Research Article

Common Fixed Point Theorems of Multivalued Maps in Fuzzy Ultrametric Spaces

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In the setting of fuzzy ultrametric spaces, we study common fixed point theorems of multivalued maps. Our results unify, extend, and generalize some related common fixed point theorems of the literature for both ultrametric spaces (Wang and Song (2013), Gajić (2002) and (2001)) and fuzzy metric spaces (Vijayaraju and Sajath (2011)).

1. Introduction

In 1965, Zadeh [1] introduced the theory of fuzzy sets. Many authors introduced the notion of fuzzy metric space in different ways. George and Veeramani [2] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [3] and defined Hausdorff topology in fuzzy metric space. Several authors [4–10] studied and developed the concept in different directions and proved fixed point theorems in fuzzy metric spaces. Vijayaraju and Sajath [11] extended some previous results and proved some common fixed points theorems for hybrid pair of single and multivalued maps under hybrid contractive conditions. Wang and Song [12] established some results on coincidence and common fixed point for two pairs of multivalued and single-valued maps in ultrametric spaces. In 2009, Savchenko and Zarichnyi [13] introduced the concept of fuzzy ultrametric space. Sedghi and Shobe [14] proved common fixed point theorems for self-maps satisfying contractive conditions on spherically complete fuzzy ultrametric spaces. In this paper, in the setting of fuzzy ultrametric spaces, we study common fixed point theorems of multivalued maps. Our results unify, extend, and generalize some related common fixed point theorems of the literature for both ultrametric spaces [12, 15, 16] and fuzzy metric spaces [11].

2. Preliminaries and Notations

Definition 1 (see [1]). Let X be any nonempty set. A fuzzy set A in X is a function with domain X and values in $[0, 1]$.

Definition 2 (see [17]). A binary operation $*$: $[0, 1]^2 \rightarrow [0, 1]$ is called a continuous triangular norm (shortly t -norm) if it satisfies the following conditions:

- (1) $*$ is associative and commutative,
- (2) $*$ is continuous,
- (3) $a * 1 = a$ for all $a, b, c, d \in [0, 1]$,
- (4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 3 (see [2]). The 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary (nonempty) set, $*$ is a continuous t -norm, and M is a fuzzy set on $X^2 \times [0, \infty)$ satisfying the following conditions, for all $x, y, z \in X$ and each t and $s > 0$:

- (1) $M(x, y, t) = 0$,
- (2) $M(x, y, t) = 1$ if and only if $x = y$,
- (3) $M(x, y, t) = M(y, x, t)$,
- (4) $M(x, y, t) * M(y, z, t) \leq M(x, z, t + s)$,
- (5) $M(x, y, \cdot) : (0, \infty) \rightarrow [0, 1]$ is continuous.

M is called a fuzzy metric on X . The functions $M(x, y, t)$ denote the degree of nearness between x and y with respect to t , respectively.

Let $(X, M, *)$ be a fuzzy metric space. For $t > 0$, the open ball $B(x, r, t)$ with center $x \in X$ and radius $0 < r < 1$ is defined by

$$B(x, r, t) = \{y \in X : M(x, y, t) > 1 - r\}. \quad (1)$$

A subset $A \subset X$ is called open if, for each $x \in A$, there exist $t > 0$ and $0 < r < 1$ such that $B(x, r, t) \subset A$. Let τ denote the family of all open subsets of X . Then τ is a topology on X induced by the fuzzy metric M . This topology is Hausdorff and first countable.

Definition 4 (see [18]). Let (X, d) be a metric space. If the metric d satisfies strong triangle inequality

$$d(x, y) \leq \max\{d(x, z), d(z, y)\}, \quad \forall x, y, z \in X, \quad (2)$$

then d is called an ultrametric on X and the pair (X, d) is called an ultrametric space.

Definition 5 (see [18]). An ultrametric space (X, d) is said to be spherically complete if every shrinking collection of balls in X has a nonempty intersection.

Definition 6 (see [14]). Let $(X, M, *)$ be a fuzzy metric space. If the fuzzy metric M satisfies strong triangle inequality

$$M(x, y, t) \geq \min\{M(x, z, t), M(z, y, t)\}, \quad \forall x, y, z \in X, t > 0, \quad (3)$$

then M is called a fuzzy ultrametric on X and the 3-tuple $(X, M, *)$ is called a fuzzy ultrametric space.

Definition 7 (see [14]). A fuzzy ultrametric space $(X, M, *)$ is said to be spherically complete if every shrinking collection of balls in X has a nonempty intersection.

Remark 8 (see [14]). (i) Let d be an ultrametric on X and $a * b = a \cdot b$ for all $a, b \in [0, 1]$. For each $t \in (0, \infty)$, define

$$M(x, y, t) = \left(\frac{1}{1 + d(x, y)} \right)^{\psi(t)}, \quad (4)$$

for all $x, y, z \in X$, $t > 0$, and $\psi \in \Psi$. Then fuzzy metric M is also a fuzzy ultrametric.

(ii) Let an ultrametric space (X, d) be spherically complete and $a * b = a \cdot b$ for all $a, b \in [0, 1]$. For each $t \in (0, \infty)$, define

$$M(x, y, t) = \left(\frac{1}{1 + d(x, y)} \right)^{\psi(t)}, \quad (5)$$

for all $x, y, z \in X$, $t > 0$, and $\psi \in \Psi$. Then fuzzy metric space $(X, M, *)$ is also spherically complete.

3. Main Results

Let $(X, M, *)$ be a fuzzy ultrametric space. Let $CB(X)$ denote the set of all nonempty closed bounded subsets of X . For every $A, B, C \in CB(X)$ and $t > 0$, we define

$$M_{\nabla}(A, B, t) = \min \left\{ \bigwedge_{a \in A} M^{\nabla}(a, B, t), \bigwedge_{b \in B} M^{\nabla}(b, A, t) \right\}, \quad (6)$$

where $M^{\nabla}(x, C, t) = \bigvee_{c \in C} M(x, c, t)$.

Remark 9 (see [8]). It is obvious that, for all $A, B \in CB(X)$ and $t > 0$, we have

- (i) $M_{\nabla}(A, B, t) \leq M^{\nabla}(a, B, t)$, whenever $a \in A$,
- (ii) $M_{\nabla}(A, B, t) = 1$ if and only if $A = B$.

It is easy to prove that $(CB(X), M_{\nabla}, *)$ is a Hausdorff ultrametric space.

Definition 10. Let $(X, M, *)$ be a fuzzy ultrametric space. An element $x \in X$ is said to be a coincidence point of a multivalued map $T : X \rightarrow CB(X)$ and a single-valued map $f : X \rightarrow X$ if $fx \in Tx$. We denote

$$C(f, T) = \{x \in X \mid fx \in Tx\} \quad (7)$$

the set of coincidence points of a multivalued map T and single-valued map f .

Definition 11. Let $(X, M, *)$ be a fuzzy ultrametric space, $T : X \rightarrow CB(X)$ a multivalued map, and $f : X \rightarrow X$ if $fx \in Tx$ is a single-valued map. T and f are said to be coincidentally computing at $z \in X$ if $fz \in Tz \Rightarrow fTz \subseteq Tfz$.

Definition 12. Let $(X, M, *)$ be a fuzzy ultrametric space. An element $x \in X$ is said to be a common fixed point of two multivalued maps $S, T : X \rightarrow CB(X)$ and a single-valued map $f : X \rightarrow X$ if

$$x = fx \in Sx \cap Tx. \quad (8)$$

Theorem 13. Let $(X, M, *)$ be a fuzzy ultrametric space. Let $S, T : X \rightarrow CB(X)$ be a pair of multivalued maps and $f, g : X \rightarrow X$ a pair of single-valued maps satisfying the following conditions:

- (a) $fg(x)$ is spherically complete;
- (b) $M_{\nabla}(Sx, Ty, t) > \min\{M(fx, gy, t), M^{\nabla}(fx, Sx, t), M^{\nabla}(gy, Ty, t)\}$, for all $x, y \in X$, with $fx \neq gy$, and for $t > 0$;
- (c) $fS = Sg$, $fg = gf$, $fT = Tf$, $gS = Sg$, $gT = Tg$, $ST = TS$;
- (d) If $S(X) \subseteq f(X)$, $T(X) \subseteq g(X)$, then there exist points u and v in X , such that

$$fu \in Su, \quad gv \in Tv, \quad fu = gv, \quad Su = Tv. \quad (9)$$

Proof. Let $B_a = (fga; \min\{M^{\nabla}(fga, Sga, t), M^{\nabla}(fga, Tfa, t)\})$ denote the closed ball with centered fga and radius

$$\min\{M^{\nabla}(fga, Sga, t), M^{\nabla}(fga, Tfa, t)\}. \quad (10)$$

Let $\mathbf{A}$ be the collection of all the spheres for all $a \in fg(X)$. Then the relation

$$B_a \leq B_b \quad \text{if } B_b \subseteq B_a \quad (11)$$

is a partial order on $\mathbf{A}$.

Consider a totally ordered subfamily $\mathbf{A}_1$ of $\mathbf{A}$. Since $fg(X)$ is spherically complete, we have

$$\bigcap_{B_a \in \mathbf{A}_1} B_a = B \neq \emptyset. \quad (12)$$

Let $fgb \in B$, where $b \in fg(X)$ and $B_a \in \mathbf{A}_1$. Then $fgb \in B_a$. Hence

$$\begin{aligned} M(fgb, fga, t) \\ \geq \min \{M^\nabla(fga, Sga, t), M^\nabla(fga, Tfa, t)\}. \end{aligned} \quad (13)$$

If $a = b$, then $B_a = B_b$. Assume that $a \neq b$. Let, for all $x \in B_b$,

$$\begin{aligned} M(x, fgb, t) \\ \geq \min \{M^\nabla(fgb, Sgb, t), M^\nabla(fgb, Tfb, t)\}. \end{aligned} \quad (14)$$

Since Sga is a nonempty closed bounded set, then there exists an element $p \in Sga$, such that

$$M^\nabla(fga, Sga, t) = M(fga, p, t). \quad (15)$$

Tfa is a nonempty closed bounded set; then there exists an element $q \in Tfa$ such that

$$\begin{aligned} & M^\nabla(fga, Tfa, t) = M(fga, q, t), \\ & \min \{M^\nabla(fgb, Sgb, t), M^\nabla(fgb, Tfb, t)\} \\ &= \min \left\{ \bigvee_{c \in Sgb} M(fgb, c, t), \bigvee_{d \in Tgb} M(fgb, d, t) \right\} \\ &\geq \min \left\{ M(fgb, fga, t), M(fga, q, t), \right. \\ &\quad \left. \bigvee_{c \in Sgb} M(q, c, t), M(fgb, fga, t), \right. \\ &\quad \left. M(fga, p, t), \bigvee_{d \in Tfb} M(p, d, t) \right\} \\ &\geq \min \{M(fgb, fga, t), M^\nabla(fga, Tfa, t), \\ &\quad M_\nabla(Tfa, Sgb, t), M(fgb, fga, t), \\ &\quad M^\nabla(fga, Sga, t), M_\nabla(Sga, Tfb, t)\} \end{aligned}$$

$$\begin{aligned} & > \min \{M(fgb, fga, t), M^\nabla(fga, Tfa, t), \\ &\quad M^\nabla(fga, Sga, t), \\ &\quad \min \{M(fgb, fga, t), M^\nabla(fga, Tfa, t), \\ &\quad M^\nabla(fgb, Sgb, t)\}\}, \\ & \min \{M(fga, fgb, t), M^\nabla(fga, Sga, t), \\ &\quad M^\nabla(fgb, Tfb, t)\} \\ &\geq \min \{M^\nabla(fga, Tfa, t), M^\nabla(fga, Sga, t)\}. \end{aligned} \quad (16)$$

From the two conditions (a), (b) and (13), we now have

$$\begin{aligned} & M(x, fga, t) \\ &\geq \min \{M(x, fgb, t), M(fgb, fga, t)\} \\ &\geq \min \{M^\nabla(fga, Sga, t), M^\nabla(fga, Tfa, t)\}. \end{aligned} \quad (17)$$

So $x \in B_a$; we have just proved that $B_b \subseteq B_a$, for every $B_a \in \mathbf{A}_1$. Thus B_b is an upper bound in $\mathbf{A}$ for the family $\mathbf{A}_1$, and hence by Zorn's lemma, there is a maximal element in $\mathbf{A}$, say B_z , $z \in fg(X)$. There exists an element $w \in X$ such that $z = fgw$.

Suppose

$$\begin{aligned} & f(gfgw) \notin S(gfgw), \\ & g(ffgw) \notin T(ffgw). \end{aligned} \quad (18)$$

Since $Sgfgw$ and $Tffgw$ are nonempty closed bounded sets, then there exist $k \in Sgfgw$ and $l \in Tffgw$ such that

$$M^\nabla(fgfgw, Sgfgw, t) = M(fgfgw, k, t), \quad (19)$$

$$M^\nabla(fgfgw, Tffgw, t) = M(fgfgw, l, t). \quad (20)$$

From the two conditions (b), (c) and (19), we have

$$\begin{aligned} & M^\nabla(Sgfgw, TSfgw, t) \\ &= \bigvee_{e \in TSfgw} M(gSfgw, e, t) \\ &\geq \min \left\{ M^\nabla(Sgfgw, fgfgw, t), \right. \\ &\quad \left. M(fgfgw, k, t), \bigvee_{e \in TSfgw} M(k, e, t) \right\} \\ &\geq \min \{M^\nabla(Sgfgw, fgfgw, t), \\ &\quad M_\nabla(Sgfgw, TSfgw, t)\} \\ &> \min \{M^\nabla(Sgfgw, fgfgw, t), \\ &\quad \min \{M(fgfgw, fSfgw, t), \end{aligned}$$

$$\begin{aligned}
& M^\nabla(fgfgw, Sgfgw, t), \\
& M^\nabla(gSfgw, TSfgw, t)\} \} \\
& = M^\nabla(fgfgw, Sgfgw, t), \\
& M^\nabla(Tffgw, STfgw, t) \\
& = \bigvee_{h \in STfgw} M(fTfgw, h, t) \\
& \geq \min \{M^\nabla(Tffgw, fgfgw, t), \\
& \quad M_\nabla(Tffgw, STfgw, t)\} \\
& > \min \{M^\nabla(Tffgw, gffgw, t), \\
& \quad \min \{(fTfgw, gffgw, t), \\
& \quad \quad M^\nabla(fTfgw, STfgw, t), \\
& \quad \quad M^\nabla(gffgw, Tffgw, t)\}\} \\
& = M^\nabla(Tffgw, fgfgw, t).
\end{aligned} \tag{22}$$

From the two conditions (b), (c) and (19)–(22), we have

$$\begin{aligned}
& M^\nabla(fggSw, SggSw, t) \\
& = \bigvee_{m \in SggSw} M(fggSw, m, t) \\
& \geq \min \left\{ M^\nabla(fggSw, TSfgw, t), \right. \\
& \quad M(TSfgw, Tffgw, t), \\
& \quad M^\nabla(Tffgw, fgfgw, t), \\
& \quad \left. M(gffgw, l, t), \bigvee_{m \in SggSw} M(l, m, t) \right\} \\
& \geq \min \{M^\nabla(fgfgw, Sgfgw, t), \\
& \quad M^\nabla(fgfgw, Tffgw, t), \\
& \quad \min \{M(fggsw, gffgw, t), \\
& \quad \quad M^\nabla(fggSw, SggSw, t), \\
& \quad \quad M^\nabla(gffgSw, Tffgw, t)\}\} \\
& \geq \min \{M^\nabla(fgfgw, Sgfgw, t), \\
& \quad M^\nabla(fgfgw, Tffgw, t)\},
\end{aligned}$$

$$\begin{aligned}
& M^\nabla(fgfTw, TffTw, t) \\
& = \bigvee_{n \in TffTw} M(fgfTw, n, t) \\
& \geq \min \left\{ M^\nabla(fgfTw, STfgw, t), \right. \\
& \quad M(STfgw, Sgfgw, t), \\
& \quad M^\nabla(Sgfgw, fgfgw, t), \\
& \quad \left. M(fgfgw, k, t), \bigvee_{n \in TffTw} M(k, n, t) \right\} \\
& \geq \min \{M^\nabla(fgfgw, Tffgw, t), \\
& \quad M^\nabla(fgfgw, Sgfgw, t), \\
& \quad M_\nabla(Sgfgw, TffTw, t)\} \\
& \geq \min \{M^\nabla(fgfgw, Tffgw, t), \\
& \quad M^\nabla(fgfgw, Sgfgw, t), \\
& \quad \min \{M(fgfgw, gffTw, t), \\
& \quad \quad M^\nabla(fgfgw, Sgfgw, t), \\
& \quad \quad M^\nabla(gffTw, TffTw, t)\}\} \\
& \geq \min \{M^\nabla(fgfgw, Tffgw, t), \\
& \quad M^\nabla(fgfgw, Sgfgw, t)\}.
\end{aligned} \tag{23}$$

From (21) and (23), we have

$$\begin{aligned}
& \min \{M(Sgfgw, TSfgw, t), M^\nabla(fggSw, SggSw, t)\} \\
& > \min \{M^\nabla(fgfgw, Sgfgw, t), \\
& \quad M^\nabla(fgfgw, Tffgw, t)\}.
\end{aligned} \tag{24}$$

From (22) and (3.7), we have

$$\begin{aligned}
& \min \{M(STfgw, Tffgw, t), M^\nabla(fgfTw, TffTw, t)\} \\
& > \min \{M^\nabla(fgfgw, Tffgw, t), \\
& \quad M^\nabla(fgfgw, Sgfgw, t)\}.
\end{aligned} \tag{25}$$

If

$$\begin{aligned}
& \min \{M^\nabla(fgfgw, Sgfgw, t), M^\nabla(fgfgw, Tffgw, t)\} \\
& = M^\nabla(fgfgw, Sgfgw, t),
\end{aligned} \tag{26}$$

then from (24), we have $fgfgw \notin B_{gSw} \Rightarrow fgz \notin B_{gSw}$.

Hence $B_z \notin B_{gSw}$. It is a contradiction to the maximality of B_z in $\mathbf{A}$, since $gSw \subseteq gf(X) = fg(X)$.

If

$$\min \{M^\nabla(fgfgw, Sgfgw, t), M^\nabla(fgfgw, Tffgw, t)\} = M^\nabla(fgfgw, Tffgw, t), \quad (27)$$

then from (25), we have $fgfgw \notin B_{fTw} \Rightarrow fgz \notin B_{fTw}$.

Hence $B_z \notin B_{fTw}$. It is a contradiction to the maximality of B_z in $\mathbf{A}$, since $fTw \subseteq fg(X)$.

So

$$f(gfgw) \in S(ffgw), \quad g(gfgw) \in T(ffgw). \quad (28)$$

In addition, $f(gfgw) = g(ffgw)$.

Using the two conditions (b), (c) and (28), we obtain

$$\begin{aligned} M_\nabla(Sgfgw, Tffgw, t) &> \min \{M(fgfgw, gffgw, t), M^\nabla(fgfgw, Sgfgw, t), \\ &M^\nabla(gffgw, Tffgw, t)\} = 1. \end{aligned} \quad (29)$$

Hence $S(gfgw) = T(ffgw)$. Then the proof is completed. $\square$

Theorem 14. Let $(X, M, *)$ be a fuzzy ultrametric space. Let $S, T : X \rightarrow CB(X)$ be a pair of multivalued maps and let $f : X \rightarrow X$ be a single-valued map satisfying the following conditions:

- (a) $f(x)$ is spherically complete;
- (b) $M_\nabla(Sx, Ty, t) > \min\{M(fx, fy, t), M^\nabla(fx, Sx, t), M^\nabla(fy, Ty, t)\}$, for all $x, y \in X$, with $x \neq y$, and for $t > 0$;
- (c) $fS = Sf$, $fT = Tf$, $ST = TS$;
- (d) If $S(X) \subseteq f(X)$, $T(X) \subseteq f(X)$.

Then f , S , and T have a coincidence point in X . Moreover, if f and S , f and T are coincidentally commuting at $z \in C(f, T)$ and $ffz = fz$, then f , S , and T have a common fixed point in X .

Proof. If $f = g$ in Theorem 13, we obtain that there exist points u and v in X such that

$$fu \in Su, \quad fv \in Tv, \quad fu = fv, \quad Su = Tv. \quad (30)$$

As $u \in C(f, S)$, f and S are coincidentally commuting at u and $ffu = fu$.

Writing $w = fu$, then $w \in Su, w \in Tv$. Then we have

$$fw = w, \quad (31)$$

$$w = fw \in f(Su) \subseteq S(fu) = Sw.$$

Now, since also $u \in C(f, T)$, f and T are coincidentally commuting at u and $ffu = fu$, so we obtain

$$w = fw \in f(Tv) \subseteq T(fv) = Tw. \quad (32)$$

Thus, we have proved that $w = fw \in Sw \cap Tw$; that is, w is a common fixed point of f , S , and T . $\square$

The proof of the following corollary is easy, so it is omitted.

Corollary 15. Let $(X, M, *)$ be a spherically complete fuzzy ultrametric space. Let $S, T : X \rightarrow CB(X)$ be a pair of multivalued maps satisfying the following conditions:

- (a) $M_\nabla(Sx, Ty, t) > \min\{M(x, y, t), M^\nabla(x, Sx, t), M^\nabla(y, Ty, t)\}$, for all $x, y \in X$, with $x \neq y$, and for $t > 0$;
- (b) $ST = TS$.

Then, there exists a point z in X such that $z \in Sz \cap Tz$ and $Sz = Tz$.

Now we give an example to illustrate Theorem 13.

Example 16. Let $(X, M, *)$ be a fuzzy ultrametric space in which $X = [0, 1]$, $a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$, and

$$M(x, y, t) = \frac{t}{t + |x - y|} \quad (33)$$

for all $t > 0$.

Define the maps S, T, f, g on X as follows:

$$Sx = Tx = 1, \quad fx = \frac{x+1}{2}, \quad gx = \frac{2x+1}{3}, \quad \forall x \in X. \quad (34)$$

Define

$$W(t_1, t_2, t_3, t_4, t_5) = t_1 - (\min\{t_2, t_3, t_4, t_5\})^h, \quad (35)$$

where W is a function from $[0, 1]^5 \rightarrow [-1, 1]$ satisfying the following conditions:

- (W₁) $W(t_1, \dots, t_5)$ is increasing in t_1 and decreasing in $t_2, \dots, t_5$
- (W₂) $W(u, v, v, v, v) \geq 0$ implies that $u > v$, for all $u \in [0, 1]$ and $v \in [0, 1]$.

Then for any $h \in (0, 1)$, the following inequality

$$\begin{aligned} M_\nabla(Sx, Ty, t) &> (\min \{M(fx, gy, t), M^\nabla(fx, Sx, t), \\ &M^\nabla(gy, Ty, t)\})^h \end{aligned} \quad (36)$$

is satisfied for all $x, y \in X$, with $x \neq y$, and for all $t > 0$ since the LHS of the inequality is 1.

Clearly all conditions in Theorem 13 are satisfied, so 1 is the unique common fixed point of S, T, f , and g .

4. Conclusion

In this paper, we get coincidence point theorems and common fixed point theorems for two pairs of multivalued and single-valued maps satisfying different contractive conditions on spherically complete fuzzy ultrametric space, which are generalized results for both ultrametric spaces [12, 15, 16] and fuzzy metric spaces [11].

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