Research Article

Some Double Sequence Spaces of Fuzzy Real Numbers of Paranormed Type

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We study different properties of convergent, null, and bounded double sequence spaces of fuzzy real numbers like completeness, solidness, sequence algebra, symmetricity, convergence-free, and so forth. We prove some inclusion results too.

1. Introduction

Throughout the paper, a double sequence is denoted by $\langle X_{nk} \rangle$, a double infinite array of elements X_{nk} , where each X_{nk} is a fuzzy real number.

The initial work on double sequences is found in Bromwich [1]. Later on, it was studied by Hardy [2], Móricz [3], Tripathy [4], Basarir and Sonalcan [5], and many others. Hardy [2] introduced the notion of regular convergence for double sequences.

The concept of paranormed sequences was studied by Nakano [6] and Simons [7] at the initial stage. Later on, it was studied by many others.

After the introduction of fuzzy real numbers, different classes of sequences of fuzzy real numbers were introduced and studied by Tripathy and Nanda [8], Choudhary and Tripathy [9], Tripathy et al. [10–13], Tripathy and Dutta [14–16], Tripathy and Borgogain [17], Tripathy and Das [18], and many others.

Let D denote the set of all closed and bounded intervals $X = [a_1, a_2]$ on R , the real line. For $X, Y \in D$, we define

$$d(X, Y) = \max(|a_1 - b_1|, |a_2 - b_2|), \quad (1)$$

where $X = [a_1, a_2]$ and $Y = [b_1, b_2]$. It is known that (D, d) is a complete metric space.

A fuzzy real number X is a fuzzy set on R , that is, a mapping $X : R \rightarrow I (= [0, 1])$ associating each real number t with its grade of membership $X(t)$.

The α -level set $[X]^\alpha$ of the fuzzy real number X , for $0 < \alpha \leq 1$, is defined as $[X]^\alpha = \{t \in R : X(t) \geq \alpha\}$.

The set of all upper semicontinuous, normal, and convex fuzzy real numbers is denoted by $R(I)$, and throughout the paper, by a fuzzy real number, we mean that the number belongs to $R(I)$.

Let $X, Y \in R(I)$, and let the α -level sets be $[X]_\alpha = [a_1^\alpha, a_2^\alpha]$, $[Y]_\alpha = [b_1^\alpha, b_2^\alpha]$, and $\alpha \in [0, 1]$; the product of X and Y is defined by

$$[X \otimes Y]_\alpha = \left[\min_{i,j \in \{1,2\}} a_i^\alpha \cdot b_j^\alpha, \max_{i,j \in \{1,2\}} a_i^\alpha \cdot b_j^\alpha \right]. \quad (2)$$

2. Definitions and Preliminaries

A fuzzy real number X is called *convex* if $X(t) \geq X(s) \wedge X(r) = \min(X(s), X(t))$, where $s < t < r$.

If there exists $t_0 \in R$ such that $X(t_0) = 1$, then the fuzzy real number X is called *normal*.

A fuzzy real number X is said to be *upper semicontinuous* if, for each $\varepsilon > 0$, $X^{-1}([0, a + \varepsilon))$, for all $a \in I$, is open in the usual topology of R .

The set R of all real numbers can be embedded in $R(I)$. For $r \in R$, $\bar{r} \in R(I)$ is defined by

$$\bar{r}(t) = \begin{cases} 1, & \text{for } t = r, \\ 0, & \text{for } t \neq r. \end{cases} \quad (3)$$

The *absolute* value, $|X|$ of $X \in R(I)$, is defined by (see, e.g., [19])

$$\begin{aligned} |X|(t) &= \max \{X(t), X(-t)\} \quad \text{if } t \geq 0, \\ |X|(t) &= 0 \quad \text{if } t < 0. \end{aligned} \quad (4)$$

A fuzzy real number X is called *nonnegative* if $X(t) = 0$, for all $t < 0$. The set of all nonnegative fuzzy real numbers is denoted by $R^*(I)$.

Let $\bar{d} : R(I) \times R(I) \rightarrow R$ be defined by

$$\bar{d}(X, Y) = \sup_{0 \leq \alpha \leq 1} d([X]^\alpha, [Y]^\alpha). \quad (5)$$

Then $\bar{d}$ defines a metric on $R(I)$.

The additive identity and multiplicative identity in $R(I)$ are denoted by $\bar{0}$ and $\bar{1}$, respectively.

A sequence (X_k) of fuzzy real numbers is said to be *convergent* to the fuzzy real number L if, for every $\varepsilon > 0$, there exists $n_0 \in N$ such that $\bar{d}(X_k, L) < \varepsilon$, for all $k \geq n_0$.

A sequence of fuzzy numbers (f_n) converges to a fuzzy number f if both $\lim_{n \rightarrow \infty} [f_n]_\alpha^- = [f]_\alpha^-$ and $\lim_{n \rightarrow \infty} [f_n]_\alpha^+ = [f]_\alpha^+$ hold for every $\alpha \in (0, 1]$ [20].

A sequence (x_n) of generalized fuzzy numbers converges weakly to a generalized fuzzy number x (and we write $x_n \xrightarrow{w} x$) if distribution functions (x_n^l) converge weakly to x^l and (x_n^r) converge weakly to x^r [21].

A double sequence (X_{nk}) of fuzzy real numbers is said to be *convergent in Pringsheim's sense* to the fuzzy real number L if, for every $\varepsilon > 0$, there exists $n_0, k_0 \in N$ such that $\bar{d}(X_{nk}, L) < \varepsilon$, for all $n \geq n_0, k \geq k_0$.

A double sequence (X_{nk}) of fuzzy real numbers is said to be *regularly convergent* if it converges in Pringsheim's sense, and the following limits exist:

$$\begin{aligned} \lim_n \bar{d}(X_{nk}, L_k) &= 0, \quad \text{for some } L_k \in R(I), \\ &\quad \text{for each } k \in N, \\ \lim_k \bar{d}(X_{nk}, J_n) &= 0, \quad \text{for some } J_n \in R(I), \\ &\quad \text{for each } n \in N. \end{aligned} \quad (6)$$

A fuzzy real number sequence (X_k) is said to be *bounded* if $\sup_k |X_k| \leq \mu$, for some $\mu \in R^*(I)$.

For $r \in R$ and $X \in R(I)$, we define

$$\begin{aligned} rX(t) &= X(r^{-1}t) \quad \text{if } r \neq 0, \\ rX(t) &= \bar{0} \quad \text{if } r = 0. \end{aligned} \quad (7)$$

Throughout the paper ${}_2w_F$, $({}_2\ell_\infty)_F$, ${}_2c_F$, $({}_2c_0)_F$, ${}_2c_F^R$, and $({}_2c_0^R)_F$ denote the classes of *all*, *bounded*, *convergent in Pringsheim's sense*, *null in Pringsheim's sense*, *regularly convergent*, and *regularly null* fuzzy real number sequences, respectively.

A double sequence space E_F is said to be *solid* (or *normal*) if $\langle Y_{nk} \rangle \in E_F$, whenever $|Y_{nk}| \leq |X_{nk}|$, for all $n, k \in N$, for some $\langle X_{nk} \rangle \in E_F$.

Let $K = \{(n_i, k_i) : i \in N; n_1 < n_2 < n_3 < \dots \text{ and } k_1 < k_2 < k_3 < \dots\} \subseteq N \times N$, and let E_F be a double sequence space. A *K-step space* of E_F is a sequence space

$$\lambda_K^E = \{ \langle X_{n_i k_i} \rangle \in {}_2w_F : \langle X_{nk} \rangle \in E_F \}. \quad (8)$$

A *canonical preimage* of a sequence $\langle X_{n_i k_i} \rangle \in E_F$ is a sequence $\langle Y_{nk} \rangle$ defined as follows:

$$Y_{nk} = \begin{cases} X_{nk} & \text{if } (n, k) \in K, \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

A *canonical preimage of a step space* λ_K^E is a set of canonical preimages of all elements in λ_K^E .

A double sequence space E_F is said to be *monotone* if E_F contains the canonical preimage of all its step spaces.

From the above definitions, we have the following remark.

Remark 1. A sequence space E_F is solid $\Rightarrow E_F$ is monotone.

A double sequence space E_F is said to be *symmetric* if $\langle X_{\pi(n,k)} \rangle \in E_F$, whenever $\langle X_{nk} \rangle \in E_F$, where π is a permutation of $N \times N$.

A double sequence space E_F is said to be *sequence algebra* if $\langle X_{nk} \otimes Y_{nk} \rangle \in E_F$, whenever $\langle X_{nk} \rangle, \langle Y_{nk} \rangle \in E_F$.

A double sequence space E_F is said to be *convergence-free* if $\langle Y_{nk} \rangle \in E_F$, whenever $\langle X_{nk} \rangle \in E_F$, and $X_{nk} = \bar{0}$ implies that $Y_{nk} = \bar{0}$.

Sequences of fuzzy real numbers relative to the para-normed sequence spaces were studied by Choudhary and Tripathy [9].

In this paper, we introduce the following sequence spaces of fuzzy real numbers.

Let $p = \langle p_{nk} \rangle$ be a sequence of positive real numbers

$$\begin{aligned} ({}_2\ell_\infty)_F(p) &= \left\{ \langle X_{nk} \rangle \in {}_2w_F : \right. \\ &\quad \left. \sup_{n,k} \{ \bar{d}(X_{nk}, \bar{0}) \}^{p_{nk}} < \infty \right\}, \end{aligned} \quad (10)$$

$$\begin{aligned} {}_2c_F(p) &= \left\{ \langle X_{nk} \rangle \in {}_2w_F : \lim_{n,k} \{ \bar{d}(X_{nk}, L) \}^{p_{nk}} = 0, \right. \\ &\quad \left. \text{for some } L \in R(I) \right\}. \end{aligned}$$

For $L = \bar{0}$, we get the class $({}_2c_F)_0(p)$.

Also a fuzzy sequence $\langle X_{nk} \rangle \in {}_2c_F^R(p)$ if $\langle X_{nk} \rangle \in {}_2c_F(p)$, and the following limits exist:

$$\begin{aligned} \lim_n \{ \bar{d}(X_{nk}, L_k) \}^{p_{nk}} &= 0, \quad \text{for some } L_k \in R(I), \\ &\quad \text{for each } k \in N, \\ \lim_k \{ \bar{d}(X_{nk}, J_n) \}^{p_{nk}} &= 0, \quad \text{for some } J_n \in R(I), \\ &\quad \text{for each } n \in N. \end{aligned} \quad (11)$$

For the class of sequences $({}_2c_F^R)_0(p)$, $L = L_k = J_n = \bar{0}$.

We define $m_F(p) = {}_2c_F^R(p) \cap ({}_2\ell_\infty)_F(p)$, $(m_0)_F(p) = ({}_2c_F^R)_0(p) \cap ({}_2\ell_\infty)_F(p)$.

3. Main Results

Theorem 2. Let $\langle p_{nk} \rangle$ be bounded. Then, the classes of sequences $({}_2\ell_\infty)_F(p)$, ${}_2c_F^R(p)$, $({}_2c_F^R)_0(p)$, $m_F(p)$, and $(m_0)_F(p)$ are complete metric spaces with respect to the metric defined by

$$f(X, Y) = \sup_{n,k} \left\{ \bar{d}(X_{nk}, Y_{nk}) \right\}^{p_{nk}/H}, \quad (12)$$

where $H = \max(1, \sup p_{nk})$.

Proof. We prove the result for ${}_2\ell_\infty(p)$. Let $\langle X_{nk}^i \rangle$ be a Cauchy sequence in $({}_2\ell_\infty)_F(p)$. Then, for a given $\varepsilon > 0$, there exists $m_0 \in N$ such that

$$\begin{aligned} f(X_{nk}^i, X_{nk}^j) &< \varepsilon < \varepsilon^{p_{nk}/H}, \quad \forall i, j \geq m_0, \\ \Rightarrow \bar{d}(X_{nk}^i, X_{nk}^j) &< \varepsilon, \quad \forall i, j \geq m_0 \\ \Rightarrow \langle X_{nk}^j \rangle_{j=1}^\infty &\text{ is a Cauchy sequence of fuzzy} \end{aligned} \quad (13)$$

real number for each $n, k \in N$.

Since $R(I)$ is complete, there exist fuzzy numbers X_{nk} such that $\lim_{j \rightarrow \infty} X_{nk}^j = X_{nk}$, for each $n, k \in N$.

Taking $j \rightarrow \infty$ in (13), we have

$$f(X_{nk}^i, X_{nk}) < \varepsilon. \quad (14)$$

Using the triangular inequality

$$f(\langle X_{nk} \rangle, \bar{0}) \leq f(\langle X_{nk} \rangle, \langle X_{nk}^j \rangle) + f(\langle X_{nk}^j \rangle, \bar{0}), \quad (15)$$

we have $\langle X_{nk} \rangle \in ({}_2\ell_\infty)_F(p)$. Hence, $({}_2\ell_\infty)_F(p)$ is complete. $\square$

Property 1. The space $({}_2\ell_\infty)_F(p)$ is symmetric, but the spaces ${}_2c_F(p)$, ${}_2c_F^R(p)$, $({}_2c_F)_0(p)$, $({}_2c_F^R)_0(p)$, $m_F(p)$, and $(m_0)_F(p)$ are not symmetric.

Proof. Obviously the space $({}_2\ell_\infty)_F(p)$ is symmetric. For the other spaces, consider the following example.

Example 3. Consider the sequence space ${}_2c_F(p)$. Let $p_{1k} = 2$, for all $k \in N$ and $p_{nk} = 3$, otherwise. Let the sequence $\langle X_{nk} \rangle$ be defined by

$$X_{1k} = \bar{1} \quad \forall k \in N, \quad (16)$$

and for $n > 1$,

$$X_{nk}(t) = \begin{cases} t+2, & \text{for } -2 \leq t \leq -1, \\ -t, & \text{for } -1 \leq t \leq 0, \\ 0, & \text{otherwise.} \end{cases} \quad (17)$$

Let $\langle Y_{nk} \rangle$ be a rearrangement of $\langle X_{nk} \rangle$ defined by

$$Y_{nm} = \bar{1}, \quad (18)$$

and for $n \neq k$,

$$Y_{nk}(t) = \begin{cases} t+2, & \text{for } -2 \leq t \leq -1, \\ -t, & \text{for } -1 \leq t \leq 0, \\ 0, & \text{otherwise.} \end{cases} \quad (19)$$

Then, $\langle X_{nk} \rangle \in {}_2c_F(p)$, but $\langle Y_{nk} \rangle \notin {}_2c_F(p)$. Hence, ${}_2c_F(p)$ is not symmetric. Similarly, it can be established that the other spaces are also not symmetric. $\square$

Theorem 4. The spaces $({}_2\ell_\infty)_F(p)$, $({}_2c_0)_F(p)$, $({}_2c_0^R)_F(p)$, and $(m_0)_F(p)$ are solid.

Proof. Consider the sequence space $({}_2\ell_\infty)_F(p)$. Let $\langle X_{nk} \rangle \in ({}_2\ell_\infty)_F(p)$, and let $\langle Y_{nk} \rangle$ be such that $\bar{d}(Y_{nk}, \bar{0}) \leq \bar{d}(X_{nk}, \bar{0})$.

The result follows from the inequality

$$\{\bar{d}(Y_{nk}, \bar{0})\}^{p_{nk}} \leq \{\bar{d}(X_{nk}, \bar{0})\}^{p_{nk}}. \quad (20)$$

Hence, the space $({}_2\ell_\infty)_F(p)$ is solid. Similarly, the other spaces are also solid. $\square$

Property 2. The spaces ${}_2c_F(p)$, $({}_2c^R)_F(p)$, and $m_F(p)$ are not monotone and hence are not solid.

Proof. The result follows from the following example.

Example 5. Consider the sequence space ${}_2c_F(p)$. Let $p_{nk} = 3$ for $n+k$ even and $p_{nk} = 2$, otherwise. Let $J = \{(n, k) : n+k \text{ is even}\} \subseteq N \times N$. Let $\langle X_{nk} \rangle$ be defined by the following: for all $n, k \in N$,

$$X_{nk}(t) = \begin{cases} t+3, & \text{for } -3 \leq t \leq -2, \\ nt(3n-1)^{-1} + 3n(3n-1)^{-1}, & \text{for } -2 \leq t \leq -1+n^{-1}, \\ 0, & \text{otherwise.} \end{cases} \quad (21)$$

Then, $\langle X_{nk} \rangle \in {}_2c_F(p)$. Let $\langle Y_{nk} \rangle$ be the canonical preimage of $\langle X_{nk} \rangle_J$ for the subsequence J of N . Then,

$$Y_{nk} = \begin{cases} X_{nk}, & \text{for } (n, k) \in J, \\ \bar{0}, & \text{otherwise.} \end{cases} \quad (22)$$

Then, $\langle Y_{nk} \rangle \notin {}_2c_F(p)$. Thus, ${}_2c_F(p)$ is not monotone. Similarly, the other spaces are also not monotone. Hence, the spaces ${}_2c_F(p)$, $({}_2c^R)_F(p)$, and $m_F(p)$ are not solid. $\square$

Property 3. The spaces $({}_2\ell_\infty)_F(p)$, ${}_2c_F(p)$, $({}_2c_0)_F(p)$, $({}_2c^R)_F(p)$, $({}_2c_0^R)_F(p)$, $m_F(p)$, and $(m_0)_F(p)$ are not convergence-free.

The result follows from the following example.

Example 6. Consider the sequence space ${}_2c_F(p)$. Let $p_{1k} = 1$, for all $k \in N$, $p_{nk} = 3$, otherwise. Consider the sequence $\langle X_{nk} \rangle$ defined by

$$X_{1k} = \bar{0}, \quad (23)$$

and for other values,

$$X_{nk}(t) = \begin{cases} t + 2, & \text{for } -2 \leq t \leq -1, \\ -nt(n+1)^{-1} + (n+1)^{-1}, & \text{for } -1 \leq t \leq n^{-1}, \\ 0, & \text{otherwise.} \end{cases} \quad (24)$$

Let the sequence $\langle Y_{nk} \rangle$ be defined by

$$Y_{1k} = \bar{0}, \quad (25)$$

and for other values,

$$Y_{nk}(t) = \begin{cases} 1, & \text{for } 0 \leq t \leq 1, \\ (n-1)(n-1)^{-1}, & \text{for } 1 \leq t \leq n, \\ 0, & \text{otherwise.} \end{cases} \quad (26)$$

Then, $\langle X_{nk} \rangle \in {}_2c_F(p)$, but $\langle Y_{nk} \rangle \notin {}_2c_F(p)$. Hence, the space ${}_2c_F(p)$ is not convergence-free. Similarly, the other spaces are also not convergence-free.

Theorem 7. $Z(p) \subseteq ({}_2\ell_\infty)_F(p)$, for $Z = ({}_2c^R)_F, ({}_2c_0^R)_F, m_F, (m_0)_F$. The inclusions are strict.

Proof. Since convergent sequences are bounded, the proof is clear. $\square$

Theorem 8. Let $0 < q_{ij} \leq p_{ij} < \infty$, for all $i, j \in N$. Then, $Z(p) \subseteq Z(q)$ for $Z = {}_2c_F, ({}_2c^R)_F, ({}_2c_F)_0, ({}_2c_0^R)_F, m_F, (m_0)_F$.

Proof. Consider the sequence spaces ${}_2c_F(p)$ and ${}_2c_F(q)$. Let $\langle X_{nk} \rangle \in {}_2c_F(p)$.

Then, $\{\bar{d}(X_{nk}, L)\}^{p_{nk}} < \varepsilon$, for all $n \geq n_0, k \geq k_0$.

The result follows from the inequality $\{\bar{d}(X_{nk}, L)\}^{q_{nk}} \leq \{\bar{d}(X_{nk}, L)\}^{p_{nk}}$. $\square$

Theorem 9. The spaces $({}_2\ell_\infty)_F(p), {}_2c_F(p), ({}_2c_0)_F(p), ({}_2c^R)_F(p), ({}_2c_0^R)_F(p), m_F(p)$, and $(m_0)_F(p)$ are sequence algebras.

Proof. Consider the sequence space $({}_2c_0)_F(p)$. Let $\langle X_{nk} \rangle, \langle Y_{nk} \rangle \in ({}_2c_0)_F(p)$. Then, the result follows immediately from the inequality

$$\{\bar{d}(X_{nk}Y_{nk}, \bar{0})\}^{p_{nk}} \leq \{\bar{d}(X_{nk}, \bar{0})\}^{p_{nk}} \{\bar{d}(Y_{nk}, \bar{0})\}^{p_{nk}}. \quad (27)$$

$\square$

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