Hindawi
Journal of Function Spaces
Volume 2018, Article ID 1462505, 9 pages
https://doi.org/10.1155/2018/1462505

Research Article

Positive Solutions for a System of Fractional Differential Equations with Two Parameters

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Received 14 September 2018; Revised 26 October 2018; Accepted 30 October 2018; Published 25 November 2018

Academic Editor: Liguang Wang

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In this paper, the existence of positive solutions in terms of different values of two parameters for a system of conformable-type fractional differential equations with the p-Laplacian operator is obtained via Guo-Krasnosel'skii fixed point theorem.

1. Introduction

In this paper, we study the existence of positive solutions for the following system of fractional differential equations:

$$\begin{aligned} D^{\alpha_1}\left(\phi_{p_1}\left(D^{\alpha_1}x(t)\right)\right) &= \lambda g\left(t, x(t), y(t)\right), \quad 0 < t < 1, \\ D^{\alpha_2}\left(\phi_{p_2}\left(D^{\alpha_2}y(t)\right)\right) &= \mu f\left(t, x(t), y(t)\right), \quad 0 < t < 1, \end{aligned} \tag{1}$$

subject to the following boundary condition:

$$\begin{aligned} x(0) &= x(1) = 0, \\ D^{\alpha_1}x(0) &= D^{\alpha_1}x(1) = 0, \\ y(0) &= y(1) = 0, \\ D^{\alpha_2}y(0) &= D^{\alpha_2}y(1) = 0, \end{aligned} \tag{2}$$

where $\alpha_1, \alpha_2 \in (1, 2]$ are real numbers; D^{α_1} and D^{α_2} are the conformable fractional derivative; $\phi_{p_i}(s) = |s|^{p_i-2}s$, $p_i > 1$, $\phi_{q_i} = \phi_{p_i}^{-1}$, $1/p_i + 1/q_i = 1$, $i = 1, 2$; $g, f : [0, 1] \times [0, +\infty) \longrightarrow [0, +\infty)$ are continuous; λ and μ are positive parameters.

Fractional differential equations have many applications in various fields such as biological science, chemistry, physics, and engineering. Many authors have made large achievements about the study of fractional differential equations boundary value problems. Most results have adopted the Riemann-Liouville and Caputo-type fractional derivatives; we can see [1–28] and the references therein; for example, in [28], by using Guo-Krasnosel'skii fixed point theorem, the authors obtained the various existence results for positive solutions about a system of Riemann-Liouville type fractional boundary value problems with two parameters and the p-Laplacian operator. As we know, there is another kind of fractional derivative which is conformable fractional derivative. Recently, in [29], the authors Khalil R. et al. first introduced a new simple well-behaved definition of the fractional derivative called conformable fractional derivative. They first presented the definition of conformable fractional derivative of order $\alpha \in (0, 1]$ and generalized the definition to include order $\alpha \in (n, n+1]$, $n \in \mathbf{N}$. In [30], Abdeljawad proceeded on to develop the definitions and set the basic concepts in this new simple interesting fractional calculus. Since then, there are a few authors to study the boundary value problems for conformable-type fractional differential equations; for example, we can see [31–33] and the reference therein. In [33], the authors applied approximation method and fixed point theorems on cone to consider the existence and multiplicity of positive solution about the following fractional differential equation with the p-Laplacian operator:

$$\begin{aligned} D^{\alpha}\left(\phi_p\left(D^{\alpha}u(t)\right)\right) &= \lambda f\left(t, u(t)\right), \quad 0 < t < 1, \\ u(0) &= u(1) = 0, \\ D^{\alpha}u(0) &= D^{\alpha}u(1) = 0, \end{aligned} \tag{3}$$

where $\alpha \in (1,2]$ is a real number, D^α is the conformable fractional derivative, $\phi_p(s) = |s|^{p-2}s$, $p > 1$, and $f : [0,1] \times [0,+\infty) \longrightarrow [0,+\infty)$ is continuous.

There are few papers about the system of fractional differential equations concerning conformable fractional derivative. System (1), (2) is a new type of conformable fractional differential equations. Motivated by the recent papers [28, 33, 34], we consider the existence of positive solutions of the system for conformable fractional differential equations (1), (2). By using Guo-Krasnosel'skii fixed point theorem, we establish some sufficient conditions on g, f, λ, μ for the existence of at least one positive solutions of system (1), (2) for appropriately chosen parameters.

The organization of this paper is as follows. In Section 2, we recall some concepts about the conformable fractional derivative and give some lemmas with respect to the corresponding Green's function. In Section 3, we give some results about the existence of positive solutions of system (1), (2). In Section 4, we summarize the main results of the third section.

2. Preliminaries

For the convenience of the reader, we give the following concepts and lemmas of conformable fractional calculus, and some auxiliary results that will be used to prove our main theorems (see [29–33]).

Definition 1. Let $\alpha \in (n, n+1]$ and f be a n-differential function at $t > 0$, then the fractional conformable derivative of order α at $t > 0$ is given by

$$D^\alpha f(t) = D^{\alpha-n} f^{(n)}(t) = \lim_{\epsilon \longrightarrow 0} \frac{f^{(n)}\left(t + \epsilon t^{n+1-\alpha}\right) - f^{(n)}(t)}{\epsilon}, \tag{4}$$

provided the limit of the right hand side exists. If f is α-differentiable in some $(0,a)$, where $a > 0$, and $\lim_{t \longrightarrow 0^+} D^\alpha f(t)$ exists, then define $D^\alpha f(0) = \lim_{t \longrightarrow 0^+} D^\alpha f(t)$.

Definition 2. Let $\alpha \in (n, n+1]$. The fractional integral of order $\alpha > 0$ at $t > 0$ of a function $f : (0,+\infty) \longrightarrow (0,+\infty)$ is given by

$$I^\alpha f(t) = I^{n+1}\left(t^{\alpha-n-1} f(t)\right) = \frac{1}{n!} \int_0^t (t-s)^n s^{\alpha-n-1} f(s)\, ds, \tag{5}$$

where I^{n+1} denotes the integration operator of order $n+1$.

Lemma 3. *Let $\alpha \in (n, n+1]$ and f, g be α-differentiable at a point $t > 0$. Then*

(i) $D^\alpha[af + bg] = aD^\alpha(f) + bD^\alpha(g)$, $\forall a, b \in R^1$.

(ii) $D^\alpha t^k = 0$, *where* $k = 0, 1, \ldots, n$.

(iii) $D^\alpha(C) = 0$, *for all constant functions* $f(t) = C$.

(iv) $D^\alpha(fg) = fD^\alpha(g) + gD^\alpha(f)$.

(v) $D^\alpha(f/g) = (gD^\alpha(f) - fD^\alpha(g))/g^2$.

(vi) *If, in addition, f is differentiable, then* $D^\alpha f(t) = t^{n+1-\alpha} f^{(n+1)}(t)$.

(vii) *If, in addition, f is differentiable at $g(t)$, then* $D^\alpha(f \circ g)(t) = f^{(n)}(g(t)) D^\alpha g(t)$.

Lemma 4. *Given $\alpha \in (n, n+1]$ and f a continuous function defined in the domain of I^α, one has that $D^\alpha I^\alpha f(t) = f(t)$ for $t > 0$.*

Lemma 5 (mean value theorem). *Let $a > 0$ and $f : [a,b] \longrightarrow R^1$ be a given function that satisfies*

(i) *f is continuous on $[a,b]$,*

(ii) *f is α-differentiable for some $\alpha \in (0,1)$.*

Then, there exists $c \in (a,b)$ such that

$$D^\alpha f(c) = \frac{f(b) - f(a)}{(1/\alpha) b^\alpha - (1/\alpha) a^\alpha}. \tag{6}$$

Lemma 6. *Given $\alpha \in (n, n+1]$ and $f : [0,+\infty) \longrightarrow R^1$ an α-differentiable function, one has that $D^\alpha f(t) = 0$ if and only if $f(t) = a_0 + a_1 t + \cdots + a_n t^n$, where $a_1, a_2, \ldots, a_n \in R^1$.*

Lemma 7. *Given $a \in (n, n+1]$ and $x \in C(0,+\infty)$ an α-differentiable function that belongs to $C(0,1) \cap L(0,1)$, one has that $I^\alpha D^\alpha x(t) = x(t) + c_0 + c_1 t + \cdots + c_n t^n$, for some $c_k \in R^1$, $k = 0, 1, \ldots, n$.*

By Lemma 2.7 in [33], we can obtain the following lemmas.

Lemma 8 (see [33]). *Let $u \in C[0,1]$ and $\alpha_1, \alpha_2 \in (1,2]$. Then the conformable fractional differential equation*

$$\begin{aligned} D^{\alpha_i}\left(\phi_p\left(D^{\alpha_i} x(t)\right)\right) &= u(t), \quad 0 < t < 1, \\ x(0) = x(1) &= 0, \\ D^{\alpha_i} x(0) = D^{\alpha_i} x(1) &= 0 \end{aligned} \tag{7}$$

has a unique solution

$$x(t) = \int_0^1 K_i(t,s)\, \phi_{q_i}\left(\int_0^1 K_i(s,\tau)\, u(\tau)\right) ds, \tag{8}$$

where

$$k_i(t,s) = \begin{cases} (1-t)\, s^{\alpha_i - 1}, & 0 \le s \le t \le 1, \\ t s^{\alpha_i - 2}(1-s), & 0 \le t \le s \le 1, \end{cases} \tag{9}$$

and $i = 1, 2$.

Lemma 9 (see [33]). *The function $K_i(t,s)$ $(i = 1,2)$ defined by (9) has the following properties:*

(i) *$K_i(t,s) > 0$, for all $t, s \in (0,1)$;*

(ii) $\min_{1/4 \le t \le 3/4} K_i(t,s) \ge (1/4)\max_{0 \le t \le 1} K_i(t,s) = (1/4) K_i(s,s)$, *for $s \in (0,1)$.*

Lemma 10 (see [35]). *Let E be a Banach space, $P \subset E$ be a cone, Ω_1 and Ω_2 be bounded open subsets of E, $\theta \in \Omega_1$, and $\overline{\Omega}_1 \subset \Omega_2$. Assume that $T : P \cap (\overline{\Omega}_2 \setminus \Omega_1) \longrightarrow P$ is a completely continuous operator such that*

(i) $\|Tu\| \leq \|u\|$, $\forall u \in P \cap \partial\Omega_1$, $\|Tu\| \geq \|u\|$, $\forall u \in P \cap \partial\Omega_2$

or

(ii) $\|Tu\| \geq \|u\|$, $\forall u \in P \cap \partial\Omega_1$, $\|Tu\| \leq \|u\|$, $\forall u \in P \cap \partial\Omega_2$.

Then the operator T has at least one fixed point in $P \cap (\overline{\Omega}_2 \setminus \Omega_1)$.

3. Main Results

Let $X = C[0,1]$ with supremum norm $\|x\| = \max_{0\leq t\leq 1} |x(t)|$. Let $E = X \times X$ with the norm $\|(x,y)\|_E = \|x\| + \|y\|$. Then E is a Banach space. We define the cone

$$P = \left\{(x,y) \in E \mid x(t) \geq 0, y(t) \geq 0, \ \forall t \in [0,1], \ \min_{1/4\leq t\leq 3/4} (x(t) + y(t)) \geq \frac{1}{4}\|(x,y)\|_E\right\}. \tag{10}$$

In the following, we define the operators $A, B : E \longrightarrow X$ and $T : E \longrightarrow E$:

$$\begin{aligned} A(x,y)(t) &= \int_0^1 K_1(t,s) \cdot \phi_{q_1}\left(\lambda \int_0^1 K_1(s,\tau)\, g(\tau, x(\tau), y(\tau))\, d\tau\right) ds, \quad t \in [0,1], \\ B(x,y)(t) &= \int_0^1 K_2(t,s) \cdot \phi_{q_2}\left(\mu \int_0^1 K_2(s,\tau)\, f(\tau, x(\tau), y(\tau))\, d\tau\right) ds, \quad t \in [0,1], \\ T(x,y) &= (A(x,y), B(x,y)), \quad (x,y) \in E, \end{aligned} \tag{11}$$

where $K_i(t,s)$ $(i = 1,2)$ is defined by (9).

Obviously, the nontrivial fixed points of the operator T in P are positive solutions of system (1), (2).

Lemma 11. *The operator $T : P \longrightarrow P$ is a completely continuous operator.*

Proof. It is obvious that $A(x,y)(t) \geq 0, B(x,y)(t) \geq 0$ for $(x,y) \in P$, $t \in [0,1]$. By Lemma 8, we have

$$\begin{aligned} A(x,y)(t) &= \int_0^1 K_1(t,s) \cdot \phi_{q_1}\left(\lambda \int_0^1 K_1(s,\tau)\, g(\tau, x(\tau), y(\tau))\, d\tau\right) ds \\ &\leq \phi_{q_1}(\lambda) \int_0^1 s(1-s)^{\alpha_1 - 1} \cdot \phi_{q_1}\left(\int_0^1 K_1(s,\tau)\, g(\tau, x(\tau), y(\tau))\, d\tau\right) ds, \end{aligned} \tag{12}$$

and

$$\begin{aligned} B(x,y)(t) &= \int_0^1 K_2(t,s) \cdot \phi_{q_2}\left(\mu \int_0^1 K_2(s,\tau)\, f(\tau, x(\tau), y(\tau))\, d\tau\right) ds \\ &\leq \phi_{q_2}(\mu) \int_0^1 s(1-s)^{\alpha_2 - 1} \cdot \phi_{q_2}\left(\int_0^1 K_2(s,\tau)\, f(\tau, x(\tau), y(\tau))\, d\tau\right) ds. \end{aligned} \tag{13}$$

Then

$$\begin{aligned} \|T(x,y)\|_E = \|A(x,y)\| + \|B(x,y)\| &\leq \phi_{q_1}(\lambda) \cdot \int_0^1 s(1-s)^{\alpha_1 - 1} \cdot \phi_{q_1}\left(\int_0^1 K_1(s,\tau)\, g(\tau, x(\tau), y(\tau))\, d\tau\right) ds \\ &+ \phi_{q_2}(\mu) \int_0^1 s(1-s)^{\alpha_2 - 1} \cdot \phi_{q_2}\left(\int_0^1 K_2(s,\tau)\, f(\tau, x(\tau), y(\tau))\, d\tau\right) ds. \end{aligned} \tag{14}$$

On the other hand, by Lemma 9, we have

$$\begin{aligned} \min_{1/4\leq t\leq 3/4} A(x,y)(t) &\geq \frac{1}{4}\phi_{q_1}(\lambda) \int_0^1 K_1(s,s) \cdot \phi_{q_1}\left(\int_0^1 K_1(s,\tau)\, g(\tau, x(\tau), y(\tau))\, d\tau\right) ds \\ &\geq \frac{1}{4}\|A(x,y)\|, \end{aligned} \tag{15}$$

and

$$\begin{aligned} \min_{1/4\leq t\leq 3/4} B(x,y)(t) &\geq \frac{1}{4}\phi_{q_2}(\mu) \int_0^1 K_2(s,s) \cdot \phi_{q_2}\left(\int_0^1 K_2(s,\tau)\, f(\tau, x(\tau), y(\tau))\, d\tau\right) ds \\ &\geq \frac{1}{4}\|B(x,y)\|. \end{aligned} \tag{16}$$

So we have

$$\begin{aligned}&\min_{1/4\le t\le 3/4}\left(A\left(x,y\right)\left(t\right)+B\left(x,y\right)\left(t\right)\right)\\&\quad\ge \min_{1/4\le t\le 3/4}A\left(x,y\right)\left(t\right)+\min_{1/4\le t\le 3/4}B\left(x,y\right)\left(t\right)\\&\quad\ge \frac{1}{4}\left\|\left(x,y\right)\right\|_E,\end{aligned}\tag{17}$$

i.e., $T(P)\subset P$.

By the paper [33], we know that A and B are completely continuous operator. It is obvious that T is completely continuous. The proof is completed. □

Denote

$$\begin{aligned}g_0&=\lim_{x+y\longrightarrow 0^+}\max_{t\in[0,1]}\frac{g\left(t,x,y\right)}{\phi_{p_1}\left(x+y\right)}.\\f_0&=\lim_{x+y\longrightarrow 0^+}\max_{t\in[0,1]}\frac{f\left(t,x,y\right)}{\phi_{p_2}\left(x+y\right)}.\\g_\infty&=\lim_{x+y\longrightarrow +\infty}\min_{1/4\le t\le 3/4}\frac{g\left(t,x,y\right)}{\phi_{p_1}\left(x+y\right)}.\\f_\infty&=\lim_{x+y\longrightarrow +\infty}\min_{1/4\le t\le 3/4}\frac{f\left(t,x,y\right)}{\phi_{p_2}\left(x+y\right)}.\\C_1&=\int_0^1 s\left(1-s\right)^{\alpha_1-1}\phi_{q_1}\left(\int_0^1 K_1\left(s,\tau\right)d\tau\right)ds,\\C_2&=\int_0^1 s\left(1-s\right)^{\alpha_2-1}\phi_{q_2}\left(\int_0^1 K_2\left(s,\tau\right)d\tau\right)ds.\\C_3&=\int_{1/4}^{3/4} s\left(1-s\right)^{\alpha_1-1}\phi_{q_1}\left(\int_{1/4}^{3/4} K_1\left(s,\tau\right)d\tau\right)ds,\\C_4&=\int_{1/4}^{3/4} s\left(1-s\right)^{\alpha_2-1}\phi_{q_2}\left(\int_{1/4}^{3/4} K_2\left(s,\tau\right)d\tau\right)ds.\end{aligned}\tag{18}$$

Theorem 12. *Assume that $g_0, f_0, g_\infty, f_\infty\in(0,+\infty)$, $M_1<M_2$ and $M_3<M_4$, then for each $\lambda\in(M_1,M_2)$ and $\mu\in(M_3,M_4)$, system (1), (2) has a positive solution $(x(t),y(t))$, $t\in[0,1]$, where*

$$\begin{aligned}M_1&=\phi_{p_1}\left(\frac{8}{C_3}\right)\frac{1}{g_\infty},\\M_2&=\phi_{p_1}\left(\frac{1}{2C_1}\right)\frac{1}{g_0},\\M_3&=\phi_{p_2}\left(\frac{8}{C_4}\right)\frac{1}{f_\infty},\\M_4&=\phi_{p_2}\left(\frac{1}{2C_2}\right)\frac{1}{f_0}.\end{aligned}\tag{19}$$

Proof. Let $\lambda\in(M_1,M_2)$ and $\mu\in(M_3,M_4)$. Then there exists a number $\epsilon>0$ such that $\epsilon<\min\{g_\infty,f_\infty\}$, and

$$\begin{aligned}\phi_{p_1}\left(\frac{8}{C_3}\right)\frac{1}{g_\infty-\epsilon}\le\lambda\le\phi_{p_1}\left(\frac{1}{2C_1}\right)\frac{1}{g_0+\epsilon},\\\phi_{p_2}\left(\frac{8}{C_4}\right)\frac{1}{f_\infty-\epsilon}\le\mu\le\phi_{p_2}\left(\frac{1}{2C_2}\right)\frac{1}{f_0+\epsilon}.\end{aligned}\tag{20}$$

For the above $\epsilon>0$, we know that there exists $R_1>0$ such that

$$\begin{aligned}g\left(t,x,y\right)&<\left(g_0+\epsilon\right)\phi_{p_1}\left(x+y\right),\\&\qquad 0\le x+y\le R_1,\ t\in[0,1].\\f\left(t,x,y\right)&<\left(f_0+\epsilon\right)\phi_{p_2}\left(x+y\right),\\&\qquad 0\le x+y\le R_1,\ t\in[0,1].\end{aligned}\tag{21}$$

Let $\Omega_1=\{(x,y)\in E\mid\|(x,y)\|_E<R_1\}$. For any $(x,y)\in P\cap\partial\Omega_1$, we have

$$\begin{aligned}A\left(x,y\right)\left(t\right)&=\int_0^1 K_1\left(t,s\right)\phi_{q_1}\left(\lambda\int_0^1 K_1\left(s,\tau\right)\right.\\&\quad\left.\cdot g\left(\tau,x\left(\tau\right),y\left(\tau\right)\right)d\tau\right)ds\le\phi_{q_1}\left(\lambda\right)\\&\quad\cdot\int_0^1 K_1\left(t,s\right)\phi_{q_1}\left(\int_0^1 K_1\left(s,\tau\right)\right.\\&\quad\left.\cdot\left[\left(g_0+\epsilon\right)\phi_{p_1}\left(x\left(\tau\right)+y\left(\tau\right)\right)d\tau\right)ds\right.\\&\le\phi_{q_1}\left(\lambda\right)\phi_{q_1}\left(g_0+\epsilon\right)\int_0^1 s\left(1-s\right)^{\alpha_1-1}\\&\quad\cdot\phi_{q_1}\left(\int_0^1 K_1\left(s,\tau\right)\phi_{p_1}\left(x\left(\tau\right)+y\left(\tau\right)\right)d\tau\right)ds\\&\le\phi_{q_1}\left(\lambda\right)\phi_{q_1}\left(g_0+\epsilon\right)\int_0^1 s\left(1-s\right)^{\alpha_1-1}\\&\quad\cdot\phi_{q_1}\left(\int_0^1 K_1\left(s,\tau\right)\phi_{p_1}\left(\left\|x\right\|+\left\|y\right\|\right)d\tau\right)ds\\&=\phi_{q_1}\left(\lambda\left(g_0+\epsilon\right)\right)C_1\left(\left\|x\right\|+\left\|y\right\|\right)\le\frac{1}{2}\left\|\left(x,y\right)\right\|_E.\end{aligned}\tag{22}$$

So

$$\left\|A\left(x,y\right)\right\|\le\frac{1}{2}\left\|\left(x,y\right)\right\|_E,\quad\forall\left(x,y\right)\in P\cap\partial\Omega_1.\tag{23}$$

Similarly, we have

$$B(x,y)(t) = \int_0^1 K_2(t,s)\,\phi_{q_2}\left(\mu\int_0^1 K_2(s,\tau)\cdot f(\tau,x(\tau),y(\tau))\,d\tau\right)ds \le \phi_{q_2}(\mu)\cdot\int_0^1 K_2(t,s)\,\phi_{q_2}\left(\int_0^1 K_2(s,\tau)\cdot\left[(f_0+\epsilon)\,\phi_{p_2}(x(\tau)+y(\tau))\,d\tau\right)ds \le \phi_{q_2}(\mu)\,\phi_{q_2}(f_0+\epsilon)\int_0^1 s(1-s)^{\alpha_2-1}\cdot\phi_{q_2}\left(\int_0^1 K_2(s,\tau)\,\phi_{p_2}(x(\tau)+y(\tau))\,d\tau\right)ds \le \phi_{q_2}(\mu)\,\phi_{q_2}(f_0+\epsilon)\int_0^1 s(1-s)^{\alpha_2-1}\cdot\phi_{q_2}\left(\int_0^1 K_2(s,\tau)\,\phi_{p_2}(\|x\|+\|y\|)\,d\tau\right)ds = \phi_{q_2}(\lambda(f_0+\epsilon))\,C_2(\|x\|+\|y\|) \le \frac{1}{2}\|(x,y)\|_E. \tag{24}$$

So

$$\|B(x,y)\| \le \frac{1}{2}\|(x,y)\|_E,\quad \forall (x,y)\in P\cap\partial\Omega_1. \tag{25}$$

Hence

$$\|T(x,y)\|_E = \|A(x,y)\| + \|b(x,y)\| \le \|(x,y)\|_E,\quad \forall (x,y)\in P\cap\partial\Omega_1. \tag{26}$$

On the other hand, for the above $\epsilon > 0$, there exists $\widetilde{R}_2 > 0$ such that

$$g(t,x,y) \ge (g_\infty-\epsilon)\,\phi_{p_1}(x+y),\quad x+y\ge\widetilde{R}_2>0,\ t\in\left[\frac{1}{4},\frac{3}{4}\right],$$
$$f(t,x,y) \ge (f_\infty-\epsilon)\,\phi_{p_2}(x+y),\quad x+y\ge\widetilde{R}_2>0,\ t\in\left[\frac{1}{4},\frac{3}{4}\right]. \tag{27}$$

Let $R_2 = \max\{2R_1, 4\widetilde{R}_2\}$. Let $\Omega_2 = \{(x,y)\in E \mid \|(x,y)\|_E < R_2\}$. For any $(x,y)\in P\cap\partial\Omega_2$, we have $\min_{1/4\le t\le 3/4}\{x(t)+y(t)\} \ge (1/4)(\|x\|+\|y\|) = (1/4)\|(x,y)\|_E = R_2/4 \ge \widetilde{R}_2$. So by Lemma 9, we have

$$A(x,y)\left(\frac{1}{4}\right) = \int_0^1 K_1\left(\frac{1}{4},s\right)\phi_{q_1}\left(\lambda\int_0^1 K_1(s,\tau)\cdot g(\tau,x(\tau),y(\tau))\,d\tau\right)ds \ge \frac{1}{4}\phi_{q_1}(\lambda)\cdot\int_0^1 s(1-s)^{\alpha_1-1}\phi_{q_1}\left(\int_{1/4}^{3/4} K_1(s,\tau)\cdot g(\tau,x(\tau),y(\tau))\,d\tau\right)ds \ge \frac{1}{4}\phi_{q_1}(\lambda)\cdot\int_{1/4}^{3/4} s(1-s)^{\alpha_1-1}\phi_{q_1}\left(\int_{1/4}^{3/4} K_1(s,\tau)\,(g_\infty-\epsilon)\cdot\phi_{p_1}(x(\tau)+y(\tau))\,d\tau\right)ds \ge \frac{1}{4}\phi_{q_1}(\lambda)\cdot\phi_{q_1}(g_\infty-\epsilon)\int_{1/4}^{3/4} s(1-s)^{\alpha_1-1}\phi_{q_1}\left(\int_{1/4}^{3/4} K_1(s,\tau)\cdot\phi_{p_1}\left(\frac{1}{4}(\|x\|+\|y\|)\right)d\tau\right)ds = \frac{1}{16}\cdot\phi_{q_1}(\lambda(g_\infty-\epsilon))\,C_3(\|(x,y)\|_E) \ge \frac{1}{2}\|(x,y)\|_E. \tag{28}$$

So

$$\|A(x,y)\| \ge \frac{1}{2}\|(x,y)\|_E,\quad \forall (x,y)\in P\cap\partial\Omega_2. \tag{29}$$

Similarly, we have

$$B(x,y)\left(\frac{3}{4}\right) = \int_0^1 K_2\left(\frac{3}{4},s\right)\phi_{q_2}\left(\mu\int_0^1 K_2(s,\tau)\cdot f(\tau,x(\tau),y(\tau))\,d\tau\right)ds \ge \frac{1}{4}\phi_{q_2}(\mu)\cdot\int_0^1 s(1-s)^{\alpha_2-1}\phi_{q_2}\left(\int_{1/4}^{3/4} K_2(s,\tau)\cdot f(\tau,x(\tau),y(\tau))\,d\tau\right)ds \ge \frac{1}{4}\phi_{q_2}(\mu)\cdot\int_{1/4}^{3/4} s(1-s)^{\alpha_2-1}\phi_{q_2}\left(\int_{1/4}^{3/4} K_2(s,\tau)\,(f_\infty-\epsilon)\cdot\phi_{p_2}(x(\tau)+y(\tau))\,d\tau\right)ds \ge \frac{1}{4}\phi_{q_2}(\mu)\cdot\phi_{q_2}(f_\infty-\epsilon)\int_{1/4}^{3/4} s(1-s)^{\alpha_2-1}\phi_{q_2}\left(\int_{1/4}^{3/4} K_2(s,\tau)\right.$$

$$\cdot \phi_{p_2}\left(\frac{1}{4}\left(\|x\|+\|y\|\right)\right)d\tau\Bigg)ds = \frac{1}{16}$$

$$\cdot \phi_{q_2}\left(\lambda\left(f_\infty-\epsilon\right)\right)C_4\left(\|(x,y)\|_E\right) \geq \frac{1}{2}\|(x,y)\|_E. \tag{30}$$

So

$$\|B(x,y)\| \geq \frac{1}{2}\|(x,y)\|_E, \quad \forall (x,y) \in P\cap\partial\Omega_2. \tag{31}$$

Hence

$$\|T(x,y)\|_E = \|A(x,y)\| + \|B(x,y)\| \geq \|(x,y)\|_E, \quad \forall (x,y) \in P\cap\partial\Omega_2. \tag{32}$$

By Lemma 10 and (26) (32), the operator T has one fixed point $(x, y) \in P\cap(\overline{\Omega}_2\backslash\Omega_1)$. That is, (x, y) is a positive solution of system (1), (2). □

Similar to the proof of Theorem 12, we can easily obtain the following results.

Theorem 13. *Assume that $g_0 = 0, g_\infty, f_0, f_\infty \in (0,+\infty)$ and $M_3 < M_4$, then for each $\lambda \in (M_1,+\infty)$ and $\mu \in (M_3, M_4)$, system (1), (2) has a positive solution $(x(t), y(t))$, $t \in [0,1]$.*

Theorem 14. *Assume that $f_0 = 0$, $g_\infty, g_0, f_\infty \in (0,+\infty)$ and $M_1 < M_2$, then for each $\lambda \in (M_1, M_2)$ and $\mu \in (M_3,+\infty)$, system (1), (2) has a positive solution $(x(t), y(t))$, $t \in [0,1]$.*

Theorem 15. *Assume that $g_0 = 0$, $f_0 = 0$, $g_\infty, f_\infty \in (0,+\infty)$, then for each $\lambda \in (M_1,+\infty)$ and $\mu \in (M_3,+\infty)$, system (1), (2) has a positive solution $(x(t), y(t))$, $t \in [0,1]$.*

Theorem 16. *Assume that $g_0, f_0 \in (0,+\infty), g_\infty = +\infty$ or $g_0, f_0 \in (0,+\infty)$, $f_\infty = +\infty$, then for each $\lambda \in (0, M_2)$ and $\mu \in (0, M_4)$, system (1), (2) has a positive solution $(x(t), y(t))$, $t \in [0,1]$.*

Theorem 17. *Assume that $g_0 \in (0,+\infty)$, $f_0 = 0$, $f_\infty = +\infty$ or $g_0 \in (0,+\infty)$, $f_0 = 0$, $g_\infty = +\infty$, then for each $\lambda \in (0, M_2)$ and $\mu \in (0,+\infty)$, system (1), (2) has a positive solution $(x(t), y(t))$, $t \in [0,1]$.*

Theorem 18. *Assume that $g_0 = f_0 = 0$, $f_\infty = +\infty$ or $g_0 = f_0 = 0$, $g_\infty = +\infty$, then for each $\lambda \in (0,+\infty)$ and $\mu \in (0,+\infty)$, system (1), (2) has a positive solution $(x(t), y(t))$, $t \in [0,1]$.*

Denote

$$\overline{g}_0 = \lim_{x+y\longrightarrow 0^+}\min_{t\in[1/4,3/4]}\frac{g(t,x,y)}{\phi_{p_1}(x+y)}.$$

$$\overline{f}_0 = \lim_{x+y\longrightarrow 0^+}\min_{t\in[1/4,3/4]}\frac{f(t,x,y)}{\phi_{p_2}(x+y)}.$$

$$\overline{g}_\infty = \lim_{x+y\longrightarrow +\infty}\max_{t\in[0,1]}\frac{g(t,x,y)}{\phi_{p_1}(x+y)}.$$

$$\overline{f}_\infty = \lim_{x+y\longrightarrow +\infty}\max_{t\in[0,1]}\frac{f(t,x,y)}{\phi_{p_2}(x+y)}. \tag{33}$$

Theorem 19. *Assume that $\overline{g}_0, \overline{f}_0, \overline{g}_\infty, \overline{f}_\infty \in (0,+\infty)$, $\overline{M}_1 < \overline{M}_2$ and $\overline{M}_3 < \overline{M}_4$, then for each $\lambda \in (\overline{M}_1, \overline{M}_2)$ and $\mu \in (\overline{M}_3, \overline{M}_4)$, system (1), (2) has a positive solution $(x(t), y(t))$, $t \in [0,1]$, where*

$$\overline{M}_1 = \phi_{p_1}\left(\frac{8}{C_3}\right)\frac{1}{\overline{g}_0},$$

$$\overline{M}_2 = \phi_{p_1}\left(\frac{1}{2C_1}\right)\frac{1}{\overline{g}_\infty},$$

$$\overline{M}_3 = \phi_{p_2}\left(\frac{8}{C_4}\right)\frac{1}{\overline{f}_0},$$

$$\overline{M}_4 = \phi_{p_2}\left(\frac{1}{2C_2}\right)\frac{1}{\overline{f}_\infty}. \tag{34}$$

Proof. Let $\lambda \in (\overline{M}_1, \overline{M}_2)$ and $\mu \in (\overline{M}_3, \overline{M}_4)$. Then there exists a number $\epsilon > 0$ such that $\epsilon < \min\{\overline{g}_0, \overline{f}_0\}$, and

$$\phi_{p_1}\left(\frac{8}{C_3}\right)\frac{1}{\overline{g}_0-\epsilon} \leq \lambda \leq \phi_{p_1}\left(\frac{1}{2C_1}\right)\frac{1}{\overline{g}_\infty+\epsilon},$$

$$\phi_{p_2}\left(\frac{8}{C_4}\right)\frac{1}{\overline{f}_0-\epsilon} \leq \mu \leq \phi_{p_2}\left(\frac{1}{2C_2}\right)\frac{1}{\overline{f}_\infty+\epsilon}. \tag{35}$$

For the above $\epsilon > 0$, we know that there exists $R_3 > 0$ such that

$$g(t,x,y) > \left(\overline{g}_0-\epsilon\right)\phi_{p_1}(x+y), \quad x \geq 0,\ y \geq 0,\ x+y \leq R_3,\ t \in \left[\frac{1}{4},\frac{3}{4}\right],$$

$$f(t,x,y) > \left(\overline{f}_0-\epsilon\right)\phi_{p_2}(x+y), \quad x \geq 0,\ y \geq 0,\ x+y \leq R_3,\ t \in \left[\frac{1}{4},\frac{3}{4}\right]. \tag{36}$$

Let $\Omega_3 = \{(x,y) \in E \mid \|(x,y)\|_E < R_3\}$. For any $(x,y) \in P \cap \partial\Omega_3$, we have $\min_{1/4\leq t\leq 3/4}\{x(t)+y(t)\} \geq (1/4)(\|x\|+\|y\|) = (1/4)\|(x,y)\|_E$. So by Lemma 9, we obtain

$$
\begin{aligned}
A(x,y)\left(\frac{1}{4}\right) &= \int_0^1 K_1\left(\frac{1}{4},s\right)\phi_{q_1}\left(\lambda\int_0^1 K_1(s,\tau)\right.\\
&\cdot g(\tau,x(\tau),y(\tau))\,d\tau\Big)\,ds \geq \frac{1}{4}\phi_{q_1}(\lambda)\\
&\cdot \int_{1/4}^{3/4} s(1-s)^{\alpha_1-1}\,\phi_{q_1}\left(\int_{1/4}^{3/4} K_1(s,\tau)\right.\\
&\cdot g(\tau,x(\tau),y(\tau))\,d\tau\Big)\,ds \geq \frac{1}{4}\phi_{q_1}(\lambda)\\
&\cdot \int_{1/4}^{3/4} s(1-s)^{\alpha_1-1}\,\phi_{q_1}\left(\int_{1/4}^{3/4} K_1(s,\tau)\left(\overline{g}_0-\epsilon\right)\right. \qquad (37)\\
&\cdot \phi_{p_1}(x(\tau)+y(\tau))\,d\tau\Big)\,ds \geq \frac{1}{4}\phi_{q_1}(\lambda)\\
&\cdot \phi_{q_1}\left(\overline{g}_0-\epsilon\right)\int_{1/4}^{3/4} s(1-s)^{\alpha_1-1}\,\phi_{q_1}\left(\int_{1/4}^{3/4} K_1(s,\tau)\right.\\
&\cdot \phi_{p_1}\left(\frac{1}{4}\left(\|x\|+\|y\|\right)\right)d\tau\Big)\,ds = \frac{1}{16}\\
&\cdot \phi_{q_1}\left(\lambda\left(\overline{g}_0-\epsilon\right)\right)C_3\left(\|(x,y)\|_E\right) \geq \frac{1}{2}\|(x,y)\|_E.
\end{aligned}
$$

So

$$
\|A(x,y)\| \geq \frac{1}{2}\|(x,y)\|_E, \quad \forall (x,y)\in P\cap\partial\Omega_3. \qquad (38)
$$

Similarly, we get

$$
\begin{aligned}
B(x,y)\left(\frac{3}{4}\right) &= \int_0^1 K_2\left(\frac{3}{4},s\right)\phi_{q_2}\left(\mu\int_0^1 K_2(s,\tau)\right.\\
&\cdot f(\tau,x(\tau),y(\tau))\,d\tau\Big)\,ds \geq \frac{1}{4}\phi_{q_2}(\mu)\\
&\cdot \int_{1/4}^{3/4} s(1-s)^{\alpha_2-1}\,\phi_{q_2}\left(\int_{1/4}^{3/4} K_2(s,\tau)\right.\\
&\cdot f(\tau,x(\tau),y(\tau))\,d\tau\Big)\,ds \geq \frac{1}{4}\phi_{q_2}(\mu)\\
&\cdot \int_{1/4}^{3/4} s(1-s)^{\alpha_2-1}\,\phi_{q_2}\left(\int_{1/4}^{3/4} K_2(s,\tau)\left(\overline{f}_0-\epsilon\right)\right. \qquad (39)\\
&\cdot \phi_{p_2}(x(\tau)+y(\tau))\,d\tau\Big)\,ds \geq \frac{1}{4}\phi_{q_2}(\mu)\\
&\cdot \phi_{q_2}\left(\overline{f}_0-\epsilon\right)\int_{1/4}^{3/4} s(1-s)^{\alpha_2-1}\,\phi_{q_2}\left(\int_{1/4}^{3/4} K_2(s,\tau)\right.\\
&\cdot \phi_{p_2}\left(\frac{1}{4}\left(\|x\|+\|y\|\right)\right)d\tau\Big)\,ds = \frac{1}{16}\\
&\cdot \phi_{q_2}\left(\mu\left(\overline{f}_0-\epsilon\right)\right)C_3\left(\|(x,y)\|_E\right) \geq \frac{1}{2}\|(x,y)\|_E.
\end{aligned}
$$

So

$$
\|B(x,y)\| \geq \frac{1}{2}\|(x,y)\|_E, \quad \forall (x,y)\in P\cap\partial\Omega_3. \qquad (40)
$$

Hence

$$
\begin{aligned}
\|T(x,y)\|_E = \|A(x,y)\| + \|B(x,y)\| \geq \|(x,y)\|_E,\\
\forall (x,y)\in P\cap\partial\Omega_3.
\end{aligned} \qquad (41)
$$

We define the functions $\widetilde{g},\widetilde{f}:[0,1]\times[0,+\infty)\longrightarrow[0,+\infty)$, $\widetilde{g}(t,u)=\max_{0\leq x+y\leq u} g(t,x,y)$, and $\widetilde{f}(t,u)=\max_{0\leq x+y\leq u} f(t,x,y)$. So it is obvious that $\widetilde{g}(t,u)$ and $\widetilde{f}(t,u)$ are nondecreasing on u for every $t\in[0,1]$; and $g(t,x,y)\leq\widetilde{g}(t,u)$, $f(t,x,y)\leq\widetilde{f}(t,u)$, $x\geq 0$, $y\geq 0$, $x+y\leq u$, $t\in[0,1]$; and they satisfy the conditions

$$
\begin{aligned}
\limsup_{u\longrightarrow+\infty}\max_{t\in[0,1]}\frac{\widetilde{g}(t,u)}{\phi_{p_1}(u)} &\leq \overline{g}_\infty,\\
\limsup_{u\longrightarrow+\infty}\max_{t\in[0,1]}\frac{\widetilde{f}(t,u)}{\phi_{p_2}(u)} &\leq \overline{f}_\infty.
\end{aligned} \qquad (42)
$$

For the above $\epsilon>0$, there exists $\widetilde{R}_4>0$ such that

$$
\begin{aligned}
\frac{\widetilde{g}(t,u)}{\phi_{p_1}(u)} &\leq \limsup_{u\longrightarrow+\infty}\max_{t\in[0,1]}\frac{\widetilde{g}(t,u)}{\phi_{p_1}(u)}+\epsilon \leq \overline{g}_\infty+\epsilon,\\
&\qquad t\in[0,1],\ u\geq\widetilde{R}_4,\\
\frac{\widetilde{f}(t,u)}{\phi_{p_2}(u)} &\leq \limsup_{u\longrightarrow+\infty}\max_{t\in[0,1]}\frac{\widetilde{f}(t,u)}{\phi_{p_2}(u)}+\epsilon \leq \overline{f}_\infty+\epsilon,\\
&\qquad t\in[0,1],\ u\geq\widetilde{R}_4.
\end{aligned} \qquad (43)
$$

So $\widetilde{g}(t,u)\leq(\overline{g}_\infty+\epsilon)\phi_{p_1}(u)$, $\widetilde{f}(t,u)\leq(\overline{f}_\infty+\epsilon)\phi_{p_2}(u)$, and $t\in[0,1], u\geq\widetilde{R}_4$.

By the definition of $\widetilde{g},\widetilde{f}$, we get $g(t,x,y)\leq\widetilde{g}(t,\|(x,y)\|_E)$, $f(t,x,y)\leq\widetilde{f}(t,\|(x,y)\|_E)$. Let $R_4=\max\{2R_3,\widetilde{R}_4\}$. Let $\Omega_4=\{(x,y)\in E\mid\|(x,y)\|_E<R_4\}$. For any $(x,y)\in P\cap\partial\Omega_4$, we have

$$
\begin{aligned}
A(x,y)(t) &= \int_0^1 K_1(t,s)\\
&\cdot\phi_{q_1}\left(\lambda\int_0^1 K_1(s,\tau)\,g(\tau,x(\tau),y(\tau))\,d\tau\right)ds\\
&\leq \phi_{q_1}(\lambda)\int_0^1 s(1-s)^{\alpha_1-1}\\
&\cdot\phi_{q_1}\left(\int_0^1 K_1(s,\tau)\,\widetilde{g}\left(\tau,\|(x,y)\|_E\right)d\tau\right)ds \qquad (44)\\
&\leq \phi_{q_1}(\lambda)\,\phi_{q_1}\left(\overline{g}_\infty+\epsilon\right)\int_0^1 s(1-s)^{\alpha_1-1}\\
&\cdot\phi_{q_1}\left(\int_0^1 K_1(s,\tau)\,\phi_{p_1}\left(\|(x,y)\|_E\right)\right)d\tau\Big)\,ds\\
&= \phi_{q_1}\left(\lambda\left(\overline{g}_\infty+\epsilon\right)\right)C_1\left(\|(x,y)\|_E\right) \leq \frac{1}{2}\|(x,y)\|_E.
\end{aligned}
$$

So

$$\|A(x,y)\| \le \frac{1}{2}\|(x,y)\|_E, \quad \forall (x,y) \in P \cap \partial\Omega_4. \tag{45}$$

Similarly, we have

$$\begin{aligned} B(x,y)(t) &= \int_0^1 K_2(t,s) \\ &\cdot \phi_{q_2}\left(\mu \int_0^1 K_2(s,\tau) f(\tau, x(\tau), y(\tau))\, d\tau\right) ds \\ &\le \phi_{q_2}(\mu) \int_0^1 s(1-s)^{\alpha_2-1} \\ &\cdot \phi_{q_2}\left(\int_0^1 K_2(s,\tau) \widetilde{f}(\tau, \|(x,y)\|_E)\, d\tau\right) ds \\ &\le \phi_{q_2}(\mu)\phi_{q_2}\left(\overline{f}_\infty + \epsilon\right) \int_0^1 s(1-s)^{\alpha_2-1} \\ &\cdot \phi_{q_2}\left(\int_0^1 K_2(s,\tau)\phi_{p_2}(\|(x,y)\|_E))\, d\tau\right) ds \\ &= \phi_{q_2}\left(\mu\left(\overline{f}_\infty + \epsilon\right)\right) C_2(\|(x,y)\|_E) \le \frac{1}{2}\|(x,y)\|_E. \end{aligned} \tag{46}$$

So

$$\|B(x,y)\| \le \frac{1}{2}\|(x,y)\|_E, \quad \forall (x,y) \in P \cap \partial\Omega_4. \tag{47}$$

Hence

$$\|T(x,y)\|_E = \|A(x,y)\| + \|B(x,y)\| \le \|(x,y)\|_E, \quad \forall (x,y) \in P \cap \partial\Omega_4. \tag{48}$$

By Lemma 10 and (41) (48), the operator T has one fixed point $(x,y) \in P \cap (\overline{\Omega}_4 \setminus \Omega_3)$. So (x,y) is a positive solution of system (1), (2). □

Similar to the proof of Theorem 19, we can easily obtain the following results.

Theorem 20. *Assume that $\overline{g}_0, \overline{g}_\infty, \overline{f}_0 \in (0,+\infty)$, $\overline{f}_\infty = 0$ and $\overline{M}_1 < \overline{M}_2$, then for each $\lambda \in (\overline{M}_1, \overline{M}_2)$ and $\mu \in (\overline{M}_3, +\infty)$, system (1), (2) has a positive solution $(x(t), y(t))$, $t \in [0,1]$.*

Theorem 21. *Assume that $\overline{g}_0, \overline{f}_\infty, \overline{f}_0 \in (0,+\infty)$, $\overline{g}_\infty = 0$ and $\overline{M}_3 < \overline{M}_4$, then for each $\lambda \in (\overline{M}_1, +\infty)$ and $\mu \in (\overline{M}_3, \overline{M}_4)$, system (1), (2) has a positive solution $(x(t), y(t))$, $t \in [0,1]$.*

Theorem 22. *Assume that $\overline{g}_0, \overline{f}_0 \in (0,+\infty)$, $\overline{g}_\infty = 0, \overline{f}_\infty = 0$, then for each $\lambda \in (\overline{M}_1, +\infty)$ and $\mu \in (\overline{M}_3, +\infty)$, system (1), (2) has a positive solution $(x(t), y(t))$, $t \in [0,1]$.*

Theorem 23. *Assume that $\overline{g}_\infty, \overline{f}_\infty \in (0,+\infty)$, $\overline{g}_0 = +\infty$ or $\overline{f}_0 = +\infty$, $\overline{g}_\infty, \overline{f}_\infty \in (0,+\infty)$, then for each $\lambda \in (0, \overline{M}_2)$ and $\mu \in (0, \overline{M}_4)$, system (1), (2) has a positive solution $(x(t), y(t))$, $t \in [0,1]$.*

Theorem 24. *Assume that $\overline{g}_0 = +\infty$, $\overline{g}_\infty \in (0,+\infty)$, $\overline{f}_\infty = 0$ or $\overline{f}_0 = +\infty$, $\overline{g}_\infty \in (0,+\infty)$, $\overline{f}_\infty = 0$, then for each $\lambda \in (0, \overline{M}_2)$ and $\mu \in (0,+\infty)$, system (1), (2) has a positive solution $(x(t), y(t))$, $t \in [0,1]$.*

Theorem 25. *Assume that $\overline{f}_\infty \in (0,+\infty)$, $\overline{g}_\infty = 0$, $\overline{g}_0 = +\infty$ or $\overline{f}_\infty \in (0,+\infty)$, $\overline{f}_0 = +\infty$, $\overline{g}_\infty = 0$, then for each $\lambda \in (0,+\infty)$ and $\mu \in (0, \overline{M}_4)$, system (1), (2) has a positive solution $(x(t), y(t))$, $t \in [0,1]$.*

Theorem 26. *Assume that $\overline{g}_0 = +\infty$, $\overline{g}_\infty = 0$, $\overline{f}_\infty = 0$ or $\overline{g}_\infty = 0$, $\overline{f}_\infty = 0$, $\overline{f}_0 = +\infty$, then for each $\lambda \in (0,+\infty)$ and $\mu \in (0,+\infty)$, system (1), (2) has a positive solution $(x(t), y(t))$, $t \in [0,1]$.*

4. Conclusion

In this paper, we investigate the existence of positive solutions for a system of conformable-type fractional differential equations with two parameters and the p-Laplacian operator. By employing Guo-Krasnosel'skii fixed point theorem, under different combinations of sublinearity and superlinearity of the nonlinearities f, g, various sufficient conditions for the existence of at least one positive solutions of system (1), (2) are derived in terms of appropriately chosen parameters λ, μ.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

Acknowledgments

The project is supported by the Youth Science Foundation of China (11801322), the National Natural Science Foundation of China (11571207), and Shandong Natural Science Foundation (ZR2018MA011).

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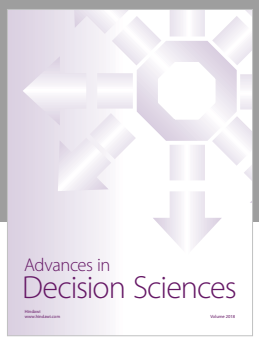

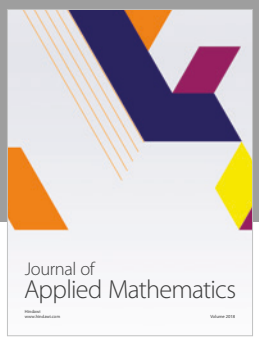

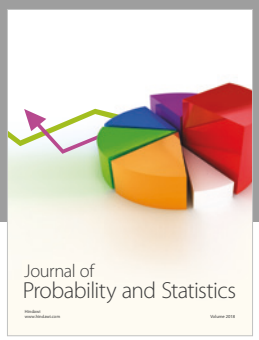

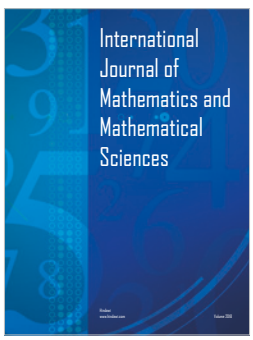

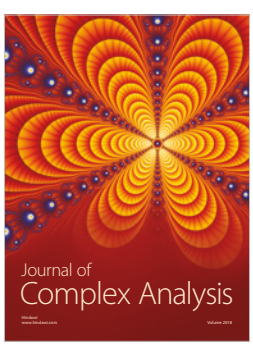

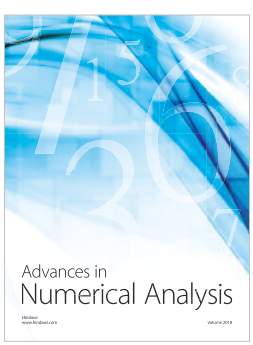

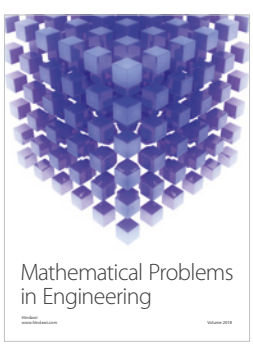

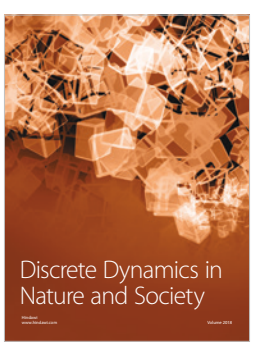

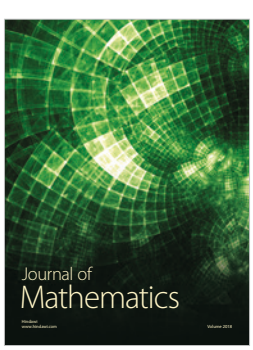

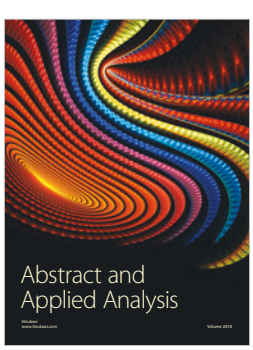

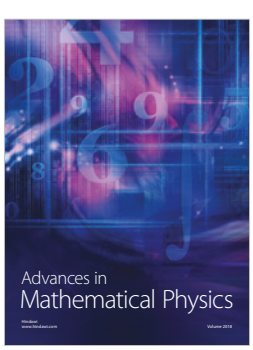

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