Research Article

On Fekete–Szegö Problems for Certain Subclasses of Analytic Functions Defined by Differential Operator Involving $q$-Ruscheweyh Operator

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In this paper, we define a new derivative operator involving $q$-Ruscheweyh differential operator using convolution. Using this new operator, we introduce two new classes of analytic functions and obtain the Fekete–Szegö inequalities.

1. Introduction

The applications of $q$-calculus are important and pivotal as they contributed worthy of noticed expansion in geometric function theory. In 1908, Jackson, was the first mathematician to develop the application of $q$-calculus in a systematic way [1, 2]. Then, Aral and Gupta [3] proposed $q$-analogue of Baskakov and Durrmeyer operator depending on $q$-calculus; Mohammed and Darus [4] defined a new operator involving the $q$-hypergeometric function. Some other applications of $q$-calculus are studied by the authors [5, 6] and Elhaddad et al. [7]. Recently, many mathematicians have worked intensively in this field (see [8–12]) and obtained various results.

Let $\mathcal{A}$ be the class of functions $f(z)$ of the form

$$f(z) = z + \sum_{i=2}^{\infty} a_i z^i, \quad (z \in \mathbb{E}),$$

(1)

which are analytic in $\mathbb{E} = \{ z : z \in \mathbb{C}, |z| < 1 \}$ and we denote by $\mathcal{S}$ the subclass of $\mathcal{A}$ that are consisted of one-to-one (univalent) functions in $\mathbb{E}$.

The convolution of functions $f$ as in (1.1) and $\Gamma(z) = z + \sum_{i=2}^{\infty} \gamma_i z^i$ will be:

$$(f * \Gamma)(z) = (\Gamma * f)(z) = z + \sum_{i=2}^{\infty} a_i \gamma_i z^i.$$  

(2)

Let $f(z)$ and $r(z)$ are analytic functions in $\mathbb{E}$, then we say that $f$ is subordinate to $r$ denoted by $f(z) \prec r(z)$ in $\mathbb{E}$, if there exists a Schwarz function $\xi(z)$ which is analytic in $\mathbb{E}$ with $\xi(0) = 0$ and $|\xi(z)| < 1 (z \in \mathbb{E})$ such that $f(z) = r(\xi(z))z \in \mathbb{E}$.

Now, we give some notations and definitions of the principal terms of $q$-calculus by assuming $0 < q < 1$, as follows:

(1) The $q$-number $[i]_q$ is defined by:

$$[i]_q = \begin{cases} \frac{1-q^i}{1-q} & (i \in \mathbb{C} \setminus \mathbb{N}), \\ 1 + q + q^2 + \ldots + q^{i-1} & (i \in \mathbb{N}). \end{cases}$$

(3)

(2) The $q$-factorial $[i]_q!$ is defined by:

$$[i]_q! = \begin{cases} [i]_q [i-1]_q \cdots [2]_q [1]_q & i = 2, 3, 4, \ldots, \\ 1 & i = 1. \end{cases}$$

(4)

(3) The $q$-derivative of a function $f$ is defined by:

$$\partial_q f(z) = \begin{cases} \frac{f(z) - f(z/q)}{z^{1-q} - z} & z \in \mathbb{C}^*, \\ f'(0), & z = 0. \end{cases}$$

(5)

where $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$ will be used throughout the article. As $f$ given by (1), then

$$\partial_q f(z) = 1 + \sum_{i=2}^{\infty} [i]_q a_i z^{i-1}.$$  

(6)
The authors in [6] introduced a $q$-differential operator $\mathcal{D}^m_{q,\alpha,\lambda,\mu} f(z)$ by:
\[
\mathcal{D}^m_{q,\alpha,\lambda,\mu} f(z) = z + \sum_{i=0}^{\infty} \binom{k-\lambda}{\alpha}(\delta - \mu)((i-\lambda)q-1 + 1)^m a_i z^i,
\]
where $(\delta, \kappa, \alpha, \mu \geq 0), \kappa > \lambda, \delta > \mu, m \in \mathbb{N}_q$.

In 2014, Aldweby and Darus in [9] introduced $q$-Ruscheweyh operator $\mathcal{R}_q^0 f(z)$ by:
\[
\mathcal{R}_q^0 f(z) = z + \sum_{i=0}^{\infty} \Omega_{\kappa,\lambda,\mu}(i\lambda q) a_i z^i,
\]
where
\[
\Omega_{\kappa,\lambda,\mu}(i\lambda q) = (k-\lambda)(\delta - \mu)((i-\lambda)q-1 + 1)^m [\delta - 1 + i\lambda q]^i [\delta - 1 + i\lambda q]! [\delta - 1 + i\lambda q]!
\]
Remarks.
(i) When $\delta = 0, \kappa = 1, \lambda = 0, \delta = 1, \mu = 0$, we get $q$-Szlagean differential operator introduced in [13].
(ii) When $\delta = 0, \kappa = 1, \lambda = 0, \delta = 1, \mu = 0, \text{and} \ q \rightarrow 1^-$, we get $q$-Szlagean differential operator introduced in [14].
(iii) When $\delta = 0, \beta = 1, \kappa = 0, \delta = 1, \mu = 0, \text{and} \ q \rightarrow 1$, then we get Al-Osbohi differential operator introduced in [15].
(iv) When $\delta = 0$ and $q \rightarrow 1^-$, we get Ramadan and Darus operator introduced in [16].
(v) When $\delta = 0$, we get Alsboh and Darus operator introduced in [6].
(vi) When $m = 0$, then we get $q$-Ruscheweyh operator introduced in [9].
(vii) When $m = 0$ and $q \rightarrow 1$, then we have Ruscheweyh operator introduced in [17].
(viii) When $m = 0, \delta = 1, \mu = 0, \lambda = 0, \text{and} \ q \rightarrow 1$, we get $D_q^0 f(z)$ [18].

Many subclases of analytic functions have been introduced by many different authors, for example, Ma and Minda [19], Ravichandran et al. [20], Seoudy and Aouf [12] and others. These works have inspired our introduction of the new subclasses $\mathcal{S}^{m,\alpha}_q(\Phi)$ and $\mathcal{C}^{m,\alpha}_q(\Phi)$ of $\mathcal{A}$, involving the differential operator $\mathcal{D}^{m,\alpha}_q,\kappa,\lambda,\mu,\lambda,\mu$ and the principle of subordination:

Definition 1. Let $\mathcal{P}$ be the subclass of functions $\Phi$ which are analytic and univalent in $\mathbb{E}$ and for which $\Phi(\mathbb{E})$ is convex with $\Phi(0) = 1$ and $\Re(\Phi(z)) < 0$ for $z \in \mathbb{E}$. A function $f \in \mathcal{A}$ is said to be in the class $f \in \mathcal{S}^{m,\alpha}_q(\Phi)$ if it satisfies the subordination condition:
\[
1 + \frac{1}{\Phi} \left( \frac{z \frac{d}{z} \mathcal{D}^{m,\alpha}_q f(z)}{\mathcal{D}^{m,\alpha}_q f(z) - 1} \right) < \Phi(z), \quad (\Re \in \mathbb{C}; \Phi \in \mathcal{P}).
\]

Definition 2. A function $f \in \mathcal{A}$ is said to be in the class $f \in \mathcal{C}^{m,\alpha}_q(\Phi)$ if it satisfies the subordination condition:
\[
1 + \frac{1}{\Phi} \left( \frac{\frac{d}{z} \mathcal{D}^{m,\alpha}_q f(z)}{\frac{d}{z} \mathcal{D}^{m,\alpha}_q f(z) - 1} \right) < \Phi(z), \quad (\Re \in \mathbb{C}; \Phi \in \mathcal{P}).
\]

2. Main Results

In this section, we obtain the Fekete–Szegö inequalities for the subclasses $\mathcal{S}^{m,\alpha}_q(\Phi)$ and $\mathcal{C}^{m,\alpha}_q(\Phi)$, by assuming $(\delta, \kappa, \lambda, \mu \geq 0), \kappa > \lambda, \delta > \mu, m \in \mathbb{N}_q, q \in \mathbb{C}$ and $\Phi \in \mathcal{P}$. In order to prove our results, we use the following lemmas of Ma and Minda [19].

Lemma 3. If $\mathcal{P}(z) = 1 + c_1 z + c_2 z^2 + c_3 z^3 + \ldots$ is a function with positive real part in the open unit disk $\mathbb{E}$ and $\sigma \in \mathbb{C}$, then
\[
|c_2 - \sigma c_1^2| \leq 2 \max\{1; |2\sigma - 1|\}.
\]

Lemma 4. If $\mathcal{P}(z) = 1 + c_1 z + c_2 z^2 + c_3 z^3 + \ldots$ is a function with positive real part in $\mathbb{E}$ and $\sigma \in \mathbb{C}$, then
\[
|c_2 - \sigma c_1^2| \leq \max\{|-\sigma + 2, \sigma \leq 0, 0 \leq \sigma \leq 1, 4\sigma - 2, \sigma \geq 1|.
\]

To get our results, we use the similar methods studied by Alsoboh and Darus [5], Elhaddad and Darus [11], and Seoudy and Aouf [12].

Theorem 5. Let $\phi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \ldots$ with $(B_1 \neq 0)$, and $f$ given by (1) belongs to $\mathcal{S}^{m,\alpha}_q(\Phi)$, then
\[
|a_3 - \sigma a_1^2| \leq \frac{\sigma B_1}{2(3q-1)\Omega^{m,\alpha}_q(3q-1)}
\]

\[
\max\left\{1, \frac{|B_2 + \sigma B_1|}{B_1} \left(1 - \left|\frac{3q-1}{12q-1}\Omega^{m,\alpha}_q(12q-1)^2 \sigma\right|\right)^2\right\}.
\]
Proof. Let \( f \in C_{\Phi}^m(\Phi) \), then there is a Schwartz function \( \xi \) which is analytic in \( \mathbb{E} \) with \( \xi(0) = 0 \), and \( |\xi(z)| < 1 \) in \( \mathbb{E} \), such that

\[
\frac{z \partial_z D_{q}^m f(z)}{D_{q}^m f(z)} = \Phi(\xi(z)).
\]

(16)

Now, we define the function \( \mathcal{P}(z) \) with \( \text{Re}(P(z)) > 0 \) and \( P(0) = 1 \) by:

\[
\mathcal{P}(z) = \frac{1 + \xi(z)}{1 - \xi(z)} = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \ldots
\]

(17)

Since \( \xi(z) \) is a Schwartz function, therefore,

\[
\Phi(\xi(z)) = \Phi\left( \frac{\mathcal{P}(z) - 1}{\mathcal{P}(z) + 1} \right) = 1 + \frac{1}{2} B_1 p_1 z
\]

\[+ \frac{1}{2} \left( B_2 \left( p_2 - \frac{p_1^2}{2} \right) + B_1 \frac{p_1^2}{2} \right) z^2 + \ldots.
\]

(18)

Now, substitute (18) in (16), we get

\[
1 + \frac{1}{\mathcal{P}(z)} \left( \frac{z \partial_z D_{q}^m f(z)}{D_{q}^m f(z)} - 1 \right) = 1 + \frac{1}{2} B_1 p_1 z
\]

\[+ \frac{1}{2} B_2 \left( p_2 - \frac{p_1^2}{2} \right) z^2 + \ldots.
\]

(19)

From Equation (19), we get

\[
a_2 = \frac{B_1 p_1 \Phi}{2 |\mathcal{P}(z) - 1| \Omega_{\kappa,\lambda,\delta,\mu}^m(\mathcal{P}(z))}.
\]

(20)

and

\[
a_3 = \frac{\Phi B_1}{2 |\mathcal{P}(z) - 1| \Omega_{\kappa,\lambda,\delta,\mu}^m(\mathcal{P}(z))} \left( p_2 - \frac{1}{2} \left( 1 - \frac{B_1}{B_2} \frac{p_1^2}{2} \right) p_1^2 \right).
\]

(21)

Therefore,

\[
a_3 - a_2^2 = \frac{\Phi B_1}{2 |\mathcal{P}(z) - 1| \Omega_{\kappa,\lambda,\delta,\mu}^m(\mathcal{P}(z))} \left( p_2 - \nu p_1^2 \right),
\]

(22)

where

\[
\nu = \frac{1}{2} \left[ 1 - \frac{B_2}{B_1} \frac{p_1^2}{2} - \nu p_1 \left( 1 - \frac{\left( |\mathcal{P}(z)| - 1 \right) \Omega_{\kappa,\lambda,\delta,\mu}^m(\mathcal{P}(z)) }{\left( |\mathcal{P}(z)| - 1 \right) \Omega_{\kappa,\lambda,\delta,\mu}^m(\mathcal{P}(z))} \right) \right],
\]

(23)

by applying Lemma 3, we get our result. This completes the proof. \( \square \)

Next, we prove the following theorem for the subclass \( C_{\Phi}^m(\Phi) \).

**Theorem 6.** Let \( \Phi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \ldots \) with \( (B_1 \neq 0) \), and \( f \) given by (1) belongs to \( C_{\Phi}^m(\Phi) \), then

\[
|a_3 - a_2^2| \leq \frac{\Phi B_1}{2 |\mathcal{P}(z) - 1| \Omega_{\kappa,\lambda,\delta,\mu}^m(\mathcal{P}(z))} \left[ 1 + \frac{B_1}{B_2} \frac{p_1^2}{2} - \frac{\Phi B_1}{2 |\mathcal{P}(z) - 1| \Omega_{\kappa,\lambda,\delta,\mu}^m(\mathcal{P}(z))} \left( p_2 - \nu p_1^2 \right) \right].
\]

(31)

The result is sharp.
Corollary 8. Let \( \phi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \ldots \) with \( B_1 \neq 0 \), and \( f \) given by (1) belong to \( \ell_{q\phi}(\phi) \), then

\[
|a_3 - a_{\sigma a_2^2}| \leq \frac{|B_1|}{[3, q]_1} \frac{|B_2|}{[3, q]_1 - 1} \left( 1 + \left( \frac{[3, q]_1 - 1}{[2, q]_1} \right)^\sigma \right)
\]

(32)

The result is sharp.

For \( m = 0, \theta = 0 \) and taking \( q \to 1^- \) in Theorem 5, we obtain the following corollary:

\[
|a_3 - a_{\sigma a_2^2}| \leq \left( \frac{\Theta_{\sigma}^m}{\Omega_{\sigma}^m([2, q]_1)} \right)^\sigma \left( \left( \frac{1}{[3, q]_1 - 1} \right)^\sigma \left( [2, q]_1 - 1 \right)^\sigma \right)
\]

where

\[
X_1 = \left( \frac{\Theta_{\sigma}^m}{\Omega_{\sigma}^m([2, q]_1)} \right)^\sigma \left( \left( \frac{1}{[3, q]_1 - 1} \right)^\sigma \left( [2, q]_1 - 1 \right)^\sigma \right)
\]

(35)

\[
X_2 = \left( \frac{\Theta_{\sigma}^m}{\Omega_{\sigma}^m([2, q]_1)} \right)^\sigma \left( \left( \frac{1}{[3, q]_1 - 1} \right)^\sigma \left( [2, q]_1 - 1 \right)^\sigma \right)
\]

(36)

Proof. Applying Lemma 4 to the Equations (22) and (23), we have three cases:

Case (1): If \( \sigma \leq X_1 \)

\[
|a_3 - a_{\sigma a_2^2}| \leq \frac{\Theta_{\sigma}^m}{\Omega_{\sigma}^m([2, q]_1)} \left( \left( \frac{1}{[3, q]_1 - 1} \right)^\sigma \left( [2, q]_1 - 1 \right)^\sigma \right)
\]

(37)

Therefore,

\[
|a_3 - a_{\sigma a_2^2}| \leq \frac{\Theta_{\sigma}^m}{\Omega_{\sigma}^m([2, q]_1)} \left( \left( \frac{1}{[3, q]_1 - 1} \right)^\sigma \left( [2, q]_1 - 1 \right)^\sigma \right)
\]

(38)

Corollary 9 (see [20]). Let \( \phi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \ldots \) with \( B_1 \neq 0 \), and \( f \) given by (1) belong to \( \ell_{q\phi}(\phi) \), then

\[
|a_3 - a_{\sigma a_2^2}| \leq \frac{|B_1|}{[3, q]_1} \left( 1 + \left( \frac{[3, q]_1 - 1}{[2, q]_1} \right)^\sigma \right)
\]

(33)

The result is sharp.

Next, by using Lemma 4, we obtain the following theorems:

Theorem 10. Let \( \phi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \ldots \) with \( B_1 > 0, B_2 \geq 0 \) and \( f \) given by (1) belong to \( \ell_{q\phi}(\phi) \), then

\[
|a_3 - a_{\sigma a_2^2}| \leq \frac{|B_1|}{[3, q]_1} \left( 1 + \left( \frac{[3, q]_1 - 1}{[2, q]_1} \right)^\sigma \right)
\]

(39)

Case (2): If \( X_1 \leq \sigma \leq X_2 \)

\[
|a_3 - a_{\sigma a_2^2}| \leq \frac{\Theta_{\sigma}^m}{\Omega_{\sigma}^m([2, q]_1)} \left( \left( \frac{1}{[3, q]_1 - 1} \right)^\sigma \left( [2, q]_1 - 1 \right)^\sigma \right)
\]

(40)

Case (3): If \( \sigma \geq X_2 \)

\[
|a_3 - a_{\sigma a_2^2}| \leq \frac{\Theta_{\sigma}^m}{\Omega_{\sigma}^m([2, q]_1)} \left( \left( \frac{1}{[3, q]_1 - 1} \right)^\sigma \left( [2, q]_1 - 1 \right)^\sigma \right)
\]

(41)

This completes our proof.

\[ \square \]

Theorem 11. Let \( \phi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \ldots \) with \( B_1 > 0, B_2 \geq 0 \) and \( f \) given by (1) belong to \( \ell_{q\phi}(\phi) \), then

\[
|a_3 - a_{\sigma a_2^2}| \leq \frac{|B_1|}{[3, q]_1} \left( 1 + \left( \frac{[3, q]_1 - 1}{[2, q]_1} \right)^\sigma \right)
\]

(42)

This completes our proof.
where
\[
\psi_1 = \frac{(2|\eta|)^2((2|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(2|\eta|))^2)(qB_1^2 + (B_2 - B_1)(2|\eta| - 1))}{[3|\eta|(3|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(3|\eta|))}.
\]
(43)
\[
\psi_2 = \frac{(2|\eta|)^2((2|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(2|\eta|))^2)(qB_1^2 + (B_2 + B_1)(2|\eta| - 1))}{[3|\eta|(3|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(3|\eta|))}.
\]
(44)

Proof. Applying Lemma 4 to the Equations (29) and (30), then we have

Case (1): If \( \sigma \leq \psi_1 \)
\[
|a_1 - \sigma a_2|^2 \leq \frac{qB_1^2}{[3|\eta|(3|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(3|\eta|))}[2 - 4v],
\]
(45)
\[
\leq \frac{qB_1^2}{[3|\eta|(3|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(3|\eta|))}\left\{ \frac{B_2}{B_1} + \frac{B_1 \sigma}{[2|\eta| - 1] \left( 1 - \left[ \frac{3|\eta|(3|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(3|\eta|))}{(2|\eta|)^2(2|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(2|\eta|))^2} \right) \sigma \right) \right\},
\]
(46)

Case (2): If \( \psi_1 \leq \sigma \leq \psi_2 \)
\[
|a_1 - \sigma a_2|^2 \leq \frac{qB_1^2}{[3|\eta|(3|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(3|\eta|))}.
\]
(47)

Case (3): If \( \sigma \geq \psi_2 \)
\[
|a_1 - \sigma a_2|^2 \leq \frac{qB_1^2}{[2(2|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(2|\eta|))}(4v - 2),
\]
(48)
\[
|a_1 - \sigma a_2|^2 \leq \frac{qB_1^2}{[3|\eta|(3|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(3|\eta|))}\left\{ \frac{B_2}{B_1} + \frac{B_1 \sigma}{[2|\eta| - 1] \left( 1 - \left[ \frac{3|\eta|(3|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(3|\eta|))}{(2|\eta|)^2(2|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(2|\eta|))^2} \right) \sigma \right) \right\},
\]
(49)
\[
\leq \frac{qB_1^2}{[2|\eta| - 1] \left( [3|\eta|(3|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(3|\eta|))\left( \sigma(3|\eta| - 1) + \frac{[2|\eta| - 1]}{[3|\eta|(3|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(3|\eta|))} \right) \right) - \frac{qB_1^2}{[3|\eta|(3|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(3|\eta|))}.
\]
(50)

This completes the proof.

When \( m = 0, \theta = 0 \) in Theorems 5 and 6, we obtain the Fekete–Szegö inequalities for the subclasses \( \mathcal{L}_{\alpha}(\phi) \) and \( \mathcal{C}_{\alpha}(\phi) \), respectively [12].

**Corollary 12.** Let \( \phi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \ldots \) with \( B_1 > 0, B_2 \geq 0 \) and \( f \) given by (1) belong to \( \mathcal{L}_{\alpha}(\phi) \), with \( q > 0 \) then
\[
|a_3 - \sigma a_2|^2 \leq \frac{\phi_{\alpha}^\theta_{\alpha} + \phi_{\alpha}^\theta_{\alpha} \left( \frac{1}{1 - \left( \frac{3|\eta| - 1}{3|\eta|} \right)^2} \right) - \frac{\phi_{\alpha}^\theta_{\alpha}}{[3|\eta|(3|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(3|\eta|))} \left( \frac{2|\eta| - 1}{[3|\eta|(3|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(3|\eta|))} \right) \right)}{[3|\eta|(3|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(3|\eta|))}.
\]
(51)

The result is sharp.

**Corollary 13.** Let \( \phi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \ldots \) with \( B_1 > 0, B_2 \geq 0 \) and \( f \) given by (1) belong to \( \mathcal{C}_{\alpha}(\phi) \), with \( q > 0 \) then
\[
|a_3 - \sigma a_2|^2 \leq \frac{\phi_{\alpha}^\theta_{\alpha} + \phi_{\alpha}^\theta_{\alpha} \left( \frac{1}{1 - \left( \frac{3|\eta| - 1}{3|\eta|} \right)^2} - \frac{\phi_{\alpha}^\theta_{\alpha}}{[3|\eta|(3|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(3|\eta|))} \right)}{[3|\eta|(3|\eta| - 1)(\Omega_{\alpha, \lambda, \omega}^\mu(3|\eta|))}.
\]
(52)

The result is sharp.

**3. Conclusion**

Fekete–Szegö problems have always been the main interests of many researchers in geometric function theory. Many studies related to Fekete-Szego revolved around classes of analytic normalised univalent functions. In this particular work, the Fekete–Szegö inequality is obtained for functions in more general classes denoted by \( \mathcal{L}^m_{\alpha}(\phi) \) and \( \mathcal{C}^m_{\alpha}(\phi) \), respectively, using a new differential operator associated with \( q \)-calculus.
Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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