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Research Article

On an Anisotropic Parabolic Equation on the Domain with a Disjoint Boundary

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Consider the anisotropic parabolic equation with the variable exponents $v_t = \sum_{i=1}^{n}(b_i(x)|v_{x_i}|^{q_i(x)-2}v_{x_i})_{x_i}$, where $b_i(x), q_i(x) \in C^1(\overline{\Omega})$, $q_i(x) > 1$, and $b_i(x) \geq 0$. If $\{b_i(x)\}$ is not degenerate on $\Sigma_p \subset \partial\Omega$, a part of the boundary, but is degenerate on the remained part $\partial\Omega \setminus \Sigma_p$, then the boundary value condition is imposed on Σ_p, but there is no boundary value condition required on $\partial\Omega \setminus \Sigma_p$. The stability of the weak solutions can be proved based on the partial boundary value condition $v|_{x\in\Sigma_p} = 0$.

1. Introduction

Recently, we had considered the anisotropic parabolic equation

$$v_t = \sum_{i=1}^{n}\left(b_i(x)\left|v_{x_i}\right|^{q_i(x)-2}v_{x_i}\right)_{x_i}, \quad (x,t) \in Q_T = \Omega \times (0,T), \tag{1}$$

with the initial-boundary value conditions

$$v(x,0) = v_0(x), \quad x \in \Omega, \tag{2}$$

$$v(x,t) = 0, \quad (x,t) \in \partial\Omega \times (0,T), \tag{3}$$

where $b_i(x), q_i(x) \in C^1(\overline{\Omega})$, $q_i(x) > 1$, $b_i(x) \geq 0$, and Ω is a bounded domain with a smooth boundary $\partial\Omega$. If $b_i(x) \equiv 1$, the existence of the weak solution had been proved by Antontsev and Shmarev [1]. If some of $b_i(x)$ are degenerate on the boundary, Zhan [2] had conjectured that, instead of the usual boundary value condition (3), only a partial boundary value condition,

$$v(x,t) = 0, \quad (x,t) \in \Sigma_p \times (0,T), \tag{4}$$

should be imposed, while Σ_p is relatively open in $\partial\Omega$. For example, if

$$\Omega = \{(x_1, x_2, \ldots, x_n) : 0 < x_1 < 1, 0 < x_2 < 1, \ldots, 0 < x_n < 1\} \tag{5}$$

and $b_i(x)$ has some special restrictions, then the explicit Σ_p is given, and the stability of the weak solutions is proved dependent on the partial boundary value condition [2]. However, for a general bounded domain Ω, how to depict out the explicit Σ_p seems very difficult. In this short paper, we give an original attempt, we first assume that

$$\partial\Omega = \Sigma_p \cup \Sigma'_p, \quad \overline{\Sigma_p} \cap \overline{\Sigma'_p} = \emptyset, \tag{6}$$

$$b_i(x) \geq c_i > 0, \quad x \in \Sigma_p, \tag{7}$$

$$b_i(x) = 0, \quad x \in \Sigma'_p, \tag{8}$$

where c_i is a suitable positive constant, and denote that

$$q_0 = \min_{x\in\overline{\Omega}}\{q_1(x), q_2(x), \ldots, q_{n-1}(x), q_n(x)\}, \quad q_0 > 1, \quad q^0 = \max_{x\in\overline{\Omega}}\{q_1(x), q_2(x), \ldots, q_{n-1}(x), q_n(x)\}. \tag{9}$$

A simple example satisfies (6) is

$$\Omega = \left\{x \in \mathbb{R}^n : 1 < x_1^2 + x_2^2 + \cdots + x_n^2 < 4\right\}. \tag{10}$$

Definition 1. If a function $v(x,t)$ satisfies that

$$v \in L^{\infty}(Q_T),$$
$$\frac{\partial v}{\partial t} \in L^2(Q_T), \tag{11}$$
$$b_i(x)\left|v_{x_i}\right|^{q_i(x)} \in L^2\left(0,T;L^1(\Omega)\right),$$

for any function $\varphi, \varphi_{x_i} \in L^2(0,T;L^{q_i(x)}(\Omega))$ and $\varphi|_{x\in\partial\Omega} = 0$,

$$\iint_{Q_T}\left[\frac{\partial v}{\partial t}\varphi + \sum_{i=1}^{n} b_i(x)\left|v_{x_i}\right|^{q_i(x)-2} v_{x_i}\varphi_{x_i}\right]dx\,dt = 0, \tag{12}$$

$$\lim_{t\to 0}\int_{\Omega}\left|u(x,t) - u_0(x)\right|dx = 0, \tag{13}$$

then we say $v(x,t)$ is a weak solution of (1) with the initial value condition (2). Besides, if the partial boundary value condition (4) is satisfied in the sense of the trace, then we say that $v(x,t)$ is a weak solution of the initial-boundary value problem (1)-(2)-(4).

The existence of the weak solution can be proved by the usual parabolically regularized method [2]. We are not ready to discuss the existence again in this paper. We mainly pay attentions on the stability.

Theorem 2. *If Ω satisfies (6), b_i satisfies (7)-(8), and for large enough m,*

$$m\left(\int_{\Omega\setminus\Omega_m} b_i(x)\left|\left(\prod_{j=1}^{n} b_j(x)\right)_{x_i}\right|^{q_i(x)} dx\right)^{1/q_i^+} \leq c, \tag{14}$$

$v(x,t)$ and $u(x,t)$ are two solutions of (1) with the same partial boundary value condition

$$v(x,t) = u(x,t) = 0, \quad (x,t) \in \Sigma_p \times (0,T), \tag{15}$$

then

$$\int_{\Omega}\left|v(x,t) - u(x,t)\right|dx \leq \int_{\Omega}\left|v(x,0) - u(x,0)\right|dx, \tag{16}$$

where $\Omega_m = \{x \in \Omega : \prod_{i=1}^{n} b_i(x) > 1/m\}$.

Remark 3. Since the domain Ω satisfies (6) and b_i satisfies (7)-(8) and when x is near to Σ_p, (1) is not degenerate, by (14),

$$\int_{\Omega_{1\delta}} |\nabla v|^{q_0}\,dx < \infty, \tag{17}$$

then we can define the trace of $v(x,t)$ on Σ_p, and condition (15) is reasonable. Here $\delta > 0$ is a small enough constant,

$$\Omega_{1\delta} = \left\{x \in \Omega : d(x) = \operatorname{dist}\left(x, \Sigma_p\right) < \delta\right\}. \tag{18}$$

However, when $\overline{\Sigma_p} \cap \overline{\Sigma_p'} \neq \emptyset$, then (17) is not clear. In this case, only if

$$\int_{\Omega}\left[b_i(x)\right]^{-1/(q_i(x)-1)}dx < \infty, \tag{19}$$

then we have a similar conclusion. This is the following theorem.

Theorem 4. *If the domain Ω satisfies*

$$\partial\Omega = \Sigma_p \cup \Sigma_p',$$
$$\Sigma_p \cap \Sigma_p' = \emptyset, \tag{20}$$
$$\overline{\Sigma_p} \cap \overline{\Sigma_p'} \neq \emptyset,$$

b_i satisfies (19) and

$$b_i(x) > 0, \quad x \in \Sigma_p, \; b_i(x) = 0, \; x \in \Sigma_p', \tag{21}$$

condition (14) is true when m is large enough, and if $v(x,t)$ and $u(x,t)$ are two solutions of (1) with the same partial boundary value condition (15), then stability (16) is true.

Let us give an example of the domain Ω and $b_i(x)$ in Theorem 4. For example, $n = 2$,

$$\Omega = D_1 = \left\{x \in \mathbb{R}^2 : x_1^2 + x_2^2 < 1\right\},$$
$$\Sigma_p = \left\{x \in \mathbb{R}^2 : x_1^2 + x_2^2 = 1, \; x_2 > 0\right\}, \tag{22}$$
$$\Sigma_p' = \left\{x \in \mathbb{R}^2 : x_1^2 + x_2^2 = 1, \; x_2 \leq 0\right\}.$$

$$v_t = \frac{\partial}{\partial x_1}\left(b_1(x)\left|v_{x_1}\right|^{q_1(x)-2} v_{x_1}\right) + \frac{\partial}{\partial x_2}\left(b_2(x)\left|v_{x_1}\right|^{q_2(x)-2} v_{x_2}\right), \tag{23}$$

$$b_1(x) = b_2(x) = x_2 + \sqrt{1 - x_1^2}. \tag{24}$$

At the end of this section, we would like to give a simple comment on the research background of this paper. Equation (1) is the generalized equation of the following equation:

$$v_t = \operatorname{div}\left(b(x)|\nabla v|^{q(x)-2}\nabla v\right), \quad (x,t) \in Q_T, \tag{25}$$

which originally comes from the electrorheological fluids theory (see [3, 4]). If $b(x) \equiv 1$, there are many related papers; one can see [5–7] and the references therein. If $b(x) > 0$ when $x \in \Omega$ but $b(x)|_{x\in\partial\Omega} = 0$, then the stability of the weak solutions without the boundary value condition had been studied by Zhan et al. [8–10], provided that the diffusion coefficient $b(x)$ satisfies some other restrictions.

2. The Stability

The concepts of the exponent variable spaces, $(L^{q(x)}(\Omega), |\cdot|_{L^{q(x)}(\Omega)})$, $(W^{1,q(x)}(\Omega), |\cdot|_{W^{1,q(x)}(\Omega)})$, and $W_0^{1,q(x)}(\Omega)$, can be found in [11–15].

Lemma 5 (see [11–13]). *If $p(x)$ and $q(x)$ are real functions with $1/p(x) + 1/q(x) = 1$ and $q(x) > 1$, then, for any $v \in L^{p(x)}(\Omega)$ and $u \in L^{q(x)}(\Omega)$, one has*

$$\left|\int_{\Omega} vu\,dx\right| \leq 2|v|_{L^{p(x)}(\Omega)}|u|_{L^{q(x)}(\Omega)}. \tag{26}$$

Moreover,

$$\begin{aligned}
&\text{if} && |v|_{L^{q(x)}(\Omega)} = 1, \\
&\text{then} && \int_\Omega |v|^{q(x)}\,dx = 1, \\
&\text{if} && |v|_{L^{q(x)}}(\Omega) > 1, \\
&\text{then} && |v|^{q^-}_{L^{q(x)}} \le \int_\Omega |v|^{q(x)}\,dx \le |v|^{q^+}_{L^{q(x)}}, \\
&\text{if} && |v|_{L^{q(x)}}(\Omega) < 1, \\
&\text{then} && |v|^{q^+}_{L^{q(x)}} \le \int_\Omega |v|^{q(x)}\,dx \le |v|^{q^-}_{L^{q(x)}}.
\end{aligned} \tag{27}$$

One lets $k_m(s)$ be an odd function, and

$$k_m(s) = \begin{cases} 1, & s > \dfrac{1}{m}, \\ m^2 s^2 e^{1-m^2 s^2}, & 0 \le s \leqslant \dfrac{1}{m}. \end{cases} \tag{28}$$

Then,

$$\lim_{m\to\infty} k_m(s) = \operatorname{sgn}(s), \quad s \in (-\infty, +\infty). \tag{29}$$

Let $\varphi(x)$ be a $C^1(\overline{\Omega})$ function satisfying

$$\begin{gathered}
\varphi(x)|_{x\in\Sigma'_p} = 0, \quad \varphi(x)|_{x\in\overline{\Omega}\setminus\Sigma'_p} > 0, \\
\Omega_m = \left\{x \in \Omega : \varphi(x) > \frac{1}{m}\right\}.
\end{gathered} \tag{30}$$

Theorem 6. *If Ω satisfies (6), b_i satisfies (7)-(8), and for large enough m,*

$$m\left(\int_{\Omega\setminus\Omega_m} b_i(x)\left|\varphi(x)_{x_i}\right|^{q_i(x)} dx\right)^{1/q_i^+} \le c, \tag{31}$$

$v(x,t)$ and $u(x,t)$ are two solutions of (1) with the same partial boundary value condition

$$v(x,t) = u(x,t) = 0, \quad (x,t) \in \Sigma_p \times (0,T), \tag{32}$$

then

$$\int_\Omega |v(x,t) - u(x,t)|\,dx \le \int_\Omega |v(x,0) - u(x,0)|\,dx. \tag{33}$$

Proof. We let $v(x,t)$ and $u(x,t)$ be two weak solutions of (1) with the partial boundary value condition (32). Then, $k_m(v-u)|_{x\in\Sigma_p} = 0$.

Let

$$\varphi_m(x) = \begin{cases} 1, & \text{if } x \in \Omega_m, \\ m\varphi(x), & \text{if } x \in \Omega \setminus \Omega_m. \end{cases} \tag{34}$$

Then $\varphi_m(x)|_{x\in\Sigma'_p} = 0$.

Let $\chi_{[\tau,s]}$ be the characteristic function of $[\tau,s) \subseteq [0,T)$. We can choose $\chi_{[\tau,s]}\varphi_m k_m(v-u)$ as the test function, then

$$\begin{aligned}
&\int_\tau^s \int_\Omega \varphi_m k_m(v-u)\frac{\partial(v-u)}{\partial t}\,dx\,dt \\
&+ \sum_{i=1}^n \int_\tau^s \int_\Omega b_i(x)\left(\left|v_{x_i}\right|^{q_i(x)-2} v_{x_i} - \left|u_{x_i}\right|^{q_i(x)-2} u_{x_i}\right) \\
&\quad \cdot \left(v_{x_i} - u_{x_i}\right) k'_m(v-u)\varphi_m(x)\,dx\,dt \\
&+ \sum_{i=1}^n \int_\tau^s \int_\Omega b_i(x)\left(\left|v_{x_i}\right|^{q_i(x)-2} v_{x_i} - \left|u_{x_i}\right|^{q_i(x)-2} u_{x_i}\right) \\
&\quad \cdot \left(v_{x_i} - u_{x_i}\right) k_m(v-u)\varphi_{mx_i}\,dx\,dt = 0.
\end{aligned} \tag{35}$$

Certainly, we have

$$\begin{aligned}
&\int_\Omega b_i(x)\left(\left|v_{x_i}\right|^{q_i(x)-2} v_{x_i} - \left|u_{x_i}\right|^{q_i(x)-2} u_{x_i}\right)\left(v_{x_i} - u_{x_i}\right) \\
&\quad \cdot k'_m(v-u)\varphi_m(x)\,dx \ge 0.
\end{aligned} \tag{36}$$

Since $v_t \in L^2(Q_T)$, by the Lebesgue dominated convergence theorem, we have

$$\begin{aligned}
&\lim_{m\to\infty} \int_\tau^s \int_\Omega \varphi_m(x) k_m(v-u)\frac{\partial(v-u)}{\partial t}\,dx\,dt \\
&\quad = \int_\Omega |v-u|(x,s)\,dx - \int_\Omega |v-u|(x,\tau)\,dx.
\end{aligned} \tag{37}$$

For the last term on the left hand side of (35), obviously, $\varphi_{mx_i} = m\varphi_{x_i}$ when $x \in \Omega\setminus\Omega_m$; in the other places, it vanishes. By condition (31), we have

$$\begin{aligned}
&\left|\int_\Omega b_i(x)\left(\left|v_{x_i}\right|^{q_i(x)-2} v_{x_i} - \left|u_{x_i}\right|^{q_i(x)-2} u_{x_i}\right)\right. \\
&\quad \left.\cdot \varphi_{mx_i} k_m(v-u)\,dx\right| = \left|\int_{\Omega\setminus\Omega_m} b_i(x)\right. \\
&\quad \cdot \left(\left|v_{x_i}\right|^{q_i(x)-2} v_{x_i} - \left|u_{x_i}\right|^{q_i(x)-2} u_{x_i}\right) \\
&\quad \left.\cdot \varphi_{mx_i} k_m(v-u)\,dx\right| \le cm\left(\int_{\Omega\setminus\Omega_m} b_i(x)\right. \\
&\quad \left.\cdot \left(\left|v_{x_i}\right|^{q_i(x)} + \left|u_{x_i}\right|^{q_i(x)}\right) dx\right)^{1/p_i^+} \left(\int_{\Omega\setminus\Omega_m} b_i(x)\right. \\
&\quad \left.\cdot \left|\varphi_{x_i}\right|^{q_i(x)} dx\right)^{1/q_i^+} \\
&\quad \le c\left[\left(\int_{\Omega\setminus\Omega_m} b_i(x)\left|v_{x_i}\right|^{q_i(x)} dx\right)^{1/p_i^+}\right.
\end{aligned}$$

$$+\left(\int_{\Omega\setminus\Omega_m} b_i(x)\left|u_{x_i}\right|^{q_i(x)} dx\right)^{1/p_i^+}\Bigg]$$
$$\cdot\left[m\left(\int_{\Omega\setminus\Omega_m} b_i(x)\left|\varphi_{x_i}\right|^{q_i(x)} dx\right)^{1/q_i^+}\right]$$
$$\leq c\left(\int_{\Omega\setminus\Omega_m} b_i(x)\left|v_{x_i}\right|^{q_i(x)} dx\right)^{1/p_i^+}$$
$$+ c\left(\int_{\Omega\setminus\Omega_m} b_i(x)\left|u_{x_i}\right|^{q_i(x)} dx\right)^{1/p_i^+}. \tag{38}$$

Here $p_i(x) = q_i(x)/(q_i(x)-1)$ and $p_i^+ = \max_{x\in\overline{\Omega}} p_i(x)$. Then

$$\lim_{m\to\infty}\left|\int_\tau^s\int_\Omega b_i(x)\left(\left|v_{x_i}\right|^{q_i(x)-2} v_{x_i} - \left|u_{x_i}\right|^{q_i(x)-2} u_{x_i}\right)\right.$$
$$\left.\cdot\,\varphi_{mx_i} k_m(v-u)\,dx\,dt\right|$$
$$\leq c\lim_{m\to\infty}\left[\left(\int_{\Omega\setminus\Omega_m} b_i(x)\left|v_{x_i}\right|^{q_i(x)} dx\right)^{1/p_i^+}\right. \tag{39}$$
$$\left.+\left(\int_{\Omega\setminus\Omega_m} b_i(x)\left|u_{x_i}\right|^{q_i(x)} dx\right)^{1/p_i^+}\right] = 0.$$

Now, let $m\to\infty$ in (35). Then

$$\int_\Omega |v(x,s)-u(x,s)|\,dx \leqslant \int_\Omega |v(x,\tau)-u(x,\tau)|\,dx, \tag{40}$$

and by the arbitrary of τ, we have (33). The theorem is proved. □

Corollary 7. *Theorem 2 is true.*

Proof. We only need to choose

$$\varphi(x) = \prod_{i=1}^n b_i(x); \tag{41}$$

in Theorem 6, the conclusion is clear.

Certainly, there are many choices of φ. For example, when x is near to the boundary, $\varphi(x) = d_{\Sigma_p'}(x) = \operatorname{dist}(x, \Sigma_p')$. □

Corollary 8. *Instead of the condition (31), if*

$$m\left(\int_{\Omega\setminus\Omega_m} b_i(x)\,dx\right)^{1/q_i^+} \leq c \tag{42}$$

and $\varphi(x) = d_{\Sigma_p'}$, then the same conclusion of Theorem 6 is true.

Only if one notices that

$$\left|\varphi_{x_i}\right| = \left|d_{x_i}\right| \leq |\nabla d| = 1, \tag{43}$$

then the corollary follows.

3. Proof of Theorem 4

Similar to the proof of Lemma 3.2 in [2], we have the following lemma.

Lemma 9. *If for any given $j \in \{1,2,\ldots,n\}$,*

$$\int_\Omega b_j^{-1/(q_j(x)-1)}(x)\,dx < \infty, \tag{44}$$

then

$$\int_\Omega \left|v_{x_j}\right| dx \leq c. \tag{45}$$

One omits the details of the proof here. By this lemma, one can see that if $b_i(x)$ satisfies (7), (8), and (44), then one can define the trace of v on the boundary $\partial\Omega$.

Proof of Theorem 4. Since $\overline{\Sigma_p} \cap \overline{\Sigma_p'} \neq \emptyset$, (17) is not true generally. But we have added another condition (44) in Theorem 4; by Lemma 9, we still can impose the partial boundary condition (15). Accordingly, we can choose $\chi_{[\tau,s]}\varphi_m k_m(v-u)$ as the test function. Thus, similar to the proof of Theorem 6, we can prove Theorem 4. □

4. Conclusion

An anisotropic parabolic equation is considered in this paper. In our previous work [2], if the diffusion coefficients are degenerate on the boundary in some directions, while in the other directions they are not degenerate, how to give a suitable partial boundary value condition to match the equation had been studied. In this short paper, we consider the problem in a different view. We assume that the all diffusion coefficients are degenerate on a part of the boundary Σ_p' but not degenerate on the remained part of the boundary Σ_p. It is clear that we should impose the boundary value condition on Σ_p. By choosing a test function associated with the domain, the stability of the weak solutions is proved in this paper based on the partial boundary value condition. The method of choosing a test function associated with the domain is an innovative method, which can be generalized to use in the other kinds of the degenerate parabolic equation.

Conflicts of Interest

The author declares that he has no conflicts of interest.

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