Hindawi
Journal of Function Spaces
Volume 2020, Article ID 7540303, 5 pages
https://doi.org/10.1155/2020/7540303

Research Article

Approximation of an Additive (ϱ_1, ϱ_2)-Random Operator Inequality

Sun Young Jang[1] and Reza Saadati[2]

[1]*Department of Mathematics, University of Ulsan, Ulsan 44610, Republic of Korea*
[2]*Department of Mathematics, Iran University of Science & Technology, Tehran, Tehran, Iran*

Correspondence should be addressed to Reza Saadati; rsaadati@eml.cc

Received 6 November 2019; Accepted 21 January 2020; Published 18 February 2020

Academic Editor: Alberto Fiorenza

We solve the additive (ϱ_1, ϱ_2)-random operator inequality $\xi_t^{T(\omega,u+v)-T(\omega,u)-T(\omega,v)} \geq \kappa_M(\xi_t^{\varrho_1(T(\omega,u+v)+T(\omega,u-v)-2T(\omega,u))}, \xi_t^{\varrho_2(2T(\omega,((u+v)/2))-T(\omega,u)-T(\omega,v))})$, in which $\varrho_1, \varrho_2 \in \mathbb{C}$ are fixed and $\max\{\sqrt{2}|\varrho_1|, |\varrho_2|\} < 1$. Finally, we get an approximation of the mentioned additive (ϱ_1, ϱ_2)-random operator inequality by direct technique.

1. Introduction

Park [1] introduced and solved the following additive (ρ_1, ρ_2)-functional inequality:

$$\|f(x+y) - f(x) - f(y)\| \leq \|\rho_1(f(x+y) + f(x-y) - 2f(x))\| + \left\|\rho_2\left(2f\left(\frac{x+y}{2}\right) - f(x) - f(y)\right)\right\|, \tag{1}$$

where ρ_1 and ρ_2 are fixed nonzero complex numbers with $\sqrt{2}|\rho_1| + |\rho_2| < 1$. Next, he proved the Hyers–Ulam stability of the additive (ρ_1, ρ_2)-functional inequality (1) in Banach spaces.

In this paper, we get a generalization from Park's results [1] in MB-spaces. Let $(\Omega, \mathscr{U}, \mu)$ be a probability measure space. Assume that $(U, \mathscr{B}_U)$ and $(V, \mathscr{B}_V)$ are Borel measureable spaces, in which U and V are MB-spaces and $T: \Omega \times U^2 \longrightarrow V$ is a random operator. In MB-spaces, first we solve the additive (ϱ_1, ϱ_2)-random operator inequality:

$$\xi_t^{T(\omega,u+v)-T(\omega,u)-T(\omega,v)} \geq \kappa_M\left(\xi_t^{\varrho_1(T(\omega,u+v)+T(\omega,u-v)-2T(\omega,u))}, \xi_t^{\varrho_2(2T(\omega,((u+v)/2))-T(\omega,u)-T(\omega,v))}\right), \tag{2}$$

in which $\varrho_1, \varrho_2 \in \mathbb{C}$ are fixed and $\max\{\sqrt{2}|\varrho_1|, |\varrho_2|\} < 1$.

Next, we get a random approximation of the additive-random operator (2).

2. Preliminaries

Let Ξ^+ be the set of distribution mappings, i.e., the set of all mappings $G: \mathbb{R} \cup \{-\infty, \infty\} \longrightarrow [0,1]$, writing G_t for $G(t)$, such that G is left continuous and increasing on $\mathbb{R}$. $O^+ \subseteq \Xi^+$ includes all mappings $G \in \Xi^+$ for which $\ell^- G_{+\infty}$ is one and $\ell^- g_x$ is the left limit of the mapping g at the point x, i.e., $\ell^- g_x = \lim_{t \longrightarrow x^-} g_t$.

In Ξ^+, we define "$\leq$" as follows:

$$H \leq F \Leftrightarrow H_s \leq F_s, \tag{3}$$

for each s in $\mathbb{R}$ (partially ordered). Note that the function ϑ^u defined by

$$\vartheta_s^u = \begin{cases} 0, & \text{if } s \leq u, \\ 1, & \text{if } s > u, \end{cases} \tag{4}$$

is an element of Ξ^+, and ϑ^0 is the maximal element in this space (for some more details, see [2–4]).

Definition 1 (see [2, 5]). Let $I = [0, 1]$. A continuous triangular norm (shortly, a ct-norm) is a continuous mapping κ from $I \times I$ to I such that

(a) $\kappa(\varsigma, \tau) = \kappa(\tau, \varsigma)$ and $\kappa(\varsigma, \kappa(\tau, \upsilon)) = \kappa(\kappa(\varsigma, \tau), \upsilon)$ for all $\varsigma, \tau, \upsilon \in [0, 1]$

(b) $\kappa(\varsigma, 1) = \varsigma$ for all $\varsigma \in I$

(c) $\kappa(\varsigma, \tau) \le \kappa(\upsilon, \iota)$ whenever $\varsigma \le \upsilon$ and $\tau \le \iota$ for all $\varsigma, \tau, \upsilon, \iota \in I$

Some examples of the t-norms are as follows:

(1) $\kappa_P(\varsigma, \tau) = \varsigma\tau$

(2) $\kappa_M(\varsigma, \tau) = \min\{\varsigma, \tau\}$

(3) $\kappa_L(\varsigma, \tau) = \max\{\varsigma + \tau - 1, 0\}$ (: the Lukasiewicz t-norm)

Definition 2 (see [3]). Suppose that κ is a ct-norm, V is a linear space and ξ is a mapping from V to O^+. In this case, the ordered tuple (V, ξ, κ) is called a *Menger normed space* (in short, MN-space) if the following conditions are satisfied:

(MN1) $\xi_t^v = \vartheta_t^0$ for all $t > 0$ if and only if $v = 0$

(MN2) $\xi_t^{\alpha v} = \xi_{(t/|\alpha|)}^v$ for all $v \in V$ and $\alpha \in \mathbb{C}$ with $\alpha \neq 0$

(MN3) $\xi_{t+s}^{u+v} \ge \kappa(\xi_t^u, \xi_s^v)$ for all $u, v \in V$ and $t, s > 0$

Let $(V, \|\cdot\|)$ be a linear normed space. Then,

$$\xi_s^v = \begin{cases} 0, & \text{if } s \le 0, \\ \exp\left(-\dfrac{\|v\|}{s}\right), & \text{if } s > 0, \end{cases} \tag{5}$$

defines a Menger norm, and the ordered tuple (V, ξ, κ_M) is an MN-space.

Let $(\Omega, \mathscr{U}, \mu)$ be a probability measure space. Assume that $(U, \mathscr{B}_U)$ and $(V, \mathscr{B}_V)$ are Borel measureable spaces, in which U and V are complete FB-spaces. A mapping $T\colon \Omega \times U \longrightarrow V$ is said to be a random operator if $\{\omega\colon T(\omega, u) \in B\} \in \mathscr{U}$ for all u in U and $B \in \mathscr{B}_V$. Also, T is a random operator, if $T(\omega, u) = v(\omega)$ be a V-valued random variable for every u in U. A random operator $T\colon \Omega \times U \longrightarrow V$ is called linear if $T(\omega, \alpha u_1 + \beta u_2) = \alpha T(\omega, u_1) + \beta T(\omega, u_2)$, almost everywhere for each u_1, u_2 in U and α, β scalers, and *Menger random bounded* (in short, MR-bounded) if there exists a nonnegative real-valued random variable $M(\omega)$ such that

$$\xi_{M(\omega)t}^{T(\omega,u_1)-T(\omega,u_2)} \ge \xi_t^{u_1-u_2}, \tag{6}$$

almost everywhere for each u_1, u_2 in U and $t > 0$.

Recently, some authors have published some papers on stability of functional equations in several spaces by the direct method and the fixed point method, for example, Banach spaces [6–8], fuzzy Menger normed algebras [9], fuzzy normed spaces [10], non-Archimedean random Lie C^*-algebras [11], non-Archimedean random normed spaces [12], random multinormed space [13], random lattice normed spaces, and random normed algebras [14, 15]. In [16, 17], the authors studied the stability problem for fractional equations. Next, Cădariu et al. [18–20] applied the fixed point method to solve the stability problem, and their work was continued by Keltouma et al. [21], Park et al. [22, 23], Jung and Lee [24], and Brzdęk and Ciepliński [25], see also [26, 27].

3. Approximation of Additive (ϱ_1, ϱ_2)-Random Operator Inequality: Direct Technique

Now, we modify and generalize Park's results [1]. First, we solve and investigate the additive (ϱ_1, ϱ_2)-random operator inequality (2) in MN-spaces.

Lemma 1. *Let (V, ξ, κ_M) be an MN-space. Let $T\colon \Omega \times U \longrightarrow V$ be a random operator satisfying $T(\omega, 0) = 0$ and (2), and then T is additive.*

Proof. Replacing v by u in (2), we get

$$\xi_t^{T(\omega,2u)-2T(\omega,u)} \ge \xi_{(t/|\varrho_1|)}^{T(\omega,2u)-2T(\omega,u)}. \tag{7}$$

Since $|\varrho_1| < 1$, $T(\omega, 2u) = 2T(\omega, u)$ for each $u \in U$ and $\omega \in \Omega$,

$$T\left(\omega, \frac{u}{2}\right) = \frac{1}{2} T(\omega, u), \tag{8}$$

almost everywhere for each $u \in U$ and $\omega \in \Omega$.

(2) and (8) imply that

$$\xi_t^{T(\omega,u+v)-T(\omega,u)-T(\omega,v)} \ge \kappa_M\left(\xi_{(t/|\varrho_1|)}^{T(\omega,u+v)+T(\omega,u-v)-2T(\omega,u)}, \xi_{(t/|\varrho_2|)}^{T(\omega,u+v)-T(\omega,u)-T(\omega,v)}\right), \tag{9}$$

and so

$$\xi_t^{T(\omega,u+v)-T(\omega,u)-T(\omega,v)} \ge \xi_{(t/|\varrho_1|)}^{T(\omega,u+v)+T(\omega,u-v)-2T(\omega,u)}, \tag{10}$$

almost everywhere for each $u, v \in U$, $\omega \in \Omega$, and $t > 0$.

Putting $z = u + v$ and $w = u - v$ in (10), we get

$$\xi_t^{T(\omega,z+w)+T(\omega,z-w)-2T(\omega,z)} \ge \xi_{(t/2|\varrho_1|)}^{T(\omega,z+w)-T(\omega,z)-T(\omega,w)}, \tag{11}$$

almost everywhere for each $z, w \in U$, $\omega \in \Omega$, and $t > 0$.

Now, (10) and (11) imply that

$$\xi_t^{T(\omega,u+v)-T(\omega,u)-T(\omega,v)} \ge \xi_{(t/2|\varrho_1|^2)}^{T(\omega,u+v)-T(\omega,u)-T(\omega,v)}, \tag{12}$$

for all $x, y \in X$. Since $|\varrho_1| < (\sqrt{2}/2)$, $T(\omega, u + v) = T(\omega, u) + T(\omega, v)$, almost everywhere for each $u, v \in U$ and $\omega \in \Omega$, which implies that T is additive.

We get an approximation of the additive (ϱ_1, ϱ_2)-random operator inequality (2) in MN-spaces, by applying the direct technique. □

Theorem 1. *Let (V, ξ, κ_M) be an MN-space. Assume that $\psi\colon U^2 \longrightarrow O^+$ be a distribution function such that there exists $\beta < 1$ with*

$$\psi_{(\beta t/2)}^{(u/2),(v/2)} \geq \psi_t^{u,v}, \quad (13)$$

$$\lim_{p\longrightarrow\infty} \psi_{(t/2^p)}^{(u/2^p),(v/2^p)} = \vartheta_t^0, \quad (14)$$

almost everywhere for each $u, v \in U$, $\omega \in \Omega$, and $t > 0$. Suppose that $T: \Omega \times U \longrightarrow V$ be a random operator satisfying $T(\omega, 0) = 0$ and

$$\xi_t^{T(\omega,u+v)-T(\omega,u)-T(\omega,v)} \geq \kappa_M\left(\xi_t^{\varrho_1(T(\omega,u+v)+T(\omega,u-v)-2T(\omega,u))}, \xi_t^{\varrho_2(2T(\omega,(u+v/2))-T(\omega,u)-T(\omega,v))}, \psi_t^{u,v}\right), \quad (15)$$

in which $\varrho_1, \varrho_2 \in \mathbb{C}$ are fixed and $max\{\sqrt{2}|\varrho_1|, |\varrho_2|\} < 1$. Therefore, there is a unique additive random operator S: $\Omega \times U \longrightarrow V$ such that

$$\xi_t^{T(\omega,u)-S(\omega,u)} \geq \psi_{((2(1-\beta)/\beta)t)}^{u,u}, \quad (16)$$

almost everywhere for each $u \in U$, $\omega \in \Omega$, and $t > 0$.

Proof. Putting $u = v$ in (15), we have that

$$\xi_t^{2T(\omega,(u/2))-T(\omega,u)} \geq \psi_t^{(u/2),(u/2)} \geq \psi_{((2/\beta)t)}^{u,u}, \quad (17)$$

almost everywhere for each $u \in U$, $\omega \in \Omega$, and $t > 0$. Replacing u by $(u/2^\ell)$ in (17) and applying (13), we get

$$\xi_t^{2^{\ell+1}T(\omega,(u/2^{\ell+1}))-2^\ell T(\omega,(u/2^\ell))} \geq \psi_{((2/\beta^{\ell+1})t)}^{u,u}, \quad (18)$$

which implies that

$$\xi_{\sum_{k=1}^{\ell}\left(\frac{\beta^k}{2}\right)t}^{2^\ell T\left(\omega,\left(\frac{u}{2^\ell}\right)\right)-T(\omega,u)} \geq \psi_t^{u,u}. \quad (19)$$

Replacing u by $(u/2^m)\pi$ in (19), we get

$$\xi_t^{2^{\ell+m}T\left(\omega,\left(\frac{u}{2^{\ell+m}}\right)\right)-2^m T\left(\omega,\left(\frac{u}{2^m}\right)\right)} \geq \psi_{\left(t/\sum_{k=m+1}^{\ell+m}(\beta^k/2)\right)}^{u,u}, \quad (20)$$

which tends to ϑ^0 when m, ℓ tend to ∞, and so the sequence $\{2^\ell T(\omega, (u/2^\ell))\}$ is Cauchy in the complete MN-space (V, ξ, κ_M) and converges to a point $S(\omega, x) \in V$. Now, for every $\varsigma > 0$, we have that

$$\xi_{t+\varsigma}^{T(\omega,u)-S(\omega,u)} \geq \kappa_M\left(\xi_t^{T(\omega,u)-2^\ell T(\omega,(u/2^\ell))}, \xi_\varsigma^{S(\omega,u)-2^\ell T(\omega,(u/2^\ell))}\right) \geq \kappa_M\left(\psi_{\left(t/\sum_{k=1}^{\ell}(\beta^k/2)\right)}^{u,u}, \xi_\varsigma^{S(\omega,u)-2^\ell T(\omega,(u/2^\ell))}\right). \quad (21)$$

When ℓ tends to ∞ in (21), we have that

$$\xi_{t+\varsigma}^{T(\omega,u)-S(\omega,u)} \geq \psi_{((2(1-\beta)/\beta)t)}^{u,u}. \quad (22)$$

Since $\varsigma > 0$ is arbitrary in (22), we have that

$$\xi_t^{T(\omega,u)-S(\omega,u)} \geq \psi_{((2(1-\beta)/\beta)t)}^{u,u}. \quad (23)$$

Replacing u and v by $(u/2^m)$ and $(v/2^m)$ in (15) and using (14) imply that S satisfies Lemma 1 and hence is an additive random operator. Now, let S' be another additive random operator satisfies (16). For an arbitrary $u \in U$ and $\omega \in \Omega$, we have that $2^m S(\omega, (u/2^m)) = S(\omega, u)$ and $2^m S'(\omega, (u/2^m)) = S'(\omega, u)$ for each natural element m. Using (16), we have that

$$\begin{aligned}
\xi_t^{S(\omega,u)-S'(\omega,u)} &= \lim_{m\longrightarrow\infty} \xi_t^{2^m S(\omega,(u/2^m))-2^m S'(\omega,(u/2^m))} \\
&\geq \lim_{m\longrightarrow\infty} \kappa_M\Big(\xi_{(t/2)}^{2^m S(\omega,(u/2^m))-2^m T(\omega,(u/2^m))}, \\
&\qquad \xi_{(t/2)}^{2^m T(\omega,(u/2^m))-2^m S'(\omega,(u/2^m))}\Big) \\
&\geq \lim_{m\longrightarrow\infty} \psi_{(((1-\beta)/2^m\beta)t)}^{(u/2^m),(u/2^m)} \\
&\geq \lim_{m\longrightarrow\infty} \psi_{(((1-\beta)/\beta^{m+1})t)}^{u,u} \\
&\longrightarrow \vartheta_t^0,
\end{aligned} \quad (24)$$

which implies that $S(\omega, u) = S'(\omega, u)$ shows the uniqueness. □

Corollary 1. *Let (V, ξ, κ_M) be an MN-space, $\rho > 1$ and $\tau > 0$. Suppose that $T: \Omega \times U \longrightarrow V$ be a random operator satisfying $T(\omega, 0) = 0$ and*

$$\xi_t^{T(\omega,u+v)-T(\omega,u)-T(\omega,v)} \geq \kappa_M\left(\xi_t^{\varrho_1(T(\omega,u+v)+T(\omega,u-v)-2T(\omega,u))}, \xi_t^{\varrho_2(2T(\omega,(u+v/2))-T(\omega,u)-T(\omega,v))}, \frac{t}{t+\tau(\|u\|^\rho+\|v\|^\rho)}\right), \quad (25)$$

in which $\varrho_1, \varrho_2 \in \mathbb{C}$ are fixed and $\max\{\sqrt{2}|\varrho_1|, |\varrho_2|\} < 1$. Then, there exists a unique additive random operator $S: \Omega \times U \longrightarrow V$ such that

$$\xi_t^{T(\omega,u)-S(\omega,u)} \geq \frac{(2^{\rho-1}-1)t}{(2^{\rho-1}-1)t + \tau\|u\|^\rho}, \tag{26}$$

for each $u \in U$, $\omega \in \Omega$, and $t > 0$.

Proof. In Theorem 1, put $\psi_t^{u,u} = (t/(t+\tau(\|u\|^\rho + \|v\|^\rho)))$ for each $u \in U$ and $t > 0$, and $\beta = 2^{1-\rho}$. □

Theorem 2. *Let (V, ξ, κ_M) be an MN-space. Assume that $\psi: U^2 \longrightarrow O^+$ be a distribution function such that there exists an $\beta < 1$ with*

$$\psi_{2\beta t}^{2u,2v} \geq \psi_t^{u,v}, \tag{27}$$

$$\lim_{p\longrightarrow\infty} \psi_{2^p t}^{2^p u, 2^p v} = \vartheta_t^0, \tag{28}$$

almost everywhere for each $u, v \in U$, $\omega \in \Omega$, and $t > 0$. Suppose that $T: \Omega \times U \longrightarrow V$ be a random operator satisfying $T(\omega, 0) = 0$ and (15). Therefore, there is a unique additive random operator $S: \Omega \times U \longrightarrow V$ such that

$$\xi_t^{T(\omega,u)-S(\omega,u)} \geq \psi_{2(1-\beta)t}^{u,u}, \tag{29}$$

almost everywhere for each $u \in U$, $\omega \in \Omega$, and $t > 0$.

Proof. Putting $u = v$ in (15), we have that

$$\xi_t^{(T(\omega,2u)/2)-T(\omega,u)} \geq \psi_{2t}^{u,u} \geq \psi_{(t/\beta)}^{(u/2),(u/2)}, \tag{30}$$

almost everywhere for each $u \in U$, $\omega \in \Omega$, and $t > 0$. Replacing u by $2^\ell u$ in (30) and applying (27), we get

$$\xi_t^{\left(T\left(\omega,2^{\ell+1}u\right)/2^{\ell+1}\right)-\left(T\left(\omega,2^\ell u\right)/2^\ell\right)} \geq \psi_{\left(\left(2/\beta^\ell\right)t\right)}^{u,u}, \tag{31}$$

which implies that

$$\xi_{\sum_{k=0}^{\ell-1}(\beta^k/2)t}^{\left(T\left(\omega,2^\ell u\right)/2^\ell\right)-T(\omega,u)} \geq \psi_t^{u,u}. \tag{32}$$

Replacing u by $(u/2^m)$ in (32), we get

$$\xi_t^{\left(T\left(\omega,2^{\ell+m}u\right)/2^{\ell+m}\right)-(T(\omega,2^m u)/2^m)} \geq \psi_{\left(t/\left(\sum_{k=m}^{\ell+m-1}(\beta^k/2)\right)\right)}^{u,u}, \tag{33}$$

which tends to ϑ^0 when m, ℓ tend to ∞, and so the sequence $\{(T(\omega, 2^\ell u)/2^\ell)\}$ is Cauchy in the complete MN-space (V, ξ, κ_M) and converges to a point $S(\omega, x) \in V$. Now, for every $\varsigma > 0$, we have that

$$\begin{aligned}\xi_{t+\varsigma}^{T(\omega,u)-S(\omega,u)} &\geq \kappa_M\left(\xi_t^{T(\omega,u)-\left(T\left(\omega,2^\ell u\right)/2^\ell\right)}, \xi_\varsigma^{S(\omega,u)-\left(T\left(\omega,2^\ell u/2^\ell\right)\right)}\right)\\ &\geq \kappa_M\left(\psi_{\left(t/\sum_{k=0}^{\ell-1}(\beta^k/2)\right)}^{u,u}, \xi_\varsigma^{S(\omega,u)-\left(T\left(\omega,2^\ell u\right)/2^\ell\right)}\right),\end{aligned} \tag{34}$$

When ℓ tends to ∞ in (34), we have

$$\xi_{t+\varsigma}^{T(\omega,u)-S(\omega,u)} \geq \psi_{2(1-\beta)t}^{u,u}. \tag{35}$$

Since $\varsigma > 0$ is arbitrary in (35), we have

$$\xi_t^{T(\omega,u)-S(\omega,u)} \geq \psi_{2(1-\beta)t}^{u,u}. \tag{36}$$

Replacing u and v by $2^m u$ and $2^m v$ in (15) and using (14) imply that S satisfies Lemma 1 and hence is an additive random operator. Now, let S' be another additive random operator satisfies (29). For an arbitrary $u \in U$ and $\omega \in \Omega$, we have that $(S(\omega, 2^m u)/2^m) = S(\omega, u)$, and for each positive integer m. Using (29), we have

$$\begin{aligned}\xi_t^{S(\omega,u)-S'(\omega,u)} &= \lim_{m\longrightarrow\infty} \xi_t^{(S(\omega,2^m u)/2^m)-\left(S'(\omega,2^m u)/2^m\right)}\\ &\geq \lim_{m\longrightarrow\infty} \kappa_M\left(\xi_{(t/2)}^{(S(\omega,2^m u)/2^m)-(T(\omega,2^m u)/2^m)},\right.\\ &\qquad \left.\xi_{(t/2)}^{(T(\omega,2^m u)/2^m)-\left(S'(\omega,2^m u)/2^m\right)}\right)\\ &\geq \lim_{m\longrightarrow\infty} \psi_{2^m(1-\beta)t}^{2^m u, 2^m u}\\ &\geq \lim_{m\longrightarrow\infty} \psi_{\left((1-\beta)/\beta^m\right)t}^{u,u}\\ &\longrightarrow \vartheta_t^0,\end{aligned} \tag{37}$$

which implies that $S(\omega, u) = S'(\omega, u)$ shows the uniqueness. □

Corollary 2. *Let (V, ξ, κ_M) be an MN-space, $\rho < 1$ and $\tau > 0$. Suppose that $T: \Omega \times U \longrightarrow V$ be a random operator satisfying $T(\omega, 0) = 0$ and (25). Therefore, there is a unique additive random operator $S: \Omega \times U \longrightarrow V$ such that*

$$\xi_t^{T(\omega,u)-S(\omega,u)} \geq \frac{(2-2^\rho)t}{(2-2^\rho)t + 2\tau\|u\|^\rho}, \tag{38}$$

for each $u \in U$, $\omega \in \Omega$, and $t > 0$.

Proof. In Theorem 2, put $\psi_t^{u,u} = (t/(t+\tau(\|u\|^\rho + \|v\|^\rho)))$ for each $u \in U$, $t > 0$and $\beta = 2^{\rho-1}$. □

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

S. Y. Jang was supported by Basic Science Research Program through the National Research Foundation of Korea funded by the Ministry of Education, Science and Technology (NRF-201807042748).

References

[1] C. Park, "The stability of an additive (ρ_1, ρ_2)-functional inequality in Banach spaces," *Journal of Mathematical Inequalities*, vol. 13, no. 1, pp. 95–104, 2019.

[2] B. Schweizer and A. Sklar, *Probabilistic Metric Spaces. North-Holland Series in Probability and Applied Mathematics*, North-Holland Publishing Co., New York, NY, USA, 1983.

[3] A. N. Šerstnev, "On the notion of a random normed space," *Proceedings of the USSR Academy of Sciences*, vol. 149, no. 2, pp. 280–283, 1963.

[4] R. Saadati, *Random Operator Theory*, Elsevier/Academic Press, London, UK, 2016.

[5] O. Hadzic and E. Pap, "Fixed point theory in probabilistic metric spaces," *Mathematics and Its Applications*, Vol. 536, Kluwer Academic Publishers, Dordrecht, Netherlands, 2001.

[6] W. Fechner, "Stability of a composite functional equation related to idempotent mappings," *Journal of Approximation Theory*, vol. 163, no. 3, pp. 328–335, 2011.

[7] G. Isac and T. M. Rassias, "Stability of Ψ-additive mappings: applications to nonlinear analysis," *International Journal of Mathematics and Mathematical Sciences*, vol. 19, no. 2, pp. 219–228, 1996.

[8] J. Rätz, "On the functional equation $x + f(y + f(x)) = y + f(x + f(y))$, II," *Aequationes Mathematicae*, vol. 89, no. 1, pp. 169–186, 2015.

[9] A. K. Mirmostafaee, "Perturbation of generalized derivations in fuzzy Menger normed algebras," *Fuzzy Sets and Systems*, vol. 195, pp. 109–117, 2012.

[10] C. Park and S. Yun, "Stability of cubic and quartic ρ-functional inequalities in fuzzy normed spaces," *Journal of Nonlinear Sciences and Applications*, vol. 9, no. 4, pp. 1693–1701, 2016.

[11] M. V. Maleki, S. M. Vaezpour, and R. Saadati, "Nonlinear stability of ρ-functional equations in latticetic random Banach lattice spaces," *Mathematics*, vol. 6, no. 2, p. 22, 2018.

[12] R. Saadati and C. Park, "Approximation of derivations and the superstability in random Banach $*$-algebras," *Advances in Difference Equations*, vol. 2018, p. 418, 2018.

[13] R. P. Agarwal, R. Saadati, and A. Salamati, "Approximation of the multiplicatives on random multi-normed space," *Journal of Inequalities and Applications*, vol. 2017, p. 204, 2017.

[14] T. Bînzar, F. Pater, and S. Nădăban, "On fuzzy normed algebras," *Journal of Nonlinear Sciences and Applications*, vol. 9, no. 9, pp. 5488–5496, 2016.

[15] R. Naeem and M. Anwar, "Jessen type functionals and exponential convexity," *Journal of Mathematics and Computer Science*, vol. 17, no. 3, pp. 429–436, 2017.

[16] M. A. El-Moneam, T. F. Ibrahim, and S. Elamody, "Stability of a fractional difference equation of high order," *Journal of Nonlinear Sciences and Applications*, vol. 12, no. 2, pp. 65–74, 2018.

[17] Y. Ding, "Ulam-Hyers stability of fractional impulsive differential equations," *Journal of Nonlinear Sciences and Applications*, vol. 11, no. 8, pp. 953–959, 2018.

[18] L. Cădariu and V. Radu, "Fixed points and the stability of Jensen's functional equation," *Journal of Inequalities in Pure and Applied Mathematics*, vol. 4, no. 1, p. 4, 2003.

[19] L. Cădariu and V. Radu, "The stability of Jensen's functional equation: a fixed point approach," *Automation, Computers, Applied Mathematics*, vol. 11, no. 1, pp. 27–32, 2002.

[20] L. Cădariu, L. Găvruţa, and P. Găvruţa, "On the stability of an affine functional equation," *Journal of Nonlinear Sciences and Applications*, vol. 6, no. 2, pp. 60–67, 2013.

[21] B. Keltouma, E. Elhoucien, T. M. Rassias, and R. Ahmed, "Superstability of Kannappan's and Van Vleck's functional equations," *Journal of Nonlinear Sciences and Applications*, vol. 11, no. 7, pp. 894–915, 2018.

[22] C. Park, Y. Jin, and X. Zhang, "Bi-additive s-functional inequalities and quasi-multipliers on Banach algebras," *Rocky Mountain Journal of Mathematics*, vol. 49, no. 2, pp. 593–607, 2019.

[23] C. Park, J. R. Lee, and X. Zhang, "Additive s-functional inequality and hom-derivations in Banach algebras," *Journal of Fixed Point Theory and Applications*, vol. 21, no. 1, p. 18, 2019.

[24] S.-M. Jung and Y.-H. Lee, "A fixed point approach to the stability of a mean value type functional equation," *Mathematics*, vol. 5, no. 4, p. 78, 2017.

[25] J. Brzdęk and K. Ciepliński, "A fixed point theorem in n-Banach spaces and Ulam stability," *Journal of Mathematical Analysis and Applications*, vol. 470, no. 1, pp. 632–646, 2019.

[26] H. Piri, S. Rahrovi, and P. Kumam, "Generalization of Khan fixed point theorem," *Journal of Mathematics and Computer Science*, vol. 17, no. 1, pp. 76–83, 2017.

[27] A. Shoaib, A. Azam, M. Arshad, and E. Ameer, "Fixed point results for multivalued mappings on a sequence in a closed ball with applications," *Journal of Mathematics and Computer Science*, vol. 17, no. 2, pp. 308–316, 2017.