Editorial

Ulam's Type Stability and Fixed Points Methods 2016

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The issue of Ulam stability has been a very popular subject of investigation for the last fifty years. It can be expressed shortly in the following way.

When must a function satisfying an equation approximately (in some sense) be closed to an exact solution to the equation?

The first known result of this type comes from a book published in 1925 by G. Pólya and G. Szegő (*Aufgaben und Lehrsätze aus der Analysis I*, Julius Springer, Berlin) and concerns real sequences. But the main motivation for the systematic study of such problems was given by the following question formulated in 1940 by S. M. Ulam, concerning approximate homomorphisms of groups.

Let G_1 be a group and (G_2, d) a metric group. Given $\varepsilon > 0$, does there exist $\delta > 0$ such that if $f : G_1 \rightarrow G_2$ satisfies

$$d(f(xy), f(x)f(y)) < \delta \quad (1)$$

for all $x, y \in G_1$, then a homomorphism $T : G_1 \rightarrow G_2$ exists with

$$d(f(x), T(x)) < \varepsilon \quad (2)$$

for all $x, y \in G_1$?

The first partial answer to this question was published in 1941 by D. H. Hyers. The method used by him (quite often called the *direct method*) has been successfully applied for study of the stability of large variety of equations. Apart from it, there are also other efficient approaches to that type of

stability, using different tools. One of such popular techniques is the *fixed point method*. It seems that it was used for the first time by J. A. Baker who in 1991 published a stability result for a functional equation in a single variable and in the proof he applied a variant of Banach's fixed point theorem. At present, numerous authors followed that way and this volume is mainly consisting of papers focussing (at least partly) on connections between Ulam stability and fixed point theory.

The volume includes 8 research articles (a survey and 7 research papers) containing the latest achievements. They have been written by 20 authors from 10 countries.

Three papers deal with the stability (also on restricted domains) of functional equations of quadratic, pexiderized quadratic, quartic, and additive-quadratic-cubic-quartic types; in particular, some orthogonal versions are considered. These investigations concern both Banach spaces and F -spaces.

One article provides some results on the stability of a first-order impulsive delay ordinary differential equation and a fixed point inclusion.

Another two papers present some fixed point theorems for new types of contractive mappings: one in classical metric spaces and the other in dislocated metric spaces. There is also an article, in which a fixed point theorem is one of the tools that have been used to prove the existence of a generalized homoclinic solution (exponentially approaching a periodic solution) of the Lotka-Volterra system under a small perturbation.

Finally, the volume contains the survey discussing the most significant results concerning approximate derivations (also generalized and/or Lie) as well as the hyperstability and superstability issues connected with them.

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