Research Article

Weak Contraction Condition Involving Cubic Terms of $d(x, y)$ under the Fixed Point Consideration

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A fixed point theorem is presented for single-valued map with using generalized φ -weak contractive condition involving various combinations of $d(x, y)$ on a complete metric space. Our result is an extension as well as a generalization of Alber and Guerre-Delabriere (1997) in particular. It also generalizes the results of Rhoades (2001), Choudhury and Dutta, (2000), and Dutta and Choudhury, (2008).

1. Introduction

Let (X, d) be a metric space. A map $T : X \rightarrow X$ is a contraction if, for each $x, y \in X$, there exists a constant $k \in (0, 1)$ such that $d(Tx, Ty) \leq k d(x, y)$.

A map $T : X \rightarrow X$ is a φ -weak contraction if, for each $x, y \in E$, there exists a function $\varphi : [0, \infty) \rightarrow [0, \infty)$, $\varphi(t) > 0$ for all $t > 0$ and $\varphi(0) = 0$ such that $d(Tx, Ty) \leq d(x, y) - \varphi(d(x, y))$.

In [1] Alber and Guerre-Delabriere introduced the concept of weak contraction in Hilbert spaces. Rhoades [2] has shown that the result which Alber and Guerre-Delabriere had proved in [2] is also valid in complete metric spaces.

In this paper we introduced generalized φ -weak contractive condition involving various combinations of $d(x, y)$. Our result is an extension as well as a generalization of Alber and Guerre-Delabriere [1] and Rhoades [2] in particular. It also generalizes the results of [3, 4].

Now, We state the result of Rhoades as follows.

Theorem 1 (see [2, Theorem 2]). *Let (X, d) be a complete metric space, and let T be a φ -weak contraction on X $\varphi : [0, +\infty) \rightarrow [0, +\infty)$ which is a continuous and nondecreasing function with $\varphi(t) > 0$ for all $t \in (0, \infty)$ and $\varphi(0) = 0$ then T has a unique fixed point in X .*

If one takes $\varphi(t) = (1 - k)t$, where $0 < k < 1$, then weak contraction reduces to contraction mapping.

In this paper, a new type of inequality is introduced with cubic terms involving $p(\geq 0)$ called a “generalized φ -weak contractive condition with cubic terms involving p ”.

Let (X, d) be a metric space and T a self-map of X satisfying the following condition:

$$\begin{aligned} & [1 + p d(x, y)] d^2(Tx, Ty) \\ & \leq p \max \left\{ \frac{1}{2} [d^2(x, Tx) d(y, Ty) + d(x, Tx) d^2(y, Ty)], \right. \\ & \quad d(x, Tx) d(x, Ty) d(y, Tx), \\ & \quad \left. d(x, Ty) d(y, Tx) d(y, Ty) \right\} \\ & \quad + m(x, y) - \varphi(m(x, y)), \end{aligned} \tag{1}$$

where

$$\begin{aligned} m(x, y) & = \max \left\{ d^2(x, y), d(x, Tx) d(y, Ty), d(x, Ty) d(y, Tx), \right. \\ & \quad \left. \frac{1}{2} [d(x, Tx) d(x, Ty) + d(y, Tx) d(y, Ty)] \right\} \end{aligned} \tag{2}$$

$p \geq 0$ is a real number and $\varphi : [0, \infty) \rightarrow [0, \infty)$ is a continuous function with $\varphi(t) = 0 \Leftrightarrow t = 0$ and $\varphi(t) > 0$ for each $t > 0$.

2. Main Result

Lemma 2. Let T be a self map of a metric space X satisfying (1). For any sequence $\{x_n\}$ in X defined by $x_{n+1} = Tx_n$, $n \geq 0$. Then the sequence $\{x_n\}$ is Cauchy in X .

Proof. Let $x_0 \in X$ be an arbitrary point. Constructing the sequence $\{x_n\}$ follows

$$x_{n+1} = Tx_n, \quad n \in N_0. \quad (3)$$

If $x_n = x_{n+1}$ for some n , then trivially T has a fixed point. We assume $x_{n+1} \neq x_n$, for all $n \in N_0$. We write $\alpha_n = d(x_n, x_{n+1})$.

First, we prove that $\{\alpha_n\}$ is a nonincreasing sequence and converges to 0.

Case 1. If n is even, taking $x = x_{2n}$ and $y = x_{2n+1}$ in (1), we get

$$\begin{aligned} & [1 + p d(x_{2n}, x_{2n+1})] d^2(Tx_{2n}, Tx_{2n+1}) \\ & \leq p \max \left\{ \frac{1}{2} [d^2(x_{2n}, Tx_{2n}) d(x_{2n+1}, Tx_{2n+1}) \right. \\ & \quad \left. + d(x_{2n}, Tx_{2n}) d^2(x_{2n+1}, Tx_{2n+1})], \right. \\ & \quad d(x_{2n}, Tx_{2n}) d(x_{2n}, Tx_{2n+1}) \\ & \quad \times d(x_{2n+1}, Tx_{2n}), d(x_{2n}, Tx_{2n+1}) \\ & \quad \times d(x_{2n+1}, Tx_{2n}) d(x_{2n+1}, Tx_{2n+1}) \left. \right\} \\ & \quad + m(x_{2n}, x_{2n+1}) - \varphi(m(x_{2n}, x_{2n+1})), \end{aligned} \quad (4)$$

where

$$\begin{aligned} & m(x_{2n}, x_{2n+1}) \\ & = \max \{d^2(x_{2n}, x_{2n+1}), d(x_{2n}, Tx_{2n}) d(x_{2n+1}, Tx_{2n+1}), \\ & \quad d(x_{2n}, Tx_{2n+1}) d(x_{2n+1}, Tx_{2n}), \\ & \quad \frac{1}{2} [d(x_{2n}, Tx_{2n}) d(x_{2n}, Tx_{2n+1}) \\ & \quad + d(x_{2n+1}, Tx_{2n}) d(x_{2n+1}, Tx_{2n+1})] \left. \right\}. \end{aligned} \quad (5)$$

By using (3), we get

$$\begin{aligned} & [1 + p d(x_{2n}, x_{2n+1})] d^2(x_{2n+1}, x_{2n+2}) \\ & \leq p \max \left\{ \frac{1}{2} [d^2(x_{2n}, x_{2n+1}) d(x_{2n+1}, x_{2n+2}) \right. \\ & \quad \left. + d(x_{2n}, x_{2n+1}) d^2(x_{2n+1}, x_{2n+2})] \right\}, \end{aligned}$$

$$\begin{aligned} & d(x_{2n}, x_{2n+1}) d(x_{2n}, x_{2n+2}) \\ & \quad \times d(x_{2n+1}, x_{2n+1}), d(x_{2n}, x_{2n+2}) \\ & \quad \times d(x_{2n+1}, x_{2n+1}) d(x_{2n+1}, x_{2n+2}) \left. \right\} \\ & \quad + m(x_{2n}, x_{2n+1}) - \varphi(m(x_{2n}, x_{2n+1})), \end{aligned} \quad (6)$$

where

$$\begin{aligned} & m(x_{2n}, x_{2n+1}) \\ & = \max \{d^2(x_{2n}, x_{2n+1}), d(x_{2n}, x_{2n+1}) d(x_{2n+1}, x_{2n+2}), \\ & \quad d(x_{2n}, x_{2n+2}) d(x_{2n+1}, x_{2n+1}), \\ & \quad \frac{1}{2} [d(x_{2n}, x_{2n+1}) d(x_{2n}, x_{2n+2}) \\ & \quad + d(x_{2n+1}, x_{2n+1}) d(x_{2n+1}, x_{2n+2})] \left. \right\}. \end{aligned} \quad (7)$$

Now consider $\alpha_{2n} = d(x_{2n}, x_{2n+1})$; then we have

$$\begin{aligned} & [1 + p \alpha_{2n}] \alpha_{2n+1}^2 \\ & \leq p \max \left\{ \frac{1}{2} [\alpha_{2n}^2 \alpha_{2n+1} + \alpha_{2n} \alpha_{2n+1}^2], 0, 0 \right\} \\ & \quad + m(x_{2n}, x_{2n+1}) - \varphi(m(x_{2n}, x_{2n+1})), \end{aligned} \quad (8)$$

where $m(x_{2n}, x_{2n+1}) = \max\{\alpha_{2n}^2, \alpha_{2n} \alpha_{2n+1}, 0, (1/2)[\alpha_{2n} d(x_{2n}, x_{2n+2}) + 0]\}$.

By triangular inequality and using property of φ , we get

$$\begin{aligned} & d(x_{2n}, x_{2n+2}) \leq d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2}) \\ & = \alpha_{2n} + \alpha_{2n+1}. \end{aligned} \quad (9)$$

Then

$$\begin{aligned} & m(x_{2n}, x_{2n+1}) \leq m'(x, y) \\ & = \max \left\{ \alpha_{2n}^2, \alpha_{2n+1} \alpha_{2n}, 0, \frac{1}{2} \alpha_{2n} (\alpha_{2n} + \alpha_{2n+1}), 0 \right\}. \end{aligned} \quad (10)$$

If $\alpha_{2n} < \alpha_{2n+1}$, then (8) reduces to $p \alpha_{2n+1}^2 \leq p \alpha_{2n+1}^2 - \varphi(\alpha_{2n+1}^2)$, a contradiction. Thus $\alpha_{2n+1}^2 \leq \alpha_{2n}^2$ implies that $\alpha_{2n+1} \leq \alpha_{2n}$.

In a similar way, if n is odd, we can obtain $\alpha_{2n+2} < \alpha_{2n+1}$.

It follows that the sequence $\{\alpha_n\}$ is decreasing.

Let $\lim_{n \rightarrow \infty} \alpha_n = r$, for some $r \geq 0$.

Suppose $r > 0$; then from inequality (1), we have

$$\begin{aligned}
 & [1 + p d(x_n, x_{n+1})] d^2(Tx_n, Tx_{n+1}) \\
 & \leq p \max \left\{ \frac{1}{2} [d^2(x_n, Tx_n) d(x_{n+1}, Tx_{n+1}) \right. \\
 & \quad \left. + d(x_n, Tx_n) d^2(x_{n+1}, Tx_{n+1})], \right. \\
 & \quad d(x_n, Tx_n) d(x_n, Tx_{n+1}) \\
 & \quad \times d(x_{n+1}, Tx_n), d(x_n, Tx_{n+1}) \\
 & \quad \left. \times d(x_{n+1}, Tx_n) d(x_{n+1}, Tx_{n+1}) \right\} \\
 & + m(x_n, x_{n+1}) - \varphi(m(x_n, x_{n+1})),
 \end{aligned} \tag{11}$$

where

$$\begin{aligned}
 & m(x_n, x_{n+1}) \\
 & = \max \{ d^2(x_n, x_{n+1}), d(x_n, Tx_n) d(x_{n+1}, Tx_{n+1}), \\
 & \quad d(x_n, Tx_{n+1}) d(x_{n+1}, Tx_n), \\
 & \quad \frac{1}{2} [d(x_n, Tx_n) d(x_n, Tx_{n+1}) \\
 & \quad + d(x_{n+1}, Tx_n) d(x_{n+1}, Tx_{n+1})] \}.
 \end{aligned} \tag{12}$$

By using (3), we get

$$\begin{aligned}
 & [1 + p d(x_n, x_{n+1})] d^2(x_{n+1}, x_{n+2}) \\
 & \leq p \max \left\{ \frac{1}{2} [d^2(x_n, x_{n+1}) d(x_{n+1}, x_{n+2}) \right. \\
 & \quad \left. + d(x_n, x_{n+1}) d^2(x_{n+1}, x_{n+2})], \right. \\
 & \quad d(x_n, x_{n+1}) d(x_n, x_{n+2}) d(x_{n+1}, x_{n+1}), \\
 & \quad \left. d(x_n, x_{n+2}) d(x_{n+1}, x_{n+1}) d(x_{n+1}, x_{n+2}) \right\} \\
 & + m(x_n, x_{n+1}) - \varphi(m(x_n, x_{n+1})),
 \end{aligned} \tag{13}$$

where

$$\begin{aligned}
 & m(x_n, x_{n+1}) \\
 & = \max \{ d^2(x_n, x_{n+1}), d(x_n, x_{n+1}) d(x_{n+1}, x_{n+2}), \\
 & \quad d(x_n, x_{n+2}) d(x_{n+1}, x_{n+1}), \\
 & \quad \frac{1}{2} [d(x_n, x_{n+1}) d(x_n, x_{n+2}) \\
 & \quad + d(x_{n+1}, x_{n+1}) d(x_{n+1}, x_{n+2})] \}.
 \end{aligned} \tag{14}$$

Using triangular inequality and property of φ and taking limits $n \rightarrow \infty$, we get

$$[1 + pr] r^2 \leq pr^3 + r^2 - \varphi(r^2). \tag{15}$$

Then $\varphi(r^2) \leq 0$, since r is positive; then by the property of φ , we get $r = 0$. We conclude that

$$\lim_{n \rightarrow \infty} \alpha_n = \lim_{n \rightarrow \infty} d(x_n, x_{n+1}) = r = 0. \tag{16}$$

Now we show that $\{x_n\}$ is a Cauchy sequence. Suppose, we assume that $\{x_n\}$ is not a Cauchy sequence; then there is $\epsilon > 0$ for which we can find two sequences of positive integers $\{m(k)\}$ and $\{n(k)\}$ such that for all positive integers k , $n(k) > m(k) > k$

$$d(x_{m(k)}, x_{n(k)}) \geq \epsilon, \quad d(x_{m(k)}, x_{n(k)-1}) < \epsilon. \tag{17}$$

Now,

$$\epsilon \leq d(x_{m(k)}, x_{n(k)}) \leq d(x_{m(k)}, x_{n(k)-1}) + d(x_{n(k)-1}, x_{n(k)}). \tag{18}$$

Letting $k \rightarrow \infty$, we get

$$\lim_{k \rightarrow \infty} d(x_{m(k)}, x_{n(k)}) = \epsilon. \tag{19}$$

Now, from the triangular inequality, we have

$$|d(x_{n(k)}, x_{m(k)+1}) - d(x_{m(k)}, x_{n(k)})| \leq d(x_{m(k)}, x_{m(k)+1}). \tag{20}$$

Taking limits as $k \rightarrow \infty$ and using (16) and (19), we have

$$\lim_{k \rightarrow \infty} d(x_{n(k)}, x_{m(k)+1}) = \epsilon. \tag{21}$$

Again from the triangular inequality, we have

$$|d(x_{m(k)}, x_{n(k)+1}) - d(x_{m(k)}, x_{n(k)})| \leq d(x_{n(k)}, x_{n(k)+1}). \tag{22}$$

Taking limits as $k \rightarrow \infty$ and using (16) and (19), we have

$$\lim_{k \rightarrow \infty} d(x_{m(k)}, x_{n(k)+1}) = \epsilon. \tag{23}$$

Again by using triangular inequality, we have

$$\begin{aligned}
 & |d(x_{m(k)+1}, x_{n(k)+1}) - d(x_{m(k)}, x_{n(k)})| \\
 & \leq d(x_{m(k)}, x_{m(k)+1}) + d(x_{n(k)}, x_{n(k)+1}).
 \end{aligned} \tag{24}$$

Taking limit $k \rightarrow \infty$ in the above inequality and using (16) and (19), we have

$$\lim_{k \rightarrow \infty} d(x_{n(k)+1}, x_{m(k)+1}) = \epsilon. \tag{25}$$

Again putting $x = x_{m(k)}$ and $y = x_{n(k)}$ in (1), we get

$$\begin{aligned}
 & [1 + p d(x_{m(k)}, x_{n(k)})] d^2(Tx_{m(k)}, Tx_{n(k)}) \\
 & \leq p \left\{ \frac{1}{2} [d^2(x_{m(k)}, Tx_{m(k)}) d(x_{n(k)}, Tx_{n(k)}) \right. \\
 & \quad \left. + d(x_{m(k)}, Tx_{m(k)}) d^2(x_{n(k)}, Tx_{n(k)})], \right. \\
 & \quad d(x_{m(k)}, Tx_{m(k)}) d(x_{m(k)}, Tx_{n(k)}) d(x_{n(k)}, Tx_{m(k)}), \\
 & \quad \left. d(x_{m(k)}, Tx_{n(k)}) d(x_{n(k)}, Tx_{m(k)}) d(x_{n(k)}, Tx_{n(k)}) \right\} \\
 & + m(x_{m(k)}, x_{n(k)}) - \varphi(m(x_{m(k)}, x_{n(k)})),
 \end{aligned} \tag{26}$$

where

$$\begin{aligned}
 & m(x_{m(k)}, x_{n(k)}) \\
 &= \max \left\{ d^2(x_{m(k)}, x_{n(k)}), \right. \\
 &\quad d(x_{m(k)}, Tx_{m(k)}) d(x_{n(k)}, Tx_{n(k)}), \\
 &\quad d(x_{m(k)}, Tx_{n(k)}) d(x_{n(k)}, Tx_{m(k)}), \\
 &\quad \frac{1}{2} [d(x_{m(k)}, Tx_{m(k)}) d(x_{m(k)}, Tx_{n(k)}) \\
 &\quad \left. + d(x_{n(k)}, Tx_{m(k)}) d(x_{n(k)}, Tx_{n(k)})] \right\}. \quad (27)
 \end{aligned}$$

Using (3), then we obtain

$$\begin{aligned}
 & [1 + pd(x_{m(k)}, x_{n(k)})] d^2(x_{m(k)+1}, x_{n(k)+1}) \\
 &\leq p \left\{ \frac{1}{2} [d^2(x_{m(k)}, x_{m(k)+1}) d(x_{n(k)}, x_{n(k)+1}) \right. \\
 &\quad \left. + d(x_{m(k)}, x_{m(k)+1}) d^2(x_{n(k)}, x_{n(k)+1})] \right. \\
 &\quad d(x_{m(k)}, x_{m(k)+1}) d(x_{m(k)}, x_{n(k)+1}) \\
 &\quad \times d(x_{n(k)}, x_{m(k)+1}) d(x_{m(k)}, x_{n(k)+1}) \\
 &\quad \left. \times d(x_{n(k)}, x_{m(k)+1}) d(x_{n(k)}, x_{n(k)+1}) \right\} \\
 &\quad + m(x_{m(k)}, x_{n(k)}) - \varphi(m(x_{m(k)}, x_{n(k)})), \quad (28)
 \end{aligned}$$

where

$$\begin{aligned}
 & m(x_{m(k)}, x_{n(k)}) \\
 &= \max \left\{ d^2(x_{m(k)}, x_{n(k)}), \right. \\
 &\quad d(x_{m(k)}, x_{m(k)+1}) d(x_{n(k)}, x_{n(k)+1}), \\
 &\quad d(x_{m(k)}, x_{n(k)+1}) d(x_{n(k)}, x_{m(k)+1}), \\
 &\quad \frac{1}{2} [d(x_{m(k)}, x_{m(k)+1}) d(x_{m(k)}, x_{n(k)+1}) \\
 &\quad \left. + d(x_{n(k)}, x_{m(k)+1}) d(x_{n(k)}, x_{n(k)+1})] \right\}. \quad (29)
 \end{aligned}$$

Letting $k \rightarrow \infty$ and using (16)–(25), we get

$$\begin{aligned}
 & [1 + p\epsilon] \epsilon^2 \\
 &\leq p \max \left\{ \frac{1}{2} [0 + 0], 0, 0 \right\} + \epsilon^2 - \varphi(\epsilon^2) \\
 &= \epsilon^2 - \varphi(\epsilon^2), \quad (30)
 \end{aligned}$$

a contradiction. Thus $\{x_n\}$ is a Cauchy in X . $\square$

Theorem 3. Let T be a self-map of a complete metric space X satisfying (1). Then T has a unique fixed point in X .

Proof. From Lemma 2 the sequence $\{x_n\}$ is a Cauchy in X . Since (X, d) is a complete metric space, then there exists a point $z \in X$ such that

$$\lim_{n \rightarrow \infty} x_n = z. \quad (31)$$

Now we prove that z is a fixed point of T .

Taking $x = x_n$ and $y = z$ in (1), we have

$$\begin{aligned}
 & [1 + pd(x_n, z)] d^2(Tx_n, Tz) \\
 &\leq p \max \left\{ \frac{1}{2} [d^2(x_n, Tx_n) d(z, Tz) \right. \\
 &\quad \left. + d(x_n, Tx_n) d^2(z, Tz)] \right. \\
 &\quad d(x_n, Tx_n) d(x_n, Tz) d(z, Tx_n), \\
 &\quad \left. d(x_n, Tz) d(z, Tx_n) d(z, Tz) \right\} \\
 &\quad + m(x_n, z) - \varphi(m(x_n, z)), \quad (32)
 \end{aligned}$$

where

$$\begin{aligned}
 & m(x_n, z) \\
 &= \max \left\{ d^2(x_n, z), \right. \\
 &\quad d(x_n, Tx_n) d(z, Tz), d(x_n, Tz) d(z, Tx_n), \\
 &\quad \frac{1}{2} [d(x_n, Tx_n) d(x_n, Tx_n) \\
 &\quad \left. + d(z, Tx_n) d(z, Tz)] \right\}. \quad (33)
 \end{aligned}$$

Using (31) and (3), we get

$$\begin{aligned}
 & [1 + pd(z, z)] d^2(z, Tz) \\
 &\leq p \max \left\{ \frac{1}{2} [d^2(z, z) d(z, Tz) + d(z, z) d^2(z, Tz)] \right. \\
 &\quad d(z, z) d(z, Tz) d(z, z), \\
 &\quad \left. d(z, Tz) d(z, z) d(z, Tz) \right\} \\
 &\quad + m(z, z) - \varphi(m(z, z)). \quad (34)
 \end{aligned}$$

Hence $d^2(z, Tz) \leq 0 \Rightarrow Tz = z$.

Then T has a fixed point in X .

To prove the uniqueness of the fixed point, we assume that z_1 and z_2 are two fixed points of T . Taking $x = z_1$ and $y = z_2$ in (1), we easily get $d(z_1, z_2) = 0$ which implies that $z_1 = z_2$. Therefore T has a unique fixed point in X . $\square$

Corollary 4. Let T be a self-map of a complete metric space X satisfying the condition

$$d^2(Tx, Ty) \leq m(x, y) - \varphi(m(x, y)), \quad (35)$$

where

$$\begin{aligned}
 m(x, y) &= \max \left\{ d^2(x, y), d(x, Tx) d(y, Ty), \right. \\
 &\quad \left. d(x, Ty) d(y, Tx), \right. \\
 &\quad \left. \frac{1}{2} [d(x, Tx) d(x, Ty) + d(y, Tx) d(y, Ty)] \right\} \\
 &\quad (36)
 \end{aligned}$$

for all $x, y \in X$ and $\varphi : [0, \infty) \rightarrow [0, \infty)$ is a continuous function with $\varphi(t) = 0 \Leftrightarrow t = 0$ and $\varphi(t) > 0$ for each $t > 0$. Then T has a unique fixed point in X .

Proof. $p = 0$ in Theorem 3; we have the result. $\square$

Now we give an example to support our result.

Example 5. Let $X = \{0, 1, 2\}$, and let d be the usual metric on X . Let $T : X \rightarrow X$ be defined by $T0 = T1 = 0$ and $T2 = 1$. And define $\varphi : [0, \infty) \rightarrow [0, \infty)$ by $\varphi(t) = t/2$. For any value of $p > 0$ and $x, y \in X$, then it is easy to verify that inequality (1) holds. Hence Theorem 3 holds well.

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