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Research Article

On a Unique Solution of a T-Maze Model Arising in the Psychology and Theory of Learning

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In the form of a T, a T-maze is an experimental design in which each trial consists of decisions between two or more options. It contains choices with particular kinds of symmetries that have gained considerable attention in psychology and learning theories. One of the simplest mazes utilized by rats is the T-maze since it requires just a single point of preference. At a T-maze base, the mouse chooses to turn right or left to get food. This paper aims at analyzing the rat's behavior in such circumstances and proposing a suitable mathematical model for it. The existence and uniqueness of a solution to the proposed T-maze model are investigated by using the appropriate fixed point method.

1. Introduction

Mathematical psychology is an approach to psychological study focused on mathematical modeling of perceptual, thinking, cognitive, and motor processes. The mathematical methods are used to develop more reliable theories and thus produce more rigorous empirical validations. The biggest issue with today's application of mathematics to psychological problems and most likely for some time to come is modeling these problems.

In an animal or human being, the learning phase may often be viewed as a series of choices between multiple possible reactions. Even in basic repetitive experiments under strictly regulated conditions, preference sequences are typically volatile, suggesting that the probability governs the choice of responses. It is also helpful to identify structural adjustments in the series of alternatives that reflect changes in trial-to-trial outcome probabilities. From this perspective, most of the learning analysis explains the probability of a trial-to-test occurrence that describes a stochastic mechanism.

In modern mathematical learning experiments, the researchers concluded that a basic learning experiment was compatible with any stochastic process. It is not a new idea (see [1] for a summary of its history). After 1950, two critical features emerged mainly in the research initiated by Bush, Estes, and Mosteller. In the first instance, the learning method egalitarian essence was a core feature of the developed model. Second, these frameworks were studied and applied in areas that did not conceal their quantitative aspects.

Several studies (Estes and Straughan [2], Grant et al. [3], Humphreys [4], and Jarvik [5]) on human actions in probability–learning scenarios have produced results aligned with the so–called event-matching hypothesis that the allocation of incentives would mirror the asymptotic distribution of answers in a two-choice setting. Conflicting findings have been reported in other studies. For example, if subjects choose the correct option in most trials, then, it would accelerate the probability close to 1 (for the detail, see [1, 6]).

Turab and Sintunavarat [7, 8] presented a functional equation to analyze Bush and Wilson's experimental study on a paradise fish [9], in which they offered the fish two options for swimming. As the starting gate was raised, swimmers had two options: swim on the right-hand side or the left-hand side of the tank's far end.

Recently, in [10], the authors discussed a particular type of traumatic avoidance learning experiment of normal dogs proposed by Solomon and Wynne [11]. They examined 30 mongrel dogs weighing between 9 and 13 kg and observed a particular form of emotional resistance performed in a tiny box with a steel grid floor. Turab and Sintunavarat [10] analyzed the dogs' behavior in such situations and proposed a mathematical model and also presented the existence of solutions of such model by using the fixed point technique.

On the other hand, the genesis of the fixed point theory was primarily for the use of successive approximations to prove the existence and uniqueness of solutions, primarily of differential and integral equations, in the second half of the nineteenth century. It is indeed a beautiful blend of pure and applied analysis, topology, and geometry. Picard's work demonstrates the fundamental concepts of a fixed point theoretic perspective. However, it is attributed to the Polish mathematician Banach for abstracting the fundamental ideas into a framework applicable to a wide variety of applications beyond ordinary differential and integral equations (see [12]). It has been generalized and extended in various directions (for the detail, see [13–16]). For more details about the fixed point theory and its applications in different spaces, we refer the reader to [17–22].

In this paper, we present a specific type of psychological learning theory experiment related to the T-maze model proposed by Brunswik and Stanley in [23, 24], and suggest a mathematical model that is appropriate for it. The existence and uniqueness of the proposed model's solution are investigated by using the suitable fixed point theorem. Later on, to check the proposed model's validity, we shall highlight some particular aspects of the T-maze model under the experimenter-subject controlled events. In the end, we raise an open problem for the interested readers.

2. A T–Maze Experiment Proposed by Brunswik and Stanley

A T-maze [23, 24] is a unique design that has gotten much attention in the past few years. It is a classic maze for rats since it has only one choice point. While experimental design modifications and generalizations have been used with mice and other subjects, we shall concentrate on the primary form of the open maze used with rats.

In Figure 1, a schematic of the apparatus can be seen. At the starting position, **s**, a rat is put and it runs to the point of

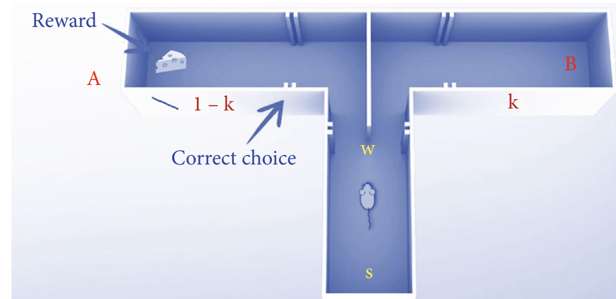

Figure 1: The behavior of a rat in a T-maze experiment.

Table 1: Alternative definitions of experimental events in a T-maze experiment depending on the placement of the food and chosen side.

Responses	Outcomes
A : left turn	$\mathbf{O}_1$: food side (reward)
B : left turn	$\mathbf{O}_1$: food side (reward)
A : right turn	$\mathbf{O}_2$: nonfood side (no reward)
B : right turn	$\mathbf{O}_2$: nonfood side (no reward)

preference, **w**. After that, the rat moves to one of the two aim bins, **A** and **B**, where it may get food.

Here, the experimental trials constitute the series of behavior. With the same rat, the process is usually performed several times. A very elaborate course of action is the overall activity of a rat during an experimental study. The rat is in a particular stimulation position when it crosses the labyrinth and is in two potential stimulus conditions after reaching the preference stage. Of course, this overall activity on a test may be divided up and appropriate measurements or indexes used by each part can be used. For example, we could ask about the starting location latency, **s** , or the running momentum between **s** and the **w**-point of preference. However, it seems to us that the part of the rat's conduct that is unique to this experiment is the behavior at the option stage, **w**.

In the study that follows, we only consider a rat's choice of the path on a trial instead of any other actions it may exhibit. The rat arrives at the decision point at a complete experimental study where the stimulation factor population is kept unchanged from trial to trial. Corresponding to the target box achieved, **A** or **B**, two groups of responses are listed in Table 1. One and only one of these response groups take part in each study trial. Then, an experimental study compares to a trial as described in [12]—a chance to select between alternatives that are mutually exclusive and exhaustive.

The condition of the organism on a specific test, according to the model, is fully specified by a probability **k** that the rat will go to goal box **A** and a probability $\mathbf{1-k}$ that it will go to goal box **B**. We have complete information about the

Table 2: The movement of a rat and its corresponding events.

Response	Outcomes	Events
A	$\mathbf{O_1}$	$\mathbf{E_1}$
B	$\mathbf{O_1}$	$\mathbf{E_2}$
A	$\mathbf{O_2}$	$\mathbf{E_3}$
B	$\mathbf{O_2}$	$\mathbf{E_4}$

learning process model when these probabilities are recognized for each trial. These probabilities can be estimated from the proportion of turns made by a single rat on several trials to goal box **A** or from the proportion of a population of rats that go on a specific trial to goal box **B**.

3. Mathematical Modeling of the Proposed T-Maze Experiment

In the above experiment, the significant interest lies in the behavior of a rat; turn left or right, "**A**" or "**B**," and get the food which is dependent on where the food is placed and the movement of a rat towards that compartment. In our view, if a rat chooses the food side, there would be an occurrence of alternative $\mathbf{O_1}$, and if a rat made a move to the other side, then, there will be an occurrence of alternative $\mathbf{O_2}$. According to the mathematical point of view, there would be four possibilities of events, depending on the movement of the rat and the placement of the food. These events are listed in Table 2.

The probability of the responses **A** and **B** are x and $(1-x)$, respectively, where $x \in [0, 1]$. The experimental pattern asks for the outcomes of the responses (whether the rat gets the food or not), trials' fixed proportion of $\varsigma \in [0, 1]$. Therefore, we get the event probabilities stated below (see Table 3).

Let us assume that $\vartheta_1, \vartheta_2 \in (0, 1)$ are the learning–rate parameters and their values measure the ineffectiveness of the events $\mathbf{E_1} - \mathbf{E_4}$ in altering the response probability. Also, $\lambda_k \in [0, 1]$, where $k = 1, 2$ is the constant of the corresponding event $\mathbf{E_1} - \mathbf{E_4}$. If ςx is the probability of response **A** with outcome $\mathbf{O_1}$ on some trial and **A** is fulfilled, the new probability of **A** with outcome $\mathbf{O_1}$ is $\vartheta_1 x + (1-\vartheta_1)\lambda_1$, and if **A** is achieved with outcome $\mathbf{O_2}$, then, the new probability will be $\vartheta_1 x + (1-\vartheta_1)(1-\lambda_1)$ with the event probability $(1-\varsigma)x$. Similarly, if **B** is performed with outcomes $\mathbf{O_1}$ and $\mathbf{O_2}$, then, the new probabilities of **B** are $\vartheta_2 x + (1-\vartheta_2)\lambda_2$ and $\vartheta_2 x + (1-\vartheta_2)(1-\lambda_2)$, having event probabilities of occurrence $(1-x)\varsigma$ and $(1-\varsigma)(1-x)$, respectively. For the four events $\mathbf{E_1} - \mathbf{E_4}$, we can define the transition operators $P_1, P_2, P_3, P_4 : [0, 1] \longrightarrow [0, 1]$ as

$$\begin{aligned} P_1 x &= \vartheta_1 x + (1-\vartheta_1)\lambda_1, \\ P_2 x &= \vartheta_2 x + (1-\vartheta_2)\lambda_2, \\ P_3 x &= \vartheta_1 x + (1-\vartheta_1)(1-\lambda_1), \\ P_4 x &= \vartheta_2 x + (1-\vartheta_2)(1-\lambda_2), \end{aligned} \tag{1}$$

for all $x \in [0, 1]$ and $0 < \vartheta_1, \vartheta_2 < 1$.

Table 3: Corresponding probabilities of the four events.

Event	Probability of occurrence
$\mathbf{E_1}$	ςx
$\mathbf{E_2}$	$(1-x)\varsigma$
$\mathbf{E_3}$	$(1-\varsigma)x$
$\mathbf{E_4}$	$(1-\varsigma)(1-x)$

It can be observed that a rat following such description, in the long run, will stop giving feedback to one of the responses and react only with the other (with probability one). Now, giving $x, \vartheta_1, \vartheta_2, \lambda_1, \lambda_2$, what is the probability that the rat stops providing **B**'s, that is, consumed by **A**? We define such probability by $P(x, \vartheta_1, \vartheta_2, \lambda_1, \lambda_2)$ as a function of x, and it depends on the path as well as the responses and outcomes. After one trial, the rat has a new probability shown in (1) depending on the events $\mathbf{E_1} - \mathbf{E_4}$ with the respective probabilities of occurrence. Thus, if its first trial is **A** with outcomes $\mathbf{O_1}$ and $\mathbf{O_2}$, its new probability of consumption by **A** will be $P(\vartheta_1 x + (1-\vartheta_1)\lambda_1, \vartheta_1, \lambda_1)$ and $P(\vartheta_1 x + (1-\vartheta_1)(1-\lambda_1), \vartheta_1, \lambda_1)$, respectively. But, if the first trial is **B** with outcomes $\mathbf{O_1}$ and $\mathbf{O_2}$, then, the new probability of absorption by **B** will be $P(\vartheta_2 x + (1-\vartheta_2)\lambda_2, \vartheta_2, \lambda_2)$ and $P(\vartheta_2 x + (1-\vartheta_2)(1-\lambda_2), \vartheta_2, \lambda_2)$, respectively.

By considering the above transition operators with their corresponding probabilities and events given in Table 3, we have the following functional equation

$$\begin{aligned} P(x, \vartheta_1, \vartheta_2, \lambda_1, \lambda_2) &= \varsigma x P(\vartheta_1 x + (1-\vartheta_1)\lambda_1, \vartheta_1, \lambda_1) \\ &+ (1-x)\varsigma P(\vartheta_2 x + (1-\vartheta_2)\lambda_2, \vartheta_2, \lambda_2) \\ &+ (1-\varsigma)x P(\vartheta_1 x + (1-\vartheta_1)(1-\lambda_1), \vartheta_1, \lambda_1) \\ &+ (1-\varsigma)(1-x) P(\vartheta_2 x + (1-\vartheta_2)(1-\lambda_2), \vartheta_2, \lambda_2). \end{aligned} \tag{2}$$

In the progression, the following noted result will be needed.

Theorem 1. *(Banach fixed point theorem [12]). Let $(\mathcal{S}, d)$ be a complete metric space and $\mathcal{M} : \mathcal{S} \longrightarrow \mathcal{S}$ be a mapping defined by*

$$d(\mathcal{M}\ell, \mathcal{M}m) \le \varpi d(\ell, m), \tag{3}$$

for some $\varpi < 1$ and for all $\ell, m \in \mathcal{S}$. Then, $\mathcal{M}$ has precisely one fixed point. Moreover, the Picard iteration $\{\ell_n\}$ in $\mathcal{S}$ which is defined by $\ell_n = \mathcal{M}\ell_{n-1}$ for all $n \in \mathbb{N}$, where $\ell_0 \in \mathcal{S}$, converges to the unique fixed point of $\mathcal{M}$.

4. Main Results

Let $\mathcal{A} = [0, 1]$. Throughout this article, we denote by $\mathcal{B}$ the class of all continuous real-valued functions $\mathcal{W} : \mathcal{A} \longrightarrow \mathbb{R}$ such that $\mathcal{W}(0) = 0$ and

$$\sup_{x \neq y} \frac{|\mathcal{W}(x) - \mathcal{W}(y)|}{|x-y|} < \infty. \tag{4}$$

It is easy to see that $(\mathscr{B}, \|\cdot\|)$ is a Banach space, where $\|\cdot\|$ is defined by

$$\|\mathscr{W}\| = \sup_{x\neq y} \frac{|\mathscr{W}(x) - \mathscr{W}(y)|}{|x-y|},$$

for all $\mathscr{W} \in \mathscr{B}$.

For the computational convenience, we define an operator $\mathscr{W} : \mathscr{A} \longrightarrow \mathbb{R}$ and write functional equation (2) as

$$\begin{aligned}\mathscr{W}(x) = {} & \varsigma x\mathscr{W}(\vartheta_1 x + (1-\vartheta_1)\lambda_1) + (1-x)\varsigma\mathscr{W}(\vartheta_2 x + (1-\vartheta_2)\lambda_2) \\ & + (1-\varsigma)x\mathscr{W}(\vartheta_1 x + (1-\vartheta_1)(1-\lambda_1)) \\ & + (1-\varsigma)(1-x)\mathscr{W}(\vartheta_2 x + (1-\vartheta_2)(1-\lambda_2)),\end{aligned} \tag{5}$$

where $0 < \vartheta_1, \vartheta_2 < 1$ and $\lambda_1, \lambda_2 \in \mathscr{A}$. Our objective is to investigate the existence and uniqueness of a solution to functional equation (5) by using the fixed point technique. We begin with the following outcome.

Theorem 2. *Let $0 < \vartheta_1, \vartheta_2 < 1$ and $\lambda_1, \lambda_2, \varsigma \in \mathscr{A}$ such that $\varpi < 1$, where*

$$\begin{aligned}\varpi \coloneqq {} & |(2\varsigma - 1)((1-\vartheta_1)\lambda_1 + (1-\vartheta_2)\lambda_2) + (1-\varsigma) \\ & \cdot ((1-\vartheta_1) + (1-\vartheta_2)) + 2(\vartheta_1 + \vartheta_2)|.\end{aligned} \tag{6}$$

If there exists a closed subset Λ of $\mathscr{B}$ such that Λ is Z invariant, that is, $Z(\Lambda) \subseteq \Lambda$, where Z is the operator from Λ defined for each $\mathscr{W} \in \Lambda$ by

$$\begin{aligned}(Z\mathscr{W})(x) = {} & \varsigma x\mathscr{W}(\vartheta_1 x + (1-\vartheta_1)\lambda_1) + \varsigma(1-x) \\ & \cdot \mathscr{W}(\vartheta_2 x + (1-\vartheta_2)\lambda_2) + (1-\varsigma) \\ & \cdot x\mathscr{W}(\vartheta_1 x + (1-\vartheta_1)(1-\lambda_1)) + (1-\varsigma) \\ & \cdot (1-x)\mathscr{W}(\vartheta_2 x + (1-\vartheta_2)(1-\lambda_2)),\end{aligned} \tag{7}$$

for all $x \in \mathscr{A}$, then, Z is a Banach contraction mapping with the metric d induced by $\|\cdot\|$.

Proof. Let $\mathscr{W}_1, \mathscr{W}_2 \in \Lambda$. For each distinct points $x, y \in \mathscr{A}$, we obtain

$$\begin{aligned}& \frac{|(Z\mathscr{W}_1 - Z\mathscr{W}_2)(x) - (Z\mathscr{W}_1 - Z\mathscr{W}_2)(y)|}{|x-y|} \\ & = \bigg|\frac{1}{x-y}[\varsigma x(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 x + (1-\vartheta_1)\lambda_1) \\ & \quad + \varsigma(1-x)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 x + (1-\vartheta_2)\lambda_2) \\ & \quad + (1-\varsigma)x(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 x + (1-\vartheta_1)(1-\lambda_1)) \\ & \quad + (1-\varsigma)(1-x)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 x + (1-\vartheta_2) \\ & \quad \cdot (1-\lambda_2)) - \varsigma y(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y + (1-\vartheta_1)\lambda_1) \\ & \quad - \varsigma(1-y)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2)\lambda_2) \\ & \quad - (1-\varsigma)y(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y + (1-\vartheta_1)(1-\lambda_1)) \\ & \quad - (1-\varsigma)(1-y)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2)(1-\lambda_2))\Big]\bigg| \\ & = \bigg|\frac{1}{x-y}[\varsigma x(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 x + (1-\vartheta_1)\lambda_1) - \varsigma x(\mathscr{W}_1 - \mathscr{W}_2) \\ & \cdot (\vartheta_1 y + (1-\vartheta_1)\lambda_1) \quad + \varsigma(1-x)(\mathscr{W}_1 - \mathscr{W}_2) \\ & \cdot (\vartheta_2 x + (1-\vartheta_2)\lambda_2) - \varsigma(1-x)(\mathscr{W}_1 - \mathscr{W}_2) \\ & \cdot (\vartheta_2 y + (1-\vartheta_2)\lambda_2) + (1-\varsigma)x(\mathscr{W}_1 - \mathscr{W}_2) \\ & \cdot (\vartheta_1 x + (1-\vartheta_1)(1-\lambda_1)) - (1-\varsigma)x(\mathscr{W}_1 - \mathscr{W}_2) \\ & \cdot (\vartheta_1 y + (1-\vartheta_1)(1-\lambda_1)) + (1-\varsigma)(1-x)(\mathscr{W}_1 - \mathscr{W}_2) \\ & \cdot (\vartheta_2 x + (1-\vartheta_2)(1-\lambda_2)) - (1-\varsigma)(1-x)(\mathscr{W}_1 - \mathscr{W}_2) \\ & \cdot (\vartheta_2 y + (1-\vartheta_2)(1-\lambda_2)) + \varsigma x(\mathscr{W}_1 - \mathscr{W}_2) \\ & \cdot (\vartheta_1 y + (1-\vartheta_1)\lambda_1) - \varsigma y(\mathscr{W}_1 - \mathscr{W}_2) \\ & \cdot (\vartheta_1 y + (1-\vartheta_1)\lambda_1) + \varsigma(1-x)(\mathscr{W}_1 - \mathscr{W}_2) \\ & \cdot (\vartheta_2 y + (1-\vartheta_2)\lambda_2) - \varsigma(1-y)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2) \\ & \cdot \lambda_2) + (1-\varsigma)x(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y + (1-\vartheta_1) \\ & \cdot (1-\lambda_1)) - (1-\varsigma)y(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y + (1-\vartheta_1) \\ & \cdot (1-\lambda_1)) + (1-\varsigma)(1-x)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2) \\ & \cdot (1-\lambda_2)) - (1-\varsigma)(1-y)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2) \\ & \cdot (1-\lambda_2))\Big]\bigg| = \bigg|\frac{1}{x-y}[\varsigma x(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 x + (1-\vartheta_1)\lambda_1) - \varsigma x \\ & \cdot (\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y + (1-\vartheta_1)\lambda_1)] + \frac{1}{x-y}[\varsigma(1-x) \\ & \cdot (\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 x + (1-\vartheta_2)\lambda_2) - \varsigma(1-x)(\mathscr{W}_1 - \mathscr{W}_2) \\ & \cdot (\vartheta_2 y + (1-\vartheta_2)\lambda_2)] + \frac{1}{x-y}[(1-\varsigma)x(\mathscr{W}_1 - \mathscr{W}_2) \\ & \cdot (\vartheta_1 x + (1-\vartheta_1)(1-\lambda_1)) - (1-\varsigma)x(\mathscr{W}_1 - \mathscr{W}_2) \\ & \cdot (\vartheta_1 y + (1-\vartheta_1)(1-\lambda_1))] + \frac{1}{x-y}[(1-\varsigma)(1-x) \\ & \cdot (\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 x + (1-\vartheta_2)(1-\lambda_2)) - (1-\varsigma)(1-x) \\ & \cdot (\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2)(1-\lambda_2))] + \frac{1}{x-y} \\ & \cdot [\varsigma x(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y + (1-\vartheta_1)\lambda_1) - \varsigma y(\mathscr{W}_1 - \mathscr{W}_2) \\ & \cdot (\vartheta_1 y + (1-\vartheta_1)\lambda_1)] + \frac{1}{x-y}[\varsigma(1-x)(\mathscr{W}_1 - \mathscr{W}_2) \\ & \cdot (\vartheta_2 y + (1-\vartheta_2)\lambda_2) - \varsigma(1-y)(\mathscr{W}_1 - \mathscr{W}_2) \\ & \cdot (\vartheta_2 y + (1-\vartheta_2)\lambda_2)] + \frac{1}{x-y}[(1-\varsigma) \\ & \cdot x(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y + (1-\vartheta_1)(1-\lambda_1)) - (1-\varsigma) \\ & \cdot y(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y + (1-\vartheta_1)(1-\lambda_1))] + \frac{1}{x-y}[(1-\varsigma) \\ & \cdot (1-x)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2)(1-\lambda_2)) - (1-\varsigma) \\ & \cdot (1-y)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2)(1-\lambda_2))]\big| \le \vartheta_1 \varsigma x \\ & \cdot \|\mathscr{W}_1 - \mathscr{W}_2\| + \vartheta_2 \varsigma(1-x)\|\mathscr{W}_1 - \mathscr{W}_2\| + \vartheta_1(1-\varsigma)x \\ & \cdot \|\mathscr{W}_1 - \mathscr{W}_2\| + \vartheta_2(1-\varsigma)(1-x)\|\mathscr{W}_1 - \mathscr{W}_2\| + |\varsigma \\ & \cdot (\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y + (1-\vartheta_1)\lambda_1) - \varsigma(\mathscr{W}_1 - \mathscr{W}_2)(0)| + |\varsigma \\ & \cdot (\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2)\lambda_2) - \varsigma \\ & \cdot (\mathscr{W}_1 - \mathscr{W}_2)(0)| + |(1-\varsigma)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y + (1-\vartheta_1) \\ & \cdot (1-\lambda_1)) - (1-\varsigma)(\mathscr{W}_1 - \mathscr{W}_2)(0)| + |(1-\varsigma)(\mathscr{W}_1 - \mathscr{W}_2) \\ & \cdot (\vartheta_2 y + (1-\vartheta_2)(1-\lambda_2)) - (1-\varsigma)(\mathscr{W}_1 - \mathscr{W}_2)(0)| = \vartheta_1 \varsigma x \\ & \cdot \|\mathscr{W}_1 - \mathscr{W}_2\| + \vartheta_2 \varsigma(1-x)\|\mathscr{W}_1 - \mathscr{W}_2\| + \vartheta_1 \\ & \cdot (1-\varsigma)x\|\mathscr{W}_1 - \mathscr{W}_2\| + \vartheta_2(1-\varsigma)(1-x) \\ & \cdot \|\mathscr{W}_1 - \mathscr{W}_2\| + \varsigma(\vartheta_1 y + (1-\vartheta_1)\lambda_1) \\ & \cdot \|\mathscr{W}_1 - \mathscr{W}_2\| + \varsigma(\vartheta_2 y + (1-\vartheta_2)\lambda_2) \\ & \cdot \|\mathscr{W}_1 - \mathscr{W}_2\| + (1-\varsigma)(\vartheta_1 y + (1-\vartheta_1)(1-\lambda_1)) \\ & \cdot \|\mathscr{W}_1 - \mathscr{W}_2\| + (1-\varsigma)(\vartheta_2 y + (1-\vartheta_2)(1-\lambda_2)) \\ & \cdot \|\mathscr{W}_1 - \mathscr{W}_2\| \le \varpi\|\mathscr{W}_1 - \mathscr{W}_2\|,\end{aligned}$$

where ϖ is defined in (6). This gives that

$$d(Z\mathscr{W}_1, Z\mathscr{W}_2) = \|Z\mathscr{W}_1 - Z\mathscr{W}_2\| \le \varpi\|\mathscr{W}_1 - \mathscr{W}_2\| = \varpi d(\mathscr{W}_1, \mathscr{W}_2).$$

It follows from $0 < \varpi < 1$ that Z is a Banach contraction mapping with the metric d induced by $\|\cdot\|$. □

We get the following conclusion from Theorem 2 about the uniqueness of a solution to functional equation (5).

Theorem 3. *The functional equation (5) has a unique solution provided that $\varpi < 1$, where ϖ is defined in (6), and there exists a closed subset Λ of $\mathscr{B}$ such that Λ is Z invariant, that is, $Z(\Lambda) \subseteq \Lambda$, where Z is the operator from Λ defined for each $\mathscr{W} \in \Lambda$ by*

$$\begin{aligned}(Z\mathscr{W})(x) = {} & \varsigma x\mathscr{W}(\vartheta_1 x + (1-\vartheta_1)\lambda_1) + \varsigma(1-x) \\ & \cdot \mathscr{W}(\vartheta_2 x + (1-\vartheta_2)\lambda_2) + (1-\varsigma) \\ & \cdot x\mathscr{W}(\vartheta_1 x + (1-\vartheta_1)(1-\lambda_1)) + (1-\varsigma) \\ & \cdot (1-x)\mathscr{W}(\vartheta_2 x + (1-\vartheta_2)(1-\lambda_2)),\end{aligned} \quad (8)$$

for all $x \in \mathscr{A}$. Moreover, the iteration $\{\mathscr{W}_n\}$ in Λ is defined by

$$\begin{aligned}(\mathscr{W}_n)(x) = {} & \varsigma x\mathscr{W}_{n-1}(\vartheta_1 x + (1-\vartheta_1)\lambda_1) + \varsigma(1-x) \\ & \cdot \mathscr{W}_{n-1}(\vartheta_2 x + (1-\vartheta_2)\lambda_2) + (1-\varsigma) \\ & \cdot x\mathscr{W}_{n-1}(\vartheta_1 x + (1-\vartheta_1)(1-\lambda_1)) \\ & + (1-\varsigma)(1-x)\mathscr{W}_{n-1}(\vartheta_2 x + (1-\vartheta_2)(1-\lambda_2)),\end{aligned} \quad (9)$$

for all $n \in \mathbb{N}$, where $\mathscr{W}_0 \in \Lambda$, converges to the unique solution of functional equation (5) in the sense of the metric d induced by $\|\cdot\|$.

Proof. We derive the result of this theorem by combining the Banach fixed point theorem with Theorem 3. □

5. A Certain Case of a T-Maze Experiment with Experimenter-Subject-Controlled Events

It has been highlighted that the examination of any experiment is based on suppositions, which are assembled about the subject. Experiments are classified as contingent and noncontingent, based on the occurrences of the result.

In the previous models on imitation problems such as T-maze experiments with fish, dogs, and humans (see [7, 10, 25]), it was already mentioned that such experiments required a contingent approach; the result of the trials was entirely dependent on the subject's choice. Such types of models required experimenter-subject-controlled events. The two responses **A** and **B** along with outcomes $\mathbf{O}_1$ and $\mathbf{O}_2$ are choosing the right or left side or pushing the right or left button, which coincides with rewarding and nonrewarding or choosing the correct and incorrect side, respectively. Now, we define the probabilities ς_1 and ς_2 which indicate the conditional probability of outcomes $\mathbf{O}_1$ and $\mathbf{O}_2$ of the given alternatives **A** and **B**, respectively, such that

$$\varsigma_1 + \varsigma_2 = 1. \quad (10)$$

Table 4: Four events under conditional probabilities of occurrence.

Events	Outcomes	Transition operators	Probability of occurrence
A	$\mathbf{O}_1$	$P_1 x = \vartheta_1 x + (1-\vartheta_1)\lambda_1$	$\varsigma_1 x$
B	$\mathbf{O}_1$	$P_2 x = \vartheta_2 x + (1-\vartheta_2)\lambda_2$	$\varsigma_2(1-x)$
A	$\mathbf{O}_2$	$P_3 x = \vartheta_1 x + (1-\vartheta_1)(1-\lambda_1)$	$(1-\varsigma_1)x$
B	$\mathbf{O}_2$	$P_4 x = \vartheta_2 x + (1-\vartheta_2)(1-\lambda_2)$	$(1-\varsigma_2)(1-x)$

With such conditions, we have Table 4.

We have the following functional equation from the data given above

$$\begin{aligned}\mathscr{W}(x) = {} & \varsigma_1 x\mathscr{W}(\vartheta_1 x + (1-\vartheta_1)\lambda_1) + \varsigma_2(1-x) \\ & \cdot \mathscr{W}(\vartheta_2 x + (1-\vartheta_2)\lambda_2) + (1-\varsigma_1) \\ & \cdot x\mathscr{W}(\vartheta_1 x + (1-\vartheta_1)(1-\lambda_1)) \\ & + (1-\varsigma_2)(1-x)\mathscr{W}(\vartheta_2 x + (1-\vartheta_2)(1-\lambda_2)),\end{aligned} \quad (11)$$

where $\mathscr{W} : \mathscr{A} \longrightarrow \mathbb{R}$ is an unknown function, $0 < \vartheta_1, \vartheta_2 < 1$, and $\lambda_1, \lambda_2, \varsigma_1, \varsigma_2 \in \mathscr{A}$ with $\varsigma_1 + \varsigma_2 = 1$. We shall begin with the following outcome.

Theorem 4. *Let $0 < \vartheta_1, \vartheta_2 < 1$ and $\lambda_1, \lambda_2, \varsigma_1, \varsigma_2 \in \mathscr{A}$ such that $\varpi^* < 1$, where*

$$\varpi^* := \left| \begin{aligned} & (2\lambda_1 - 1)(\varsigma_1(1-\vartheta_1)) + (2\lambda_2 - 1)(\varsigma_2(1-\vartheta_2)) \\ & + ((1-\vartheta_1)(1-\lambda_1) + (1-\vartheta_2)(1-\lambda_2)) + 2(\vartheta_1 + \vartheta_2) \end{aligned} \right|. \quad (12)$$

If there exists a closed subset Λ of $\mathscr{B}$ such that Λ is Z invariant, that is, $Z(\Lambda) \subseteq \Lambda$, where Z is the operator from Λ defined for each $\mathscr{W} \in \Lambda$ by

$$\begin{aligned}(Z\mathscr{W})(x) = {} & \varsigma_1 x\mathscr{W}(\vartheta_1 x + (1-\vartheta_1)\lambda_1) + \varsigma_2(1-x) \\ & \cdot \mathscr{W}(\vartheta_2 x + (1-\vartheta_2)\lambda_2) + (1-\varsigma_1) \\ & \cdot x\mathscr{W}(\vartheta_1 x + (1-\vartheta_1)(1-\lambda_1)) \\ & + (1-\varsigma_2)(1-x)\mathscr{W}(\vartheta_2 x + (1-\vartheta_2)(1-\lambda_2)),\end{aligned} \quad (13)$$

for all $x \in \mathscr{A}$, then, Z is a Banach contraction mapping with the metric d induced by $\|\cdot\|$.

Proof. Let $\mathscr{W}_1, \mathscr{W}_2 \in \Lambda$. For each distinct points $x, y \in \mathscr{A}$, we obtain

$$
\begin{aligned}
&\frac{|(Z\mathscr{W}_1 - Z\mathscr{W}_2)(x) - (Z\mathscr{W}_1 - Z\mathscr{W}_2)(y)|}{|x-y|} \\
&= \left|\frac{1}{x-y}[\varsigma_1 x(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 x + (1-\vartheta_1)\lambda_1) + \varsigma_2(1-x)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 x + (1-\vartheta_2)\lambda_2)\right. \\
&\quad + (1-\varsigma_1)x(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 x + (1-\vartheta_1)(1-\lambda_1)) + (1-\varsigma_2)(1-x)(\mathscr{W}_1 - \mathscr{W}_2) \\
&\quad \cdot (\vartheta_2 x + (1-\vartheta_2)(1-\lambda_2)) - \varsigma_1 y(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y + (1-\vartheta_1) \\
&\quad \cdot \lambda_1) - \varsigma_2(1-y)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2)\lambda_2) - (1-\varsigma_1) \\
&\quad \cdot y(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y + (1-\vartheta_1)(1-\lambda_1)) - (1-\varsigma_2)(1-y) \\
&\quad \left. \cdot (\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2)(1-\lambda_2)]\right| \\
&= \left|\frac{1}{x-y}[\varsigma_1 x(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 x + (1-\vartheta_1)\lambda_1) - \varsigma_1 x(\mathscr{W}_1 - \mathscr{W}_2)\right. \\
&\quad \cdot (\vartheta_1 y + (1-\vartheta_1)\lambda_1) + \varsigma_2(1-x)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 x + (1-\vartheta_2) \\
&\quad \cdot \lambda_2) - \varsigma_2(1-x)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2)\lambda_2) + (1-\varsigma_1) \\
&\quad \cdot x(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 x + (1-\vartheta_1)(1-\lambda_1)) - (1-\varsigma_1) \\
&\quad \cdot x(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y + (1-\vartheta_1)(1-\lambda_1)) + (1-\varsigma_2)(1-x) \\
&\quad \cdot (\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 x + (1-\vartheta_2)(1-\lambda_2)) - (1-\varsigma_2)(1-x) \\
&\quad \cdot (\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2)(1-\lambda_2)) + \varsigma_1 x(\mathscr{W}_1 - \mathscr{W}_2) \\
&\quad \cdot (\vartheta_1 y + (1-\vartheta_1)\lambda_1) - \varsigma_1 y(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y + (1-\vartheta_1)\lambda_1) \\
&\quad + \varsigma_2(1-x)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2)\lambda_2) - \varsigma_2(1-y) \\
&\quad \cdot (\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2)\lambda_2) + (1-\varsigma_1)x(\mathscr{W}_1 - \mathscr{W}_2) \\
&\quad \cdot (\vartheta_1 y + (1-\vartheta_1)(1-\lambda_1)) - (1-\varsigma_1)y(\mathscr{W}_1 - \mathscr{W}_2) \\
&\quad \cdot \vartheta_1 y + (1-\vartheta_1)(1-\lambda_1)) + (1-\varsigma_2)(1-x)(\mathscr{W}_1 - \mathscr{W}_2) \\
&\quad \cdot (\vartheta_1 y + (1-\vartheta_1)(1-\lambda_1)) - (1-\varsigma_2)(1-y)(\mathscr{W}_1 - \mathscr{W}_2) \\
&\quad \left. \cdot (\vartheta_1 y + (1-\vartheta_1)(1-\lambda_1))]\right| \\
&= \left|\frac{1}{x-y}[\varsigma_1 x(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 x + (1-\vartheta_1)\lambda_1) - \varsigma_1 x(\mathscr{W}_1 - \mathscr{W}_2)\right. \\
&\quad \cdot (\vartheta_1 y + (1-\vartheta_1)\lambda_1)] + \frac{1}{x-y}[\varsigma_2(1-x)(\mathscr{W}_1 - \mathscr{W}_2) \\
&\quad \cdot (\vartheta_2 x + (1-\vartheta_2)\lambda_2) - \varsigma_2(1-x)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2)\lambda_2)] \\
&\quad + \frac{1}{x-y}[(1-\varsigma_1)x(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 x + (1-\vartheta_1)(1-\lambda_1)) - (1-\varsigma_1) \\
&\quad \cdot x(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y + (1-\vartheta_1)(1-\lambda_1))] + \frac{1}{x-y}[(1-\varsigma_2)(1-x) \\
&\quad \cdot (\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 x + (1-\vartheta_2)(1-\lambda_2)) - (1-\varsigma_2)(1-x) \\
&\quad \cdot (\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2)(1-\lambda_2))] + \frac{1}{x-y}[\varsigma_1 x(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y \\
&\quad + (1-\vartheta_1)\lambda_1) - \varsigma_1 y(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y + (1-\vartheta_1)\lambda_1)] + \frac{1}{x-y} \\
&\quad \cdot [\varsigma_2(1-x)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2)\lambda_2) - \varsigma_2(1-y)(\mathscr{W}_1 - \mathscr{W}_2) \\
&\quad \cdot (\vartheta_2 y + (1-\vartheta_2)\lambda_2)] + \frac{1}{x-y}[(1-\varsigma_1)x(\mathscr{W}_1 - \mathscr{W}_2) \\
&\quad \cdot (\vartheta_1 y + (1-\vartheta_1)(1-\lambda_1)) - (1-\varsigma_1)y(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y + (1-\vartheta_1) \\
&\quad \cdot (1-\lambda_1))] + \frac{1}{x-y}[(1-\varsigma_2)(1-x)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2)(1-\lambda_2)) \\
&\quad \left. - (1-\varsigma_2)(1-y)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2)(1-\lambda_2))]\right| \le \vartheta_1\varsigma_1 x\|\mathscr{W}_1 - \mathscr{W}_2\| + \vartheta_2\varsigma_2(1-x) \\
&\quad \cdot \|\mathscr{W}_1 - \mathscr{W}_2\| + \vartheta_1(1-\varsigma_1)x\|\mathscr{W}_1 - \mathscr{W}_2\| + \vartheta_2(1-\varsigma_2)(1-x)\|\mathscr{W}_1 - \mathscr{W}_2\| + \\
&\quad \cdot |\varsigma_1(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y + (1-\vartheta_1)\lambda_1) - \varsigma_1(\mathscr{W}_1 - \mathscr{W}_2)(0)| + |\varsigma_2(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2)\lambda_2) \\
&\quad - \varsigma_2(\mathscr{W}_1 - \mathscr{W}_2)(0)| + |(1-\varsigma_1)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_1 y + (1-\vartheta_1)(1-\lambda_1)) - (1-\varsigma_1)(\mathscr{W}_1 - \mathscr{W}_2)(0)| \\
&\quad + |(1-\varsigma_2)(\mathscr{W}_1 - \mathscr{W}_2)(\vartheta_2 y + (1-\vartheta_2)(1-\lambda_2)) - (1-\varsigma_2)(\mathscr{W}_1 - \mathscr{W}_2)(0)| = \vartheta_1\varsigma_1 x\|\mathscr{W}_1 - \mathscr{W}_2\| \\
&\quad + \vartheta_2\varsigma_2(1-x)\|\mathscr{W}_1 - \mathscr{W}_2\| + \vartheta_1(1-\varsigma_1)x\|\mathscr{W}_1 - \mathscr{W}_2\| + \vartheta_2(1-\varsigma_2)(1-x)\|\mathscr{W}_1 - \mathscr{W}_2\| \\
&\quad + \varsigma_1(\vartheta_1 y + (1-\vartheta_1)\lambda_1)\|\mathscr{W}_1 - \mathscr{W}_2\| + \varsigma_2(\vartheta_2 y + (1-\vartheta_2)\lambda_2)\|\mathscr{W}_1 - \mathscr{W}_2\| + (1-\varsigma_1) \\
&\quad \cdot (\vartheta_1 y + (1-\vartheta_1)(1-\lambda_1))\|\mathscr{W}_1 - \mathscr{W}_2\| + (1-\varsigma_2)(\vartheta_2 y + (1-\vartheta_2)(1-\lambda_2))\|\mathscr{W}_1 - \mathscr{W}_2\| \le \varpi^\star \\
&\quad \cdot \|\mathscr{W}_1 - \mathscr{W}_2\|,
\end{aligned}
$$

where $\varpi^\star$ is defined in (12). This gives that

$$
\begin{aligned}
d(Z\mathscr{W}_1, Z\mathscr{W}_2) = \|Z\mathscr{W}_1 - Z\mathscr{W}_2\| \le \varpi^\star \|\mathscr{W}_1 - \mathscr{W}_2\| \\
= \varpi^\star d(\mathscr{W}_1, \mathscr{W}_2).
\end{aligned}
$$

It follows from $0 < \varpi^\star < 1$ that Z is a Banach contraction mapping with the metric d induced by $\|\cdot\|$. □

We get the following conclusion from Theorem 4 about the uniqueness of a solution to functional equation (11).

Theorem 5. *The functional equation (11) has a unique solution provided that $\varpi^\star < 1$, where $\varpi^\star$ is given in (12), and there exists a closed subset Λ of $\mathscr{B}$ such that Λ is Z invariant, that is, $Z(\Lambda) \subseteq \Lambda$, where Z is the operator from Λ defined for each $\mathscr{W} \in \Lambda$ by*

$$
\begin{aligned}
(Z\mathscr{W})(x) = \varsigma_1 x\mathscr{W}(\vartheta_1 x + (1-\vartheta_1)\lambda_1) + \varsigma_2(1-x) \\
\cdot \mathscr{W}(\vartheta_2 x + (1-\vartheta_2)\lambda_2) + (1-\varsigma_1) \\
\cdot x\mathscr{W}(\vartheta_1 x + (1-\vartheta_1)(1-\lambda_1)) + (1-\varsigma_2) \\
\cdot (1-x)\mathscr{W}(\vartheta_2 x + (1-\vartheta_2)(1-\lambda_2)),
\end{aligned} \tag{14}
$$

for all $x \in \mathscr{A}$. Moreover, the iteration $\{\mathscr{W}_n\}$ in Λ which is defined by

$$
\begin{aligned}
(\mathscr{W}_n)(x) = \varsigma_1 x\mathscr{W}_{n-1}(\vartheta_1 x + (1-\vartheta_1)\lambda_1) + \varsigma_2(1-x) \\
\cdot \mathscr{W}_{n-1}(\vartheta_2 x + (1-\vartheta_2)\lambda_2) + (1-\varsigma_1) \\
\cdot x\mathscr{W}_{n-1}(\vartheta_1 x + (1-\vartheta_1)(1-\lambda_1)) + (1-\varsigma_2) \\
\cdot (1-x)\mathscr{W}_{n-1}(\vartheta_2 x + (1-\vartheta_2)(1-\lambda_2)),
\end{aligned} \tag{15}
$$

for all $n \in \mathbb{N}$, where $\mathscr{W}_0 \in \Lambda$, converges to the unique solution of functional equation (11) in the sense of the metric d induced by $\|\cdot\|$.

Proof. We derive the result of this theorem by combining the Banach fixed point theorem with Theorem 4. □

6. Some Particular Aspects of the Proposed T-Maze Model

In this section, we have discussed some particular cases of the proposed T-maze model.

6.1. Events with Equal Lambda Conditions. This condition (sometimes called commutative condition) says that the transition operators $P_1 - P_4$ (none of them is an identity operator) have the same lambda conditions ($\lambda_1 = \lambda = \lambda_2$). These conditions reduce our transition operators (1) to

$$
\begin{aligned}
P_1 x &= \vartheta_1 x + (1-\vartheta_1)\lambda, \\
P_2 x &= \vartheta_2 x + (1-\vartheta_2)\lambda, \\
P_3 x &= \vartheta_1 x + (1-\vartheta_1)(1-\lambda), \\
P_4 x &= \vartheta_2 x + (1-\vartheta_2)(1-\lambda).
\end{aligned} \tag{16}
$$

Now, we can write our functional equation (5) as

$$
\begin{aligned}
\mathscr{W}(x) = \varsigma x\mathscr{W}(\vartheta_1 x + (1-\vartheta_1)\lambda) + \varsigma(1-x)\mathscr{W}(\vartheta_2 x + (1-\vartheta_2)\lambda) \\
+ (1-\varsigma)x\mathscr{W}(\vartheta_1 x + (1-\vartheta_1)(1-\lambda)) \\
+ (1-\varsigma)(1-x)\mathscr{W}(\vartheta_2 x + (1-\vartheta_2)(1-\lambda)),
\end{aligned} \tag{17}
$$

where $\mathscr{W} : \mathscr{A} \longrightarrow \mathbb{R}$ is an unknown function, $0 < \vartheta_1, \vartheta_2 < 1$ and $\lambda, \varsigma \in \mathscr{A}$. The following conclusions are drawn as a result of Theorem 3.

Corollary 6. *Let $0 < \vartheta_1, \vartheta_2 < 1$ and $\lambda, \varsigma \in \mathscr{A}$ with*

$$|((1-\varsigma)(1-\lambda)+\lambda\varsigma)((1-\vartheta_1)+(1-\vartheta_2))+2(\vartheta_1+\vartheta_2)|<1. \tag{18}$$

If there exists a closed subset Λ of $\mathscr{B}$ such that Λ is Z invariant, that is, $Z(\Lambda) \subseteq \Lambda$, where Z is the operator from Λ defined for each $\mathscr{W} \in \Lambda$ by

$$\begin{aligned}(Z\mathscr{W})(x) = {} & \varsigma x\mathscr{W}(\vartheta_1 x+(1-\vartheta_1)\lambda)+\varsigma(1-x) \\ & \cdot \mathscr{W}(\vartheta_2 x+(1-\vartheta_2)\lambda)+(1-\varsigma) \\ & \cdot x\mathscr{W}(\vartheta_1 x+(1-\vartheta_1)(1-\lambda))+(1-\varsigma) \\ & \cdot (1-x)\mathscr{W}(\vartheta_2 x+(1-\vartheta_2)(1-\lambda)),\end{aligned} \tag{19}$$

for all $x \in \mathscr{A}$, then, Z is a Banach contraction mapping with the metric d induced by $\|\cdot\|$.

Corollary 7. *The functional equation (17) has a unique solution provided that*

$$|((1-\varsigma)(1-\lambda)+\lambda\varsigma)((1-\vartheta_1)+(1-\vartheta_2))+2(\vartheta_1+\vartheta_2)|<1. \tag{20}$$

Also, there exists a closed subset Λ of $\mathscr{B}$ such that Λ is Z invariant, that is, $Z(\Lambda) \subseteq \Lambda$, where Z is the operator from Λ defined for each $\mathscr{W} \in \Lambda$ by

$$\begin{aligned}(Z\mathscr{W})(x) = {} & \varsigma x\mathscr{W}(\vartheta_1 x+(1-\vartheta_1)\lambda)+\varsigma(1-x) \\ & \cdot \mathscr{W}(\vartheta_2 x+(1-\vartheta_2)\lambda)+(1-\varsigma) \\ & \cdot x\mathscr{W}(\vartheta_1 x+(1-\vartheta_1)(1-\lambda))+(1-\varsigma) \\ & \cdot (1-x)\mathscr{W}(\vartheta_2 x+(1-\vartheta_2)(1-\lambda)),\end{aligned} \tag{21}$$

for all $x \in \mathscr{A}$. Moreover, the iteration $\{\mathscr{W}_n\}$ in Λ which is defined by

$$\begin{aligned}(\mathscr{W}_n)(x) = {} & \varsigma x\mathscr{W}_{n-1}(\vartheta_1 x+(1-\vartheta_1)\lambda)+\varsigma(1-x) \\ & \cdot \mathscr{W}_{n-1}(\vartheta_2 x+(1-\vartheta_2)\lambda)+(1-\varsigma) \\ & \cdot x\mathscr{W}_{n-1}(\vartheta_1 x+(1-\vartheta_1)(1-\lambda)) \\ & +(1-\varsigma)(1-x)\mathscr{W}_{n-1}(\vartheta_2 x+(1-\vartheta_2)(1-\lambda)),\end{aligned} \tag{22}$$

for all $n \in \mathbb{N}$, where $\mathscr{W}_0 \in \Lambda$, converges to the unique solution the functional equation (17) in the sense of the metric d induced by $\|\cdot\|$.

6.2. Extinction of an Operant Response. In some cases, nonfood side responses (turning right or left) frequently by the mouse decrease the probability of that event towards an asymptote to zero. In this situation, we have $\lambda_1 = 0 = \lambda_2$. These conditions reduce our operators (1) to

$$\begin{aligned}P_1 x &= \vartheta_1 x, \\ P_2 x &= \vartheta_2 x, \\ P_3 x &= \vartheta_1 x+(1-\vartheta_1), \\ P_4 x &= \vartheta_2 x+(1-\vartheta_2).\end{aligned} \tag{23}$$

Now, we can write our functional equation (5) as

$$\begin{aligned}\mathscr{W}(x) = {} & \varsigma x\mathscr{W}(\vartheta_1 x)+\varsigma(1-x)\mathscr{W}(\vartheta_2 x)+(1-\varsigma) \\ & \cdot x\mathscr{W}(\vartheta_1 x+(1-\vartheta_1))+(1-\varsigma)(1-x) \\ & \cdot \mathscr{W}(\vartheta_2 x+(1-\vartheta_2)),\end{aligned} \tag{24}$$

where $\mathscr{W}: \mathscr{A} \longrightarrow \mathbb{R}$ is an unknown function such that, $0 < \vartheta_1, \vartheta_2 < 1$, and $\varsigma \in \mathscr{A}$. We have the following corollaries of Theorem 3.

Corollary 8. *For $0 < \vartheta_1, \vartheta_2 < 1$ and $\varsigma \in \mathscr{A}$ with*

$$|(1-\varsigma)((1-\vartheta_1)+(1-\vartheta_2))+2(\vartheta_1+\vartheta_2)|<1. \tag{25}$$

If there exists a closed subset Λ of $\mathscr{B}$ such that Λ is Z invariant, that is, $Z(\Lambda) \subseteq \Lambda$, where Z is the operator from Λ defined for each $\mathscr{W} \in \Lambda$ by

$$\begin{aligned}(Z\mathscr{W})(x) = {} & \varsigma x\mathscr{W}(\vartheta_1 x)+\varsigma(1-x)\mathscr{W}(\vartheta_2 x)+(1-\varsigma) \\ & \cdot x\mathscr{W}(\vartheta_1 x+(1-\vartheta_1))+(1-\varsigma)(1-x) \\ & \cdot \mathscr{W}(\vartheta_2 x+(1-\vartheta_2)),\end{aligned} \tag{26}$$

for all $x \in \mathscr{A}$, then, Z is a Banach contraction mapping with the metric d induced by $\|\cdot\|$.

Corollary 9. *The functional equation (24) has a unique solution provided that*

$$|(1-\varsigma)((1-\vartheta_1)+(1-\vartheta_2))+2(\vartheta_1+\vartheta_2)|<1. \tag{27}$$

Also, there exists a closed subset Λ of $\mathscr{B}$ such that Λ is Z invariant, that is, $Z(\Lambda) \subseteq \Lambda$, where Z is the operator from Λ defined for each $\mathscr{W} \in \Lambda$ by

$$\begin{aligned}(Z\mathscr{W})(x) = {} & \varsigma x\mathscr{W}(\vartheta_1 x)+\varsigma(1-x)\mathscr{W}(\vartheta_2 x) \\ & +(1-\varsigma)x\mathscr{W}(\vartheta_1 x+(1-\vartheta_1))+(1-\varsigma) \\ & \cdot (1-x)\mathscr{W}(\vartheta_2 x+(1-\vartheta_2)),\end{aligned} \tag{28}$$

for all $x \in \mathscr{A}$. Moreover, the iteration $\{\mathscr{W}_n\}$ in Λ which is defined by

$$\begin{aligned}(\mathscr{W}_n)(x) = {} & \varsigma x\mathscr{W}_n(\vartheta_1 x)+\varsigma(1-x)\mathscr{W}_n(\vartheta_2 x)+(1-\varsigma) \\ & \cdot x\mathscr{W}_n(\vartheta_1 x+(1-\vartheta_1))+(1-\varsigma)(1-x) \\ & \cdot \mathscr{W}_n(\vartheta_2 x+(1-\vartheta_2)),\end{aligned} \tag{29}$$

for all $n \in \mathbb{N}$, where $\mathscr{W}_0 \in \Lambda$, converges to the unique solution *of functional equation (24) in the sense of the metric d induced by* $\|\cdot\|$.

Similarly, if the mouse chooses the food side repeatedly, then, the probability of that specific event will increase. Thus, we have $\lambda_1 = 1 = \lambda_2$. *In this situation, our four operators (1) will be*

$$\begin{aligned} P_1 x &= \vartheta_1 x + (1-\vartheta_1), \\ P_2 x &= \vartheta_2 x + (1-\vartheta_2), \\ P_3 x &= \vartheta_1 x, \\ P_4 x &= \vartheta_2 x. \end{aligned} \tag{30}$$

Now, we can write our functional equation (5) as

$$\begin{aligned} \mathscr{W}(x) &= \varsigma x \mathscr{W}(\vartheta_1 x + (1-\vartheta_1)) + \varsigma(1-x) \\ &\quad \cdot \mathscr{W}(\vartheta_2 x + (1-\vartheta_2)) + (1-\varsigma) x \mathscr{W}(\vartheta_1 x) \\ &\quad + (1-\varsigma)(1-x)\mathscr{W}(\vartheta_2 x), \end{aligned} \tag{31}$$

where $\mathscr{W} : \mathscr{A} \longrightarrow \mathbb{R}$ *is an unknown function,* $0 < \vartheta_1, \vartheta_2 < 1$, *and* $\varsigma \in \mathscr{A}$. *Now, we have the following corollaries of Theorem 3.*

Corollary 10. *For* $0 < \vartheta_1, \vartheta_2 < 1$ *and* $\varsigma \in \mathscr{A}$ *with*

$$|\varsigma((1-\vartheta_1) + (1-\vartheta_2)) + 2(\vartheta_1 + \vartheta_2)| < 1. \tag{32}$$

If there exists a closed subset Λ *of* $\mathscr{B}$ *such that* Λ *is Z invariant, that is,* $Z(\Lambda) \subseteq \Lambda$, *where Z is the operator from* Λ *defined for each* $\mathscr{W} \in \Lambda$ *by*

$$\begin{aligned} (Z\mathscr{W})(x) &= \varsigma x \mathscr{W}(\vartheta_1 x + (1-\vartheta_1)) + \varsigma(1-x) \\ &\quad \cdot \mathscr{W}(\vartheta_2 x + (1-\vartheta_2)) + (1-\varsigma) x \mathscr{W}(\vartheta_1 x) \\ &\quad + (1-\varsigma)(1-x)\mathscr{W}(\vartheta_2 x), \end{aligned} \tag{33}$$

for all $x \in \mathscr{A}$, *then, Z is a Banach contraction mapping with the metric d induced by* $\|\cdot\|$.

Corollary 11. *The functional equation (31) has a unique solution provided that*

$$|\varsigma((1-\vartheta_1) + (1-\vartheta_2)) + 2(\vartheta_1 + \vartheta_2)| < 1. \tag{34}$$

Also, there exists a closed subset Λ *of* $\mathscr{B}$ *such that* Λ *is Z invariant, that is,* $Z(\Lambda) \subseteq \Lambda$, *where Z is the operator from* Λ *defined for each* $\mathscr{W} \in \Lambda$ *by*

$$\begin{aligned} (Z\mathscr{W})(x) &= \varsigma x \mathscr{W}(\vartheta_1 x + (1-\vartheta_1)) + \varsigma(1-x) \\ &\quad \cdot \mathscr{W}(\vartheta_2 x + (1-\vartheta_2)) + (1-\varsigma) \\ &\quad \cdot x \mathscr{W}(\vartheta_1 x) + (1-\varsigma)(1-x)\mathscr{W}(\vartheta_2 x), \end{aligned} \tag{35}$$

for all $x \in \mathscr{A}$. *Moreover, the iteration* $\{\mathscr{W}_n\}$ *in* Λ *which is defined by*

$$\begin{aligned} (\mathscr{W}_n)(x) &= \varsigma x \mathscr{W}_n(\vartheta_1 x + (1-\vartheta_1)) + \varsigma(1-x) \\ &\quad \cdot \mathscr{W}_n(\vartheta_2 x + (1-\vartheta_2)) + (1-\varsigma) x \mathscr{W}_n(\vartheta_1 x) \\ &\quad + (1-\varsigma)(1-x)\mathscr{W}_n(\vartheta_2 x), \end{aligned} \tag{36}$$

for all $n \in \mathbb{N}$, *where* $\mathscr{W}_0 \in \Lambda$, *converges to the unique solution of functional equation (31) in the sense of the metric d induced by* $\|\cdot\|$.

6.3. Attraction towards the Specific Choice. In some specific cases, it is possible that the mouse always follows the $\mathbf{O_1}$ outcome and never choose $\mathbf{O_2}$. For such a case, we choose $\lambda_1 = 1$. Similarly, if the mouse chooses $\mathbf{O_2}$ again and again, then, the probability of that event should turn towards zero. It means that $\lambda_2 = 0$. These conditions reduce our four operators (1) to

$$\begin{aligned} P_1 x &= \vartheta_1 x + (1-\vartheta_1), \\ P_2 x &= \vartheta_2 x, \\ P_3 x &= \vartheta_1 x, \\ P_4 x &= \vartheta_2 x + (1-\vartheta_2). \end{aligned} \tag{37}$$

Now, we can write our functional equation (5) as

$$\begin{aligned} \mathscr{W}(x) &= \varsigma x \mathscr{W}(\vartheta_1 x + 1 - \vartheta_1) + \varsigma(1-x)\mathscr{W}(\vartheta_2 x) \\ &\quad + (1-\varsigma) x \mathscr{W}(\vartheta_1 x) + (1-\varsigma)(1-x)\mathscr{W}(\vartheta_2 x + 1 - \vartheta_2), \end{aligned} \tag{38}$$

where $\mathscr{W} : \mathscr{A} \longrightarrow \mathbb{R}$ is an unknown function, $0 < \vartheta_1, \vartheta_2 < 1$, and $\varsigma \in \mathscr{A}$. We have the following results of Theorem 3.

Corollary 12. *For* $0 < \vartheta_1, \vartheta_2 < 1$ *and* $\varsigma \in \mathscr{A}$ *with*

$$|(2\varsigma - 1)(1-\vartheta_1) + (1-\varsigma)((1-\vartheta_1) + (1-\vartheta_2)) + 2(\vartheta_1 + \vartheta_2)| < 1. \tag{39}$$

If there exists a closed subset Λ *of* $\mathscr{B}$ *such that* Λ *is Z invariant, that is,* $Z(\Lambda) \subseteq \Lambda$, *where Z is the operator from* Λ *defined for each* $\mathscr{W} \in \Lambda$ *by*

$$\begin{aligned} (Z\mathscr{W})(x) &= \varsigma x \mathscr{W}(\vartheta_1 x + 1 - \vartheta_1) + \varsigma(1-x)\mathscr{W}(\vartheta_2 x) \\ &\quad + (1-\varsigma) x \mathscr{W}(\vartheta_1 x) + (1-\varsigma) \\ &\quad \cdot (1-x)\mathscr{W}(\vartheta_2 x + 1 - \vartheta_2), \end{aligned} \tag{40}$$

for all $x \in \mathscr{A}$, *then Z is a Banach contraction mapping with the metric d induced by* $\|\cdot\|$.

Corollary 13. *The functional equation (38) has a unique solution provided that*

$$|(2\varsigma - 1)(1-\vartheta_1) + (1-\varsigma)((1-\vartheta_1) + (1-\vartheta_2)) + 2(\vartheta_1 + \vartheta_2)| < 1, \tag{41}$$

and there exists a closed subset Λ of $\mathscr{B}$ such that Λ is Z invariant, that is, $Z(\Lambda) \subseteq \Lambda$, where Z is the operator from Λ defined for each $\mathscr{W} \in \Lambda$ by

$$\begin{aligned}(Z\mathscr{W})(x) = \varsigma x\mathscr{W}(\vartheta_1 x + 1 - \vartheta_1) + \varsigma(1-x)\mathscr{W}(\vartheta_2 x) \\ + (1-\varsigma)x\mathscr{W}(\vartheta_1 x) + (1-\varsigma)(1-x) \\ \cdot \mathscr{W}(\vartheta_2 x + 1 - \vartheta_2),\end{aligned} \tag{42}$$

for all $x \in \mathscr{A}$. Moreover, the iteration $\{\mathscr{W}_n\}$ in Λ which is defined by

$$\begin{aligned}(\mathscr{W}_n)(x) = \varsigma x\mathscr{W}_n(\vartheta_1 x + 1 - \vartheta_1) + \varsigma(1-x)\mathscr{W}_n(\vartheta_2 x) \\ + (1-\varsigma)x\mathscr{W}_n(\vartheta_1 x) + (1-\varsigma)(1-x) \\ \cdot \mathscr{W}_n(\vartheta_2 x + 1 - \vartheta_2),\end{aligned} \tag{43}$$

for all $n \in \mathbb{N}$, where $\mathscr{W}_0 \in \Lambda$, converges to the unique solution of functional equation (31) in the sense of the metric d induced by $\|\cdot\|$.

7. Conclusion

In an animal or a human being, the learning phase may also be analyzed through a sequence of choices between multiple possible answers. The choice sequence is usually unpredictable, even in basic experiments conducted under highly regulated conditions, indicating that probabilities govern the selection of responses. Thus, it is helpful to think of the sequential changes in a sequence of choices in response probabilities from trial to trial. In this paper, we have discussed a particular type of stochastic process related to the T-maze experiment [23, 24], which plays a vital role in observing the behavior of the mouse in a two-choice situation. We analyzed the rat's behavior in such situations and proposed a mathematical model for it. The existence and uniqueness of a solution to the proposed model have been investigated by using the Banach fixed point theorem. To observe the flexibility of the T-maze model, we examined it under the experimenter-subject-controlled events. Furthermore, the proposed model depends only on the contingent reinforcement behavior of rats in which a rat gets the reward for choosing the food side. However, in general, a natural question arises, which we present here to make this interaction more interesting.

7.1. Question. What happens if a mouse does not move to any side (left or right) on a specific trial k and remains sticking to its starting position?

Moreover, one of the critical issues in functional equations is to find out its stability regarding Hyers-Ulam- and Hyers-Ulam-Rassias-type stability (see for the detail, [26–30]). We leave the stability question to the following functional equation as an open problem:

$$\begin{aligned}\mathscr{W}(x) = \varsigma x\mathscr{W}(\vartheta_1 x + (1-\vartheta_1)\lambda_1) + \varsigma(1-x) \\ \cdot \mathscr{W}(\vartheta_2 x + (1-\vartheta_2)\lambda_2) + (1-\varsigma)x\mathscr{W}(\vartheta_1 x + (1-\vartheta_1) \\ \cdot (1-\lambda_1)) + (1-\varsigma)(1-x)\mathscr{W}(\vartheta_2 x + (1-\vartheta_2)(1-\lambda_2)),\end{aligned} \tag{44}$$

where $\mathscr{W} : \mathscr{A} \longrightarrow \mathbb{R}$ is an unknown function, $0 < \vartheta_1, \vartheta_2 < 1$, and $\lambda_1, \lambda_2, \varsigma \in \mathscr{A}$.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare no conflict of interest.

Authors' Contributions

All authors contributed equally to the manuscript and typed, read, and approved the final manuscript.

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