Research Article

On Positive Periodic Solutions to Nonlinear Fifth-Order Differential Equations with Six Parameters

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We study the existence and multiplicity of positive periodic solutions to the nonlinear differential equation: $u^{(5)}(t) + ku^{(4)}(t) - \beta u^{(3)} - \xi u''(t) + \alpha u'(t) + \omega u(t) = \lambda h(t)f(u)$, in $0 \leq t \leq 1$, $u^i(0) = u^i(1)$, $i = 0, 1, 2, 3, 4$, where $k, \alpha, \omega, \lambda > 0$, $\beta, \xi \in R$, $h \in C(R, R)$ is a 1-periodic function. The proof is based on the Krasnoselskii fixed point theorem.

1. Introduction

Fifth-order boundary value problems (BVPs) are known to arise in the mathematical modeling of viscoelastic flow and other branches of mathematical, physical, and engineering sciences; see [1–4]. So the existence, multiplicity, and nonexistence of solutions to the fifth-order BVPs are noticed by many authors. For example, Agarwal and Odda [5, 6] gave some theorems which list conditions for the existence and uniqueness of solutions to the fifth-order BVPs by the topological methods; Gamel and Lv [7, 8] focused their attention on the study of numerical solution to the fifth-order BVPs.

On the other hand, all kinds of topological methods, such as the method of upper and lower solutions, degree theory, and some fixed point theorems in cones, have been widely applied to study the singular and regular periodic boundary value problems; see [9–19].

Inspired by the above references, we establish the existence and multiplicity results of the nonlinear differential equation in this paper:

$$\begin{aligned} &u^{(5)}(t) + ku^{(4)}(t) - \beta u^{(3)} - \xi u''(t) + \alpha u'(t) + \omega u(t) \\ &= \lambda h(t)f(u), \quad \text{in } 0 \leq t \leq 1, \\ &u^i(0) = u^i(1), \quad i = 0, 1, 2, 3, 4, \end{aligned} \quad (1)$$

where $k, \alpha, \omega, \lambda > 0$, $\beta, \xi \in R$, and $h \in C(R, R)$ is a 1-periodic function.

Both of regular and singular cases are considered. The proof is based on the Krasnoselskii fixed point theorem in a cone.

Lemma 1 (see [20]). *Let E be a Banach space, and let $K \subset E$ be a cone in E . Assume Ω_1 and Ω_2 are open subsets of E with $0 \in \Omega_1$, $\overline{\Omega_1} \subset \Omega_2$, and let $T : K \cap (\overline{\Omega_2} \setminus \Omega_1) \rightarrow K$ be a completely continuous operator such that either*

- (i) $\|Tu\| \leq \|u\|$, $u \in K \cap \partial\Omega_1$, and $\|Tu\| \geq \|u\|$, $u \in K \cap \partial\Omega_2$ or
- (ii) $\|Tu\| \geq \|u\|$, $u \in K \cap \partial\Omega_1$, and $\|Tu\| \leq \|u\|$, $u \in K \cap \partial\Omega_2$.

Then, T has a fixed point in $K \cap (\overline{\Omega_2} \setminus \Omega_1)$.

This paper is organized as follows: in Section 2, some preliminaries are given; in Section 3, we give the main results; in Section 4, we give an example to illustrate our main results.

2. Preliminaries

If $k, \alpha, \omega, \beta, \xi$ satisfy the assumption

$$(H1) \quad k > 0, \beta > -2\pi, 0 < \alpha < ((1/2)\beta + 2\pi^2)^2, \alpha/\pi^4 + \beta/\pi^2 + 1 > 0, \xi = k\beta, \omega = k\alpha,$$

then (1) can be rewritten as

$$u^{(5)}(t) + ku^{(4)}(t) - \beta u^{(3)} - k\beta u''(t) + \alpha u'(t) + k\alpha u(t) = \lambda h(t) f(u). \quad (2)$$

Furthermore, we have

$$\begin{aligned} & \left(u^{(4)}(t) - \beta u^{(2)}(t) + \alpha u(t) \right)' \\ & + k \left(u^{(4)}(t) - \beta u''(t) + \alpha u(t) \right) = \lambda h(t) f(u). \end{aligned} \quad (3)$$

From Lemma 3 in [12], the fourth-order linear problem

$$\begin{aligned} u^{(4)}(t) - \beta u''(t) + \alpha u(t) &= 0, \quad \text{in } 0 \leq t \leq 1, \\ u^i(0) &= u^i(1), \quad i = 0, 1, 2; \\ u^{(3)}(0) - u^{(3)}(1) &= 1 \end{aligned} \quad (4)$$

has a unique continuous solution $\tilde{u}(t)$ and $\tilde{u}(t) > 0$, $0 \leq t \leq 1$. Let

$$\begin{aligned} m_1 &= \min_{0 \leq t \leq 1} \tilde{u}(t), \\ M_1 &= \max_{0 \leq t \leq 1} \tilde{u}(t). \end{aligned} \quad (5)$$

For given constants α and β , the minimum m_1 and maximum M_1 can be computed explicitly. Let $C[0, 1]$ denote the Banach space $\{u \in C(R, R) : u(t) = u(t+1)\}$ with the maximum norm $\|u\| = \max_{t \in [0, 1]} |u(t)|$.

Lemma 2 (see [12]). Assume that (H1) holds. For $\varphi(t) \in C[0, 1]$, the linear boundary value problem

$$\begin{aligned} u^{(4)}(t) - \beta u''(t) + \alpha u(t) &= \varphi(t), \quad \text{in } 0 \leq t \leq 1, \\ u^i(0) &= u^i(1), \quad i = 0, 1, 2, 3 \end{aligned} \quad (6)$$

has a unique solution $u(t)$, which is given by expression

$$u(t) = \int_0^1 G_1(t, s) \varphi(s) ds, \quad (7)$$

where

$$G_1(t, s) = \begin{cases} \tilde{u}(t-s), & 0 \leq s \leq t \leq 1, \\ \tilde{u}(1+t-s), & 0 \leq t \leq s \leq 1. \end{cases} \quad (8)$$

Lemma 3 (see [13]). For any $k > 0$ and $\psi(t) \in C[0, 1]$, the linear boundary value problem

$$\begin{aligned} v'(t) + kv(t) &= \psi(t), \quad \text{in } 0 \leq t \leq 1, \\ v(0) &= v(1) \end{aligned} \quad (9)$$

has a unique solution $v(t)$, which can be expressed as

$$v(t) = \int_0^1 G_2(t, s) \varphi(s) ds, \quad (10)$$

where

$$G_2(t, s) = \begin{cases} \frac{e^{-k(t-s)}}{1-e^{-k}}, & 0 \leq s \leq t \leq 1, \\ \frac{e^{-k(1+t-s)}}{1-e^{-k}}, & 0 \leq t \leq s \leq 1, \end{cases} \quad (11)$$

satisfying the estimates

$$m_2 = \frac{e^{-k}}{1-e^{-k}} \leq G_2(t, s) \leq \frac{1}{1-e^{-k}} = M_2. \quad (12)$$

From Lemmas 2 and 3, it follows that $u(t)$ is the solution of (1) if and only if $u(t)$ satisfies

$$u(t) = \lambda \int_0^1 \int_0^1 G_1(t, s) G_2(s, \tau) h(\tau) f(u(\tau)) d\tau ds. \quad (13)$$

We define an operator T on $C[0, 1]$ by

$$\begin{aligned} T(u)(t) &= \lambda \int_0^1 \int_0^1 G_1(t, s) G_2(s, \tau) h(\tau) f(u(\tau)) d\tau ds. \end{aligned} \quad (14)$$

Let K be a cone in $C[0, 1]$ defined as

$$\begin{aligned} K &= \{u(t) \in C[0, 1] : u(t) \geq 0, t \in [0, 1], u(t) \\ &\geq \theta \|u\|\}, \end{aligned} \quad (15)$$

where $\theta = m_1 m_2 / M_1 M_2$. For $r > 0$, let $\Omega_r = \{u \in C[0, 1] : \|u\| < r\}$. Note that $\partial\Omega_r = \{u \in C[0, 1] : \|u\| = r\}$.

Lemma 4. Assume that (H1) holds. In addition, we suppose the following conditions hold:

(H2) $h(t) \in C[0, 1]$ and $\int_0^1 h(t) dt > 0$.

(H3) $f : [0, \infty) \rightarrow [0, \infty)$ is continuous.

(H4) $f(u) > 0$, for $u > 0$.

Then, $T(K) \subset K$ and $T : K \rightarrow K$ is completely continuous.

Proof. For any $u \in K$, on one hand, we have

$$\begin{aligned} T(u)(t) &= \lambda \int_0^1 \int_0^1 G_1(t, s) G_2(s, \tau) h(\tau) f(u(\tau)) d\tau ds \\ &\geq \lambda m_1 m_2 \int_0^1 h(\tau) f(u(\tau)) d\tau. \end{aligned} \quad (16)$$

On the other hand, we have

$$\begin{aligned} \|T(u)\| &= \max_{t \in [0, 1]} \lambda \int_0^1 \int_0^1 G_1(t, s) G_2(s, \tau) h(\tau) f(u(\tau)) d\tau ds \\ &\leq \lambda M_1 M_2 \int_0^1 h(\tau) f(u(\tau)) d\tau. \end{aligned} \quad (17)$$

Thus,

$$T(u)(t) \geq \frac{m_1 m_2}{M_1 M_2} \|Tu\| = \theta \|Tu\|. \quad (18)$$

Therefore, $T(K) \subset K$. Using the Ascoli-Arzelà theorem (Chapter 2 [21]), it is easy to show that $T : K \rightarrow K$ is compact and continuous. $\square$

From Lemma 4 and [18], it follows that

Lemma 5. Assume that (H1) and (H2) hold. In addition, we suppose the following conditions hold:

(H5) $\lim_{u \rightarrow 0^+} f(u) = \infty$.

(H6) $f : R^+ \setminus \{0\} \rightarrow (0, \infty)$ is continuous.

Then, $T(K \setminus \{0\}) \subset K$ and $T : K \setminus \{0\} \rightarrow K$ is completely continuous.

3. Main Results

For convenience, we give the notations

$$\begin{aligned} f_0 &= \lim_{u \rightarrow 0^+} \frac{f(u)}{u}, \\ f_\infty &= \lim_{u \rightarrow +\infty} \frac{f(u)}{u}. \end{aligned} \quad (19)$$

Theorem 6 (regular case). Assume that (H1)–(H4) hold.

- (a) If $f_0 = 0$ and $f_\infty = \infty$, then, for all $\lambda > 0$, (1) has a positive solution.
- (b) If $f_0 = \infty$ and $f_\infty = 0$, then, for all $\lambda > 0$, (1) has a positive solution.
- (c) If $f_0 = 0$ or $f_\infty = 0$, then there exists a $\lambda_0 > 0$ such that, for all $\lambda > \lambda_0$, (1) has a positive solution.
- (d) If $f_0 = \infty$ or $f_\infty = \infty$, then there exists a $\lambda_0 > 0$ such that, for all $0 < \lambda < \lambda_0$, (1) has a positive solution.
- (e) If $f_0 = f_\infty = 0$, then there exists a $\lambda_0 > 0$ such that, for all $\lambda > \lambda_0$, (1) has two positive solutions.
- (f) If $f_0 = f_\infty = \infty$, then there exists a $\lambda_0 > 0$ such that, for all $0 < \lambda < \lambda_0$, (1) has two positive solutions.

Proof. (a) Since $f_0 = 0$, there exists a $r > 0$ such that $f(u) \leq \epsilon u$, for any $u \in [0, r]$, where ϵ satisfies $\epsilon \lambda M_1 M_2 \int_0^1 h(t) dt < 1$. Then, for any $u \in \partial\Omega_r \cap K$, we have

$$\begin{aligned} \|T(u)\| &= \max_{t \in [0,1]} \lambda \int_0^1 G_1(t, s) G_2(s, \tau) h(\tau) f(u(\tau)) d\tau ds \\ &\leq \lambda M_1 M_2 \int_0^1 h(\tau) f(u(\tau)) d\tau \\ &\leq \lambda M_1 M_2 \int_0^1 h(\tau) \epsilon u(\tau) d\tau \\ &\leq \epsilon \lambda M_1 M_2 \|u\| \int_0^1 h(\tau) d\tau < \|u\|. \end{aligned} \quad (20)$$

Since $f_\infty = \infty$, there exists a $\bar{R} > 0$ such that $f(u) \geq \eta u$, for any $u \geq \bar{R}$, where η satisfies $\eta \lambda \theta m_1 m_2 \int_0^1 h(t) dt > 1$. Take $R = \max\{\bar{R}/\theta, 2r\} + 1$. Then, for any $u \in \partial\Omega_R \cap K$, we have $u \geq \theta \|u\| = \theta R > \bar{R}$ and

$$\begin{aligned} \|T(u)\| &\geq T(u)(t) \\ &= \lambda \int_0^1 G_1(t, s) G_2(s, \tau) h(\tau) f(u(\tau)) d\tau ds \\ &\geq \lambda m_1 m_2 \int_0^1 h(\tau) f(u(\tau)) d\tau \\ &\geq \lambda m_1 m_2 \int_0^1 h(\tau) \eta u(\tau) d\tau \\ &\geq \eta \lambda m_1 m_2 \theta \|u\| \int_0^1 h(\tau) d\tau > \|u\|. \end{aligned} \quad (21)$$

By the above inequalities and Lemma 1, T has a fixed point in $K \cap (\bar{\Omega}_R \setminus \Omega_r)$.

(b) Since $f_0 = \infty$, there exists a $r > 0$ such that $f(u) \geq \eta u$, for any $u \in [0, r]$, where η satisfies $\eta \lambda \theta m_1 m_2 \int_0^1 h(t) dt > 1$. Then, for any $u \in \partial\Omega_r \cap K$, we have

$$\begin{aligned} \|T(u)\| &\geq T(u)(t) \\ &= \lambda \int_0^1 G_1(t, s) G_2(s, \tau) h(\tau) f(u(\tau)) d\tau ds \\ &\geq \lambda m_1 m_2 \int_0^1 h(\tau) f(u(\tau)) d\tau \\ &\geq \lambda m_1 m_2 \int_0^1 h(\tau) \eta u(\tau) d\tau \\ &\geq \eta \lambda m_1 m_2 \theta \|u\| \int_0^1 h(\tau) d\tau > \|u\|. \end{aligned} \quad (22)$$

Since $f_\infty = 0$, there exists a $\bar{R} > 0$ such that $f(u) \leq \epsilon u$, for any $u \geq \bar{R}$, where ϵ satisfies $\epsilon \lambda M_1 M_2 \int_0^1 h(t) dt < 1$.

Take $R = \max\{\bar{R}/\theta, 2r\} + 1$. Then, for any $u \in \partial\Omega_R \cap K$, we have $u \geq \theta \|u\| = \theta R > \bar{R}$ and

$$\begin{aligned} \|T(u)\| &= \max_{t \in [0,1]} \lambda \int_0^1 G_1(t, s) G_2(s, \tau) h(\tau) f(u(\tau)) d\tau ds \\ &\leq \lambda M_1 M_2 \int_0^1 h(\tau) f(u(\tau)) d\tau \\ &\leq \lambda M_1 M_2 \int_0^1 h(\tau) \epsilon u(\tau) d\tau \\ &\leq \epsilon \lambda M_1 M_2 \|u\| \int_0^1 h(\tau) d\tau < \|u\|. \end{aligned} \quad (23)$$

By the above inequalities and Lemma 1, T has a fixed point in $K \cap (\bar{\Omega}_R \setminus \Omega_r)$.

(c) For any $\rho > 0$, let $m(\rho) = \min\{f(u) : u \geq 0, \theta\rho \leq u \leq \rho\}$. Then, there exists a $\lambda_0 > 0$ such that

$$\begin{aligned} \|T(u)\| &\geq T(u)(t) \\ &= \lambda \int_0^1 G_1(t, s) G_2(s, \tau) h(\tau) f(u(\tau)) d\tau ds \\ &\geq \lambda m_1 m_2 \int_0^1 h(\tau) f(u(\tau)) d\tau \\ &\geq \lambda m_1 m_2 m(\rho) \int_0^1 h(\tau) d\tau > \|u\| \end{aligned} \quad (24)$$

for any $u \in \partial\Omega_\rho \cap K$, $\lambda > \lambda_0$.

If $f_0 = 0$, from the proof of (a), it follows that there exists a $r < \rho$ such that

$$\|T(u)\| < \|u\|, \quad \text{for } u \in \partial\Omega_r \cap K. \quad (25)$$

If $f_\infty = 0$, from the proof of (b), it follows that there exists a $R > 0$ such that

$$\|T(u)\| < \|u\|, \quad \text{for } u \in \partial\Omega_R \cap K. \quad (26)$$

By Lemma 1, T has a fixed point in $K \cap (\overline{\Omega}_\rho \setminus \Omega_r)$ or $K \cap (\overline{\Omega}_R \setminus \Omega_\rho)$.

(d) For any $\rho > 0$, let $M(\rho) = \max\{f(u) : u \geq 0, \theta\rho \leq u \leq \rho\}$. Then, there exists a $\lambda_0 > 0$ such that

$$\begin{aligned} \|T(u)\| &= \max_{t \in [0,1]} \lambda \int_0^1 G_1(t, s) G_2(s, \tau) h(\tau) f(u(\tau)) d\tau ds \\ &\leq \lambda M_1 M_2 \int_0^1 h(\tau) f(u(\tau)) d\tau \\ &\leq \lambda M_1 M_2 M(\rho) \int_0^1 h(\tau) d\tau < \|u\| \end{aligned} \quad (27)$$

for any $u \in \partial\Omega_R \cap K$, $0 < \lambda < \lambda_0$.

From the part proof of (a) and (b), the results of (d) follow.

(e) From the proof of (c), it is easy to obtain the result.

(f) From the proof of (d), it is easy to obtain the result. $\square$

Theorem 7 (singular case). *Assume that (H1), (H2), (H5), and (H6) hold.*

- (A) *There exists a $\lambda_0 > 0$ such that (1) has a positive solution for $0 < \lambda < \lambda_0$.*
- (B) *If $\lim_{u \rightarrow \infty} (f(u)/u) = 0$, then, for any $\lambda > 0$, (1) has a positive solution.*
- (C) *If $\lim_{u \rightarrow \infty} (f(u)/u) = \infty$, then (1) has two positive solutions for all sufficiently small $\lambda > 0$.*

Proof. (A) On one hand, from the proof of (d) in Theorem 6, it is clear to see that there exists a $\lambda_0 > 0$ such that

$\|Tu\| < \|u\|$, for $u \in \partial\Omega_\rho$, $0 < \lambda < \lambda_0$. On the other hand, since $\lim_{u \rightarrow 0^+} f(u) = \infty$, there exists a $r > 0$ with $r < \rho$ such that $f(u) \geq \mu u$, for $0 < u \leq r$, where μ satisfies $\lambda \mu \theta m_1 m_2 \int_0^1 h(t) dt > 1$. For any $u \in \partial\Omega_r \cap K$, we have $\|Tu\| > \|u\|$. Therefore, by Lemma 1, T has a fixed point in $K \cap (\overline{\Omega}_\rho \setminus \Omega_r)$.

Finally, from (A) and (b), result (B) follows. From (A) and (d), result (C) follows. $\square$

4. Examples

Example 1. Consider the fourth-order nonlinear problems

$$\begin{aligned} u^{(5)}(t) + 2u^{(4)}(t) - 5u^{(3)}(t) - 10u''(t) + 4u'(t) + 8u(t) \\ = \lambda \sin^2(2\pi t) f(u), \\ u^i(0) = u^i(1), \quad i = 0, 1, 2, 3, 4, \end{aligned} \quad (28)$$

where

$$\begin{aligned} k &= 2, \\ \alpha &= 5, \\ \omega &= 10, \\ \beta &= 4, \\ \xi &= 8, \end{aligned} \quad (29)$$

$$h(t) = \sin^2(2\pi t),$$

$$f(u) = \begin{cases} u^\mu, & 0 < u \leq 1, \\ u^\nu, & u > 1. \end{cases}$$

First, via some simple computations, it is easy to see that (H1) and (H2) hold. Second, if $\mu, \nu > 0$, then (H3) and (H4) obviously hold. Now we illustrate the cases of Theorem 6 via the discussions of μ and ν .

- (a) If $\mu > 1$ and $\nu > 1$, then, for all $\lambda > 0$, (28) has a positive solution.
- (b) If $0 < \mu < 1$ and $0 < \nu < 1$, then, for all $\lambda > 0$, (28) has a positive solution.
- (c) If $\mu > 1$ or $0 < \nu < 1$, then there exists a $\lambda_0 > 0$ such that, for all $\lambda > \lambda_0$, (28) has a positive solution.
- (d) If $0 < \mu < 1$ or $\nu > 1$, then there exists a $\lambda_0 > 0$ such that, for all $0 < \lambda < \lambda_0$, (28) has a positive solution.
- (e) If $\mu > 1$ and $0 < \nu < 1$, then there exists a $\lambda_0 > 0$ such that, for all $\lambda > \lambda_0$, (28) has two positive solutions.
- (f) If $0 < \mu < 1$ and $\nu > 1$, then there exists a $\lambda_0 > 0$ such that, for all $0 < \lambda < \lambda_0$, (28) has two positive solutions.

Finally, if $\mu < 0$, $\nu \geq 0$, then (H5) and (H6) hold. Now we illustrate the cases of Theorem 7 via the discussions of μ and ν .

- (A) If $\mu < 0$, then there exists a $\lambda_0 > 0$ such that (28) has a solution for $0 < \lambda < \lambda_0$.
- (B) If $\mu < 0$, $0 \leq \nu < 1$, then, for any $\lambda > 0$, (28) has a positive solution.
- (C) If $\mu < 0$, $\nu > 1$, then (28) has two positive solutions for all sufficient small $\lambda > 0$.

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Conflict of Interests

The authors declare that they have no competing interests.

Authors' Contribution

Both authors contributed to each part of this work equally and read and approved the final paper.

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