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Research Article

A Subfamily of Univalent Functions Associated with q-Analogue of Noor Integral Operator

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The main objective of the present paper is to define a new subfamily of analytic functions using subordinations along with the newly defined q-Noor integral operator. We investigate a number of useful properties such as coefficient estimates, integral representation, linear combination, weighted and arithmetic means, and radius of starlikeness for this class.

1. Introduction and Definitions

In recent years, q-analysis (q-calculus) has motivated the researchers a lot due to its numerous applications in mathematics and physics. Jackson [1, 2] was the first to give some application of q-calculus and also introduced the q-analogue of derivative and integral operator. Later on, Aral and Gupta [3, 4] defined the q-Baskakov-Durrmeyer operator by using q-beta function while in papers [5, 6] the authors discussed the q-generalization of complex operators known as q-Picard and q-Gauss-Weierstrass singular integral operators. Using convolution of normalized analytic functions, Kanas and Raducanu [7] defined q-analogue of Ruscheweyh differential operator and studied some of its properties. The application of this differential operator was further studied by Aldweby and Darus [8] and Mahmood and Sokół [9]. The aim of the current paper is to define a q-analogue of the Noor integral operator involving convolution concepts and then give some interesting applications of this operator.

Let us denote the open unit disk by $\mathfrak{D} = \{z \in \mathbb{C} : |z| < 1\}$ and the symbol $\mathfrak{A}$ denotes the family of those analytic functions f which has the following Taylor series representation:

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \quad (z \in \mathfrak{D}). \tag{1}$$

For two functions f and g that are analytic in $\mathfrak{D}$ and have the form (1), we define the convolution of these functions by

$$f(z) * g(z) = z + \sum_{k=2}^{\infty} a_k b_k z^k, \quad (z \in \mathfrak{D}). \tag{2}$$

For $0 < q < 1$, the q-derivative of a function $f \in \mathfrak{A}$ is defined by

$$\partial_q f(z) = \frac{f(qz) - f(z)}{z(q-1)}, \quad (z \neq 0). \tag{3}$$

It can easily be seen that for $n \in \mathbb{N} := \{1, 2, 3, \ldots\}$ and $z \in \mathfrak{D}$

$$\partial_q \left\{ \sum_{n=1}^{\infty} a_n z^n \right\} = \sum_{n=1}^{\infty} [n, q] a_n z^{n-1}, \tag{4}$$

where

$$[n, q] = \frac{1 - q^n}{1 - q} = 1 + \sum_{l=1}^{n-1} q^l, \quad [0, q] = 0. \tag{5}$$

For any nonnegative integer n, the q-number shift factorial is defined by

$$[n, q]! = \begin{cases} 1, & n = 0, \\ [1, q][2, q][3, q] \cdots [n, q], & n \in \mathbb{N}. \end{cases} \tag{6}$$

Also the q-generalized Pochhammer symbol for $x > 0$ is given by

$$[x,q]_n = \begin{cases} 1, & n = 0, \\ [x,q][x+1,q]\cdots[x+n-1,q], & n \in \mathbb{N}. \end{cases} \quad (7)$$

For $\mu > -1$, we define the function $\mathcal{F}_{q,\mu+1}^{-1}(z)$ by

$$\mathcal{F}_{q,\mu+1}^{-1}(z) * \mathcal{F}_{q,\mu+1}(z) = z\partial_q f(z), \quad (8)$$

where the function $\mathcal{F}_{q,\mu+1}(z)$ is given by

$$\mathcal{F}_{q,\mu+1}(z) = z + \sum_{n=2}^{\infty} \frac{[\mu+1,q]_{n-1}}{[n-1,q]!} z^n, \quad (z \in \mathfrak{D}). \quad (9)$$

It is quite clear that the series defined in (9) is convergent absolutely in $\mathfrak{D}$. Using the definition of q-derivative along with the idea of convolutions, we now define the integral operator $\mathfrak{I}_q^\mu : \mathfrak{A} \to \mathfrak{A}$ by

$$\mathfrak{I}_q^\mu f(z) = \mathcal{F}_{q,\mu+1}^{-1}(z) * f(z) = z + \sum_{n=2}^{\infty} \psi_{n-1} a_n z^n, \quad (z \in \mathfrak{D}), \quad (10)$$

with

$$\psi_{n-1} = \frac{[n,q]!}{[\mu+1,q]_{n-1}}. \quad (11)$$

From (10), we can easily get the identity

$$[\mu+1,q]\,\mathfrak{I}_q^\mu f(z) = [\mu,q]\,\mathfrak{I}_q^{\mu+1} f(z) + q^\mu z\partial_q\left(\mathfrak{I}_q^{\mu+1} f(z)\right). \quad (12)$$

We note that $\mathfrak{I}_q^0 f(z) = z\partial_q f(z)$, $\mathfrak{I}_q^1 f(z) = f(z)$, and

$$\lim_{q\to 1^-} \mathfrak{I}_q^\mu f(z) = z + \sum_{n=2}^{\infty} \frac{n!}{(\mu+1)_{n-1}} a_n z^n. \quad (13)$$

This shows that, by taking $q \to 1^-$, the operator defined in (10) reduces to the familiar Noor integral operator introduced in [10, 11]. Also for more details on the q-analogue of differential and integral operators, see the work [12–14].

Motivated from the work studied in [7, 15–17], we now define subfamilies of the set $\mathfrak{A}$ by using the operator $\mathfrak{I}_q^\mu$ as follows.

Definition 1. Let $-1 \le B < A \le 1$ and $0 < q < 1$. Then the function $f \in \mathfrak{A}$ is in the class $\mathcal{Q}_q(\mu, A, B)$ if it satisfies

$$\frac{z\partial_q\left(\mathfrak{I}_q^\mu f(z)\right)}{\mathfrak{I}_q^\mu f(z)} \prec \frac{1+Az}{1+Bz}, \quad (z \in \mathfrak{D}), \quad (14)$$

where the notion "$\prec$" denotes the familiar subordinations.

Equivalently, a function $f \in \mathfrak{A}$ is in the class $\mathcal{Q}_q(\mu, A, B)$, if and only if

$$\left| \frac{z\partial_q\left(\mathfrak{I}_q^\mu f(z)\right)/\mathfrak{I}_q^\mu f(z) - 1}{A - B\left(z\partial_q\left(\mathfrak{I}_q^\mu f(z)\right)/\mathfrak{I}_q^\mu f(z)\right)} \right| < 1, \quad (z \in \mathfrak{D}). \quad (15)$$

We will assume throughout our discussion, unless otherwise stated, that

$$\begin{gathered} \mu > -1, \\ -1 \le B < A \le 1, \\ 0 < q < 1 \end{gathered} \quad (16)$$

and all coefficients a_k are positive.

We need the following result in the proof of a result.

Lemma 2 (see [18]). *Let $-1 \le B_2 \le B_1 < A_1 \le A_2 \le 1$. Then*

$$\frac{1+A_1 z}{1+B_1 z} \prec \frac{1+A_2 z}{1+B_2 z}. \quad (17)$$

2. Main Results

Theorem 3. *Let $f \in \mathfrak{A}$ be given by (1). Then the function f is in the family $\mathcal{Q}_q(\mu, A, B)$, if and only if*

$$\sum_{n=2}^{\infty} \{[n,q](1-B) - 1 + A\} a_n \psi_{n-1} < (A-B). \quad (18)$$

Proof. Let us assume first that inequality (18) holds. To show $f \in \mathcal{Q}_q(\mu, A, B)$, we only need to prove the inequality (15). For this, consider

$$\begin{aligned} &\left| \frac{z\partial_q\left(\mathfrak{I}_q^\mu f(z)\right)/\mathfrak{I}_q^\mu f(z) - 1}{A - B\left(z\partial_q\left(\mathfrak{I}_q^\mu f(z)\right)/\mathfrak{I}_q^\mu f(z)\right)} \right| \\ &\quad = \left| \frac{\sum_{n=2}^{\infty} \psi_{n-1}\left[[n,q]-1\right] a_n z^n}{(A-B)z + \sum_{n=2}^{\infty} \psi_{n-1}\left[A - B[n,q]\right] a_n z^n} \right| \\ &\quad \le \frac{\sum_{n=2}^{\infty} \psi_{n-1}\left[[n,q]-1\right] a_n}{(A-B) - \sum_{n=2}^{\infty} \psi_{n-1}\left[A - B[n,q]\right] a_n} < 1, \end{aligned} \quad (19)$$

where we have used (4), (10), and (18) and this completes the direct part. Conversely, let $f \in \mathcal{Q}_q(\mu, A, B)$ be of the form (1). Then from (15) along with (10), we have, for $z \in \mathfrak{D}$,

$$\begin{aligned} &\left| \frac{z\partial_q\left(\mathfrak{I}_q^\mu f(z)\right)/\mathfrak{I}_q^\mu f(z) - 1}{A - B\left(z\partial_q\left(\mathfrak{I}_q^\mu f(z)\right)/\mathfrak{I}_q^\mu f(z)\right)} \right| \\ &\quad = \left| \frac{\sum_{n=2}^{\infty} \psi_{n-1}\left[[n,q]-1\right] a_n z^n}{(A-B)z + \sum_{n=2}^{\infty} \psi_{n-1}\left[A - B[n,q]\right] a_n z^n} \right| < 1. \end{aligned} \quad (20)$$

Since $|\mathfrak{Re}\, z| < |z|$, we have

$$\mathfrak{Re}\left\{ \frac{\sum_{n=2}^{\infty} \psi_{n-1}\left[[n,q]-1\right] a_n z^n}{(A-B) + \sum_{n=2}^{\infty} \psi_{n-1}\left[A - B[n,q]\right] a_n z^n} \right\} < 1. \quad (21)$$

Now we choose values of z on the real axis such that $z\partial_q(\mathfrak{I}_q^{\mu} f(z))/\mathfrak{I}_q^{\mu} f(z)$ is real. Upon clearing the denominator in (21) and letting $z \to 1^-$ through real values, we obtain the required inequality (18). □

Theorem 4. *Let $f \in \mathcal{Q}_q(\mu, A, B)$. Then*

$$\mathfrak{I}_q^{\mu} f(z) = \exp \int_0^z \frac{1}{t} \left(\frac{1 - A\phi(t)}{1 - B\phi(t)} \right) d_q t, \tag{22}$$

with $|\phi(z)| < 1$ and $z \in \mathfrak{D}$.

Proof. Let $f \in \mathcal{Q}_q(\mu, A, B)$ and setting

$$\frac{z\partial_q\left(\mathfrak{I}_q^{\mu} f(z)\right)}{\mathfrak{I}_q^{\mu} f(z)} = v(z), \tag{23}$$

with

$$v(z) \prec \frac{1 + Az}{1 + Bz}, \tag{24}$$

equivalently, we can write

$$\left| \frac{v(z) - 1}{A - Bv(z)} \right| < 1, \tag{25}$$

or in other way, we have

$$\begin{gathered} \frac{v(z) - 1}{A - Bv(z)} = \phi(z), \\ |\phi(z)| < 1, \\ (z \in \mathfrak{D}). \end{gathered} \tag{26}$$

Thus we can rewrite

$$\frac{z\partial_q\left(\mathfrak{I}_q^{\mu} f(z)\right)}{\mathfrak{I}_q^{\mu} f(z)} = \left(\frac{1 - A\phi(t)}{1 - B\phi(t)} \right), \tag{27}$$

and further by simple computation of integration, the proof is completed. □

Theorem 5. *Let $f_i \in \mathcal{Q}_q(\mu, A, B)$ and have the form*

$$f_i(z) = z + \sum_{k=1}^{\infty} a_{k,i} z^k, \quad \text{for } i = 1, 2, \ldots, l. \tag{28}$$

Then $F \in \mathcal{Q}_q(\mu, A, B)$, where

$$F(z) = \sum_{i=1}^{l} c_i f_i(z) \quad \text{with } \sum_{i=1}^{l} c_i = 1. \tag{29}$$

Proof. By the virtue of Theorem 3, one can write

$$\sum_{n=2}^{\infty} \left\{ \frac{([n, q](1 - B) - 1 + A)\psi_{n-1}}{A - B} \right\} a_{n,i} < 1. \tag{30}$$

Therefore

$$\begin{aligned} F(z) &= \sum_{i=2}^{l} c_i \left(z + \sum_{n=2}^{\infty} a_{n,i} z^n \right) = z + \sum_{i=2}^{l} \sum_{n=2}^{\infty} c_i a_{n,i} z^n \\ &= z + \sum_{n=2}^{\infty} \left(\sum_{i=2}^{l} c_i a_{n,i} \right) z^n; \end{aligned} \tag{31}$$

however

$$\begin{aligned} &\sum_{n=2}^{\infty} \frac{([n, q](1 - B) - 1 + A)\psi_{n-1}}{A - B} \left(\sum_{i=2}^{l} a_{n,i} c_i \right) \\ &\quad = \sum_{i=2}^{l} \left[\sum_{n=2}^{\infty} \frac{([n, q](1 - B) - 1 + A)\psi_{n-1}}{A - B} a_{n,i} \right] c_i \le 1; \end{aligned} \tag{32}$$

then $F \in \mathcal{Q}_q(\mu, A, B)$. Hence the proof is complete. □

Theorem 6. *If f and g belong to $\mathcal{Q}_q(\mu, A, B)$, then their weighted mean h_j is also in $\mathcal{Q}_q(\mu, A, B)$, where h_j is defined by*

$$h_j(z) = \left\{ \frac{(1 - j) f(z) + (1 + j) g(z)}{2} \right\}. \tag{33}$$

Proof. From (33), we can easily write

$$h_j(z) = z + \sum_{n=2}^{\infty} \left\{ \frac{(1 - j) a_n + (1 + j) b_n}{2} \right\} z^n. \tag{34}$$

To prove that $h_j \in \mathcal{Q}_q(\mu, A, B)$, we need to show that

$$\begin{aligned} &\sum_{n=2}^{\infty} \left\{ \frac{[n, q](1 - B) - 1 + A}{A - B} \right\} \\ &\quad \cdot \left\{ \frac{(1 - j) a_n + (1 + j) b_n}{2} \right\} \psi_{n-1} < 1. \end{aligned} \tag{35}$$

For this, consider

$$\begin{aligned} &\sum_{n=2}^{\infty} \left\{ \frac{[n, q](1 - B) - 1 + A}{A - B} \right\} \\ &\quad \cdot \left\{ \frac{(1 - j) a_n + (1 + j) b_n}{2} \right\} \psi_{n-1} = \frac{(1 - j)}{2} \\ &\quad \cdot \sum_{n=2}^{\infty} \left\{ \frac{[n, q](1 - B) - 1 + A}{A - B} \right\} \psi_{n-1} a_n + \frac{(1 + j)}{2} \\ &\quad \cdot \sum_{n=2}^{\infty} \left\{ \frac{[n, q](1 - B) - 1 + A}{A - B} \right\} \psi_{n-1} b_n < \frac{(1 - j)}{2} \\ &\quad + \frac{(1 + j)}{2} = 1, \end{aligned} \tag{36}$$

where we have used inequality (18). Hence the result follows. □

Theorem 7. *Let f_i with $i = 1, 2, \ldots, \lambda$ belong to the class $\mathcal{Q}_q(\mu, A, B)$. Then the arithmetic mean h of f_i is given by*

$$h(z) = \frac{1}{\lambda}\sum_{i=1}^{\lambda} f_i(z) \tag{37}$$

and is also in the class $\mathcal{Q}_q(\mu, A, B)$.

Proof. From (37), we can write

$$h(z) = \frac{1}{\lambda}\sum_{i=1}^{\lambda}\left(z + \sum_{n=2}^{\infty} a_{n,i} z^n\right) = z + \sum_{n=2}^{\infty}\left(\frac{1}{\lambda}\sum_{i=1}^{\lambda} a_{n,i}\right) z^n. \tag{38}$$

Since $f_i \in \mathcal{Q}_q(\mu, A, B)$ for every $i = 1, 2, \ldots, \lambda$, using (38) and (18), we have

$$\begin{aligned}\sum_{n=2}^{\infty}\psi_{n-1}\left\{[n,q](1-B) - 1 + A\right\}\left(\frac{1}{\lambda}\sum_{i=1}^{\lambda} a_{n,i}\right) &= \frac{1}{\lambda}\sum_{i=1}^{\lambda}\left(\sum_{n=2}^{\infty}\psi_{n-1}\left\{[n,q](1-B) - 1 + A\right\} a_{n,i}\right) \\ &\leq \frac{1}{\lambda}\sum_{i=1}^{\lambda}(A - B) = (A - B),\end{aligned} \tag{39}$$

and this completes the proof. $\square$

Theorem 8. *Let $f \in \mathcal{Q}_q(\mu, A, B)$. Then f is in the family $\mathcal{S}^*(\beta)$ of starlike functions of order β $(0 \leq \beta < 1)$ for $|z| < r_1$, where*

$$r_1 = \left(\frac{(1-\beta)([n,q](1-B) - 1 + A)[n,q]!}{(n-\beta)(A-B)[\mu+1,q]_{n-1}}\right)^{1/(n-1)}. \tag{40}$$

Proof. Let $f \in \mathcal{Q}_q(\mu, A, B)$. To prove $f \in \mathcal{S}^*(\beta)$, we only need to show

$$\left|\frac{zf'(z)/f(z) - 1}{zf'(z)/f(z) + 1 - 2\beta}\right| < 1. \tag{41}$$

Using (1) along with some simple computation yields

$$\sum_{n=2}^{\infty}\left(\frac{n-\beta}{1-\beta}\right)|a_n||z|^{n-1} < 1. \tag{42}$$

Since $f \in \mathcal{Q}_q(\mu, A, B)$, from (18), we can easily obtain

$$\sum_{n=2}^{\infty}\frac{[n,q]!}{[\mu+1,q]_{n-1}}\left(\frac{[n,q](1-B) - 1 + A}{(A-B)}\right)|a_n| < 1. \tag{43}$$

Now inequality (42) will be true, if the following holds:

$$\sum_{n=2}^{\infty}\left(\frac{n-\beta}{1-\beta}\right)|a_n||z|^{n-1} < \sum_{n=2}^{\infty}\frac{[n,q]!}{[\mu+1,q]_{n-1}}\left(\frac{[n,q](1-B) - (1-A)}{A-B}\right)|a_n|, \tag{44}$$

which implies that

$$|z|^{n-1} < \frac{(1-\beta)([n,q](1-B) - 1 + A)[n,q]!}{(n-\beta)(A-B)[\mu+1,q]_{n-1}}, \tag{45}$$

and thus we get the needed result. $\square$

Theorem 9. *Let $-1 \leq B_2 \leq B_1 < A_1 \leq A_2 \leq 1$ and $\mathfrak{I}_q^{\mu+1} f(z) \neq 0$ in $\mathfrak{D}$, and this satisfies*

$$\frac{[\mu+1,q]}{q^{\mu}}\left\{\frac{\mathfrak{I}_q^{\mu} f(z)}{\mathfrak{I}_q^{\mu+1} f(z)} - \frac{[\mu,q]}{[\mu+1,q]}\right\} \prec \frac{1 + A_1 z}{1 + B_1 z}. \tag{46}$$

Then $f \in \mathcal{Q}_q(\mu+1, A_2, B_2)$.

Proof. Since $\mathfrak{I}_q^{\mu+1} f(z) \neq 0$ in $\mathfrak{D}$, therefore let us define the function $p(z)$ by

$$\frac{z\partial_q\left(\mathfrak{I}_q^{\mu+1} f(z)\right)}{\mathfrak{I}_q^{\mu+1} f(z)} = p(z) \quad (z \in \mathfrak{D}). \tag{47}$$

By the virtue of identity (12), we obtain

$$\frac{1}{q^{\mu}}\left\{[\mu+1,q]\frac{\mathfrak{I}_q^{\mu} f(z)}{\mathfrak{I}_q^{\mu+1} f(z)} - [\mu,q]\right\} = p(z). \tag{48}$$

Therefore, using (46), we have

$$\frac{z\partial_q\left(\mathfrak{I}_q^{\mu+1} f(z)\right)}{\mathfrak{I}_q^{\mu+1} f(z)} = p(z) \prec \frac{1 + A_1 z}{1 + B_1 z}, \tag{49}$$

and now, using Lemma 2, we have $f \in \mathcal{Q}_q(\mu+1, A_2, B_2)$. $\square$

Conflicts of Interest

The authors agree with the contents of the manuscript and there are no conflicts of interest among the authors.

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