Research Article

Fixed Point Theorems for Multivalued Nonself G -Almost Contractions in Banach Spaces Endowed with Graphs

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In this paper, we prove some fixed point theorems for multivalued nonself G -almost contractions in Banach spaces with a directed graph and give some examples to illustrate our main results. The main results in this paper extend and generalize many known results in the literature therein.

1. Introduction

Fixed point theory plays a very important role in nonlinear analysis, economics, and applications. The important tool for solving existence problems in many branches of mathematics and applied sciences is Banach contraction principle [1]. It has been generalized in many directions by many authors ([2–15]).

On the other hand, the fixed point theory for nonself mappings has been studied by many authors ([3–5]). In 2013, Berinde and Păcurar [5] introduced the new type of contraction for nonself single valued mappings.

Definition 1. Let K be a nonempty subset of a normed space X . A mapping $T : K \rightarrow X$ is said to be almost contractive if there exist $\delta \in [0, 1)$ and $L \geq 0$ such that

$$\|Tx - Ty\| \leq \delta \|x - y\| + L \|y - Tx\| \quad (1)$$

for all $x, y \in K$

By the above definition, they proved some fixed point theorem for almost contractive mappings in Banach spaces.

Later, Alghamdi et al. [4] extended the almost contractive mappings to a multivalued nonself almost contractions.

Definition 2. Let (X, d) be a complete convex metric space and K a nonempty subset of X . A mapping $T : K \rightarrow CB(X)$

is said to be a *multivalued almost contraction* if there exist $\delta \in [0, 1)$ and $L \geq 0$ such that

$$H(Tx, Ty) \leq \delta d(x, y) + Ld(y, Tx) \quad (2)$$

for all $x, y \in K$, where $H(\cdot, \cdot)$ is the Pompeiu-Hausdorff metric induced by d .

They proved a fixed point theorem for multivalued nonself almost contraction on convex metric spaces.

Let $G = (V(G), E(G))$ be a directed graph, where $V(G)$ is a set of vertices of graph and $E(G)$ is a set of its edges. Assume that G has no parallel edges. We denote by G^{-1} the directed graph obtained from G by reversing the direction of edges: that is,

$$E(G^{-1}) = \{(x, y) : (y, x) \in E(G)\}. \quad (3)$$

Let x and y be two vertices in G ; a *path* in G from x to y of length $n \in \mathbb{N} \cup \{0\}$ is a sequence $\{x_i\}_{i=0}^n$ of $n+1$ vertices such that $x_0 = x$, $x_n = y$, $(x_{i-1}, x_i) \in E(G)$ for each $i = 1, 2, \dots, n$. A graph G is said to be *connected* if there exists a (directed) path between any two vertices of G . We denote

$$[x]_G^N = \{y \in X : \quad (4)$$

there is a path in G of length N from x to $y\}$.

On the other hand, in 2008, Jachymski [6] extended Banach contraction principle by introducing the notion of G -contraction and proved a fixed point theorem for G -contractions in a metric space endowed with a directed graph. These results have been generalized by some authors in several ways (see [7–14]).

Motivated by the previous results, we introduced the concept of multivalued nonself G -almost contractions in normed spaces and establish some fixed point theorems for this type of mappings in normed spaces endowed with a directed graph. We give some examples to illustrate our main results.

2. Preliminaries

In this section, we give some basic and useful definitions and known results that are useful for the main results in this paper.

Let X be a normed space. We denote by $CB(X)$ the class of all nonempty closed and bounded subsets of X . For any $A, B \in CB(X)$, define the function $H : CB(X) \times CB(X) \rightarrow \mathbb{R}^+$ by

$$H(A, B) = \max \{d(A, B), d(B, A)\}, \quad (5)$$

where

$$\begin{aligned} d(A, B) &= \sup \{d(a, B) : a \in A\}, \\ d(B, A) &= \sup \{d(b, A) : b \in B\}, \\ d(a, C) &= \inf \{\|a - x\| : x \in C\}. \end{aligned} \quad (6)$$

Note that H is called the *Pompeiu-Hausdorff metric* induced by metric d .

The following lemma is very useful for our main results.

Lemma 3 (see [2]). *Let (X, d) be a metric space. If $A, B \in CB(X)$ and $a \in A$, then, for each $\epsilon > 0$, there exists $b \in B$ such that $d(a, b) \leq H(A, B) + \epsilon$.*

Let X be a normed space, K a nonempty closed subset of X , and $T : K \rightarrow CB(X)$ a nonself mapping. If $x \in K$ with $Tx \subset K$, then we can always choose an $y \in \partial K$ such that $y = (1 - \lambda)x + \lambda z$, where $0 < \lambda < 1$ and $z \in Tx \setminus K$, which expresses the fact that

$$\|x - z\| = \|x - y\| + \|y - z\|. \quad (7)$$

We denote

$$\begin{aligned} P_T(x) &= \{y \in \partial K : \|x - z\| = \|x - y\| \\ &\quad + \|y - z\| \text{ where } z \in Tx\}. \end{aligned} \quad (8)$$

The following property was introduced by Jachymski [6]. It is very useful for our main results.

Property A (see [7]). With any sequence (x_n) in X , if $x_n \rightarrow x$ and $(x_n, x_{n+1}) \in E(G)$ for all $n \in \mathbb{N}$, then $(x_n, x) \in E(G)$ for any $n \in \mathbb{N}$.

In order to obtain our main results, we need the following definition of domination in graphs [16, 17].

Let $G = (V(G), E(G))$ be a directed graph. A set $X \subseteq V(G)$ is called a dominating set if, for all $v \in V(G) \setminus X$, there exists $x \in X$ such that $(x, v) \in E(G)$ and we say that x dominates v or v is dominated by x . For any $v \in V$, a set $X \subseteq V$ is *dominated* by v if $(v, x) \in E(G)$ for any $x \in X$ and we say that X *dominates* v if $(x, v) \in E(G)$ for all $x \in X$.

3. Main Results

In this section, we prove some fixed point theorems for multivalued nonself G -almost contractions in a Banach space endowed with a directed graph. First, we recall some definitions which are useful for our main results.

Definition 4. Let K be a nonempty subset of a normed space X and $G = (V(G), E(G))$ be a directed graph such that $V(G) = K$. A mapping $T : K \rightarrow CB(X)$ is said to be *G -almost contraction*

- (1) if there exist $\delta \in (0, 1)$ and $L \geq 0$ with $\delta(1 + L) < 1$ such that, for any $x, y \in K$,

$$H(Tx, Ty) \leq \delta \|x - y\| + L \|y - Tx\| \quad (9)$$

whenever $(x, y) \in E(G)$,

- (2) if $u \in Tx$ and $v \in Ty$ are such that $d(u, v) \leq d(x, y)$; then $(u, v) \in E(G)$.

First, we prove a fixed point theorem for a G -almost contraction in a Banach space endowed with a directed graph.

Theorem 5. *Let K be a nonempty closed subset of a Banach space X and $G = (V(G), E(G))$ be a directed graph such that $V(G) = K$. Let $T : K \rightarrow CB(X)$ be a G -almost contraction. Suppose that*

- (1) *T satisfies Rothe's boundary condition: that is, $x \in \partial K \Rightarrow Tx \subset K$,*
- (2) *K has Property A,*
- (3) *there is $x_0 \in X$ such that $(x_0, y) \in E(G)$ for some $y \in Tx_0$*
- (4) *$P_T(x)$ is dominated by x for all $x \in C$ with $Tx \not\subset K$.*

Then T has a fixed point in K .

Proof. Let $x_0 \in K$ be such that $(x_0, y_1) \in E(G)$, where $y_1 \in Tx_0$. If $y_1 \in K$, we denote $x_1 = y_1$; that is, $(x_0, x_1) \in E(G)$. If $y_1 \notin K$, there exists $x_1 \in \partial K$ such that

$$\|x_0 - x_1\| + \|x_1 + y_1\| = \|x_0 - y_1\|. \quad (10)$$

Since $P_T(x_0)$ is dominated by x_0 , we obtain $(x_0, x_1) \in E(G)$. If $x_0 = x_1$ then $x_0 \in Tx_0$. Suppose $x_0 \neq x_1$; we can choose $k_0 \in \mathbb{N}$ such that

$$\delta^{k_0} < (1 - \delta) \|x_0 - x_1\|. \quad (11)$$

By Lemma 3, we can choose $y_2 \in Tx_1$ such that

$$\begin{aligned} \|y_1 - y_2\| &\leq H(Tx_0, Tx_1) + \delta^{k_0} \\ &\leq \delta \|x_0 - x_1\| + Ld(x_1, Tx_0) + \delta^{k_0} \\ &= \delta \|x_0 - x_1\| + \delta^{k_0} < \|x_0 - x_1\|. \end{aligned} \quad (12)$$

Hence, $(y_1, y_2) \in E(G)$. If $y_2 \in K$, we denote $x_2 = y_2$. If $y_2 \notin K$, then there exists $x_2 \in \partial K$ such that

$$\|x_1 - x_2\| + \|x_2 - y_2\| = \|x_1 - y_2\|. \quad (13)$$

If $x_1 = x_2$ then $x_1 \in Tx_1$. Suppose $x_1 \neq x_2$; we can choose $k_1 \in \mathbb{N}$ with $k_1 > k_0$ such that

$$\delta^{k_1} < (1 - \delta) \|x_1 - x_2\|. \quad (14)$$

By Lemma 3, we can choose $y_3 \in Tx_2$ such that

$$\begin{aligned} \|y_2 - y_3\| &\leq H(Tx_1, Tx_2) + \delta^{k_1} \\ &\leq \delta \|x_1 - x_2\| + Ld(x_2, Tx_1) + \delta^{k_1} \\ &= \delta \|x_1 - x_2\| + \delta^{k_1} < \|x_1 - x_2\|. \end{aligned} \quad (15)$$

Hence, $(y_1, y_2) \in E(G)$. By induction, we obtain two sequences $\{x_n\}$ and $\{y_n\}$ such that

- (i) $y_{n+1} \in Tx_n$;
- (ii) $\|y_n - y_{n+1}\| \leq H(Tx_{n-1}, Tx_n) + \delta^{k_n}$, where
 - (a) $y_n = x_n$ if $y_n \in K$,
 - (b) $y_n \neq x_n$ whenever $y_n \notin K$, and then $x_n \in \partial K$ is such that

$$\|x_{n-1} - x_n\| + \|x_n - y_n\| = \|x_{n-1} - y_n\|, \quad (16)$$
 - (c) $\{k_n\}$ is a sequence of positive integers with $\delta^{k_n} < (1 - \delta)\|x_n - x_{n+1}\|$.

Suppose that

$$\begin{aligned} P &= \{x_k \in \{x_n\} : x_k = y_k\}, \\ Q &= \{x_k \in \{x_n\} : x_k \neq y_k\}. \end{aligned} \quad (17)$$

Note that $\{y_n\} \subset K$ and $(y_n, y_{n+1}) \in E(G)$ for all $n \in \mathbb{N}$. Moreover, if $x_k \in Q$, then both x_{k-1} and x_k belong to the set P . Since x_{k-1} and x_k are not in the set Q for all k , we cannot have two consecutive terms of $\{x_k\}$ in the set Q .

Now, we claim that $\{x_n\}$ is a Cauchy sequence. We conclude that there are three possibilities.

Case 1. If $x_n, x_{n+1} \in P$, then $x_n \in Tx_{n-1}$ and $x_{n+1} \in Tx_n$. Since T is a G -almost contraction and $(x_n, x_{n+1}) \in E(G)$, we have

$$\begin{aligned} \|x_n - x_{n+1}\| &= H(Tx_{n-1}, Tx_n) + \delta^{k_{n-1}} \\ &\leq \delta \|x_{n-1} - x_n\| + Ld(x_n, Tx_{n-1}) + \delta^{k_{n-1}} \\ &= \delta \|x_{n-1} - x_n\| + \delta^{k_{n-1}}. \end{aligned} \quad (18)$$

Case 2. If $x_n \in P$ and $x_{n+1} \in Q$, then $x_n \in Tx_{n-1}$ and $y_{n+1} \neq x_{n+1}$; that is,

$$\|x_n - x_{n+1}\| + \|x_{n+1} - Tx_n\| = \|x_n - Tx_n\|. \quad (19)$$

Since T is a G -almost contraction and $(x_n, x_{n+1}) \in E(G)$, we have

$$\begin{aligned} \|x_n - x_{n+1}\| &\leq \|x_n - x_{n+1}\| + \|x_{n+1} - y_{n+1}\| \\ &= \|x_n - y_{n+1}\| = \|y_n - y_{n+1}\| \\ &\leq H(Tx_{n-1}, Tx_n) + \delta^{k_{n-1}} \\ &\leq \delta \|x_{n-1} - x_n\| + Ld(x_n, Tx_{n-1}) + \delta^{k_{n-1}} \\ &= \delta \|x_{n-1} - x_n\| + \delta^{k_{n-1}}. \end{aligned} \quad (20)$$

Case 3. If $x_n \in Q$ and $x_{n+1} \in P$, then $x_{n-1} \in P$ and $y_n \neq x_n$ such that

$$\|x_{n-1} - x_n\| + \|x_n - Tx_{n-1}\| = \|x_{n-1} - Tx_{n-1}\|. \quad (21)$$

Moreover, $y_{n-1} = x_{n-1}$ and $y_n \in Tx_{n-1}$. Since T is a G -almost contraction, we have

$$\begin{aligned} \|x_n - x_{n+1}\| &= \|x_n - x_{n+1}\| \leq \|x_n - y_n\| + \|y_n, y_{n+1}\| \\ &\leq \|x_n - y_n\| + H(Tx_{n-1}, Tx_n) + \delta^{k_{n-1}} \\ &\leq \|x_n - y_n\| + \delta \|x_{n-1} - x_n\| \\ &\quad + Ld(x_n, Tx_{n-1}) + \delta^{k_{n-1}} \\ &\leq \|x_n - y_n\| + \|x_{n-1} - x_n\| \\ &\quad + Ld(x_n, Tx_{n-1}) + \delta^{k_{n-1}} \\ &= \|x_{n-1} - y_n\| + Ld(x_n, Tx_{n-1}) + \delta^{k_{n-1}} \\ &\leq \|x_{n-1} - y_n\| + L \|x_n, y_n\| + \delta^{k_{n-1}} \\ &= \|x_{n-1} - y_n\| + L \|x_{n-1}, y_n\| \\ &\quad - L \|x_{n-1}, x_n\| + \delta^{k_{n-1}} \\ &= (1 + L) \|x_{n-1} - y_n\| - L \|x_{n-1}, x_n\| \\ &\quad + \delta^{k_{n-1}} \leq (1 + L) \|y_{n-1} - y_n\| + \delta^{k_{n-1}} \\ &\leq (1 + L) H(x_{n-2}, x_{n-1}) + (1 + L) \delta^{k_{n-2}} \\ &\quad + \delta^{k_{n-1}} \\ &\leq (1 + L) \delta \|x_{n-2} - x_{n-1}\| \\ &\quad + (1 + L) Ld(x_{n-1}, Tx_{n-2}) \\ &\quad + (1 + L) \delta^{k_{n-2}} + \delta^{k_{n-1}} \\ &= (1 + L) \delta \|x_{n-2} - x_{n-1}\| + (1 + L) \delta^{k_{n-2}} \\ &\quad + \delta^{k_{n-1}}. \end{aligned} \quad (22)$$

We denote $h = (1 + L)\delta < 1$, so

$$\|x_n - x_{n+1}\| \leq h \|y_{n-2} - y_{n-1}\| + h\delta^{k_{n-3}} + \delta^{k_{n-1}}. \quad (23)$$

Therefore, combining Cases 1, 2, and 3, we obtain

$$\begin{aligned} \|x_n - x_{n+1}\| &\leq h \max \{\|x_{n-1} - x_n\|, \|x_{n-2} - x_{n-1}\|\} \\ &\quad + h\delta^{k_{n-3}} + \delta^{k_{n-1}} \\ &\leq h \max \{\|x_{n-1} - x_n\|, \|x_{n-2} - x_{n-1}\|\} \\ &\quad + h^{k_{n-3}+1} + h^{k_{n-1}} \\ &\leq h \max \{\|x_{n-1} - x_n\|, \|x_{n-2} - x_{n-1}\|\} \\ &\quad + h^{n-3} + h^{n-1} \end{aligned} \quad (24)$$

for all $n \geq 2$. By inductivity, it follows that, for any $n > 1$,

$$\|x_n - x_{n+1}\| \leq h^{n/2} \max \{\|x_0 - x_1\|, \|x_1 - x_2\|\} + h^{(n-1)/2} n. \quad (25)$$

Now, for any $n > m$, we obtain

$$\begin{aligned} \|x_m - x_n\| &\leq \|x_m - x_{m+1}\| + \|x_{m+1} - x_{m+2}\| + \cdots \\ &\quad + \|x_{n-1} - x_n\| \leq \left(h^{(m-1)/2} + h^{m/2} + \cdots + h^{(n-2)/2}\right) \\ &\quad \cdot \max \{\|x_0 - x_1\|, \|x_1 - x_2\|\} + h^{(m-1)/2} m \\ &\quad + h^{m/2} (m+1) + \cdots + h^{(n-1)/2} n. \end{aligned} \quad (26)$$

This implies that $\{x_n\}$ is a Cauchy sequence. Since C is closed and $\{x_n\} \subset C$, there exists $x \in C$ such that $\lim_{n \rightarrow \infty} x_n = x$. Let $\{x_{n_k}\} \subset P$ be an infinite subsequence of $\{x_n\}$. Since X has Property A, $(x_{n_k}, x) \in E(G)$ for all $k \in \mathbb{N}$. Then we have

$$\begin{aligned} d(x, Tx) &\leq \|x - x_{n_k+1}\| + d(x_{n_k+1}, Tx) \\ &= \|x - x_{n_k+1}\| + H(Tx_{n_k}, Tx) \\ &\leq \|x - x_{n_k+1}\| + \delta \|x_{n_k} - x\| + Ld(x, Tx_{n_k}) \\ &= (1 + L) \|x - x_{n_k+1}\| + \delta \|x_{n_k} - x\|. \end{aligned} \quad (27)$$

Letting $k \rightarrow \infty$, we obtain

$$d(x, Tx) = 0 \quad (28)$$

and hence $x \in Tx$: that is, x is a fixed point of T . This completes the proof. $\square$

As a consequence of Theorem 5, by putting $E(G) = C \times C$, we obtain the following corollary.

Corollary 6. *Let K be a nonempty closed subset of a Banach space X and $T : K \rightarrow CB(X)$ be an almost contractive mapping with Rothe's boundary condition. Then T has a fixed point.*

Remark 7. (i) In Theorem 5, if we take $E(G) = X \times X$, we obtain Theorem 9 of Alghmdid et al. [4] in Banach spaces.

(ii) In Theorem 5, if we take $E(G) = X \times X$ and $T : K \rightarrow X$, we obtain the existence result which is similar to Berinde and Păcurar's result [5].

(iii) In Theorem 5, if we take $T : X \rightarrow X$ and $L = 0$, we obtain the existence result which is similar to Jachymski's result [5] in Banach spaces.

(iv) In Theorem 5, if we take $E(G) = X \times X$ and $L = 0$, we obtain the existence result which is similar to Nadler's theorem [5] in Banach spaces.

Next, we give an example to illustrate Theorem 5.

Example 8. Let $X = \mathbb{R}$ and $K = [0, \infty)$ with the usual norm $\|x - y\| = |x - y|$ and $G = (V(G), E(G))$ be such that $V(G) = K$,

$$\begin{aligned} E(G) &= \left\{ \left(\frac{1}{2^n}, \frac{1}{2^{2n+1}} \right), \left(\frac{1}{2^n}, 0 \right), \left(\frac{1}{2^{2n+1}}, \frac{1}{2^{2n+2}} \right) : n \geq 2 \right\} \\ &\quad \cup \left\{ \left(\frac{1}{2}, 0 \right), (0, 0) \right\}. \end{aligned} \quad (29)$$

Notice that K has Property A. Define a mapping $T : K \rightarrow CB(X)$ by

$$Tx = \begin{cases} \{0\} & \text{if } x = 0, \\ \left\{ -\frac{1}{8} \right\} & \text{if } x = \frac{1}{2}, \\ \left\{ \frac{1}{2^{2k}}, \frac{1}{2^{2k+1}} \right\} & \text{if } x = \frac{1}{2^{2k+1}}, k \in \mathbb{N} \cup \{0\} \\ \left\{ \frac{1}{2^{2k+2}}, \frac{1}{2^{2k+3}} \right\} & \text{if } x = \frac{1}{2^{2k}}, k \in \mathbb{N} \cup \{0\} \setminus \left\{ \frac{1}{2} \right\}. \end{cases} \quad (30)$$

We see that the only $x \in K$ with $Tx \notin K$ is $x = 1/2$, and we see that $P_T(1/2) = \{0\}$ and $(1/2, 0) \in E(G)$; hence, $P_T(x)$ is dominated by x . By direct computation, we can show that T is G -almost contraction with $\delta = 1/4$ and $L = 2$. Moreover, we see that

$$\begin{aligned} T(\partial K) &= T(\{0, 1\}) = \{0, 1\} \\ &\implies T0 \text{ and } T1 \text{ are subset of } K, \end{aligned} \quad (31)$$

which implies T satisfies Rothe's boundary condition. Choose $x_0 = 0 \in \partial C$, we see that $(x_0, Tx_0) = (0, 0) \in E(G)$. Therefore, all conditions of Theorem 5 are satisfied and we see that $F(T) = \{0\}$.

Competing Interests

The authors declare no competing interests.

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