Research Article

Estimates of Certain Integral Inequalities on Time Scales

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The main objective of this paper is to establish explicit bounds on certain integral inequalities on time scales, which can be used as tools in the study of certain classes of integral equations on time scales. Some applications of our results are also given.

1. Introduction

Recently, many authors have studied various aspects of dynamic inequalities on time scales using various techniques [1–4]. In this paper, we obtain explicit bounds on certain integral inequalities on time scales. In the present paper we offer some fundamental dynamic integral inequalities on time scales which can be used as tools for handling the qualitative behavior of solutions of certain dynamic equations on time scales. Excellent information about introduction to time scales can be found in [5, 6]. In what follows, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{Z}$ the set of integers, and $\mathbb{T}$ denotes arbitrary time scales. Now following [3, 4] we give some basic definitions about calculus on time scales in two variables.

We say that $f : \mathbb{T} \rightarrow \mathbb{R}$ is rd-continuous provided f is continuous at each right-dense point of $\mathbb{T}$ and has a finite left sided limit at each left-dense point of $\mathbb{T}$ and will be denoted by C_{rd} . Let $\mathbb{T}_1$ and $\mathbb{T}_2$ be two time scales with at least two points and consider the time scales intervals $\overline{\mathbb{T}}_1 = [x_0, \infty) \cap \mathbb{T}_1$ and $\overline{\mathbb{T}}_2 = [y_0, \infty) \cap \mathbb{T}_2$ for $x_0 \in \mathbb{T}_1$ and $y_0 \in \mathbb{T}_2$ and $\Omega = \mathbb{T}_1 \times \mathbb{T}_2$. Let $\sigma_1, \rho_1, \Delta_1$ and $\sigma_2, \rho_2, \Delta_2$ denote the forward jump operators, backward jump operators and the delta differentiation operator, respectively, on $\mathbb{T}_1$ and $\mathbb{T}_2$. Let $a < b$ are points in $\mathbb{T}_1$, $c < d$ are points in $\mathbb{T}_2$, $[a, b]$ is the half closed bounded interval in $\mathbb{T}_1$, and $[c, d]$ is the half closed bounded interval in $\mathbb{T}_2$.

We say that a real-valued function f on $\mathbb{T}_1 \times \mathbb{T}_2$ at $(t_1, t_2) \in \overline{\mathbb{T}}_1 \times \overline{\mathbb{T}}_2$ has a Δ_1 partial derivative $f^{\Delta_1}(t_1, t_2)$ with

respect to t_1 if for each $\epsilon > 0$ there exists a neighborhood U_{t_1} of t_1 such that

$$\begin{aligned} & \left| f(\sigma_1(t_1), t_2) - f(s, t_2) - f^{\Delta_1}(t_1, t_2)(\sigma_1(t_1) - s) \right| \\ & \leq \epsilon |\sigma_1(t_1) - s|, \end{aligned} \quad (1)$$

for all $s \in U_{t_1}$. We say that f on $\mathbb{T}_1 \times \mathbb{T}_2$ at $(t_1, t_2) \in \overline{\mathbb{T}}_1 \times \overline{\mathbb{T}}_2$ has a Δ_2 partial derivative $f^{\Delta_2}(t_1, t_2)$ with respect to t_2 if for each $\eta > 0$ there exists a neighborhood U_{t_2} of t_2 such that

$$\begin{aligned} & \left| f(t_1, \sigma_2(t_2)) - f(t_1, l) - f^{\Delta_2}(t_1, t_2)(\sigma_2(t_2) - l) \right| \\ & \leq \eta |\sigma_2(t_2) - l|, \end{aligned} \quad (2)$$

for all $l \in U_{t_2}$. The function f is called rd-continuous in t_2 if for every $\alpha_1 \in \mathbb{T}_1$, the function $f(\alpha_1, t_2)$ is rd-continuous on $\mathbb{T}_2$. The function f is called rd-continuous in t_1 if for every $\alpha_2 \in \mathbb{T}_2$ the function $f(t_1, \alpha_2)$ is rd-continuous on $\mathbb{T}_1$.

We require that the following lemma proved in [1, 2] holds.

Lemma 1. Let $u, f \in C_{rd}(\mathbb{T}, \mathbb{R})$, $k(t, s), k^{\Delta_1}(t, s) \in C_{rd}(\Omega, \mathbb{R})$ and c nonnegative constants. If

$$u(t) \leq c + \int_{t_0}^t f(s) \left[u(s) + \int_{s_0}^s k(s, \tau) u(\tau) \Delta \tau \right] \Delta s, \quad (3)$$

for $t \in \mathbb{T}$, then

$$u(t) \leq c \left[1 + \int_{t_0}^t f(s) e_{f+A}(s, s_0) \Delta s \right], \quad (4)$$

for $t \in \mathbb{T}$, where

$$A(t) = k(t, \sigma(t)) + \int_{t_0}^t k^{\Delta_1}(t, \eta) \Delta \eta, \quad (5)$$

for $t \in \mathbb{T}$.

Lemma 2 (see [1, Lemma]). Let $u, a, f \in C'_{\text{rd}}(\overline{\mathbb{T}}_1 \times \overline{\mathbb{T}}_2, \mathbb{R}_+)$ and a is nondecreasing in each of its variables. If

$$u(x, y) \leq a(x, y) + \int_{x_0}^x \int_{y_0}^y f(s, t) u(s, t) \Delta t \Delta s, \quad (6)$$

for $(x, y) \in \overline{\mathbb{T}}_1 \times \overline{\mathbb{T}}_2$, then

$$u(x, y) \leq a(x, y) + e_{\int_{y_0}^y f(x, t) \Delta t}(x, x_0), \quad (7)$$

for $(x, y) \in \overline{\mathbb{T}}_1 \times \overline{\mathbb{T}}_2$.

2. Main Results

Our main results are given in the following theorems.

Theorem 3. Let $u, f, a \in C_{\text{rd}}(\mathbb{T}, \mathbb{R}_+)$, $k(t, s), k^{\Delta}(t, s) \in C_{\text{rd}}(\Omega, \mathbb{R}_+)$. If

$$u(t) \leq a(t) + \int_{t_0}^t f(s) \left[u(s) + \int_{s_0}^s k(s, \tau) u(\tau) \Delta \tau \right] \Delta s, \quad (8)$$

for $t \in \mathbb{T}$, then

$$u(t) \leq a(t) + h(t) \left[1 + \int_{t_0}^t f(s) e_{f+A}(s, s_0) \Delta s \right], \quad (9)$$

for $t \in \mathbb{T}$, where

$$h(t) = \int_{t_0}^t f(s) \left[a(s) + \int_{s_0}^s k(s, \tau) a(\tau) \Delta \tau \right] \Delta s, \quad (10)$$

for $t \in \mathbb{T}$ and $A(t)$ defined by (5).

Proof. Define a function $z(t)$ by

$$z(t) = \int_{t_0}^t f(s) \left[u(s) + \int_{s_0}^s k(s, \tau) u(\tau) \Delta \tau \right] \Delta s, \quad (11)$$

then from (8), $u(t) \leq a(t) + z(t)$, and using this in (11) we get

$$\begin{aligned} z(t) &\leq \int_{t_0}^t f(s) \left[a(s) + z(s) + \int_{s_0}^s k(s, \tau) (a(\tau) + z(\tau)) \Delta \tau \right] \Delta s \\ &= h(t) + \int_{t_0}^t f(s) \left[z(s) + \int_{s_0}^s k(s, \tau) z(\tau) \Delta \tau \right] \Delta s, \end{aligned} \quad (12)$$

where $h(t)$ is defined by (10). Clearly $h(t)$ is nonnegative, rd-continuous and nondecreasing in t , $t \in \mathbb{T}$ for $a(t)$ nondecreasing. First we assume that $h(t) > 0$ for $t \in \mathbb{T}$. From (12), we have

$$\frac{z(t)}{h(t)} \leq 1 + \int_{t_0}^t f(s) \left[\frac{z(s)}{h(s)} + \int_{s_0}^s k(s, \tau) \frac{z(\tau)}{h(\tau)} \Delta \tau \right] \Delta s. \quad (13)$$

Now as an application of Lemma 1 we have

$$\frac{z(t)}{h(t)} \leq \left[1 + \int_{t_0}^t f(s) e_{f+A}(s, s_0) \Delta s \right]. \quad (14)$$

The desired inequality (9) follows from (14) and the fact that $u(t) \leq a(t) + z(t)$. If $h(t) \geq 0$, we carry out the above procedure with $h(t) + \epsilon$ instead of $h(t)$, where $\epsilon > 0$ is an arbitrary small constants, and then subsequently pass to the limit as $\epsilon \rightarrow 0$ to obtain (9). The following theorem deals with two independent versions of inequalities established in Theorem 3. $\square$

Theorem 4. Let $u, f \in C_{\text{rd}}(\Omega, \mathbb{R}_+)$, $k(x, y, s, t), k^{\Delta_1}(x, y, s, t), k^{\Delta_2}(x, y, s, t), k^{\Delta_2 \Delta_1}(x, y, s, t) \in C_{\text{rd}}(\Omega \times \Omega, \mathbb{R}_+)$, and c be a nonnegative constants. If

$$\begin{aligned} u(x, y) &\leq c + \int_{x_0}^x \int_{y_0}^y f(s, t) \\ &\times \left[u(s, t) + \int_{s_0}^s \int_{t_0}^t k(s, t, \tau, \xi) u(\tau, \xi) \Delta \xi \Delta \tau \right] \Delta t \Delta s, \end{aligned} \quad (15)$$

for $(x, y) \in \Omega$, then

$$u(x, y) \leq c \left[1 + \int_{x_0}^x \int_{y_0}^y f(s, t) e_{\int_{t_0}^t [f(\tau, \xi) + A(\tau, \xi)] \Delta \xi \Delta \tau}(x, x_0) \right] \Delta t \Delta s, \quad (16)$$

where

$$\begin{aligned} A(x, y) &= k(x, y, \sigma(x), \sigma(y)) + \int_{x_0}^x k^{\Delta_1}(x, y, \tau, \sigma(y)) \Delta \tau \\ &+ \int_{y_0}^y k^{\Delta_1}(x, y, \sigma(x), \eta) \Delta \eta \\ &+ \int_{x_0}^x \int_{y_0}^y k^{\Delta_2 \Delta_1}(x, y, \tau, \eta) \Delta \eta \Delta \tau, \end{aligned} \quad (17)$$

for $(x, y) \in \Omega$.

Proof. Let $c > 0$ and define a function $z(x, y)$ by the right-hand side of (15); then $z(x_0, y) = z(x, y_0) = c$, $u(x, y) \leq z(x, y)$, and

$$\begin{aligned} z^{\Delta_2 \Delta_1}(x, y) &= f(x, y) \left[u(x, y) + \int_{x_0}^x \int_{y_0}^y k(x, y, \tau, \xi) u(\tau, \xi) \Delta \xi \Delta \tau \right] \\ &\leq f(x, y) \left[z(x, y) + \int_{x_0}^x \int_{y_0}^y k(x, y, \tau, \xi) z(\tau, \xi) \Delta \xi \Delta \tau \right]. \end{aligned} \quad (18)$$

Define a function $v(x, y)$ by

$$v(x, y) = z(x, y) + \int_{x_0}^x \int_{y_0}^y k(x, y, \tau, \xi) z(\tau, \xi) \Delta \xi \Delta \tau, \quad (19)$$

then $v(x_0, y) = z(x_0, y) = c$, $v(x, y_0) = z(x, y_0) = c$, $z(x, y) \leq v(x, y)$, $z^{\Delta_1 \Delta_2}(x, y) \leq f(x, y)v(x, y)$, $v(x, y)$ is nondecreasing for $(x, y) \in \Omega$, and

$$\begin{aligned} v^{\Delta_1 \Delta_2}(x, y) &= z^{\Delta_1 \Delta_2}(x, y) + k(x, y, \sigma(x), \sigma(y)) z(x, y) \\ &\quad + \int_{x_0}^x k^{\Delta_1}(x, y, \tau, \sigma(y)) z(\tau, y) \Delta \tau \\ &\quad + \int_{y_0}^y k^{\Delta_2}(x, y, \sigma(x), \xi) z(x, \xi) \Delta \xi \\ &\quad + \int_{x_0}^x \int_{y_0}^y k^{\Delta_2 \Delta_1}(x, y, \tau, \xi) z(\tau, \xi) \Delta \xi \Delta \tau \\ &\leq f(x, y) v(x, y) + k(x, y, \sigma(x), \sigma(y)) v(x, y) \\ &\quad + \int_{x_0}^x k^{\Delta_1}(x, y, \tau, \sigma(y)) v(\tau, y) \Delta \tau \\ &\quad + \int_{y_0}^y k^{\Delta_2}(x, y, \sigma(x), \xi) v(x, \xi) \Delta \xi \\ &\quad + \int_{x_0}^x \int_{y_0}^y k^{\Delta_2 \Delta_1}(x, y, \tau, \xi) v(\tau, \xi) \Delta \xi \Delta \tau \\ &\leq |f(x, y) + A(x, y)| v(x, y), \end{aligned} \quad (20)$$

where $A(x, y)$ is defined by (17).

Using the fact that $z(x, y) > 0$, $z^{\Delta_1}(x, y) \geq 0$, $z^{\Delta_2}(x, y) \geq 0$, and we have

$$\frac{v^{\Delta_2 \Delta_1}(x, y)}{v(x, y)} \leq [f(x, y) + A(x, y)] + \frac{v^{\Delta_1}(x, y) v^{\Delta_2}(x, y)}{v(x, y) v(x, \sigma(y))}. \quad (21)$$

Now keeping x fixed and delta integrating with respect to t from y_0 to y , we obtain estimates

$$\frac{v^{\Delta_1}(x, y)}{v(x, y)} \leq \int_{y_0}^y [f(x, \xi) + A(x, y)] \Delta_2 \xi, \quad (22)$$

keeping y fixed in (22) and delta integrating s from x_0 to x , we get

$$v(x, y) \leq c + \int_{x_0}^x \int_{y_0}^y [f(\tau, \xi) + A(\tau, \xi)] v(x, y) \Delta_2 \xi \Delta_1 \tau. \quad (23)$$

Then Lemma 2 implies

$$v(x, y) \leq c + e^{\int_{y_0}^y [f(\tau, \xi) + A(\tau, \xi)] \Delta \xi} (x, x_0). \quad (24)$$

Using (24) in (18) and integrating the resulting inequality from 0 to y and then from 0 to x for $(x, y) \in \Omega$, we get

$$z(x, y) \leq c \left[1 + \int_{x_0}^x \int_{y_0}^y f(s, \tau) e^{\int_{t_0}^t [f(\tau, \xi) + A(\tau, \xi)] \Delta \xi \Delta \tau} (s, s_0) \Delta t \Delta s \right]. \quad (25)$$

Using (25) in $u(x, y) \leq z(x, y)$, we get the required inequality in (16). $\square$

Theorem 5. Let $u, f, a \in C_{rd}(\Omega \times \Omega, R_+)$ and $k(x, y, s, t), k^{\Delta_1}(x, y, s, t), k^{\Delta_2}(x, y, s, t), k^{\Delta_2 \Delta_1}(x, y, s, t) \in C_{rd}(\Omega, R_+)$. If

$$\begin{aligned} u(x, y) &\leq a(x, y) + \int_{x_0}^x \int_{y_0}^y f(s, t) \\ &\quad \times \left[u(s, t) + \int_{s_0}^s \int_{t_0}^t k(s, t, \tau, \xi) u(\tau, \xi) \Delta \xi \Delta \tau \right] \Delta t \Delta s, \end{aligned} \quad (26)$$

For $(x, y) \in \Omega$, then

$$\begin{aligned} u(x, y) &\leq a(x, y) + \bar{h}(x, y) \\ &\quad \times \left[1 + \int_{x_0}^x \int_{y_0}^y f(s, t) e^{\int_{t_0}^t [f(\tau, \xi) + A(\tau, \xi)] \Delta \xi \Delta \tau} (x, x_0) \right] \Delta t \Delta s, \end{aligned} \quad (27)$$

where

$$\begin{aligned} \bar{h}(x, y) &= \int_{x_0}^x \int_{y_0}^y f(s, t) \\ &\quad \times \left[a(s, t) + \int_{s_0}^s \int_{t_0}^t k(s, t, \sigma(x), \xi) a(\tau, \xi) \Delta \xi \Delta \tau \right] \Delta t \Delta s, \end{aligned} \quad (28)$$

for $(x, y) \in \Omega$ and $A(x, y)$ as defined in (17).

Proof. Define a function $z(x, y)$ by the right-hand side of (26)

$$\begin{aligned} z(x, y) &\leq \int_{x_0}^x \int_{y_0}^y f(s, t) \\ &\quad \times \left[u(s, t) + \int_{s_0}^s \int_{t_0}^t k(s, t, \tau, \xi) u(\tau, \xi) \Delta \xi \Delta \tau \right] \Delta t \Delta s, \end{aligned} \quad (29)$$

then from (26) we have $u(x, y) \leq a(x, y) + z(x, y)$, and using this in (28) we get

$$\begin{aligned}
 & z(x, y) \\
 & \leq \int_{x_0}^x \int_{y_0}^y f(s, t) \left[a(s, t) + z(s, t) \right. \\
 & \quad \left. + \int_{s_0}^s \int_{t_0}^t k(s, t, \tau, \xi) \right. \\
 & \quad \left. \times [a(\tau, \xi) + z(\tau, \xi)] \Delta \xi \Delta \tau \right] \Delta t \Delta s \\
 & \leq \bar{h}(x, y) + \int_{x_0}^x \int_{y_0}^y f(s, t) \left[z(s, t) + \int_{s_0}^s \int_{t_0}^t k(s, t, \tau, \xi) \right. \\
 & \quad \left. \times z(\tau, \xi) \Delta \xi \Delta \tau \right] \Delta t \Delta s,
 \end{aligned} \tag{30}$$

where $\bar{h}(x, y)$ is defined by (28). Here $\bar{h}(x, y)$ nondecreasing for $(x, y) \in \Omega$, then from (30) we have

$$\begin{aligned}
 & \frac{z(x, y)}{\bar{h}(x, y)} \\
 & \leq 1 + \int_{x_0}^x \int_{y_0}^y f(s, t) \\
 & \quad \times \left[\frac{z(s, t)}{\bar{h}(s, t)} + \int_{s_0}^s \int_{t_0}^t k(s, t, \tau, \xi) \frac{z(\tau, \xi)}{\bar{h}(\tau, \xi)} \Delta \xi \Delta \tau \right] \Delta t \Delta s.
 \end{aligned} \tag{31}$$

Now as an application of inequality in Theorem 3 to (31) we get

$$\frac{z(x, y)}{\bar{h}(x, y)} \leq \left[1 + \int_{x_0}^x \int_{y_0}^y f(s, t) e_{[f(\tau, \xi) + A(\tau, \xi)] \Delta \xi \Delta \tau} (x, x_0) \right] \Delta t \Delta s. \tag{32}$$

The required inequality in (27) follows from (32) and using the fact that $u(x, y) \leq a(x, y) + z(x, y)$. $\square$

3. Applications

In this section, we give application of inequality in Theorem 4 to study certain properties of solution of nonlinear partial integrodifferential equation on time scales,

$$\begin{aligned}
 & u^{\Delta_2 \Delta_1}(x, y) \\
 & = F\left(x, y, u(x, y), \int_{x_0}^x \int_{y_0}^y h(x, y, \tau, \xi, u(\tau, \xi)) \Delta \xi \Delta \tau\right),
 \end{aligned} \tag{33}$$

with the initial boundary condition

$$\begin{aligned}
 & u(x, y_0) = \alpha_1(x), \\
 & u(x_0, y) = \alpha_2(y), \\
 & \alpha_1(0) = \alpha_2(0) = 0,
 \end{aligned} \tag{34}$$

where $u \in C_{rd}(\Omega, R_+)$, $h \in C_{rd}(\Omega \times R, R_+)$, and $F \in C_{rd}(\Omega \times \Omega, R_+)$.

The following theorem deals with the estimate on the solution of (33)-(34).

Theorem 6. Assume that

$$\begin{aligned}
 & |h(x, y, s, t, u(s, t))| \leq k(x, y, s, t) |u(s, t)|, \\
 & |F(x, y, u, v)| \leq f(x, y) [|u| + |v|], \\
 & |\alpha_1(x) + \alpha_2(y)| \leq c,
 \end{aligned} \tag{35}$$

where k, f , and c are as defined in (9). If $u(x, y)$, $(x, y) \in \Omega$ is any solution of (33)-(34), then

$$\begin{aligned}
 & |u(x, y)| \\
 & \leq c \left[1 + \int_{x_0}^x \int_{y_0}^y f(s, t) e_{\int_{t_0}^t [f(\tau, \xi) + A(\tau, \xi)] \Delta t} (s, s_0) \Delta t \Delta s \right]
 \end{aligned} \tag{36}$$

for $(x, y) \in \Omega$, where $A(x, y)$ is defined by (17).

Proof. The solution $u(x, y)$ of (33)-(34) can be written as

$$\begin{aligned}
 & u(x, y) = \alpha_1(x) + \alpha_2(y) \\
 & \quad + \int_{x_0}^x \int_{y_0}^y F\left(s, t, u(s, t), \right. \\
 & \quad \left. \int_{s_0}^s \int_{t_0}^t h(s, t, \tau, \xi, u(\tau, \xi)) \Delta \xi \Delta \tau \right) \Delta t \Delta s.
 \end{aligned} \tag{37}$$

Using (35) in (37), we have

$$\begin{aligned}
 & |u(x, y)| \\
 & \leq c + \int_{x_0}^x \int_{y_0}^y f(s, t) \\
 & \quad \times \left[|u(s, t)| + \left(\int_{s_0}^s \int_{t_0}^t k(s, t, \tau, \xi, |u(\tau, \xi)|) \right. \right. \\
 & \quad \left. \left. \times \Delta \xi \Delta \tau \right) \right] \Delta t \Delta s.
 \end{aligned} \tag{38}$$

Now an application of inequality in Theorem 4 yields the estimate in (36). $\square$

In the next result, we give uniqueness of solutions of (33)-(34).

Theorem 7. Suppose that the functions h, F in (33) satisfy the conditions

$$\begin{aligned} |h(x, y, s, t, u_1) - h(x, y, s, t, u_2)| &\leq k(x, y, s, t) |u_1 - u_2|, \\ |F(x, y, u_1, u_2) - F(x, y, v_1, v_2)| \\ &\leq f(x, y) [|u_1 - v_1| + |u_2 - v_2|], \end{aligned} \quad (39)$$

where k and f are as in Theorem 4. Then the problem (33)-(34) has at most one solution on Ω .

Proof. Let $u_1(x, y)$ and $u_2(x, y)$ be two solutions of (33)-(34) on Ω , then we have

$$\begin{aligned} u_1(x, y) - u_2(x, y) &= \int_{x_0}^x \int_{y_0}^y \left[F\left(s, t, u_1(s, t), \int_{s_0}^s \int_{t_0}^t h(s, t, \tau, \xi, u_1(\tau, \xi)) \Delta\xi \Delta\tau\right) \right. \\ &\quad \left. - F\left(s, t, u_2(s, t), \int_{s_0}^s \int_{t_0}^t h(s, t, \tau, \xi, u_2(\tau, \xi)) \right. \right. \\ &\quad \left. \left. \times \Delta\xi \Delta\tau\right) \right] \Delta t \Delta s. \end{aligned} \quad (40)$$

From (39) and (40), we have

$$\begin{aligned} |u_1(x, y) - u_2(x, y)| &\leq \int_{x_0}^x \int_{y_0}^y \left[f(s, t) |u_1(s, t) - u_2(s, t)| \right. \\ &\quad \left. + \int_{s_0}^s \int_{t_0}^t k(s, t, \tau, \xi) |u_1(\tau, \xi) - u_2(\tau, \xi)| \right. \\ &\quad \left. \times \Delta\xi \Delta\tau \right] \Delta t \Delta s. \end{aligned} \quad (41)$$

As an application of inequality in Theorem 4 with $c = 0$ yields $|u_1(x, y) - u_2(x, y)| \leq 0$, therefore $u_1(x, y) = u_2(x, y)$; that is, there is at most one solution of (33)-(34) on Ω . $\square$

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