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Research Article

Exact Traveling Wave Solutions and Bifurcation of a Generalized $(3+1)$-Dimensional Time-Fractional Camassa-Holm-Kadomtsev-Petviashvili Equation

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In this paper, we study the $(3+1)$-dimensional time-fractional Camassa-Holm-Kadomtsev-Petviashvili equation with a conformable fractional derivative. By the fractional complex transform and the bifurcation method for dynamical systems, we investigate the dynamical behavior and bifurcation of solutions of the traveling wave system and seek all possible exact traveling wave solutions of the equation. Furthermore, the phase portraits of the dynamical system and the remarkable features of the solutions are demonstrated via interesting figures.

1. Introduction

Fractional differential equations have wide applications in physics, biology, geology, and other fields. Fu and Yang [1] derived a $(3+1)$-dimensional time-space fractional ZK equation which describes the fractal process of nonlinear dust acoustic waves and obtained some exact solutions via the extended Kudryashov method. The Lump solution of fractional KP equation in dusty plasma is studied in [2]. Guo et al. [3] derived a time-fractional mZK equation to study gravity solitary waves. And some new model equations were derived to study the dynamics of nonlinear Rossby waves in [4, 5]. In recent years, many powerful methods for solving fractional differential equations have been proposed, for instance, the first integral method [6, 7], the variational iterative method [8–10], the exp-function method [11, 12], the fractional complex transform [13], and the (G'/G)-expansion method [14–16]. It is rather remarkable that the exact traveling wave solutions of the fractional differential equations were obtained by using the bifurcation theory in [17–19].

Recently, by the semi-inverse method, the Euler-Lagrange equation, and Agrawal's method, Lu et al. [20] derived a generalized $(3+1)$-dimensional time-fractional Camassa-Holm-Kadomtsev-Petviashvili (gCH-KP) equation:

$$(D_t^\alpha u + au_x + buu_x + cD_t^\alpha u_{xx})_x + c_1 u_{yy} + c_2 u_{zz} = 0, \qquad (1)$$

which describes the role of dispersion in the formation of patterns in liquid drops, where a, b, c, c_1, and c_2 are nonzero constants and $D_t{}^\alpha$ is the Riemann-Liouville fractional derivative of $u(t, x, y, z)$, $0 < \alpha < 1$ [21]. Some exact solutions of Equation (1) are constructed in [20] by a bilinear method and the radial basis function meshless approach. When $\alpha = 1$, Equation (1) becomes the generalized Camassa-Holm-Kadomtsev-Petviashvili equation, which has been extensively studied in [22–26].

In this paper, we use the conformable fractional derivative proposed by Khalil et al. [27] which is a natural definition and can be applied in optical fiber, physics, and so on. By using the bifurcation method of dynamical systems [17–19, 28–31], we investigate the dynamical behavior and bifurcation of solutions of the traveling wave system and try to construct all possible exact traveling wave solutions of Equation (1).

This paper is structured as follows. A brief description on the conformable fractional derivative is given in Section 2. In

Section 3, by the fractional complex transformation [13], we transform the time-fractional gCH-KP Equation (1) into an ordinary differential equation. Then, applying the bifurcation method of dynamical systems [17–19, 28–31], we give the bifurcations and phase portraits of Equation (1). In Section 4, we give all possible exact traveling wave solutions of Equation (1) under different parameter conditions. In Section 5, we state the main conclusion of this paper.

2. Overview on Fractional Derivative

Since L'Hospital in 1695 asked what does it mean $d^n f/dx^n$ if $n=(1/2)$, a lot of definitions of fractional derivative have been introduced in different senses, such as Riemann-Liouville, Caputo, and Weyl fractional derivatives [21]. In this paper, we consider the conformable fractional derivative proposed by Khalil et al. [27]. Given a function $f:(0,+\infty)\longrightarrow \mathbf{R}$. Then, the conformable fractional derivative of f of order α is defined as

$$D_t^{\alpha} f(t)=\lim_{\varepsilon\to 0}\frac{f\left(t+\varepsilon t^{1-\alpha}\right)-f(t)}{\varepsilon} \tag{2}$$

for all $t>0$, $\alpha\in(0,1]$. And the conformable fractional derivative has the following properties (see [27]). Let $\alpha\in(0,1]$ and f,g be α-differentiable at a point $t>0$. Then,

$$\begin{aligned} &D_t^{\alpha} t^s = s t^{s-\alpha}, \quad s\in\mathbf{R},\\ &D_t^{\alpha}[f(t)g(t)] = g(t)D_t^{\alpha} f(t)+f(t)D_t^{\alpha} g(t). \end{aligned} \tag{3}$$

In addition, if f is differentiable, then

$$D_t^{\alpha} f(t)=t^{1-\alpha}\frac{df(t)}{dt}. \tag{4}$$

3. Bifurcations and Phase Portraits of the Time-Fractional gCH-KP Equation

Introduce the following fractional transformation [13]:

$$\begin{aligned} &\xi = kx+ly+mz-\frac{n}{\alpha}t^{\alpha},\\ &u(t,x,y,z)=U(\xi), \end{aligned} \tag{5}$$

where k, l, m, and n are arbitrary constants. By (3)–(4), it infers

$$D_t^{\alpha} u(t)=t^{1-\alpha}\frac{dU(\xi)}{dt}=t^{1-\alpha}\frac{dU(\xi)}{d\xi}\cdot\frac{d\xi}{dt}=-n\frac{dU(\xi)}{d\xi}. \tag{6}$$

Substituting Equation (5) into Equation (1), we have

$$\begin{aligned} &k\left(-nU'+akU'+bkUU'-cnk^2U'''\right)'\\ &\quad + c_1 l^2 U''+c_1 m^2 U''=0, \end{aligned} \tag{7}$$

where $'$ is the derivative with respect to ξ. Then, we integrate Equation (7) twice and obtain

$$cnk^3U''=\frac{bk^2}{2}U^2+\left(ak^2-nk+c_1l^2+c_2m^2\right)U. \tag{8}$$

Following [17–19, 28–30], Equation (8) is equivalent to the planar Hamiltonian system

$$\begin{aligned} &\frac{dU}{d\xi}=V,\\ &\frac{dV}{d\xi}=AU^2+BU, \end{aligned} \tag{9}$$

with the Hamiltonian

$$H(U,V)=\frac{1}{2}V^2-\frac{A}{3}U^3-\frac{B}{2}U^2=h, \tag{10}$$

where $A+(b/2cnk)$, $B=(ak^2-nk+c_1l^2+c_2m^2)/cnk^3$. Clearly, $A\neq 0$. We shall investigate the bifurcations of phase portraits of the system (9) in the (U,V)-phase plane depending on the parameters a,b,c,c_1,c_2,k,l,m,n. It is well known that a smooth homoclinic orbit of the system (9) gives rise to a smooth solitary wave solution of Equation (1). For a continuous solution of $U(kx+ly+mz-(n/\Gamma(1+\alpha))t^{a})=U(\xi)(-\infty<\xi<+\infty)$ of the system (9) satisfying $\lim_{\xi}\longrightarrow +\infty U(\xi)=\beta$ and $\lim_{\xi}\longrightarrow -\infty\, U(\xi)=\gamma$, $U(\xi)$ is called a homoclinic orbit if $\beta=\gamma$; $U(\xi)$ is called a heteroclinic orbit if $\beta\neq\gamma$. Usually, a homoclinic orbit of the system (9) corresponds to a solitary wave solution of Equation (1), and a heteroclinic orbit of the system (9) corresponds to a kink (or antikink) wave solution. Similarly, a periodic orbit of the system (9) corresponds to a periodic traveling wave solution of Equation (1). Thus, we try to seek all homoclinic orbits, heteroclinic orbits, and periodic orbits of the system (9), which depend on the parameters a,b,c,c_1,c_2,k,l,m,n.

In order to study the phase portraits of the system (9), we first study the equilibrium points of the system (9). It is easy to see that when $B\neq 0$, the system (9) has two equilibrium points $E_0(0,0)$ and $E_1(-(B/A),0)$. When $B=0$, the system (9) has only one equilibrium point $E_0(0,0)$. Let $M(U_e,V_e)$ be the coefficient matrix of the linearized system of the system (9) at an equilibrium point $E_j\,(j=0,1)$. And let $J=\det(M(U_e,V_e))$. We have

$$\begin{aligned} J(E_0)&=-B,\\ J(E_1)&=B,\\ \mathrm{Trace}(M(E_0))&=0,\\ \mathrm{Trace}(M(E_1))&=0. \end{aligned} \tag{11}$$

According to the bifurcation method of dynamical systems [28–31] and the above analysis, it infers the following.

Case 1. When $B>0$, the origin $E_0(0,0)$ is a saddle point and $E_1(-(B/A),0)$ is a center point. The bifurcations of phase portraits of system (9) are shown in Figures 1(a) and 1(b).

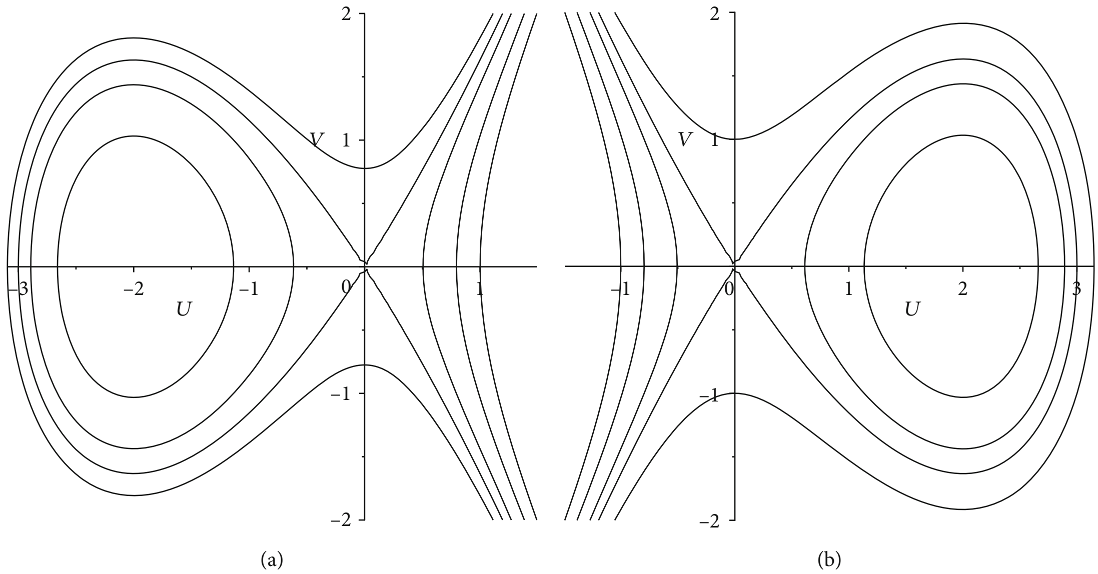

Figure 1: The phase portraits of the system (9) for $B > 0$: (a) $A > 0$; (b) $A < 0$.

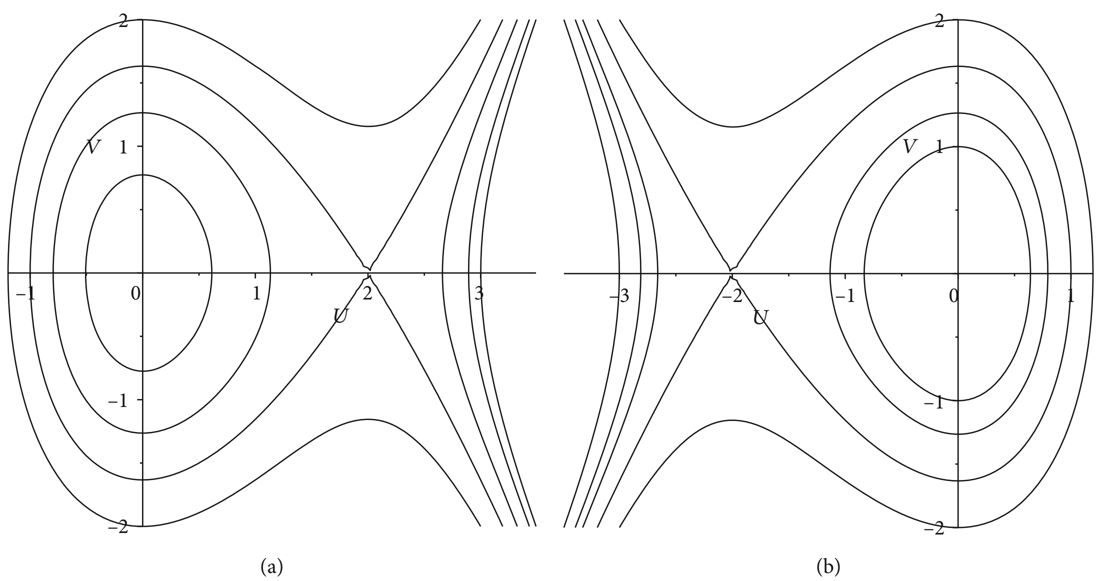

Figure 2: The phase portraits of the system (9) for $B < 0$: (a) $A > 0$; (b) $A < 0$.

Case 2. When $B < 0$, the origin $E_0(0,0)$ is a center point and $E_1(-(B/A),0)$ is a saddle point. The bifurcations of phase portraits of the system (9) are shown in Figures 2(a) and 2(b).

Case 3. When $B = 0$, the origin $E_0(0,0)$ is a cusp point. The bifurcations of phase portraits of the system (9) are shown in Figures 3(a) and 3(b).

4. Explicit Parametric Expressions of the Solutions of Equation (1)

In this section, by using the elliptic integral theory in Byrd and Fridman [32] and the direct integration method, we give all possible explicit parametric representations of the traveling wave solutions of Equation (1). For convenience, denote

$$\begin{aligned} h_0 &= H(0,0) = 0, \\ h_1 &= H\left(-\frac{B}{A},0\right) = -\frac{B^3}{6A^2}. \end{aligned} \tag{12}$$

4.1. Consider Case 1 in Section 3 (See Figure 1)

(1) If $B > 0, A > 0$, corresponding to the homoclinic orbit to the origin $E_0(0,0)$ defined by $H(U,V) = h_0$, Equation (1) has a smooth solitary wave solution of

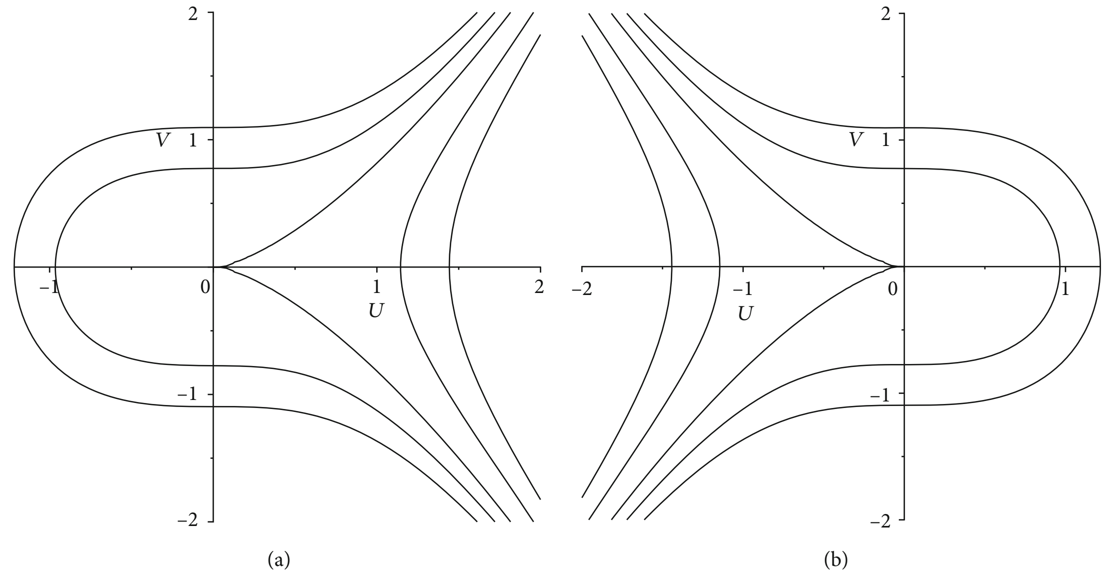

Figure 3: The phase portraits of the system (9) for $B = 0$: (a) $A > 0$; (b) $A < 0$.

valley type shown in Figure 1(a). By $H(U, V) = h_0 = 0$, it obtains

$$V = \mp\sqrt{\frac{2A}{3}}U\sqrt{U + \frac{3B}{2A}}. \quad (13)$$

Then, using the first equation of the system (9) and Equation (13), we obtain the parametric expression of the smooth solitary wave solution of valley type as follows:

$$u(t, x, y, z) = \left|\frac{3B}{2A}\right|\left(\tanh^2\left(\frac{\sqrt{B}}{2}|\xi|\right) - 1\right), \quad (14)$$

where $\xi = kx + ly + mz - (n/\Gamma(1+\alpha))t^\alpha, A = (b/2cnk)$, and $B = (ak^2 - nk + c_1 l^2 + c_2 m^2)/cnk^3$. The profile of the valley solitary wave solution (14) is shown in Figure 4(a) with $A = 1$, $B = 2$, $k = 1$, $n = 1$, $\alpha = 0.5$, and $y = z = 0$.

(2) If $B > 0$, $A < 0$, corresponding to the homoclinic orbit to the origin $E_0(0, 0)$ defined by $H(U, V) = h_0$, Equation (1) has a smooth solitary wave solution of peak type shown in Figure 1(b). By $H(U, V) = h_0 = 0$, it infers

$$V = \pm\sqrt{-\frac{2A}{3}}U\sqrt{-\frac{3B}{2A} - U}. \quad (15)$$

Combining $dU/V = d\xi$, we obtain the exact solitary wave solution of peak type as follows:

$$u(t, x, y, z) = \left|\frac{3B}{2A}\right|\left(1 - \tanh^2\left(\frac{\sqrt{B}}{2}|\xi|\right)\right), \quad (16)$$

where $\xi = kx + ly + mz - (n/\Gamma(1+\alpha))t^\alpha$. The profile of the peak solitary wave solution (16) is shown in Figure 4(b) with $A = -1$, $B = 2$, $k = 1$, $n = 1$, $\alpha = 0.5$, and $y = z = 0$.

(3) If $B > 0$, $A > 0$ or $B > 0$, $A < 0$, Equation (1) has a family of smooth periodic wave solutions defined by $H(U, V) = h$, $h \in (h_1, 0)$ (see Figures 1(a) and 1(b)). When $B > 0$, $A > 0$, the phase portrait of the system (9) is shown in Figure 1(a). In this case, the expressions of the closed orbits can be written

$$V = \pm\sqrt{\frac{2A}{3}}\sqrt{(U - U_1)(U_2 - U)(U_3 - U)}, \quad (17)$$

where $(U_1, 0)$, $(U_2, 0)$, and $(U_3, 0)$ are the intersections of the curve defined by $H(U, V) = h$, $h \in (h_1, 0)$, and the U-axis, and the relation $U_1 < U < U_2 < U_3$ holds. Thus, by using the first equation of the system (9) and Equation (17), we obtain the parametric expression of the periodic solutions as follows:

$$u(t, x, y, z) = U_1 + (U_2 - U_1)\mathrm{Sn}^2 \cdot \left(\sqrt{\frac{A(U_3 - U_1)}{6}}|\xi|, \sqrt{\frac{U_2 - U_1}{U_3 - U_1}}\right). \quad (18)$$

The profile of the periodic solution (18) is shown in Figure 4(c) with $A = 1/2$, $B = 1$, $k = 1$, $n = 1$, $h = -(1/3)$, $\alpha = 0.5$, and $y = z = 0$. When $B > 0$, $A < 0$, the similar analysis can be applied to Figure 1(b). Assume that $(U_4, 0)$, $(U_5, 0)$, and $(U_6, 0)$ are the intersections

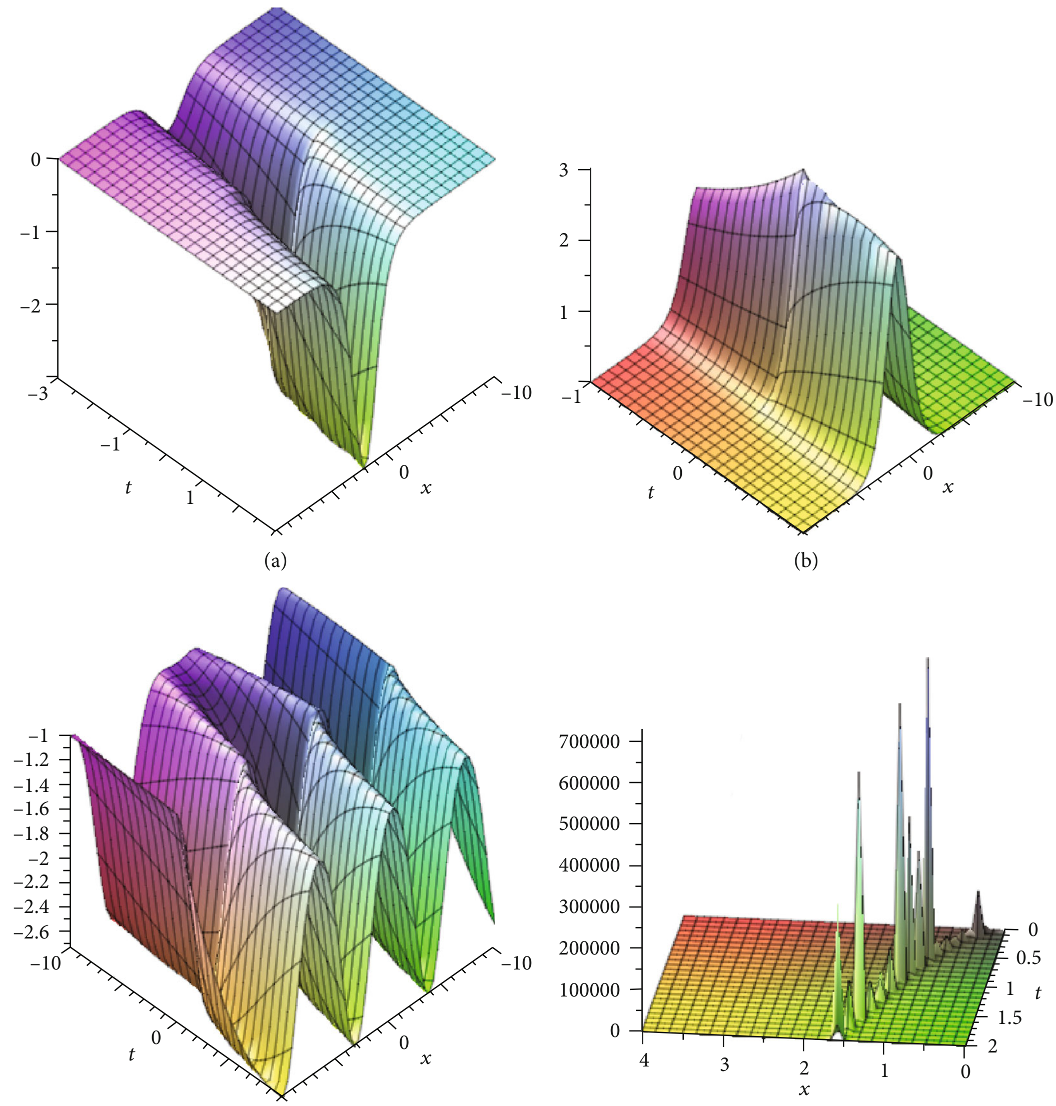

Figure 4: The profile of the solutions of Equation (1): (a) $A=1, b=2, k=1, n=1, \alpha=0.5$, and$y=z=0$;.(b)$A=-1, B=2, k=1, n=1, \alpha=0.5$, and $y=z=0$; (c)$A=1/2, B=1, k=1, n=1, h=-(1/3), \alpha=0.5$, and $y=z=0$; (d) $A=1, B=0, k=1, n=1, \alpha=0.5$, and $y=z=0$.

of the curve defined by $H(U,V)=h$ and the U-axis, and the relation $U_4<U_5<U<U_6$ holds, we obtain the parametric expression of the periodic solutions as follows:

$$u(t,x,y,z)=U_4+\frac{(U_4-U_5)(U_6-U_4)}{(U_6-U_5)\mathrm{Sn}^2\left(\sqrt{A(U_4-U_6)/6}|\xi|,\sqrt{(U_6-U_5)/(U_6-U_4)}\right)-(U_6-U_4)}. \tag{19}$$

4.2. Consider Case 2 in Section 3 (See Figure 2)

(1) If $B<0$, $A>0$, corresponding to the homoclinic orbit to the equilibrium point $E_1(-(\mathrm{B/A}),0)$ defined by $H(U,V)=h_1$, Equation (1) has a smooth solitary wave solution of valley type shown in Figure 2(a). The expressions of the homoclinic orbits defined by $H(U,V)=h_1$ can be written:

$$V^2 = \frac{2A}{3}\left(U + \frac{B}{A}\right)^2\left(U - \frac{B}{2A}\right). \quad (20)$$

Then, using the first equation of the system (9) and Equation (20), we obtain the parametric expression of the smooth solitary wave solution of valley type as follows:

$$u(t,x,y,z) = \left|\frac{B}{2A}\right|\left(3\tanh^2\left(\frac{\sqrt{-B}}{2}|\xi|\right) - 1\right), \quad (21)$$

where $\xi = kx + ly + mz - (n/\Gamma(1+\alpha))t^\alpha$.

(2) If $B > 0, A < 0$, corresponding to the homoclinic orbit to the equilibrium point $E_1(-(B/A), 0)$ defined by $H(U, V) = h_1$, Equation (1) has a smooth solitary wave solution of peak type shown in Figure 2(b). The expressions of the homoclinic orbits defined by $H(U, V) = h_1$ can be written:

$$V^2 = -\frac{2A}{3}\left(U + \frac{B}{A}\right)^2\left(\frac{B}{2A} - U\right). \quad (22)$$

Combining $dU/V = d\xi$, we obtain the exact solitary wave solution of peak type as follows:

$$u(t,x,y,z) = \left|\frac{B}{2A}\right|\left(1 - 3\tanh^2\left(\frac{\sqrt{-B}}{2}|\xi|\right)\right), \quad (23)$$

where $\xi = kx + ly + mz - (n/\Gamma(1+\alpha))t^\alpha$.

(3) If $B < 0, A > 0$ or $B < 0, A < 0$, Equation (1) has a family of smooth periodic wave solutions defined by $H(U, V) = h$, $h \in (0, h_1)$ (see Figures 2(a) and 2(b)). In this case, the parametric expression of the periodic solutions is similar to Equations (18) and (19).

4.3. Consider Case 3 in Section 3 (See Figure 3)

(1) If $B = 0, A > 0$, there is an open orbit with the same Hamiltonian as the origin point (0,0) (see Figure 3(a)). The open orbit can be written as follows:

$$V^2 = -\frac{2A}{3}U^3. \quad (24)$$

Then, by $dU/d\xi = V$, it infers the periodic cusp wave solution:

$$u(t,x,y,z) = \frac{6}{A\xi^2}, \quad (25)$$

where $\xi = kx + ly + mz - (n/\Gamma(1+\alpha))t^\alpha$. The profile of the periodic cusp wave solution (25) is shown in Figure 4(d) with $A = 1, B = 0, k = 1, n = 1, \alpha = 0.5$, and $y = z = 0$.

(2) The similar analysis can be applied to Figure 3(b). In this case, we obtain the periodic cusp wave solution:

$$u(t,x,y,z) = -\frac{6}{A\xi^2}, \quad (26)$$

where $\xi = kx + ly + mz - (n/\Gamma(1+\alpha))t^\alpha$.

5. Conclusion

In this study, using the fractional complex transform [13] and the bifurcation method of dynamical systems [17–19, 28–31], we obtain the bifurcations and phase portraits of the traveling wave system and construct all possible exact traveling wave solutions of Equation (1) with the conformable fractional derivative under different parameter conditions. The profile of the solitary wave solutions, periodic solutions, and periodic cusp wave solutions is given.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

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