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Research Article

EOQ Models with Varying Holding Cost

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Models of inventory management contain different parameters. An issue is observable in the classical models which can be related to the determination of the quantity of the economic order and the quantity of the economic production. In these models, the parameters like setup and holding costs and also the rate of demands are fixed. This matter causes the quantity of the economic ordering in classic model to have some differences in comparison with the real-world conditions. It should be stated that holding cost of spoiled and useless products is not always fixed and so the costs increase by passing the time. This paper is an attempt to develop classical EOQ models by considering holding cost as an increasing function of the ordering cycle length. So the classical EOQ models are developed, and the related optimum quantity to the ordering cycle length, economic ordering quantity, and the optimum total cost are determined.

1. Introduction

Models of inventory management contain different parameters. An issue that is observable in the classical models can be related to the determination of the quantity of the economical ordering and the quantity of the economical production. In these models the parameters like setup cost, holding cost, and also the rate of demand are fixed. This matter causes that the economic ordering quantity and the economic production quantity in classic models to have some differences in comparison with the real-world conditions. Thus, it seems necessary that the parameters, assumptions, and limitations should be considered in a manner to approach the real condition as much as possible [1].

It should be stated that the costs of holding spoiled and useless products are not always fixed and similar, and the rate of cost is increased by passing the time. Also, the rate of holding costs which is related to rental equipment can get discounts for long-term usages and can be decreasing by time passing [2].

The aim of this paper is to present some models similar to the real-world conditions. In this paper, the related parameters to the holding cost are not considered as the fixed ones, and it is regarded as the increasing function of the ordering cycle length for determining the economic ordering cycle length and the economic ordering quantity in two cases. The first case does not allow backorders and the other one permits backorders. It means that in a situation where the holding cost is a function of the ordering cycle length, we want to minimize the total amount of the costs. Holding cost should be considered as a kind of increasing function of ordering cycle length, and the optimum inventory cycle and economic order quantity are decision variables.

Inventory control models are used widely in different fields for production and sale since the introduction of the models on economic order quantity and economical production quantity. Some researchers have questioned and investigated the practical usages of these models with reference to the unreal assumptions, as well as the parameters such as setup and holding costs and also the rate of demand. Darwish developed an EPQ classic model with reference to the setup cost as a function of production time [1]. He suggested two models such that in the first model the shortages were not allowed and in the second model the shortages were allowed. He indicates that the function of cost is convex in both cases, and the optimum quantity of the main function is observable in both models. The results show that the relationship between the cost function and the production time can have a significant effect on the economical production quantity and the average amount of the total cost in EPQ classic model.

The holding cost in some few models has been considered as a variable one. Alfares investigated the inventory policy for an item with a stock-level-dependent demand rate and a storage-time-dependent holding cost [2]. The holding cost per unit of the item per unit time is assumed to be an increasing function of the time spent in storages. Two time-dependent holding cost step functions are considered: retroactive holding cost increasing function and incremental holding cost increasing function. Procedures are developed for determining the optimal order quantity and the optimal cycle time for both cost structures.

Giri et al. developed a generalized EOQ model for deteriorating items with shortages, in which both the demand rate and the holding cost are continuous functions of time. The optimal inventory policy is derived assuming a finite planning horizon and constant replenishment cycles [3].

Ray and Chaudhuri take the time value of money into account in analyzing an inventory system with stock-dependent demand rate and shortages. Two types of inflation rates are considered: internal (company) inflation, and external (general economy) inflation [4].

Goh developed a classic model for the fund related demand and the holding expense. He considered the holding cost in two forms. In the first case, it was in the shape of a nonlinear function of time, and the second case was in the shape of a nonlinear function of inventory level [5]. Baker and Urban developed a deterministic inventor system with an inventory-level-dependent rate [6]. Barron considered economic production quantity with rework process at a single-stage manufacturing system with planned backorders [7]. Hou considers an EPQ model with imperfect production processes, in which the setup cost and process quality are functions of capital expenditure [8]. Pan et al. developed optimal reorder point inventory models with variable lead time and backorder discount considerations [9]. Sphicas considered EOQ and EPQ with linear and fixed backorder costs: two cases identified and models analyzed without calculus [10]. Urban presented a comprehensive review and unifying theory for inventory models with inventory-level-dependent demand [11].

2. Problem Description

The main objective of this paper is developing classical EOQ models with considering holding cost as an increasing continuous function of ordering cycle length.

2.1. Assumptions. Consider the following.

(i) The rate of demand is fixed.

(ii) There is no discount.

(iii) The delivery of the product is wholesale.

(iv) All parameters are fixed and deterministic.

(v) Holding cost is an increasing function of period length.

2.2. Parameters. Consider the following.

D: the rate of demands.

Q: the quantity of order (decision variable).

k: the cost of ordering or setup in the period of main orders.

h_0: the constant unit holding cost before T'.

P: the rate of production (the rate of production import).

B: the quantity of backorder (decision variable).

π: the unit backorder cost per unit time.

T: inventory cycle length.

TC: annual total cost.

TOC: annual setup cost.

THC: annual holding cost.

TSC: annual backorder cost.

3. Models Development

In this part, EOQ models in two cases are developed in a condition where the holding cost is an increasing continuous function of ordering cycle length. At first, model backorders are not permitted, and in second model backorders are allowed. We assume that the holding cost will be fixed till a definite time (T') and then will be increased according to a function of ordering cycle length. So for holding cost (h), we have

$$h = \begin{cases} h_0 T^{\varepsilon} & T > T' \\ h_0 & T \le T' \end{cases} \quad 0 < \varepsilon < 1, \tag{1}$$

where T' is a time moment before which holding cost is constant. It is clear that in (1) while $\varepsilon = 0$, the presented models will reflect the results of classic models.

3.1. Model 1. In this model, it has been assumed that the total amount of the ordered products is delivered wholesale. Behavior of inventory level is presented in Figure 1.

$TC^*, Q^*,$ and T^* for $T > T'$ were calculated based on the following formulas:

$$\begin{gathered} T^* = \sqrt[\varepsilon+2]{\frac{2k}{(\varepsilon+1) h_0 D}}, \\ Q^* = D \sqrt[2+\varepsilon]{\frac{2k}{(\varepsilon+1) h_0 D}}, \\ TC^* = K \sqrt[2+\varepsilon]{\frac{(\varepsilon+1) h_0 D}{2k}} + \frac{h_0 D}{2} \left(\sqrt[2+\varepsilon]{\frac{2k}{(\varepsilon+1) h_0 D}} \right)^{\varepsilon+1}. \end{gathered} \tag{2}$$

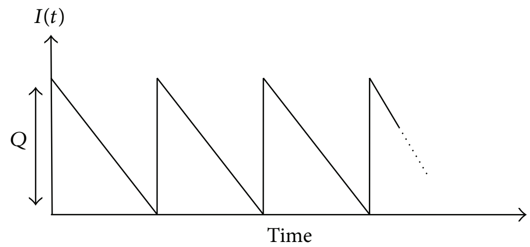

Figure 1: EOQ model without backorders.

Proof. The annual total cost is calculated based on the following procedures:

$$TC = TOC + THC, \tag{3}$$

$$TC = \frac{Dk}{Q} + \frac{hQ}{2}. \tag{4}$$

With replacing (1) in (4), we have

$$TC = \begin{cases} \dfrac{Dk}{Q} + \dfrac{h_0 T^{\varepsilon} Q}{2} & T > T', \\ \dfrac{Dk}{Q} + \dfrac{h_0 Q}{2} & T \le T'. \end{cases} \tag{5}$$

We know from classical inventory models that

$$Q = DT. \tag{6}$$

So we can state that

$$TC = \begin{cases} \dfrac{k}{T} + \dfrac{h_0 t^{\varepsilon} DT}{2} & T > T', \\ \dfrac{k}{T} + \dfrac{h_0 DT}{2} & T \le T'. \end{cases} \tag{7}$$

For $T \ge T'$, we have

$$TC = \frac{k}{T} + \frac{h_0 D T^{\varepsilon+1}}{2}. \tag{8}$$

By calculating first derivative of total cost function TC with respect to T, we have

$$\frac{dTC}{dT} = -\frac{k}{T^2} + \frac{(\varepsilon+1) h_0 D T^{\varepsilon}}{2}; \tag{9}$$

Finally, we can find the optimal period length as follows:

$$\begin{aligned} -\frac{k}{T^2} + \frac{(\varepsilon+1) h_0 D T^{\varepsilon}}{2} &= 0, \\ \frac{k}{T^2} &= \frac{(\varepsilon+1) h_0 D T^{\varepsilon}}{2}, \\ T^{\varepsilon+2} &= \frac{2k}{(\varepsilon+1) h_0 D}, \\ \Longrightarrow T^* &= \sqrt[\varepsilon+2]{\frac{2k}{(\varepsilon+1) h_0 D}}. \end{aligned} \tag{10}$$

Table 1: Optimal solution for different values of ε when backorder is not allowed.

ε	T^*	Q^*	TC^*	Loss (%)
0	0.3536	8485.3	169706	0
0.1	0.3550	8520.4	161320	4.943728
0.2	0.3577	8584.7	153760	9.398385
0.3	0.3613	8670.1	146920	13.42879
0.4	0.3654	8770.8	140730	17.07619
0.5	0.3701	8882.6	135100	20.39361
0.6	0.3751	9002.5	129960	23.42231
0.7	0.3803	9128.1	125280	26.17995
0.8	0.3857	9257.9	120980	28.71369
0.9	0.3913	9390.3	117030	31.04119
1	0.3969	9524.4	113390	33.18602

The second derivative of TC with respect to T is

$$\frac{d^2 TC}{dT^2} = \frac{2k}{T^3} + \frac{\varepsilon(\varepsilon+1) h_0 D T^{\varepsilon-1}}{2}. \tag{11}$$

As $0 < \varepsilon < 1$, the previous equation is always positive so that function TC is a convex function and T^* will be the minimum of function TC; by replacing T^* in (6), the Q^* can be calculated as follows:

$$Q^* = D\,T^* = D \sqrt[2+\varepsilon]{\frac{2k}{(\varepsilon+1) h_0 D}}. \tag{12}$$

Then, by replacing T^* in (8), we get

$$\begin{aligned} TC^* &= \frac{k}{\sqrt[2+\varepsilon]{2k/(\varepsilon+1) h_0 D}} \\ &\quad + \frac{h_0 D}{2} \left(\sqrt[2+\varepsilon]{\frac{2k}{(\varepsilon+1) h_0 D}} \right)^{\varepsilon+1} \\ &= K \sqrt[2+\varepsilon]{\frac{(\varepsilon+1) h_0 D}{2k}} \\ &\quad + \frac{h_0 D}{2} \left(\sqrt[2+\varepsilon]{\frac{2k}{(\varepsilon+1) h_0 D}} \right)^{\varepsilon+1}. \end{aligned} \tag{13}$$

□

Numerical Example 1. Assume that the annual demand of a material is 24000, the setup cost is 30000, and the holding cost is 20 annually. The amounts of T^*, Q^*, and TC^* for $0 \le \varepsilon \le 1$ are summarized in Table 1. Figure 2 demonstrates total cost versus cycle length. Now define the percent loss due to using the classical EPQ model instead of the proposed model.

It is clear from Table 1 that by increasing ε, T^* and Q^* are increased whereas TC^* is decreased.

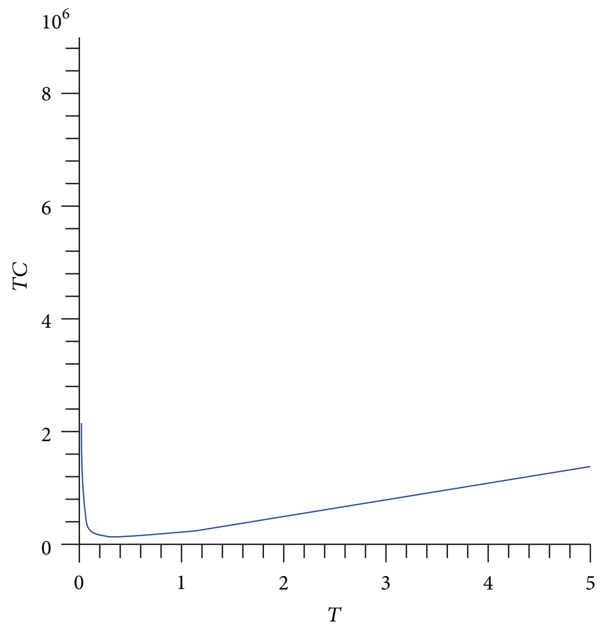

Figure 2: Total cost versus cycle length for $\varepsilon = 0.1$ (Example 1).

3.2. Model 2. In this model, we assume that the total amount of the ordered materials will be delivered to the store in a wholesale where shortage is allowed. Behavior of inventory level is presented in Figure 3. t_2 is the section of cycle for which we have shortage.

In this case, the optimum amounts of T^*, Q^*, and B^* for $T > T'$ are calculated based on the following formulas:

$$B^* = \frac{h_0 D\, T^{\varepsilon+1}}{h_0 T^{\varepsilon} + \pi},$$
$$\pi h_0^2 D^2 T^{2\varepsilon+2} + (\varepsilon + 1)\,\pi^2 h_0 D^2 T^{\varepsilon+2} - 2kDh_0^2 T^{2\varepsilon} - 4k\pi D h_0 T^{\varepsilon} - 2\pi^2 kD = 0. \quad (14)$$

Proof. See the appendix. □

Also, we can calculate TC^* by replacing T^* and B^* in

$$TC(T, B) = \frac{k}{T} + \frac{h_0 D T^{\varepsilon+1}}{2} - h_0 B T^{\varepsilon} + \frac{h_0 B^2 T^{\varepsilon-1}}{2D} + \frac{\pi B^2}{2DT}. \quad (15)$$

Proof of the convexity of TC and calculation of optimal solution is mentioned in the appendix. The overall convexity of total cost is not guaranteed, but in special cases, convexity can be approved. These cases are considered in the appendix too.

Numerical Example 2. Assume that the annual need for a product is 24000; the setup cost is 95000. The holding cost is 50 annually and the backorder cost for each product equals 250.

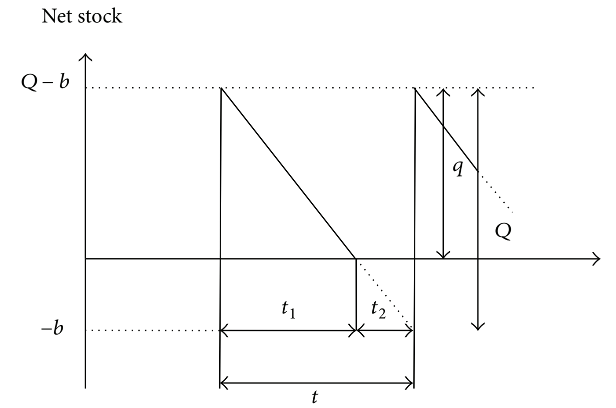

Figure 3: EOQ model with backorders.

Table 2: Optimal solutions for different values of ε when backorder is allowed.

ε	T^*	B^*	Q^*	TC^*	Loss (%)
0	0.4359	1743.6	10461	435890	0
0.1	0.4335	1616.6	10404	421225	3.364381
0.2	0.4324	1501.0	10377	407360	6.545229
0.3	0.4322	1396.0	10373	394300	9.541398
0.5	0.4343	1213.8	10423	370470	15.00837
0.6	0.4362	1135.0	10470	359640	17.49295
0.7	0.4387	1063.3	10528	349470	19.8261
0.8	0.4415	998.09	10596	339930	22.01473
0.9	0.4447	938.67	10672	330980	24.068
1	0.4481	884.54	10754	322570	25.99738

Optimal values of T^*, T_p^*, Q^*, and TC^* for $0 \le \varepsilon \le 1$ are summarized in Table 2.

It is clear from Table 2 that by increasing ε, T^* and Q^* are increased and TC^* is decreased. Figure 4 demonstrates total cost versus cycle length and amount of backorder.

4. Conclusions and Suggestions

In this paper, the classical EOQ models have been developed by the consideration of holding cost as an increasing continuous function of ordering cycle length. It was assumed that the holding cost will be fixed till a definite time and then will be increased as a function of ordering cycle length. Two models were developed. The first case does not allow backorders, and other one permits backorders. Economic ordering quantity, the optimum cycle length, and the optimum total cost were determined in both cases. From the numerical results, we could clearly see that loss due to using the classical EOQ model is significant. It is suggested for the future research that the holding cost be considered as different functions of the ordering cycle length depend on reality.

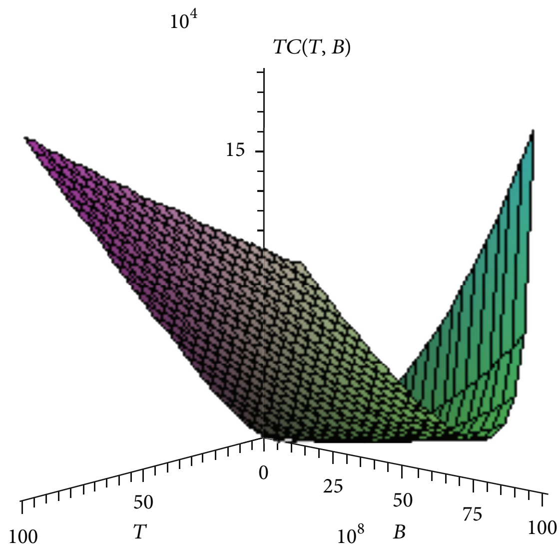

Figure 4: Result of Numerical Example 2 for $\varepsilon = 0.1$.

Appendix

The annual total cost can be proved as follows:

$$TC(T, B) = TOC + THC + TSC, \tag{A.1}$$

$$TC(T, B) = \frac{Dk}{Q} + \frac{h(Q-B)^2}{2Q} + \frac{\pi B^2}{2Q}. \tag{A.2}$$

With replacing h from (1) in (A.2), we have

$$TC(T, B) = \begin{cases} \dfrac{Dk}{Q} + \dfrac{h_0 T^{\varepsilon}(Q-B)^2}{2Q} + \dfrac{\pi B^2}{2Q}, & T > T', \\ \dfrac{Dk}{Q} + \dfrac{h_0 (Q-B)^2}{2Q} + \dfrac{\pi B^2}{2Q}, & T \leq T'. \end{cases} \tag{A.3}$$

With replacing Q from (6) in (A.3), we have

$$\begin{aligned} TC(T, B) &= \begin{cases} \dfrac{k}{T} + \dfrac{h_0 T^{\varepsilon}(DT-B)^2}{2DT} + \dfrac{\pi B^2}{2DT} & T > T', \\ \dfrac{k}{T} + \dfrac{h(DT-B)^2}{2DT} + \dfrac{\pi B^2}{2DT} & T \leq T', \end{cases} \\ TC(T, B) &= \frac{k}{T} + \frac{h_0 T^{\varepsilon}\left(D^2T^2 - 2DTB + B^2\right)}{2DT} + \frac{\pi B^2}{2DT} \\ &= \frac{k}{T} + \frac{h_0 D^2 T^{\varepsilon+2} - 2h_0 DBT^{\varepsilon+1} + h_0 B^2 T^{\varepsilon}}{2DT} + \frac{\pi B^2}{2DT} \\ &= \frac{k}{T} + \frac{h_0 D T^{\varepsilon+1}}{2} - h_0 B T^{\varepsilon} + \frac{h_0 B^2 T^{\varepsilon-1}}{2D} + \frac{\pi B^2}{2DT}. \end{aligned} \tag{A.4}$$

When $T > T'$, the gradient of $TC(T, B)$ function is in the following form:

$$\begin{aligned} \nabla TC(T, B) &= \left(\frac{\partial C(T, B)}{\partial T}, \frac{\partial C(T, B)}{\partial B}\right) \\ &= \left(\frac{-k}{T^2} + \frac{h_0 D(\varepsilon+1)T^{\varepsilon}}{2} - h_0 \varepsilon\, B\, T^{\varepsilon-1}\right. \\ &\quad + \frac{h_0(\varepsilon+1)B^2T^{\varepsilon-2}}{2D} - \frac{\pi B^2}{2DT^2}, \\ &\quad \left. -h_0 T^{\varepsilon} + \frac{h_0 T^{\varepsilon-1}}{D} B + \frac{\pi B}{DT}\right). \end{aligned} \tag{A.5}$$

The Hessian matrix of $TC(T, B)$ function can be calculated as follows:

$$\begin{aligned} H &= \begin{bmatrix} \dfrac{\partial^2 TC(T, B)}{\partial T^2} & \dfrac{\partial^2 TC(T, B)}{\partial B \partial T} \\ \dfrac{\partial^2 TC(T, B)}{\partial T \partial B} & \dfrac{\partial^2 TC(T, B)}{\partial B^2} \end{bmatrix}, \\ \frac{\partial^2 TC(T, B)}{\partial T^2} &= \frac{2K}{T^3} + \frac{h_0 D\varepsilon(\varepsilon+1)T^{\varepsilon-1}}{2} - h_0\varepsilon(\varepsilon-1)BT^{\varepsilon-2} \\ &\quad + \frac{h_0(\varepsilon-1)(\varepsilon-2)B^2T^{\varepsilon-3}}{2D} + \frac{\pi B^2}{DT^3}, \\ \frac{\partial^2 TC(T, B)}{\partial B^2} &= \frac{h_0 T^{\varepsilon-1}}{D} + \frac{\pi}{DT}, \\ \frac{\partial^2 TC(T, B)}{\partial T \partial B} &= -h_0\varepsilon T^{\varepsilon-1} + \frac{h_0(\varepsilon-1)BT^{\varepsilon-2}}{D} - \frac{\pi B}{DT^2}, \\ \frac{\partial^2 TC(T, B)}{\partial B \partial T} &= -h_0\varepsilon T^{\varepsilon-1} + \frac{h_0(\varepsilon-1)BT^{\varepsilon-2}}{D} - \frac{\pi B}{DT^2}. \end{aligned} \tag{A.6}$$

Since $0 < \varepsilon < 1$, then $\delta^2 TC(T, B)/\delta T^2 > 0$ is always true.

The determinant of the H matrix is computed as follows:

$$\begin{aligned} |H| &= \left(\frac{\partial^2 TC(T, B)}{\partial T^2}\right)\left(\frac{\partial^2 TC(T, B)}{\partial B^2}\right) \\ &\quad - \left(\frac{\partial^2 TC(T, B)}{\partial B \partial T}\right)^2 \end{aligned}$$

$$= \frac{2kh_0 T^{\varepsilon-4}}{D} + \frac{h_0^2 \varepsilon(\varepsilon+1) T^{2\varepsilon-2}}{2} - \frac{h_0^2 \varepsilon(\varepsilon-1) B T^{2\varepsilon-3}}{D} + \frac{h_0^2 (\varepsilon-1)(\varepsilon-2) B^2 T^{2\varepsilon-4}}{2D^2} + \frac{h_0 \pi B^2 T^{\varepsilon-4}}{D^2} + \frac{2\pi k}{DT^4} + \frac{\pi h_0 \varepsilon(\varepsilon+1) T^{\varepsilon-2}}{2} - \frac{\pi h_0 \varepsilon(\varepsilon-1) B T^{\varepsilon-3}}{D} + \frac{\pi h_0 (\varepsilon-1)(\varepsilon-2) B^2 T^{\varepsilon-4}}{2D^2} + \frac{\pi^2 B^2}{D^2 T^4} - h_0^2 \varepsilon^2 T^{2\varepsilon-2} + \frac{h_0^2 \varepsilon(\varepsilon-1) B T^{2\varepsilon-3}}{D} - \frac{\pi h_0 \varepsilon B T^{\varepsilon-3}}{D} + \frac{h_0^2 \varepsilon(\varepsilon-1) B T^{2\varepsilon-3}}{D} - \frac{h_0^2 (\varepsilon-1)^2 B^2 T^{2\varepsilon-4}}{D} + \frac{\pi h_0 (\varepsilon-1) B^2 T^{\varepsilon-4}}{D^2} - \frac{\pi h_0 \varepsilon B\ T^{\varepsilon-3}}{D} + \frac{\pi h_0 (\varepsilon-1) B^2 T^{\varepsilon-4}}{D^2} - \frac{\pi^2 B^2}{D^2 T^4}$$

$$= \frac{2Kh_0 T^{\varepsilon-4}}{D} + \frac{h_0^2 \varepsilon(\varepsilon-1) T^{2\varepsilon-2}}{2} + \frac{h_0^2 (\varepsilon-1)(\varepsilon-2) B^2 T^{2\varepsilon-4}}{2D^2} + \frac{h_0 \pi B^2 T^{\varepsilon-4}}{D^2} + \frac{2\pi K}{DT^4} + \frac{\pi h_0 \varepsilon(\varepsilon+1) T^{\varepsilon-2}}{2} - \frac{\pi h_0 \varepsilon(\varepsilon-1) B T^{\varepsilon-3}}{D} + \frac{\pi h_0 (\varepsilon-1)(\varepsilon-2) B^2 T^{\varepsilon-4}}{2D^2} - h_0^2 \varepsilon^2 T^{2\varepsilon-2} - \frac{2\pi h_0 \varepsilon B T^{\varepsilon-3}}{D} + \frac{h_0^2 \varepsilon(\varepsilon-1) B T^{\varepsilon-3}}{D} - \frac{h_0^2 (\varepsilon-1)^2 B^2 T^{2\varepsilon-4}}{D^2} + \frac{2\pi h_0 (\varepsilon-1) B^2 T^{\varepsilon-4}}{D^2}. \tag{A.7}$$

Verifying the sign of mentioned relation (A.7) seems impossible, as while $\varepsilon = 1$, we have

$$|H| = \frac{2kh_0 T^{-3}}{D} + h_0^2 + \frac{h_0 \pi B^2 T^{-3}}{D^2} + \frac{2\pi k}{DT^4} + \pi h_0 T^{-1} - h_0^2 - \frac{2\pi h_0 B T^{-2}}{D} = \frac{2kh_0}{DT^3} + \frac{h_0 \pi B^2}{D^2 T^3} + \frac{2\pi k}{DT^4} + \frac{\pi h_0}{T} - \frac{2\pi h_0 B}{DT^2} = \left(4kh_0 DT + 2h_0 \pi B^2 T + 4\pi k D + 2\pi h_0 D^2 T^3 - 4\pi h_0 B D T^2\right)\left(2D^2 T^4\right)^{-1}. \tag{A.8}$$

While $\varepsilon = 1$, if one of the following conditions (A.9) is realized, the determinant of H would not be negative and $TC(T, B)$ will be convex. It should be mentioned that this condition is not the most essential condition but is an adequate condition. Consider

$$\begin{gathered} 2kh_0 DT + h_0 \pi B^2 T + 2\pi k D + \pi h_0 D^2 T^3 - 2\pi h_0 BD\ T^2 > 0, \\ 2kh_0 DT > 2\pi h_0 BDT^2 \Longrightarrow B > 2DT, \\ h_0 \pi B^2 T > 2\pi h_0 BDT^2 \Longrightarrow k > \pi BT, \\ 2\pi k D > 2\pi h_0 BDT^2 \Longrightarrow k > h_0 BT^2, \\ \pi h_0 D^2 T^3 > 2\pi h_0 BDT^2 \Longrightarrow 2DT > B. \end{gathered} \tag{A.9}$$

For finding minimum points of TC, we survey its condition as follows.

TC is positive. TC is always continuous for $T > 0$ and $B > 0$. Also, there is a section for which function TC is convex (when $T \to 0^+, TC \to +\infty$).

The gradient vector is always available when T is positive. $(\partial^2 TC(T, B)/\partial T^2)$ is always positive, so the TC is not concave and the extremum points will be in types of minimum points. Consider the following:

TC does not have a local maximum point or a total maximum point. When $T \to 0^+$, then $TC \to +\infty$; when $B \to 0^+$, then $TC \to +\infty$; when $T \to +\infty$, then $TC \to +\infty$.

So these cases provide just the following two conditions for function $TC(T, B)$.

(i) Function $TC(T, B)$ has a local minimum point that can be considered as an optimal point which is obtained from solving (A.10). In this case, (A.10) has only one real root.

(ii) Function $TC(T, B)$ has some local minimum points and some saddle points such that the maximum number of these points is $\lceil 2\varepsilon + 2 \rceil$. One of these points will be the optimal minimum point and the other points will be the local minimum or saddle points. Based on the aforementioned conditions, the $TC(T, B)$ function will certainly have an optimal minimum point. In this case, (A.10) will have more than one real root. If the determinant of Hessian matrix is positive, then the point will be minimum, and if the determinant of Hessian matrix is negative, then the point will be saddle. The optimal solution is one of minimum points which has the lowest amount of TC.

For finding the optimal solution, we have

$$\nabla TC(T,B)=0, \tag{A.10}$$

$$\left(\frac{\partial TC(T,B)}{\partial T},\frac{\partial TC(T,B)}{\partial B}\right)=0, \tag{A.11}$$

$$\begin{aligned}\Bigg(&\frac{-k}{T^2}+\frac{h_0D(\varepsilon+1)T^{\varepsilon}}{2}-h_0\varepsilon BT^{\varepsilon-1}\\&+\frac{h_0(\varepsilon+1)B^2T^{\varepsilon-2}}{2D}\\&-\frac{\pi B^2}{2DT^2},-h_0T^{\varepsilon}+\frac{h_0T^{\varepsilon-1}}{D}B+\frac{\pi B}{DT}\Bigg)=0,\end{aligned} \tag{A.12}$$

$$\begin{aligned}&\frac{-k}{T^2}+\frac{h_0D(\varepsilon+1)T^{\varepsilon}}{2}-h_0\varepsilon BT^{\varepsilon-1}\\&\quad+\frac{h_0(\varepsilon+1)B^2T^{\varepsilon-2}}{2D}-\frac{\pi B^2}{2DT^2}=0,\end{aligned} \tag{A.13}$$

$$-h_0T^{\varepsilon}+\frac{h_0T^{\varepsilon-1}}{D}B+\frac{\pi B}{DT}=0, \tag{A.14}$$

$$\Longrightarrow B=\frac{h_0DT^{\varepsilon+1}}{h_0T^{\varepsilon}+\pi}. \tag{A.15}$$

With replacing (A.15) in (A.14), we have

$$\begin{aligned}&\pi h_0^2D^2T^{2\varepsilon+2}+(\varepsilon+1)\pi^2h_0D^2T^{\varepsilon+2}-2kDh_0^2T^{2\varepsilon}\\&\quad-4k\pi Dh_0T^{\varepsilon}-2\pi^2kD=0.\end{aligned} \tag{A.16}$$

Finally, we can obtain optimal solution for T by solving (A.16).

References

[1] M. A. Darwish, "EPQ models with varying setup cost," *International Journal of Production Economics*, vol. 113, no. 1, pp. 297–306, 2008.

[2] H. K. Alfares, "Inventory model with stock-level dependent demand rate and variable holding cost," *International Journal of Production Economics*, vol. 108, no. 1-2, pp. 259–265, 2007.

[3] B. C. Giri, A. Goswami, and K. S. Chaudhuri, "An EOQ model for deteriorating items with time varying demand and costs," *Journal of the Operational Research Society*, vol. 47, no. 11, pp. 1398–1405, 1996.

[4] J. Ray and K. S. Chaudhuri, "An EOQ model with stock-dependent demand, shortage, inflation and time discounting," *International Journal of Production Economics*, vol. 53, no. 2, pp. 171–180, 1997.

[5] M. Goh, "Some results for inventory models having inventory level dependent demand rate," *International Journal of Production Economics*, vol. 27, no. 2, pp. 155–160, 1992.

[6] R. C. Baker and T. L. Urban, "A deterministic inventor system with an inventory level dependent rate," *Journal of the Operation Research Society*, vol. 39, no. 9, pp. 823–831, 1988.

[7] L. E. C. Barron, "Economic production quantity with rework process at a single-stage manufacturing system with planned backorders," *Computers and Industrial Engineering*, vol. 57, no. 3, pp. 1105–1113, 2009.

[8] K. L. Hou, "An EPQ model with setup cost and process quality as functions of capital expenditure," *Applied Mathematical Modelling*, vol. 31, no. 1, pp. 10–17, 2007.

[9] J. C. H. Pan, M. C. Lo, and Y. C. Hsiao, "Optimal reorder point inventory models with variable lead time and backorder discount considerations," *European Journal of Operational Research*, vol. 158, no. 2, pp. 488–505, 2004.

[10] G. P. Sphicas, "EOQ and EPQ with linear and fixed backorder costs: two cases identified and models analyzed without calculus," *International Journal of Production Economics*, vol. 100, no. 1, pp. 59–64, 2006.

[11] T. L. Urban, "Inventory models with inventory-level-dependent demand: a comprehensive review and unifying theory," *European Journal of Operational Research*, vol. 162, no. 3, pp. 792–804, 2005.