Research Article

b -Metric Generalization of Some Fixed Point Theorems

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We prove common fixed point theorems in terms of b -metric spaces with new contractions. The results presented in this paper include b -metric generalizations of some fixed point theorems of Fisher, Pachpatte, and Sahu and Sharma.

1. Introduction

Fixed point theory is a fascinating topic for research in both analysis and topology. In this direction the Banach contraction mapping theorem of 1922 popularly known as Banach contraction mapping principle is a rewarding result in fixed point theory. It has widespread applications in both pure and applied mathematics. The well-known Banach [1] contraction mapping principle states that if “ X is a complete metric space and $T : X \rightarrow X$ is a contraction mapping of X into itself then T has unique fixed point in X .” This celebrated principle has been generalized by several authors. In 1989, Bakhtin [2] introduced the concept of b -metric space which is generalization of renowned Banach contraction mapping principle. Czerwik [3, 4] extended the concept of b -metric space in 1993. Bakhtin’s concept of b -metric spaces has been extensively generalized and improved by several mathematicians for fixed points in several different ways, namely, Boriceanu [5], Bota et al. [6], Chen et al. [7], Hussain and Shah [8], Kutbi et al. [9], and Shukla [10] to name a few. In this paper, our main concern is to study common fixed point theorems in complete b -metric spaces for three self-mappings. The obtained results are generalizations of b -metric variant of fixed point theorems of Fisher, Pachpatte, and Sahu and Sharma.

The following fixed point theorems were proved in [11–13].

Theorem 1 (see [11]). Let T be a mapping of the complete metric space X into itself satisfying the inequality

$$(d(Tx, Ty))^2 \leq \alpha_1 (d(x, Tx) d(y, Ty)) + \alpha_2 (d(x, Ty) d(y, Tx)), \quad (1)$$

$\forall x, y \in X$, $0 \leq \alpha_1 < 1$, $0 \leq \alpha_2$ then T has a fixed point in X .

Theorem 2 (see [12]). If T is a mapping of the complete metric space X into itself satisfying the inequality

$$[d(Tx, Ty)]^2 \leq \alpha_1 [d(x, Tx) d(y, Ty) + d(x, Ty) d(y, Tx)] + \alpha_2 [d(x, Tx) d(y, Tx) + d(x, Ty) d(y, Ty)], \quad (2)$$

for all $x, y \in X$, where $\alpha_1, \alpha_2 \geq 0$ and $\alpha_1 + 2\alpha_2 < 1$ then T has a unique fixed point.

Theorem 3 (see [13]). If T is a mapping of the complete metric space X into itself satisfying the inequality

$$[d(Tx, Ty)]^2 \leq \alpha_1 [d(x, Tx) d(y, Ty) + d(x, Ty) d(y, Tx)] + \alpha_2 [d(x, Tx) d(y, Tx) + d(x, Ty) d(y, Ty)] + \alpha_3 [\{d(y, Tx)\}^2 + \{d(y, Ty)\}^2], \quad (3)$$

for all x, y in X , where $\alpha_1, \alpha_2, \alpha_3 \geq 0$ and $\alpha_1 + 2\alpha_2 + \alpha_3 < 1$, then T has a unique fixed point.

2. Preliminaries

In this section we recall some basic definitions and necessary results from existing literature that will be used in the sequel.

Definition 4 (see [3]). Let X be a nonempty set and $s \geq 1$ be a given real number. A function $d : X \times X \rightarrow \mathbb{R}^+$ is said to be a b -metric on X if the following conditions hold:

- (i) $d(x, y) = 0$ if and only if $x = y$.
- (ii) $d(x, y) = d(y, x)$ for all $x, y \in X$.
- (iii) $d(x, y) \leq s(d(x, z) + d(z, y))$ for all $x, y, z \in X$.

The pair (X, d) is called b -metric space.

It is clear from the definition of b -metric that every metric space is b -metric for $s = 1$, but the converse need not be true. The following example illustrates the above remarks.

Example 5 (see [5]). Let $X = \{0, 1, 2\}$. Define $d : X \times X \rightarrow \mathbb{R}^+$ by $d(0, 0) = d(1, 1) = d(2, 2) = 0$, $d(1, 2) = d(2, 1) = d(1, 0) = d(0, 1) = 1$, and $d(0, 2) = d(2, 0) = m \geq 2$ for $s = m/2$. The function defined above is a b -metric space but is not a metric space for $m > 2$.

Proposition 6 (see [14]). Let X be a nonempty set and the mappings $f, g, h : X \rightarrow X$ have a unique point of coincidence in X . If $\{f, h\}$ and $\{g, h\}$ are weakly compatible self-maps of X , then f, g, h have a unique common fixed point.

Definition 7. A sequence $\{x_n\}$ in a b -metric space (X, d) is called Cauchy sequence if and only if $\lim_{m, n \rightarrow \infty} d(x_n, x_m) = 0$.

Definition 8. A sequence $\{x_n\}$ in a b -metric space (X, d) is said to converge to a point $x \in X$ if and only if $\lim_{n \rightarrow \infty} d(x_n, x) = 0$. We denote this by $\lim_{n \rightarrow \infty} x_n = x$.

Definition 9. A b -metric space (X, d) is said to be complete if and only if every Cauchy sequence in X converges to a point of X .

Definition 10 (see [15]). Let f and g be self-mappings of a set X . If $z = fx = gx$ for some x in X , then x is called a coincidence point of f and g and z is called a point of coincidence of f and g .

Definition 11 (see [16]). The mappings $f, g : X \rightarrow X$ are weakly compatible, if, for every $x \in X$, the following holds: $f(gx) = g(fx)$ whenever $fx = gx$.

Definition 12. A point $x \in X$ is said to be a fixed point of a self-map $T : X \rightarrow X$ if $T(x) = x$.

3. Main Results

In this section we obtain coincidence points and common fixed point theorems for three maps in complete b -metric spaces. In order to start our main results we begin with a simple but useful Lemma.

Lemma 13. Let (X, d) be a complete b -metric space with the coefficient $s \geq 1$ and let $f, g, h : X \rightarrow X$ be self-mappings from X into itself satisfying the following conditions:

- (i) $f(X) \cup g(X) \subseteq h(X)$.
- (ii)

$$\begin{aligned} [d(fx, gy)]^2 &\leq \alpha_1 [d(hx, fx) d(hy, gy) \\ &\quad + d(hx, gy) d(hy, fx)] + \alpha_2 [d(hy, gx) d(hy, fx) \\ &\quad + d(hx, gy) d(hy, gy)] + \alpha_3 \{[d(hy, fx)]^2 \\ &\quad + [d(hy, gy)]^2\}, \end{aligned} \quad (4)$$

for all $x, y \in X$; $\alpha_1, \alpha_2, \alpha_3 \geq 0$, such that

$$s\alpha_1 + (s^2 + s)\alpha_2 + \alpha_3 < 1. \quad (5)$$

Then every sequence $\{y_n\}$ with initial point x_0 is a Cauchy sequence in X .

Proof. Let $x_0 \in X$ and choose a point $x_1 \in X$ such that $hx_1 = fx_0$ and for x_1 there exists $x_2 \in X$ such that $hx_2 = gx_1$, and continuing this process we construct sequences $\{x_n\}$ and $\{y_n\}$ in X given by

$$\begin{aligned} y_{2n} &= hx_{2n+1} = fx_{2n}, \\ y_{2n+1} &= hx_{2n+1} = gx_{2n+1} \quad \forall n \geq 0. \end{aligned} \quad (6)$$

Suppose that there exists $h \in [0, 1/s)$ such that

$$d(y_n, y_{n+1}) \leq hd(y_{n-1}, y_n) \quad \forall n \geq 1. \quad (7)$$

We show that $\{y_n\}$ is a Cauchy sequence in X . Using (4), we have

$$\begin{aligned} [d(y_{2n}, y_{2n+1})]^2 &= [d(hx_{2n+1}, hx_{2n+2})]^2 \\ &= [d(fx_{2n}, gx_{2n+1})]^2 \\ &\leq \alpha_1 [d(hx_{2n}, fx_{2n}) d(hx_{2n+1}, gx_{2n+1}) \\ &\quad + d(hx_{2n}, gx_{2n+1}) d(hx_{2n+1}, fx_{2n})] \\ &\quad + \alpha_2 [d(hx_{2n+1}, gx_{2n}) d(hx_{2n+1}, fx_{2n}) \\ &\quad + d(hx_{2n}, gx_{2n+1}) d(hx_{2n+1}, gx_{2n+1})] \\ &\quad + \alpha_3 \{[d(hx_{2n+1}, fx_{2n})]^2 + [d(hx_{2n+1}, gx_{2n+1})]^2\} \\ &\leq \alpha_1 [d(hx_{2n}, hx_{2n+1}) d(hx_{2n+1}, hx_{2n+2}) \\ &\quad + d(hx_{2n}, hx_{2n+2}) d(hx_{2n+1}, hx_{2n+1})] \\ &\quad + \alpha_2 [d(hx_{2n}, hx_{2n+1}) d(hx_{2n+1}, hx_{2n+1}) \\ &\quad + d(hx_{2n} hx_{2n+2}) d(hx_{2n+1}, hx_{2n+2})] \\ &\quad + \alpha_3 \{[d(hx_{2n+1}, hx_{2n+1})]^2 + [d(hx_{2n+1}, hx_{2n+2})]^2\} \end{aligned}$$

$$\begin{aligned}
&\leq \alpha_1 [d(y_{2n-1}, y_{2n}) d(y_{2n}, y_{2n+1}) \\
&+ d(y_{2n-1}, y_{2n+1}) d(y_{2n}, y_{2n})] \\
&+ \alpha_2 [d(y_{2n-1}, y_{2n}) d(y_{2n}, y_{2n}) \\
&+ d(y_{2n-1}, y_{2n+1}) d(y_{2n}, y_{2n+1})] \\
&+ \alpha_3 [\{d(y_{2n}, y_{2n})\}^2 + \{d(y_{2n}, y_{2n+1})\}^2] \\
&\leq \alpha_1 d(y_{2n-1}, y_{2n}) + \alpha_2 d(y_{2n-1}, y_{2n+1}) \\
&+ \alpha_3 d(y_{2n}, y_{2n+1}) \leq \alpha_1 d(y_{2n-1}, y_{2n}) \\
&+ s\alpha_2 [d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n+1})] \\
&+ \alpha_3 d(y_{2n}, y_{2n+1}) \leq (\alpha_1 + s\alpha_2) d(y_{2n-1}, y_{2n}) \\
&+ (s\alpha_2 + \alpha_3) d(y_{2n}, y_{2n+1}) \\
&[1 - (s\alpha_2 + \alpha_3)] d(y_{2n}, y_{2n+1}) \leq (\alpha_1 \\
&+ s\alpha_2) d(y_{2n-1}, y_{2n}) \implies \\
&d(y_{2n}, y_{2n+1}) \leq \frac{(\alpha_1 + s\alpha_2)}{[1 - (s\alpha_2 + \alpha_3)]} d(y_{2n-1}, y_{2n}) \\
&\leq \lambda d(y_{2n-1}, y_{2n}),
\end{aligned}$$

where

$$\begin{aligned}
\lambda &= \frac{(\alpha_1 + s\alpha_2)}{[1 - (s\alpha_2 + \alpha_3)]} < 1, \\
d(y_{2n}, y_{2n+1}) &\leq \lambda d(y_{2n-1}, y_{2n}), \\
d(y_{2n}, y_{2n+1}) &\leq \lambda^2 d(y_{2n-1}, y_{2n-2}).
\end{aligned} \tag{9}$$

Therefore, for all $n \in \mathbb{N}$, we can write

$$d(y_{n+1}, y_{n+2}) \leq \lambda d(y_n, y_{n+1}) \leq \dots \leq \lambda^{n+1} d(y_0, y_1). \tag{10}$$

Now, for any $m, n \in \mathbb{N}$, $m > n$, we have

$$\begin{aligned}
d(y_n, y_m) &\leq sd(y_n, y_{n+1}) + sd(y_{n+1}, y_m) \\
&\leq sd(y_n, y_{n+1}) + s^2 d(y_{n+1}, y_{n+2}) + s^3 d(y_{n+2}, y_m) \\
&\leq sd(y_n, y_{n+1}) + s^2 d(y_{n+1}, y_{n+2}) + s^3 d(y_{n+2}, y_{n+3}) \\
&\quad + \dots + s^{m-n-1} d(y_{m-2}, y_{m-1}) + s^{m-n} d(y_{m-1}, y_m) \\
&\leq sd(y_n, y_{n+1}) + s^2 d(y_{n+1}, y_{n+2}) + s^3 d(y_{n+2}, y_{n+3}) \\
&\quad + \dots + s^{m-n-1} d(y_{m-2}, y_{m-1}) + s^{m-n} d(y_{m-1}, y_m) \\
&\leq [s\lambda^n + s^2 \lambda^{n+1} + s^3 \lambda^{n+2} + \dots + s^{m-n} \lambda^{m-1}] d(y_1, y_0) \\
&\leq \frac{s\lambda^n}{(1 - s\lambda)} d(y_1, y_0).
\end{aligned} \tag{11}$$

Therefore, we have

$$d(y_n, y_m) \leq \frac{s\lambda^n}{(1 - s\lambda)} d(y_0, y_1) \longrightarrow 0 \quad \text{as } n \longrightarrow \infty. \tag{12}$$

Thus,

$$d(y_n, y_m) \longrightarrow 0 \quad \text{as } n \longrightarrow \infty. \tag{13}$$

Hence $\{y_n\}$ is a Cauchy sequence in b -metric space X . $\square$

The next theorem is b -metric variant of Theorem 1.3 in [13].

Theorem 14. Let (X, d) be a complete b -metric space with the coefficient $s \geq 1$. Suppose the self-maps $f, g, h : X \rightarrow X$ satisfy the condition

$$\begin{aligned}
[d(fx, gy)]^2 &\leq \alpha_1 [d(hx, fx) d(hy, gy) \\
&+ d(hx, gy) d(hy, fx)] \\
&+ \alpha_2 [d(hy, gx) d(hy, fx) \\
&+ d(hx, gy) d(hy, gy)] + \alpha_3 [\{d(hy, fx)\}^2 \\
&+ \{d(hy, gy)\}^2],
\end{aligned} \tag{14}$$

where $\alpha_1, \alpha_2, \alpha_3 \geq 0$ are nonnegative reals with

$$s\alpha_1 + (s^2 + s)\alpha_2 + \alpha_3 < 1. \tag{15}$$

If $f(X) \cup g(X) \subseteq h(X)$ and $h(X)$ is a complete subspace of X , then the maps f, g , and h have a coincidence point u in X . Moreover, if $\{f, h\}$ and $\{g, h\}$ are weakly compatible pairs. Then f, g , and h have a unique common fixed point in X .

Proof. Let x_0 be an arbitrary point in X and define the sequence $\{y_n\}$ in X such that $y_{2n} = hx_{2n+1} = fx_{2n}$ and $y_{2n+1} = hx_{2n+2} = gx_{2n+1} \quad \forall n = 0, 1, 2, \dots$. Now, we show that $\{y_n\}$ is a Cauchy sequence. So, by (4), we have

$$\begin{aligned}
[d(y_{2n}, y_{2n+1})]^2 &= [d(fx_{2n}, gx_{2n+1})]^2 \\
&\leq \alpha_1 [d(hx_{2n}, fx_{2n}) d(hx_{2n+1}, gx_{2n+1}) \\
&+ d(hx_{2n}, gx_{2n+1}) d(hx_{2n+1}, fx_{2n})] \\
&+ \alpha_2 [d(hx_{2n+1}, gx_{2n}) d(hx_{2n+1}, fx_{2n}) \\
&+ d(hx_{2n}, gx_{2n+1}) d(hx_{2n+1}, gx_{2n+1})] \\
&+ \alpha_3 [\{d(hx_{2n+1}, fx_{2n})\}^2 + \{d(hx_{2n+1}, gx_{2n+1})\}^2] \\
&\leq \alpha_1 [d(y_{2n-1}, y_{2n}) d(y_{2n}, y_{2n+1}) \\
&+ d(y_{2n-1}, y_{2n}) d(y_{2n}, y_{2n})] \\
&+ \alpha_2 [d(y_{2n}, y_{2n}) d(y_{2n}, y_{2n}) \\
&+ d(y_{2n-1}, y_{2n+1}) d(y_{2n}, y_{2n+1})] \\
&+ \alpha_3 [\{d(y_{2n}, y_{2n})\}^2 + \{d(y_{2n}, y_{2n+1})\}^2]
\end{aligned}$$

$$\begin{aligned}
&\leq \alpha_1 [d(y_{2n-1}, y_{2n}) d(y_{2n}, y_{2n+1})] \\
&+ \alpha_2 [d(y_{2n-1}, y_{2n+1}) d(y_{2n}, y_{2n+1})] \\
&+ \alpha_3 \{d(y_{2n}, y_{2n+1})\}^2 \Rightarrow \\
d(y_{2n}, y_{2n+1}) &\leq \alpha_1 d(y_{2n-1}, y_{2n}) + \alpha_2 d(y_{2n-1}, y_{2n+1}) \\
&+ \alpha_3 d(y_{2n}, y_{2n+1}) \leq \alpha_1 d(y_{2n-1}, y_{2n}) \\
&+ s\alpha_2 [d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n+1})] \\
&+ \alpha_3 d(y_{2n}, y_{2n+1}) \leq (\alpha_1 + s\alpha_2) d(y_{2n}, y_{2n+1}) \\
&+ (s\alpha_2 + \alpha_3) d(y_{2n-1}, y_{2n}) \Rightarrow \\
d(y_{2n}, y_{2n+1}) &\leq \frac{(\alpha_1 + s\alpha_2)}{[1 - (s\alpha_2 + \alpha_3)]} d(y_{2n-1}, y_{2n}) \\
&\leq \lambda d(y_{2n-1}, y_{2n}) \Rightarrow \\
d(y_{2n}, y_{2n+1}) &\leq \lambda d(y_{2n-1}, y_{2n}).
\end{aligned} \tag{16}$$

Similarly, we can show that

$$d(y_{2n+1}, y_{2n+2}) \leq \lambda^2 d(y_{2n}, y_{2n+1}), \tag{17}$$

where

$$\lambda = \frac{(\alpha_1 + s\alpha_2)}{[1 - (s\alpha_2 + \alpha_3)]} < \frac{1}{s}. \tag{18}$$

Therefore, for all $m, n \in \mathbb{N}$ with $m > n$, we can get

$$\begin{aligned}
(y_{n+1}, y_{n+2}) &\leq \lambda d(y_n, y_{n+1}) \leq \dots \leq \lambda^{n+1} d(y_0, y_1) \\
d(y_n, y_m) &\leq sd(y_n, y_{n+1}) + sd(y_{n+1}, y_m) \\
&\leq sd(y_n, y_{n+1}) + s^2 d(y_{n+1}, y_{n+2}) \\
&\quad + s^3 d(y_{n+2}, y_m) \\
&\leq sd(y_n, y_{n+1}) + s^2 d(y_{n+1}, y_{n+2}) \\
&\quad + s^3 d(y_{n+2}, y_{n+3}) + s^4 d(y_{n+3}, y_m) \\
&\leq sd(y_n, y_{n+1}) + s^2 d(y_{n+1}, y_{n+2}) \\
&\quad + s^3 d(y_{n+2}, y_{n+3}) + \dots \\
&\quad + s^{m-n-2} d(y_{m-3}, y_{m-2}) \\
&\quad + s^{m-n-1} d(y_{m-2}, y_{m-1}) \\
&\quad + s^{m-n} d(y_{m-1}, y_m) \Rightarrow \\
d(y_n, y_m) &\leq \frac{s\lambda^n}{(1 - s\lambda)} d(y_0, y_1).
\end{aligned} \tag{19}$$

Thus, as $n \rightarrow \infty$, $d(y_n, y_m) \rightarrow 0$. It follows from Lemma 13 that $\{y_n\}$ is a Cauchy sequence and, by the completeness of X , $\{y_n\}$ converges to some $y \in X$. Therefore,

$$\begin{aligned}
\lim_{n \rightarrow \infty} fx_{2n} &= \lim_{n \rightarrow \infty} gx_{2n+1} = \lim_{n \rightarrow \infty} hx_{2n+1} = \lim_{n \rightarrow \infty} hx_{2n+2} \\
&= y.
\end{aligned} \tag{20}$$

Thus, $f(X) \cup g(X) \subseteq h(X)$ implies either $f(X) \subseteq h(X)$ or $g(X) \subseteq h(X)$.

Case 1. Let $f(X) \subseteq h(X)$. Since $h(X)$ is a complete subspace of $f(X) \cup g(X)$ and $f(X) \subseteq h(X)$ implies $h(X)$ is closed, hence, there exist $v, y \in X$ such that $hv = y$. If $gv \neq y$, then, by using (14), we get

$$\begin{aligned}
[d(fx_{2n}, gv)]^2 &\leq \alpha_1 [d(hx_{2n}, fx_{2n}) d(hv, gv) \\
&\quad + d(hx_{2n}, gv) d(hv, fx_{2n})] \\
&+ \alpha_2 [d(hv, gx_{2n}) d(hv, fx_{2n}) \\
&\quad + d(hx_{2n}, gv) d(hv, gv)] + \alpha_3 [d(hv, fx_{2n})]^2 \\
&\quad + [d(hv, gv)]^2].
\end{aligned} \tag{21}$$

Taking limit as $n \rightarrow \infty$ yields

$$\begin{aligned}
[d(fx_{2n}, gv)]^2 &\leq \alpha_1 [d(hx_{2n}, fx_{2n}) d(hv, gv) \\
&\quad + d(hx_{2n}, gv) d(hv, fx_{2n})] \\
&+ \alpha_2 [d(hv, gx_{2n}) d(hv, fx_{2n}) \\
&\quad + d(hx_{2n}, gv) d(hv, gv)] + \alpha_3 [d(hv, fx_{2n})]^2 \\
&\quad + [d(hv, gv)]^2] \\
[d(y, gv)]^2 &\leq \alpha_1 [d(y, y) d(y, gv) \\
&\quad + d(y, gv) d(y, y)] \\
&+ \alpha_2 [d(y, y) d(y, y) + d(y, gv) d(y, gv)] \\
&\quad + \alpha_3 [d(y, y)]^2 + [d(y, gv)]^2 \leq (\alpha_2 + \alpha_3) \\
&\quad \cdot [d(y, gv)]^2 \Rightarrow \\
[1 - (\alpha_2 + \alpha_3)] [d(y, gv)]^2 &\leq 0,
\end{aligned} \tag{22}$$

and the above inequality is possible only if $[d(y, gv)]^2 = 0$ which implies that $y = gv$. It follows that

$$hv = y = gv. \tag{23}$$

Since g and h are weakly compatible, we have $hgv = ghv$ and so

$$gy = hy. \tag{24}$$

If $y \neq gy$, then by (14) we have

$$\begin{aligned} & [d(fx_{2n}, gy)]^2 \\ & \leq \alpha_1 [d(hx_{2n}, fx_{2n})d(hy, gy) + d(hx_{2n}, gy)d(hy, fx_{2n})] \\ & \quad + \alpha_2 [d(hy, gx_{2n})d(hy, fx_{2n}) + d(hx_{2n}, gy)d(hy, gy)] \\ & \quad + \alpha_3 [\{d(hy, fx_{2n})\}^2 + \{d(hy, gy)\}^2]. \end{aligned} \quad (25)$$

As $n \rightarrow \infty$, we have

$$\begin{aligned} & [d(y, gy)]^2 \\ & \leq \alpha_1 [d(y, y)d(hy, gy) + d(y, gy)d(hy, y)] \\ & \quad + \alpha_2 [d(hy, y)d(hy, y) + d(y, gy)d(hy, gy)] \\ & \quad + \alpha_3 [\{d(hy, y)\}^2 + \{d(hy, gy)\}^2] \\ & \leq \alpha_1 [d(y, y)d(gy, gy) + d(y, gy)d(gy, y)] \\ & \quad + \alpha_2 [d(gy, y)d(gy, y) + d(y, gy)d(gy, gy)] \\ & \quad + \alpha_3 [\{d(gy, y)\}^2 + \{d(gy, gy)\}^2] \\ & \leq (\alpha_1 + \alpha_2 + \alpha_3) [d(y, gy)]^2 \Rightarrow [d(y, gy)]^2 \\ & \leq (\alpha_1 + \alpha_2 + \alpha_3) [d(y, gy)]^2 \Rightarrow \end{aligned} \quad (26)$$

$$[1 - (\alpha_1 + \alpha_2 + \alpha_3)] [d(y, gy)]^2 \leq 0,$$

and the inequality is possible only if

$$\begin{aligned} d(y, gy) &= 0 \Rightarrow \\ y &= gy. \end{aligned} \quad (27)$$

By using (24), we have

$$hy = gy = y. \quad (28)$$

Case 2. If $g(X) \subseteq h(X)$, again there exists points $v, y \in X$ such that $y = hv$ if $y \neq fv$, and then by using (14) we get

$$\begin{aligned} & [d(fv, gy)]^2 \leq \alpha_1 [d(hv, fv)d(hy, gy) \\ & \quad + d(hv, gy)d(hy, fv)] \\ & \quad + \alpha_2 [d(hy, gv)d(hy, fv) \\ & \quad + d(hv, gy)d(hy, gy)] + \alpha_3 [\{d(hy, fv)\}^2 \\ & \quad + \{d(hy, gy)\}^2]. \end{aligned} \quad (29)$$

It follows that

$$\begin{aligned} & [d(fv, y)]^2 \\ & \leq \alpha_1 [d(y, fv)d(y, y) + d(y, y)d(y, fv)] \\ & \quad + \alpha_2 [d(y, y)d(y, fv) + d(y, y)d(y, y)] \\ & \quad + \alpha_3 [\{d(y, fv)\}^2 + \{d(y, y)\}^2] \Rightarrow \\ & (1 - \alpha_3) [d(y, fv)]^2 \leq 0, \end{aligned} \quad (30)$$

which is possible only if $d(y, fv) = 0 \Rightarrow y = fv$:

$$\Rightarrow fv = hv = y. \quad (31)$$

Since f and h are weakly compatible and hence $fhv = hfv$ and so $fy = hy$ and by (28), we have

$$y = fy = gy = hy. \quad (32)$$

Thus, y is the common fixed point of self-mappings f, g , and h . This completes the proof of the theorem. $\square$

Uniqueness. In order to prove uniqueness, let $y_1 \neq y_2$ be two distinct common fixed points of the self-maps f, g , and h then we have by (14)

$$\begin{aligned} & [d(y_1, y_2)]^2 = [d(fy_1, gy_2)]^2 \\ & \leq \alpha_1 [d(hy_1, fy_1)d(hy_2, gy_2) \\ & \quad + d(hy_1, gy_2)d(hy_2, fy_1)] \\ & \quad + \alpha_2 [d(hy_2, gy_1)d(hy_2, fy_1) \\ & \quad + d(hy_1, gy_2)d(hy_2, gy_2)] + \alpha_3 [\{d(hy_2, fy_1)\}^2 \\ & \quad + \{d(hy_2, gy_2)\}^2] \leq \alpha_1 [d(y_1, y_1)d(y_2, y_2) \\ & \quad + d(y_1, y_2)d(y_1, y_2)] + \alpha_2 [d(y_2, y_1)d(y_2, y_1) \\ & \quad + d(y_1, y_2)d(y_2, y_2)] + \alpha_3 [\{d(y_1, y_2)\}^2 \\ & \quad + \{d(y_2, y_2)\}^2] \leq (\alpha_1 + \alpha_2 + \alpha_3) [d(y_1, y_2)]^2 \Rightarrow \\ & [1 - (\alpha_1 + \alpha_2 + \alpha_3)] [d(y_1, y_2)]^2 \leq 0, \end{aligned} \quad (33)$$

which is possible only if $d(y_1, y_2) = 0 \Rightarrow y_1 = y_2$ which gives us uniqueness of y .

Remark 15. If we put $\alpha_3 = 0$, then we get Theorem 1.2 in [12].

Now we present the modified form of Theorem 3 in terms of b -metric spaces.

Theorem 16. Let (X, d) be a complete b -metric space with the coefficient $s \geq 1$ and suppose the self-maps $f, g, h : X \rightarrow X$ satisfy the condition

$$\begin{aligned} & [d(fx, gy)]^2 \\ & \leq \alpha_1 [d(hx, fx)d(hy, gy)] \\ & \quad + \alpha_2 [d(hx, gy)d(hy, gy)(1 + d(fx, hy))] \\ & \quad + \alpha_3 [d(hx, hy)d(hy, gy)(1 + d(fx, hy))] \\ & \quad + \alpha_4 [d(hx, gy)d(hy, gy)] \\ & \quad + \alpha_5 [\{d(hy, fx)\}^2 + \{d(hy, gy)\}^2], \end{aligned} \quad (34)$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5 \geq 0$ are nonnegative reals with

$$s\alpha_1 + (s^2 + s)\alpha_2 + s\alpha_3 + (s^2 + s)\alpha_4 + \alpha_5 < 1. \quad (35)$$

If $f(X) \cup g(X) \subseteq h(X)$ and $h(X)$ is a complete subspace of X , then the maps f, g , and h have a coincidence point u in X . Moreover, if $\{f, h\}$ and $\{g, h\}$ are weakly compatible, then f, g , and h have a unique common fixed point in X .

Proof. Let $x_0 \in X$ and define a sequence of points $\{y_n\}$ in X as follows, $y_{2n} = hx_{2n+1} = fx_{2n}$ and $y_{2n+1} = hx_{2n+2} = gx_{2n+1} \forall n = 0, 1, 2, \dots$. By using (34), we have

$$\begin{aligned}
 [d(y_{2n}, y_{2n+1})]^2 &= [d(fx_{2n}, gx_{2n+1})]^2 \leq \alpha_1 [d(hx_{2n}, \\
 &\quad fx_{2n}) d(hx_{2n+1}, gx_{2n+1})] \\
 &\quad + \alpha_2 [d(hx_{2n}, \\
 &\quad gx_{2n+1}) d(hx_{2n+1}, gx_{2n+1}) (1 + d(fx_{2n}, hx_{2n+1}))] \\
 &\quad + \alpha_3 [d(hx_{2n}, \\
 &\quad hx_{2n+1}) d(hx_{2n+1}, gx_{2n+1}) (1 + d(fx_{2n}, hx_{2n+1}))] \\
 &\quad + \alpha_4 [d(hx_{2n}, gx_{2n+1}) d(hx_{2n+1}, gx_{2n+1})] \\
 &\quad + \alpha_5 [\{d((hx_{2n+1}, fx_{2n}))\}^2 \\
 &\quad + \{d(hx_{2n+1}, gx_{2n+1})\}^2] \leq \alpha_1 [d(y_{2n-1}, \\
 &\quad y_{2n}) d(y_{2n}, y_{2n+1})] + \alpha_2 [d(y_{2n-1}, \\
 &\quad y_{2n+1}) d(y_{2n}, y_{2n+1}) (1 + d(y_{2n}, y_{2n}))] \\
 &\quad + \alpha_3 [d(y_{2n-1}, \\
 &\quad y_{2n}) d(y_{2n}, y_{2n+1}) (1 + d(y_{2n}, y_{2n}))] \\
 &\quad + \alpha_4 [d(y_{2n-1}, y_{2n+1}) d(y_{2n+1}, y_{2n})] \\
 &\quad + \alpha_5 [\{d(y_{2n}, y_{2n})\}^2 + \{d(y_{2n}, y_{2n+1})\}^2].
 \end{aligned} \tag{36}$$

Or

$$\begin{aligned}
 d(y_{2n}, y_{2n+1}) &\leq \alpha_1 d(y_{2n-1}, y_{2n}) + \alpha_2 d(y_{2n-1}, y_{2n+1}) \\
 &\quad + \alpha_3 d(y_{2n-1}, y_{2n}) + \alpha_4 d(y_{2n-1}, y_{2n+1}) \\
 &\quad + \alpha_5 d(y_{2n}, y_{2n+1}) \\
 &\leq \alpha_1 d(y_{2n-1}, y_{2n}) \\
 &\quad + s\alpha_2 [d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n+1})] \\
 &\quad + \alpha_3 d(y_{2n-1}, y_{2n}) \\
 &\quad + s\alpha_4 [d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n+1})] \\
 &\quad + \alpha_5 d(y_{2n}, y_{2n+1}) \\
 &\leq (s\alpha_2 + s\alpha_4 + \alpha_5) d(y_{2n}, y_{2n+1}) \\
 &\quad + (\alpha_1 + s\alpha_2 + \alpha_3 + s\alpha_4) d(y_{2n}, y_{2n-1}) \\
 &\quad [1 - (s\alpha_2 + s\alpha_4 + \alpha_5)] d(y_{2n}, y_{2n+1})
 \end{aligned}$$

$$\begin{aligned}
 &\leq (\alpha_1 + s\alpha_2 + \alpha_3 + s\alpha_4) d(y_{2n}, y_{2n-1}) \implies \\
 d(y_{2n+1}, y_{2n}) &\leq \frac{(\alpha_1 + s\alpha_2 + \alpha_3 + s\alpha_4)}{[1 - (s\alpha_2 + s\alpha_4 + \alpha_5)]} d(y_{2n}, y_{2n-1}) \implies \\
 d(y_{2n+1}, y_{2n}) &\leq \lambda d(y_{2n}, y_{2n-1}),
 \end{aligned} \tag{37}$$

where

$$\lambda = \frac{(\alpha_1 + s\alpha_2 + \alpha_3 + s\alpha_4)}{[1 - (s\alpha_2 + s\alpha_4 + \alpha_5)]} < \frac{1}{s}. \tag{38}$$

Similarly, it can be shown that

$$d(y_{2n+1}, y_{2n+2}) \leq \lambda d(y_{2n}, y_{2n+1}). \tag{39}$$

Therefore, for all $n \in N$ we can get

$$d(y_{n+1}, y_{n+2}) \leq \lambda d(y_n, y_{n+1}) \leq \dots \leq \lambda^{n+1} d(y_0, y_1). \tag{40}$$

Now, for any $m > n$, we have

$$\begin{aligned}
 d(y_n, y_m) &\leq d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \dots \\
 &\quad + d(y_{m-1}, y_m) \\
 &\leq [\lambda^n + \lambda^{n+1} + \dots + \lambda^{m-1}] d(y_1, y_0) \\
 &\leq \frac{\lambda^n}{(1 - \lambda)} d(y_1, y_0).
 \end{aligned} \tag{41}$$

Therefore, from Lemma 13, we have

$$d(y_n, y_m) \leq \frac{s\lambda^n}{(1 - s\lambda)} d(y_1, y_0) \longrightarrow 0 \tag{42}$$

as $m, n \longrightarrow \infty$,

where $s\lambda < 1$. It follows that the sequence $\{y_n\}$ is a Cauchy sequence and, by the completeness of X , $\{y_n\}$ converges to some $y \in X$.

Therefore,

$$\begin{aligned}
 \lim_{n \rightarrow \infty} y_n &= \lim_{n \rightarrow \infty} fx_{2n} = \lim_{n \rightarrow \infty} gx_{2n+1} = \lim_{n \rightarrow \infty} hx_{2n+1} \\
 &= \lim_{n \rightarrow \infty} hx_{2n+2} = y.
 \end{aligned} \tag{43}$$

Since $h(X)$ is a complete subspace of X , there exists $u, y \in X$ such that $hu = y$. If $gu \neq y$, using (34), we get

$$\begin{aligned}
 [d(fx_{2n}, gu)]^2 &\leq \alpha_1 [d(hx_{2n}, fx_{2n}) d(hu, gu)] \\
 &\quad + \alpha_2 [d(hx_{2n}, gu) d(hu, gu) (1 + d(fx_{2n}, hu))] \\
 &\quad + \alpha_3 [d(hx_{2n}, hu) d(hu, gu) (1 + d(fx_{2n}, hu))] \\
 &\quad + \alpha_4 [d(hx_{2n}, gu) d(hu, gu)] \\
 &\quad + \alpha_5 [\{d(hu, fx_{2n})\}^2 + \{d(hu, gu)\}^2],
 \end{aligned} \tag{44}$$

and, on taking limit as $n \rightarrow \infty$,

$$\begin{aligned}
 & [d(y, gu)]^2 \\
 & \leq \alpha_1 [d(y, y) d(y, gu)] \\
 & \quad + \alpha_2 [d(y, gu) d(y, gu) (1 + d(y, y))] \\
 & \quad + \alpha_3 [d(y, y) d(y, gu) (1 + d(y, y))] \\
 & \quad + \alpha_4 [d(y, gu) d(y, gu)] \\
 & \quad + \alpha_5 [\{d(y, y)\}^2 + \{d(y, gu)\}^2] \\
 & \leq (\alpha_2 + \alpha_4 + \alpha_5) [d(y, gu)]^2 \Rightarrow \\
 & [1 - (\alpha_2 + \alpha_4 + \alpha_5)] [d(y, gu)]^2 \leq 0,
 \end{aligned} \tag{45}$$

which is possible only if $[d(y, gu)]^2 = 0 \Rightarrow y = gu$. It follows that $hu = y = gu$. Since the pair of mappings $\{g, h\}$ is weakly compatible, we have $ghu = hgu$ and so

$$gy = hy. \tag{46}$$

If $y \neq gy$, by (34), we get the following.

As $n \rightarrow \infty$, we have

$$\begin{aligned}
 & \lim_{n \rightarrow \infty} [d(fx_{2n}, gy)]^2 \\
 & \leq \lim_{n \rightarrow \infty} \alpha_1 [d_1(hx_{2n}, fx_{2n}) d(hy, gy)] \\
 & \quad + \alpha_2 [d(hx_{2n}, gy) d(hy, gy) (1 + d(fx_{2n}, hy))] \\
 & \quad + \alpha_3 [d(hx_{2n}, hy) d(hy, gy) (1 + d(y, y))] \\
 & \quad + \alpha_4 [d(hx_{2n}, gy) d(hy, gy)] \\
 & \quad + \alpha_5 [\{d(hy, fx_{2n})\}^2 + \{d(hy, gy)\}^2] \\
 & \leq \alpha_1 [d(y, y) d(gy, gy)] \\
 & \quad + \alpha_2 [d(y, gy) d(y, gy) (1 + d(y, y))] \\
 & \quad + \alpha_3 [d(y, y) d(y, gy) (1 + d(y, y))] \\
 & \quad + \alpha_4 [d(y, gy) d(y, gy)] \\
 & \quad + \alpha_5 [\{d(y, y)\}^2 + \{d(y, gy)\}^2] \\
 & \leq (\alpha_2 + \alpha_4 + \alpha_5) [d(y, g)]^2 \Rightarrow \\
 & [1 - (\alpha_2 + \alpha_4 + \alpha_5)] [d(y, gy)]^2 \leq 0,
 \end{aligned} \tag{47}$$

and the above inequality is possible only if $[d(y, gy)]^2 = 0 \Rightarrow y = gy$. Therefore, by (46) we get

$$y = gy = hy. \tag{48}$$

Again, if $y \neq fu$, by (34) we have the following.

Taking limit as $n \rightarrow \infty$, we have

$$\begin{aligned}
 & [d(fu, gx_{2n+1})]^2 = [d(y, fu)]^2 \leq \alpha_1 [d(hu, \\
 & \quad fu) d(hx_{2n+1}, gx_{2n+1})] \\
 & \quad + \alpha_2 [d(hu, \\
 & \quad gx_{2n+1}) d(hx_{2n+1}, gx_{2n+1}) (1 + d(fu, hx_{2n+1}))] \\
 & \quad + \alpha_3 [d(hu, \\
 & \quad hx_{2n+1}) d(hx_{2n+1}, gx_{2n+1}) (1 + d(fu, hx_{2n+1}))] \\
 & \quad + \alpha_4 [d(hu, gx_{2n+1}) d(hx_{2n+1}, gx_{2n+1})] \\
 & \quad + \alpha_5 [\{d(hx_{2n+1}, fu)\}^2 + \{d(hx_{2n+1}, gx_{2n+1})\}^2] \\
 & \leq \alpha_1 [d(y, fu) d(y, y)] + \alpha_2 [d(y, \\
 & \quad y) d(y, y) (1 + d(fu, y))] \\
 & \quad + \alpha_3 [d(y, y) d(y, y) (1 + d(fu, y))] + \alpha_4 [d(y, \\
 & \quad y) d(y, y)] + \alpha_5 [\{d(y, fu)\}^2 \\
 & \quad + \{d(y, y)\}^2] \leq \alpha_5 [d(y, fu)]^2 \Rightarrow \\
 & (1 - \alpha_5) [d(y, fu)]^2 \leq 0,
 \end{aligned} \tag{49}$$

and this is possible only if $[d(fu, y)]^2 = 0 \Rightarrow y = fu$. Since f and h are weakly compatible, $fhu = hfu$:

$$\Rightarrow fy = gy. \tag{50}$$

By (48) and (50) we have

$$fy = gy = hy = y. \tag{51}$$

Thus y is the unique common fixed point of f, g , and h .

Uniqueness. In order to see the uniqueness of the common fixed point, let y_1 and y_2 be two distinct common fixed points of f, g , and h such that $y_1 \neq y_2$. Then by using (34) we get

$$\begin{aligned}
 & [d(y_1, y_2)]^2 = [d(fy_1, gy_2)]^2 \\
 & \leq \alpha_1 [d(hy_1, fy_1) d(hy_2, gy_2)] \\
 & \quad + \alpha_2 [d(hy_1, gy_2) d(hy_2, gy_2) (1 + d(fy_1, hy_2))] \\
 & \quad + \alpha_3 [d(hy_1, hy_2) d(hy_2, gy_2) (1 + d(fy_1, hy_2))] \\
 & \quad + \alpha_4 [d(hy_1, gy_2) d(hy_2, gy_2)] \\
 & \quad + \alpha_5 [\{d(hy_2, fy_1)\}^2 + \{d(hy_2, gy_2)\}^2] \\
 & \leq \alpha_1 [d(y_1, y_1) d(y_2, y_2)] \\
 & \quad + \alpha_2 [d(y_1, y_2) d(y_2, y_2) (1 + d(y_1, y_2))] \\
 & \quad + \alpha_3 [d(y_1, y_2) d(y_2, y_2) (1 + d(y_1, y_2))]
 \end{aligned}$$

$$\begin{aligned}
& + \alpha_4 [d(y_1, y_2) d(y_2, y_2)] \\
& + \alpha_5 [\{d(y_2, y_1)\}^2 + \{d(y_2, y_2)\}^2] \\
& \leq \alpha_5 [d(y_1, y_2)]^2 \implies \\
& (1 - \alpha_5) [d(y_1, y_2)]^2 \leq 0,
\end{aligned} \tag{52}$$

and the inequality is possible only if $[d(y_1, y_2)]^2 = 0$ which implies that $y_1 = y_2$ and the common fixed point is unique. $\square$

Theorem 16 yields the following corollaries.

Corollary 17. Let (X, d) be a complete b -metric space with coefficient $s \geq 1$ and f, g, h be a self-mapping of X into itself satisfying

$$\begin{aligned}
[d(fx, gy)]^2 & \leq \alpha_1 [d(hx, fx) d(hy, gy) \\
& + d(hx, gy) d(hy, gy) (1 + d(fx, hy))] \\
& + \alpha_3 [d(hx, hy) d(hy, gy) (1 + d(fx, hy)) \\
& + d(hx, gy) d(hy, gy)],
\end{aligned} \tag{53}$$

for all $x, y \in X$; and $\alpha_1, \alpha_3 \geq 0$ such that $\alpha_1 + (s^2 + 1)\alpha_3 < 1$ then f, g , and h have a unique common fixed point in X .

Proof. Putting $\alpha_5 = 0$ and $\alpha_2 = \alpha_1$, and $\alpha_4 = \alpha_3$ in Theorem 16, we get the required result. $\square$

Corollary 18. Let (X, d) be a complete b -metric space and $f, g, h : X \rightarrow X$ be self-maps of X into itself satisfying the inequality

$$\begin{aligned}
[d(fx, gy)]^2 & \leq \alpha_1 [d(hx, fx) d(hy, gy)] \\
& + \alpha_2 [d(hx, gy) d(hy, gy) (1 + d(fx, hy))],
\end{aligned} \tag{54}$$

for all $x, y \in X$, such that $\alpha_1 + (s^2 + s)\alpha_2 < 1$, and then f, g, h have a unique common fixed point in X .

Proof. Putting $\alpha_3 = \alpha_4 = \alpha_5 = 0$ in Theorem 16 we get the required result. $\square$

Remark 19. Corollary 17, is the result of Pachpatte [12].

Remark 20. Corollary 18, is the result of Fisher [11].

Example 21. Let $X = \{3, 4, 5\}$, and $d : X \times X \rightarrow R^+$ is defined as follows:

$$\begin{aligned}
d(3, 4) & = d(4, 3) = 1, \\
d(5, 4) & = d(4, 5) = \frac{20}{25}, \\
d(3, 5) & = d(5, 3) = \frac{1}{25}, \\
d(3, 3) & = d(4, 4) = d(5, 5) = 0.
\end{aligned} \tag{55}$$

Then, it is easy to verify that (X, d) is a b -metric space with parameter $s = 4/5$. Define the mappings $f, g, h : X \rightarrow X$ by

$$\begin{aligned}
f_3 & = f_5 = 4, \\
f_4 & = 5, \\
g_3 & = g_4 = g_5 = 5, \\
h_3 & = 4, \\
h_4 & = 3, \\
h_5 & = 5.
\end{aligned} \tag{56}$$

Then, for $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 1/2$, the self-maps f, g , and h satisfy all the conditions of Theorems 14 and 16 with 5 as the only common fixed point in X .

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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