2, Z_t^1 - Z_t^2)$, and β is constant:

$$\begin{aligned}
& e^{\beta t} \Psi_M(\bar{Y}_t) + \beta \int_t^T e^{\beta s} \Psi_M(\bar{Y}_{s-}) ds \\
& + \int_t^T e^{\beta s} 1_{\{-M \leq \bar{Y}_s \leq M\}} \|\bar{Z}_s\|^2 ds = \int_t^T e^{\beta s} \Psi'_M(\bar{Y}_{s-}) \\
& \cdot (f(s, Y_{s-}^1, Z_s^1) - f(s, Y_{s-}^2, Z_s^2)) ds \\
& + \sum_{j=1}^{\infty} \int_t^T e^{\beta s} 1_{\{-M \leq \bar{Y}_s \leq M\}} |g_j(s, Y_{s-}^1, Z_s^1) \\
& - g_j(s, Y_{s-}^2, Z_s^2)|^2 ds - \sum_{j=1}^{\infty} \int_t^T e^{\beta s} \Psi'_M(\bar{Y}_{s-}) \\
& \cdot (g_j(s, Y_{s-}^1, Z_s^1) - g_j(s, Y_{s-}^2, Z_s^2)) d\beta_j(s) \\
& - \sum_{i=1}^{\infty} \int_t^T e^{\beta s} (Z_s^{1(i)} - Z_s^{2(i)}) dH_s^{(i)} \\
& - \int_t^T \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} e^{\beta s} (Z_s^{1(i)} - Z_s^{1(j)}) \\
& \cdot (Z_s^{2(i)} - Z_s^{2(j)}) d[H^{(i)}, H^{(j)}]_s.
\end{aligned} \tag{37}$$

Taking expectation on both sides,

$$\begin{aligned}
& E e^{\beta t} \Psi_M(\bar{Y}_t) + \beta E \int_t^T e^{\beta s} \Psi_M(\bar{Y}_{s-}) ds \\
& + E \int_t^T e^{\beta s} 1_{\{-M \leq \bar{Y}_s \leq M\}} \|\bar{Z}_s\|^2 ds \\
& = E \int_t^T e^{\beta s} \Psi'_M(\bar{Y}_{s-}) (f(s, Y_{s-}^1, Z_s^1) \\
& - f(s, Y_{s-}^2, Z_s^2)) ds \\
& + \sum_{j=1}^{\infty} E \int_t^T e^{\beta s} 1_{\{-M \leq \bar{Y}_s \leq M\}} |g_j(s, Y_{s-}^1, Z_s^1) \\
& - g_j(s, Y_{s-}^2, Z_s^2)|^2 ds \leq \left(\frac{2C}{1-\alpha} + \sum_{j=1}^{\infty} C_j \right. \\
& \left. + \frac{1-\alpha}{2} \right) E \int_t^T e^{\beta s} |\bar{Y}_{s-}|^2 ds + \frac{1+\alpha}{2} \\
& \cdot E \int_t^T e^{\beta s} \|\bar{Z}_s\|^2 ds.
\end{aligned} \tag{38}$$

Let $M \rightarrow \infty$, β is large enough, and we have

$$\begin{aligned}
& E e^{\beta t} |\bar{Y}_t|^2 \\
& + \left(\beta - \frac{2C}{1-\alpha} - \sum_{j=1}^{\infty} C_j - \frac{1-\alpha}{2} \right) E \int_t^T e^{\beta s} |\bar{Y}_{s-}|^2 ds \\
& + \frac{1-\alpha}{2} E \int_t^T e^{\beta s} \|\bar{Z}_s\|^2 ds \leq 0.
\end{aligned} \tag{39}$$

So, we complete the proof of uniqueness. $\square$

4. Application to SPDIEs

In this section, we consider the application of BSDEs driven by Lévy processes and countably many Brownian motions to the solution of a class of SPDIEs. Suppose that our Lévy processes have the form of $L_t = bt + \int_{|z| \leq 1} (z(N_t(\cdot, dz)) - t\nu(dz))$, where $N_t(\omega, dz)$ denotes the random measure such that $\int_{\Lambda} N_t(\cdot, dz)$ is a Poisson process with parameter $\nu(\Lambda)$ for all the set Λ where $0 \notin \bar{\Lambda}$.

Consider the following SDE:

$$X_t = x + \int_0^t \sigma(X_{s-}) dL_s, \quad t \in [0, T]. \tag{40}$$

Under adequate conditions, there exists a unique solution of (40).

In order to get the main result, we give a technical lemma that appears in [17].

Lemma 6. Let $c : \Omega \times [0, T] \times \mathbb{R} \rightarrow \mathbb{R}$ be a measurable function such that

$$c(s, y) \leq a_s (y^2 \wedge |y|), \quad a.s., \tag{41}$$

where $\{a_s : s \in [0, T]\}$ is a nonnegative predictable process such that $E \int_0^T a_s^2 ds < \infty$. Then, for each $0 \leq t \leq T$, we have

$$\begin{aligned}
\sum_{t \leq s \leq T} c(s, \Delta L_s) &= \sum_{i=1}^{\infty} \int_t^T \langle c(s, \cdot), p_i \rangle_{L^2(\nu)} dH_s^{(i)} \\
&+ \int_t^T \int_{\mathbb{R}} c(s, y) \nu(dy) ds.
\end{aligned} \tag{42}$$

Consider the following BSDEs driven by Lévy processes and countably many Brownian motions:

$$\begin{aligned}
Y_t &= h(L_T) + \int_t^T f(s, X_{s-}, Y_{s-}, Z_s) ds \\
&+ \sum_{j=1}^{\infty} \int_t^T g_j(s, X_{s-}, Y_{s-}, Z_s) d\beta_j(s) \\
&- \sum_{i=1}^{\infty} \int_t^T Z_s^{(i)} dH_s^{(i)}, \quad t \in [0, T],
\end{aligned} \tag{43}$$

where $E|h(L_T)|^2 < \infty$.

Define

$$u^1(t, x, y) = u(t, x + y) - u(t, x) - \frac{\partial u}{\partial x}(t, x) y, \tag{44}$$

where u is the solution of the following SPDIEs:

$$\begin{aligned}
& \frac{\partial u}{\partial t}(t, x) + \frac{1}{2} \sigma^2(x) \frac{\partial^2 u}{\partial x^2}(t, x) + a' \frac{\partial u}{\partial x}(t, x) \\
& + \int_{\mathbb{R}} u^1(t, x, y) \nu(dy) \\
& + f\left[t, u(t, x), \frac{\partial u}{\partial x}(t, x), \{u^{(i)}(t, x)\}_{i=1}^{\infty}\right] \\
& + \sum_{j=1}^{\infty} g_j\left[t, u(t, x), \frac{\partial u}{\partial x}(t, x), \{u^{(i)}(t, x)\}_{i=1}^{\infty}\right] \dot{\beta}_j(t) dt \\
& = 0,
\end{aligned} \tag{45}$$

with $u(T, x) = h(x)$, $a' = a + \int_{|y| \leq 1} y \nu(dy)$, and

$$u^1(t, x) = \int_{\mathbb{R}} u^1(t, x, y) p_1(y) \nu(dy) + \frac{\partial u}{\partial x}(t, x) \left(\int_{\mathbb{R}} y^2 \nu(dy) \right)^{1/2}, \quad (46)$$

and for $i \geq 2$

$$u^{(i)}(t, x) = \int_{\mathbb{R}} u^1(t, x, y) p_i(y) \nu(dy). \quad (47)$$

In order to give the meaning of $\beta_j(t)dt$, we write the above SPDIEs in the following integral form:

$$\begin{aligned} u(t, x) = & h(x) + \int_t^T \left[\frac{1}{2} \sigma^2(x) \frac{\partial^2 u}{\partial x^2}(s, x) \right. \\ & + \int_{\mathbb{R}} u^1(s, x, y) \nu(dy) + a' \frac{\partial u}{\partial x}(s, x) \Big] ds \\ & + \int_t^T f \left[s, u(s, x), \frac{\partial u}{\partial x}(s, x), \{u^{(i)}(s, x)\}_{i=1}^{\infty} \right] ds \\ & + \sum_{j=1}^{\infty} \int_t^T g_j \left[s, u(s, x), \frac{\partial u}{\partial x}(s, x), \right. \\ & \left. \{u^{(i)}(s, x)\}_{i=1}^{\infty} \right] d\beta_j(s). \end{aligned} \quad (48)$$

Suppose that u is $\mathcal{C}^{1,2}$ function that $\partial u / \partial t$ and $\partial^2 u / \partial x^2$ are bounded by a polynomial function of x , uniformly in t . Next, we give the main result of this section.

Theorem 7. *The unique adapted solution of (43) is given by*

$$\begin{aligned} Y_t &= u(t, L_t), \\ Z_t^{(i)} &= \int_{\mathbb{R}} u^1(t, L_{t-}, y) p_i(y) \nu(dy), \quad i \geq 2, \\ Z_t^{(1)} &= \int_{\mathbb{R}} u^1(t, L_{t-}, y) p_1(y) \nu(dy) \\ &+ \frac{\partial u}{\partial x}(t, L_{t-}) \left(\int_{\mathbb{R}} y^2 \nu(dy) \right)^{1/2}. \end{aligned} \quad (49)$$

Proof. Applying Itô formula to $u(s, L_s)$, we obtain

$$\begin{aligned} u(T, L_T) - u(t, L_t) &= \int_t^T \frac{\partial u}{\partial s}(s, L_s) ds + \frac{1}{2} \int_t^T \sigma^2(L_s) \frac{\partial^2 u}{\partial x^2}(s, L_s) ds \\ &+ \int_t^T \frac{\partial u}{\partial x}(s, L_{s-}) dL_s \\ &+ \sum_{t \leq s \leq T} \left[u(s, L_s) - u(s, L_{s-}) - \frac{\partial u}{\partial x}(s, L_{s-}) \Delta L_s \right]. \end{aligned} \quad (50)$$

We apply Lemma 6 to $u(s, L_{s-} + y) - u(s, L_{s-}) - (\partial u / \partial x)(s, L_{s-})y$, and then

$$\begin{aligned} & \sum_{t \leq s \leq T} \left[u(s, L_s) - u(s, L_{s-}) - \frac{\partial u}{\partial x}(s, L_{s-}) \Delta L_s \right] \\ &= \sum_{i=1}^{\infty} \int_t^T \left(\int_{\mathbb{R}} u^1(s, L_{s-}, y) p_i(y) \nu(dy) \right) dH_s^{(i)} \\ &+ \int_t^T \int_{\mathbb{R}} u^1(s, L_{s-}, y) \nu(dy) ds. \end{aligned} \quad (51)$$

Note that

$$L_t = Y_t^{(1)} + tEL_1 = \left(\int_{\mathbb{R}} y^2 \nu(dy) \right)^{1/2} H_t^{(1)} + tEL_1, \quad (52)$$

where $EL_1 = a + \int_{|y| \geq 1} y \nu(dy)$.

Substituting (51) and (52) into (50), we obtain

$$\begin{aligned} h(L_T) - u(t, L_t) &= \int_t^T \frac{\partial u}{\partial s}(s, L_{s-}) ds + \frac{1}{2} \\ &\cdot \int_t^T \sigma^2(L_s) \frac{\partial^2 u}{\partial x^2}(s, L_s) ds \\ &+ \sum_{i=1}^{\infty} \int_t^T \left(\int_{\mathbb{R}} u^1(s, L_{s-}, y) p_i(y) \nu(dy) \right) dH_s^{(i)} \\ &+ \int_t^T \int_{\mathbb{R}} u^1(s, L_{s-}, y) \nu(dy) ds + \int_t^T \frac{\partial u}{\partial s}(s, L_{s-}) \\ &\cdot \left[\left(\int_{\mathbb{R}} y^2 \nu(dy) \right)^{1/2} dH_s^{(1)} + a' ds \right] \end{aligned} \quad (53)$$

$$\begin{aligned} h(L_T) - u(t, L_t) &= \int_t^T \left[\frac{\partial u}{\partial s}(s, L_{s-}) + \frac{1}{2} \right. \\ &\cdot \sigma^2(L_s) \frac{\partial^2 u}{\partial x^2}(s, L_s) + a' \frac{\partial u}{\partial s}(s, L_{s-}) \Big] ds \\ &+ \int_t^T \int_{\mathbb{R}} u^1(s, L_{s-}, y) \nu(dy) ds + \int_t^T \left[\frac{\partial u}{\partial s}(s, L_{s-}) \right. \\ &\cdot \int_{\mathbb{R}} y^2 \nu(dy)^{1/2} \\ &+ \int_{\mathbb{R}} u^1(s, L_{s-}, y) p_1(y) \nu(dy) \Big] dH_s^{(1)} \\ &+ \sum_{i=2}^{\infty} \int_t^T \int_{\mathbb{R}} u^1(s, L_{s-}, y) p_i(y) \nu(dy) dH_s^{(i)}. \end{aligned} \quad (54)$$

From (54), we can derive the result. $\square$

In the following, we give an example of SPDIEs.

Example 8. Suppose that the Lévy process L has the form of $L_t = at + \sum_{i=1}^{\infty} (N_t^{(i)} - \alpha_i t)$, where $\{N_t^{(i)}\}_{i=1}^{\infty}$ is a sequence of independent Poisson processes with parameters $\{\alpha_i\}_{i=1}^{\infty}$ ($\alpha_i > 0$).

Its Lévy measure is $\nu(dx) = \sum_{i=1}^{\infty} \alpha_i \delta_{\beta_i}(dx)$, where $\delta_{\beta_i}(dx)$ denotes the positive mass measure at $\beta_i \in \mathbb{R}$ of size 1. Moreover, we assume that $\sum_{i=1}^{\infty} \alpha_i |\beta_i|^2 < \infty$. Note that $H_t^{(1)} = \sum_{i=1}^{\infty} (\beta_i / \sqrt{\alpha_i})(N_t^{(i)} - \alpha_i t)$ and $H_t^{(i)} = 0$, $i \geq 2$ (see [17]). Let (Y, Z) be the solution of the following equation:

$$\begin{aligned} Y_t &= h(L_T) + \int_t^T f(s, X_{s-}, Y_{s-}, Z_s) ds \\ &+ \sum_{j=1}^{\infty} \int_t^T g_j(s, X_{s-}, Y_{s-}, Z_s) d\beta_j(s) \\ &- \sum_{i=1}^{\infty} \int_t^T Z_s^{(i)} d(N_s^{(i)} - \alpha_i s). \end{aligned} \quad (55)$$

Then

$$\begin{aligned} Y_t &= u(t, L_t), \\ Z_t^{(1)} &= \alpha_1 u^1(t, L_{t-}, \beta_1) p_1(\beta_1) \\ &+ \left(\sum_{i=1}^{\infty} \alpha_i |\beta_i|^2 \right)^{1/2} \frac{\partial u}{\partial x}(t, L_{t-}), \\ Z_t^{(i)} &= \alpha_i u^i(t, L_{t-}, \beta_i) p_i(\beta_i), \quad i \geq 2, \end{aligned} \quad (56)$$

where u is the solution of the following SPDIEs:

$$\begin{aligned} &\frac{\partial u}{\partial t}(t, x) + \frac{1}{2} \sigma^2(x) \frac{\partial^2 u}{\partial x^2}(t, x) + a' \frac{\partial u}{\partial x}(t, x) \\ &+ \int_{\mathbb{R}} u^1(t, x, y) \nu(dy) \\ &+ f\left[t, u(t, x), \frac{\partial u}{\partial x}(t, x), \{u^{(i)}(t, x)\}_{i=1}^{\infty}\right] \\ &+ \sum_{j=1}^{\infty} g_j\left[t, u(t, x), \frac{\partial u}{\partial x}(t, x), \{u^{(i)}(t, x)\}_{i=1}^{\infty}\right] \\ &\cdot \dot{\beta}_j(t) dt = 0, \\ &u(T, x) = h(x). \end{aligned} \quad (57)$$

Competing Interests

The author declares that he has no competing interests.

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