Hindawi Publishing Corporation
Journal of Mathematics
Volume 2016, Article ID 7308609, 11 pages
http://dx.doi.org/10.1155/2016/7308609

Research Article

Stability Analysis of the Periodic Solutions of Some Kinds of Predator-Prey Dynamical Systems

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Received 29 September 2016; Accepted 18 October 2016

Academic Editor: Ram Jiwari

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Analysis of predator-prey dynamical systems that have the functional response which generalizes the other types of functional responses in two dimensions is mainly studied in this paper. The main problems for this study are to detect the if and only if conditions for attaining the periodic solution of the considered system and to find the condition for global asymptotic stability of this solution for some different types of predator-prey systems that are obtained from that system. To get the desired results, some aspects of semigroup theory for stability analysis and coincidence degree theory are used.

1. Introduction

Predator-prey dynamic systems mainly investigate the relationships between the species and the relation of the species with the outer environment. Analysis of such kind of mathematical models is really important because, by using these analytical results, one can see the future of the species. The analytical results for these systems change according to two main issues. The first one is the functional response, which shows the effect of predator on prey and the effect of prey on predator. The second one is being in the periodic environment. In this study, by using these two issues, the necessary and sufficient conditions to have globally attractive or globally asymptotically stable solution for some different types of predator-prey systems are found.

First, let us give some information about the meaning of functional response and the types of functional responses. As it is remembered above, functional responses show how and how much predator gets benefit from the prey and how and how much prey is affected by predator. There are many types of functional responses. Some of them are Holling type, Beddington-DeAngelis type, ratio type, monotype, semiratio type, and so on. The following studies are some examples about predator-prey models with Holling type functional response: [1–3]. Studies that are about Beddington-DeAngelis type functional response are [4–9]. Predator-prey systems with other types of functional responses are studied in [10–13]. In this study, the main aim is to formulize the functional responses in a general form. In other words, the essential object for this study is to be able to express different types of functional responses in one functional response. Therefore, the dynamical properties that are valid for this system become also valid and useful for the other types.

The second issue in that study is to be in a periodic environment. In nature, periodicity can be seen in many different circumstances. For example, many animals ovulate periodically or many insects have periodic life cycle. Therefore, the analysis of the predator-prey dynamic system in a periodic environment is very significant. On the other hand, in population growth model, the significantly studied problem is the stability and global existence of a positive periodic solution in that system. In an autonomous model, the globally stable equilibrium point is the same as the notion globally asymptotically stable positive periodic solution in a nonautonomous system. Hence, the main problem in this study is to determine under which conditions globally attractive positive periodic solution is attained for the systems with different functional responses that are obtained from the system with generalized functional response. Additionally the importance of this issue can be seen from previous studies, since this subject is investigated in those ones. Some of the examples can be given as follows: [3, 10–20].

In study [21], some kinds of impulsive predator-prey systems have been investigated on time scales. Nevertheless, the functional response that affects predator is not used in [21]. Only its effect on prey is considered. Hence, the results of that paper only related to generalized type of semiratio dependent functional responses. However, in this study, the main aim is to obtain a model with a generalized functional response both on prey and on predator. Because of the above reason, this study has important contributions.

On the other hand, the systems with Holling type I and II functional responses which are obtained from the system with generalized functional response are also studied. For the system with Holling type I functional responses, in paper [22], an if and only if condition has been found to obtain the permanence of the given system. However, by using the main result of our study, an if and only if condition can be given for the globally asymptotically stable periodic solution of the considered system. Also in the study of [22], in the predator part, to bound the solution of predator from above an extra term has been used, but we can get rid of that extra term in this study. These are some points that show the importance of our study.

As a result, the primary objective of that study is to generalize the functional responses that act both on predator and on prey. Additionally, the second aim is to find the if and only if condition for the globally asymptotically stable periodic solution of the considered systems with some different types of functional responses.

2. Preliminaries

As preliminary, we use Definition 1, Lemma 1, Theorem 1, and necessary information that is needed for Theorem 1 from [4].

3. Main Result

The following is the main equation for this study:

$$\begin{aligned}\tilde{x}'(t) &= a(t)\tilde{x}(t) - b(t)\tilde{x}^2(t) \\ &\quad - \Phi_1(t, \tilde{x}(t), \tilde{y}(t))\tilde{y}(t), \\ \tilde{y}'(t) &= -d(t)\tilde{y}(t) + \Phi_2(t, \tilde{x}(t), \tilde{y}(t))\tilde{x}(t).\end{aligned} \tag{1}$$

In system (1), Φ_1 satisfies the following: $\Phi_1(t, 0, y) = 0$, is continuous and w-periodic, and satisfies the following inequality:

$$\begin{aligned}\gamma_0(t)x^{m_1}(t) + \cdots + \gamma_{m_1-1}(t)x(t) &\le \Phi_1(t, x(t), y(t)) \\ &\le \alpha_0(t)x^{m_1}(t) + \cdots + \alpha_{m_1-1}(t)x(t).\end{aligned} \tag{2}$$

Assumptions on Φ_2 are as follows: $\Phi_2(t, x, 0) = 0$, is continuous and w-periodic, and satisfies the following inequality:

$$\Phi_2(t, x(t), y(t)) \le \beta_0(t)y^{m_2} + \cdots + \beta_{m_2-1}(t)y(t). \tag{3}$$

For the above inequalities, α_i, β_j, and γ_j are positive, w-periodic coefficient functions for $i = 1, 2, \ldots, m_1 - 1$ and $j = 1, 2, \ldots, m_2 - 1$. Also, $\Phi_1(t, x(t), y(t))$ and $\Phi_2(t, x(t), y(t)) > 0$ if $x(t)$ and $y(t) > 0$. In addition to these conditions, $\Phi_2(t, \exp(x), \exp(y)) - d\Phi_2(t, \exp(x), \exp(y))/dy > 0$ and $a(t)$, $b(t)$, and $d(t)$ are positive and w-periodic coefficient functions.

(1) $a(t)\tilde{x}(t) - b(t)\tilde{x}^2(t)$ is the specific growth rate of the prey in the absence of predator;

(2) $d(t)$ is the death rate of predator;

(3) $-\Phi_1(t, \tilde{x}(t), \tilde{y}(t))\tilde{y}(t)$ is generalized effect of predator on prey;

(4) $\Phi_2(t, \tilde{x}(t), \tilde{y}(t))\tilde{x}(t)$ is the generalized effect of prey on predator.

Assume that $\tilde{y}(t) = \exp(y(t))$ and $\tilde{x}(t) = \exp(x(t))$. Therefore, system (1) becomes equal to the following system:

$$\begin{aligned}&x'(t) \\ &= a(t) - b(t)\exp(x(t)) \\ &\quad - \Phi_1(t, \exp(x(t)), \exp(y(t)))\exp(y(t) - x(t)), \\ &y'(t) \\ &= -d(t) \\ &\quad + \Phi_2(t, \exp(x(t)), \exp(y(t)))\exp(x(t) - y(t)).\end{aligned} \tag{4}$$

Theorem 1. *In system (4), all of the coefficient functions satisfy the above conditions. System (4) has at least one w-periodic solution if and only if $\int_0^w a(t)dt > 0$ and $\int_0^w d(t)dt > 0$.*

Proof. When there is at least one w-periodic solution for system (4), then it is apparent that $\int_0^w a(t)dt > 0$ and $\int_0^w d(t)dt > 0$. Since

$$\begin{aligned}\int_0^w a(t)\,dt &= \int_0^w b(t)\exp(x(t)) \\ &\quad + \Phi_1(t, \exp(x(t)), \exp(y(t))) \\ &\quad \cdot \exp(y(t) - x(t))\,dt > 0, \\ \int_0^w d(t)\,dt &= \int_0^w \Phi_2(t, \exp(x(t)), \exp(y(t))) \\ &\quad \cdot \exp(x(t) - y(t))\,dt > 0,\end{aligned} \tag{5}$$

now, for the converse part, assume that $\int_0^w a(t)dt, \int_0^w d(t)dt > 0$ and try to find the fact that system (4) has at least one w-periodic solution.

Let X, Y be the normed vector spaces and let the operators L, N be the same as they are defined in the proof of Theorem 2 in [4]. By using the similar proof techniques of Theorem 2 from [4], then for any open bounded set $\Omega \subset X$, we can obtain that N is L-compact on $\overline{\Omega}$.

The below system is investigated by using the application of Theorem 1 from [4]:

$$\begin{aligned} x'(t) &= \lambda\left[a(t) - b(t)\exp(x(t))\right. \\ &\quad \left. - \Phi_1(t, \exp(x(t)), \exp(y(t)))\exp(y(t) - x(t))\right], \\ y'(t) &= \lambda\left[-d(t)\right. \\ &\quad \left. + \Phi_2(t, \exp(x(t)), \exp(y(t)))\exp(x(t) - y(t))\right]. \end{aligned} \tag{6}$$

Let $\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] \in X$ solve system (6). If system (6) is integrated from 0 to w, then

$$\begin{aligned} \int_0^w a(t)\,dt &= \int_0^w b(t)\exp(x(t)) \\ &\quad + \Phi_1(t, \exp(x(t)), \exp(y(t))) \\ &\quad \cdot \exp(y(t) - x(t))\,dt, \\ \int_0^w d(t)\,dt &= \int_0^w \Phi_2(t, \exp(x(t)), \exp(y(t))) \\ &\quad \cdot \exp(x(t) - y(t))\,dt. \end{aligned} \tag{7}$$

By using the first equations of (6) and (7), we get

$$\int_0^w \left|x'(t)\right| dt \le \lambda\left[2\int_0^w a(t)\,dt\right] \le S_1, \tag{8}$$

where $S_1 := 2\int_0^w a(t)dt$.

By using the second equations of (6) and (7), we have

$$\int_0^w \left|y'(t)\right| dt \le \lambda\left[2\int_0^w d(t)\,dt\right] \le S_2; \tag{9}$$

here $S_2 := 2\int_0^w d(t)dt$.

On compact sets, continuous functions attain its maximum and minimum. Therefore, there exist ψ_i, θ_i, $i = 1, 2$, for $\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] \in X$ such that

$$\begin{aligned} x(\theta_1) &= \min_{t\in[0,w]} x(t), \\ x(\psi_1) &= \max_{t\in[0,w]} x(t), \end{aligned} \tag{10}$$

$$\begin{aligned} y(\theta_2) &= \min_{t\in[0,w]} y(t), \\ y(\psi_2) &= \max_{t\in[0,w]} y(t). \end{aligned} \tag{11}$$

The following is obtained by using the first equations of (7) and (11):

$$x(\theta_1) < \tilde{l}_1, \quad \text{where } \tilde{l}_1 := \ln\left(\frac{\int_0^w a(t)\,dt}{\int_0^w b(t)\,dt}\right). \tag{12}$$

Using the first inequality in Lemma 1 from [4] and inequality (8) we have

$$\begin{aligned} x(t) &\le x(\theta_1) + \int_0^w \left|x'(t)\right| dt \\ &\le x(\theta_1) + \left(2\int_0^w a(t)\,dt\right) < P_1 := \tilde{l}_1 + S_1. \end{aligned} \tag{13}$$

Using the first equation of (7), the below result is obtained:

$$\begin{aligned} \int_0^w a(t)\,dt &\ge \int_0^w \left[\gamma_0(t)\exp(x(t))^{m_1} + \cdots\right. \\ &\quad \left. + \gamma_{m_1-1}(t)\exp(x(t))\right]\exp(y(t) - x(t))\,dt \\ &\ge \int_0^w \gamma_{m_1-1}(t)\exp(y(t) - x(t))\,dt \\ &\ge \exp(y(\theta_2))\int_0^w \gamma_{m_1-1}(t)\,dt \ge \exp(y(\theta_2)) \\ &\quad \cdot \int_0^w \gamma_{m_1-1}(t)\,dt. \end{aligned} \tag{14}$$

Therefore,

$$\exp(y(\theta_2)) \le \frac{\int_0^w d(t)\,dt}{\int_0^w \gamma_{m_1-1}(t)\,dt} := \tilde{l}_2. \tag{15}$$

Using the first inequality in Lemma 1 from [4] and inequality (9), we get

$$\begin{aligned} y(t) &\le y(\theta_2) + \int_0^w \left|y'(t)\right| dt \\ &\le y(\theta_2) + \left(2\int_0^w d(t)\,dt\right) < P_2 := \tilde{l}_2 - S_2. \end{aligned} \tag{16}$$

From the second equation of (7), we have

$$\begin{aligned} \int_0^w d(t)\,dt &\le \int_0^w \left[\beta_0(t)\exp(y(t))^{m_2} + \cdots\right. \\ &\quad \left. + \beta_{m_2-1}\exp(y(t))\right]\exp(x(t) - y(t))\,dt \\ &\le \int_0^w \left[\beta_0(t)\exp(y(t))^{m_2-1} + \cdots + \beta_{m_2-1}\right] \\ &\quad \cdot \exp(x(t))\,dt \le \exp(x(\psi_1)) \\ &\quad \cdot \int_0^w \left[\beta_0(t)\exp(P_2)^{m_2-1} + \cdots + \beta_{m_2-1}\right] dt. \end{aligned} \tag{17}$$

Hence,

$$\begin{aligned} &x(\psi_1) \\ &\quad \ge \ln\left(\frac{\int_0^w d(t)\,dt}{\int_0^w \left[\beta_0(t)\exp(P_2)^{m_2-1} + \cdots + \beta_{m_2-1}\right] dt}\right) \\ &\quad := \tilde{l}_3. \end{aligned} \tag{18}$$

Using the second inequality in Lemma 1 from [4] and inequality (8), we have

$$\begin{aligned} x(t) &\geq x(\psi_1) + \int_0^w |x'(t)|\,dt \\ &\geq x(\psi_1) + \left(2\int_0^w a(t)\,dt\right) > P_3 := \tilde{l}_3 + S_1. \end{aligned} \tag{19}$$

Again by using the first equations of (7) and (13), we get

$$\begin{aligned} \int_0^w a(t)\,dt &\leq \int_0^w \left[b(t)\exp(x(t)) \right. \\ &\quad \left. + \alpha_0(t)\exp(x(t))^{m_1} + \cdots + \alpha_{m_1-1}(t)\exp(x(t))\right] \\ &\quad \cdot \exp(y(t)-x(t))\,dt \\ &\leq \int_0^w \left[\alpha_0(t)\exp(x(t))^{m_1-1} + \cdots \right. \\ &\quad \left. + \left(\alpha_{m_1-1}(t)+b(t)\right)\right]\exp(y(t))\,dt \\ &\leq \exp(y(\psi_2))\int_0^w \left[\alpha_0(t)\exp(x(t))^{m_1-1} + \cdots \right. \\ &\quad \left. + \left(\alpha_{m_1-1}(t)+b(t)\right)\right]dt \leq \exp(y(\psi_2)) \\ &\quad \cdot \int_0^w \left[\alpha_0(t)\exp(P_1)^{m_1-1} + \cdots \right. \\ &\quad \left. + \left(\alpha_{m_1-1}(t)+b(t)\right)\right]dt. \end{aligned} \tag{20}$$

Therefore the following is obtained:

$$\begin{aligned} &\exp(y(\psi_2)) \\ &\geq \frac{\int_0^w a(t)\,dt}{\int_0^w \left[\alpha_0(t)\exp(P_1)^{m_1-1} + \cdots + \left(\alpha_{m_1-1}(t)+b(t)\right)\right]dt} \\ &:= \tilde{l}_4. \end{aligned} \tag{21}$$

Using the second inequality in Lemma 1 from [4] and (9), we get

$$\begin{aligned} y(t) &\geq y(\psi_2) - \int_0^w |y'(t)|\,dt \\ &\geq y(\psi_2) - \left(2\int_0^w d(t)\,dt\right) > P_4 := \tilde{l}_4 - S_2. \end{aligned} \tag{22}$$

By (13) and (19), we have $\sup_{t\in[0,w]}|x(t)| \leq C_1 := \max\{|P_1|, |P_3|\}$ and by (16) and (22), we have $\sup_{t\in[0,w]}|y(t)| \leq C_2 := \max\{|P_2|, |P_4|\}$. Here, C_1 and C_2 are independent from λ. Let $S = C_1 + C_2 + 1$. In addition to these,

$$\max_{t\in[0,w]}\left\|\begin{bmatrix} x \\ y \end{bmatrix}\right\| < S. \tag{23}$$

Suppose that $\Omega = \{\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] \in X : \|\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]\| < S\}$ verifies the first requirement of Theorem 1 from [4]. When $\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] \in \operatorname{Ker} L \cap \partial\Omega$, then we have $\|\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]\| = S$, where S is a constant. Thus,

$$QN\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)\left(\begin{bmatrix} \int_0^w a(s) - b(s)\exp(x) - \Phi_1(s,\exp(x),\exp(y))\exp(y-x)\,ds \\ \int_0^w -d(s) + \Phi_2(t,\exp(x),\exp(y))\exp(x-y)\,ds \end{bmatrix}\right). \tag{24}$$

The operator $J : \operatorname{Im} V \to \operatorname{Ker} L$ is taken as the identity operator and the homotopy is defined as

$$H_\nu = \nu(JQN) + (1-\nu)G, \tag{25}$$

where

$$\begin{aligned} &G\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \\ &= \begin{bmatrix} \int_0^w a(s) - b(s)\exp(x)\,ds \\ \int_0^w d(s) - \Phi_2(t,\exp(x),\exp(y))\exp(x-y)\,ds \end{bmatrix}. \end{aligned} \tag{26}$$

Let DJ_G be the determinant of the Jacobian of G. We know that $\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] \in \operatorname{Ker} L$; then the Jacobian of G is

$$\begin{bmatrix} -e^x\int_0^w b(s)\,ds & 0 \\ \int_0^w e^{(x-y)}\left[\Phi_2(t,\exp(x),\exp(y)) + \dfrac{d\Phi_2(t,\exp(x),\exp(y))}{dx}\right]ds & -\int_0^w e^{(x-y)}\left[\Phi_2(t,\exp(x),\exp(y)) - \dfrac{d\Phi_2(t,\exp(x),\exp(y))}{dy}\right]ds \end{bmatrix}. \tag{27}$$

Here sign DJ_G is always positive and this is obtained by the assumption of the theorem. Hence

$$\deg(JN, \Omega \cap \operatorname{Ker} L, 0) = \deg(G, \Omega \cap \operatorname{Ker} L, 0) = \sum_{\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] \in G^{-1}\left(\left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right]\right)} \operatorname{sign} DJ_G\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \neq 0. \tag{28}$$

Thus, all the conditions of Theorem 1 from [4] are satisfied. Therefore, system (1) has at least one positive w-periodic solution. □

3.1. Predator-Prey Systems with Different Types of Functional Responses

3.1.1. Predator-Prey System with Holling Type Functional Response. One of the applications of the above result for Theorem 1 is the Holling type functional response from [1].

System (3.13) from [1] for continuous case is

$$\begin{aligned} \tilde{x}'(t) &= a(t)\tilde{x}(t) - b(t)\tilde{x}^2(t) - \frac{c(t)\tilde{x}(t)\tilde{y}(t)}{1+m(t)\tilde{x}(t)}, \\ \tilde{y}'(t) &= -d(t)\tilde{y}(t) + \frac{f(t)\tilde{y}(t)\tilde{x}(t)}{1+m(t)\tilde{x}(t)}. \end{aligned} \tag{29}$$

When $\tilde{x}(t) = \exp(x(t))$ and $\tilde{y}(t) = \exp(y(t))$, we obtain the transformed form of this system as

$$\begin{aligned} x'(t) &= a(t) - b(t)\exp(x(t)) - \frac{c(t)\exp(y(t))}{1+m(t)\exp(x(t))}, \\ y'(t) &= -d(t) + \frac{f(t)\exp(x(t))}{1+m(t)\exp(x(t))}. \end{aligned} \tag{30}$$

After assuming the system has positive coefficient functions, it is obvious that we can take

$$\begin{aligned} \Phi_1(t, x(t), y(t)) &= \frac{c(t)x(t)}{1+m(t)x(t)}, \\ \Phi_2(t, x(t), y(t)) &= \frac{f(t)y(t)}{1+m(t)x(t)}. \end{aligned} \tag{31}$$

It is also apparent that $\Phi_1(t, x(t), y(t)) \leq c(t)x(t)$ and $\Phi_2(t, x(t), y(t)) \leq f(t)y(t)$. Here if prey does not go to extinction, then there exists $k \in \mathbb{N}$ such that for sufficiently large k, $\Phi_1(t, x(t), y(t)) \geq (1/k)c(t)x(t)$, since $c(t)x(t)/(1+m(t)x(t)) > 0$. Therefore, Theorem 1 has equal statement with the following remark for the system with Holling type functional response.

Remark 2. For system (3.13) from [1] for continuous case, there exists at least one w-periodic solution if and only if the prey does not become extinct.

Remark 3. Here by using proof techniques of Lemma 3 from [4], it can be seen that if predator does not become extinct, then prey also does not become extinct. Therefore, Remark 2 becomes as follows: system (3.13) from [1] for continuous case has w-periodic solution if and only if predator does not become extinct.

Lemma 4. *For system (3.13) from [1] for continuous case, predator becomes extinct if and only if*

$$\int_0^w -d(t) + \frac{f(t)x^*(t)}{1+m(t)x^*(t)} \leq 0, \tag{32}$$

where $x^*(t) = (1-\exp(-\int_0^w a(s)ds))/\int_0^w b(t-s)\exp(-\int_0^s a(t-\tau)d\tau)ds$. *Here* $x^*(t)$ *is the globally attractive unique solution of the system*

$$x(t) = a(t)x(t) - b(t)x^2(t). \tag{33}$$

Proof. Assume that predator goes to extinction. The first equation of the predator-prey system with Holling type functional response is

$$\tilde{x}'(t) = a(t)\tilde{x}(t) - b(t)\tilde{x}^2(t) - \frac{c(t)\tilde{x}(t)\tilde{y}(t)}{1+m(t)\tilde{x}(t)}. \tag{34}$$

Since as t tends to infinity, predator or $\tilde{y}(t) = \exp(y(t))$ tends to zero. Hence, equality (34) tends to the following equation as t tends to infinity:

$$\tilde{x}'(t) = a(t)\tilde{x}(t) - b(t)\tilde{x}^2(t). \tag{35}$$

This means that as t tends to infinity the solution of $\tilde{x}(t)$ tends to $x^*(t)$.

Since predator goes to extinction, when we take $\exp(y(t)) = \tilde{y}(t)$, then from the second equation of this transformed version of the system with Holling type functional response the following inequality is obtained as

$$y(t) = \int_0^w -d(t) + \frac{f(t)\exp(x(t))}{1+m(t)\exp(x(t))} \leq 0. \tag{36}$$

Since t tends to infinity, the solution of $\tilde{x}(t) = \exp(x(t))$ tends to $x^*(t)$. Thus, equality (36) tends to

$$y(t) = \int_0^w -d(t) + \frac{f(t)x^*(t)}{1+m(t)x^*(t)}. \tag{37}$$

Since $\int_0^w -d(t) + f(t)\exp(x(t))/(1+m(t)\exp(x(t))) \leq 0$, then also $\int_0^w -d(t) + f(t)x^*(t)/(1+m(t)x^*(t)) \leq 0$.

For the converse, let us assume that $\int_0^w -d(t) + f(t)x^*(t)/(1+m(t)x^*(t)) \leq 0$. Consider the first equation of the system with Holling type functional response, then we obtain

$$\begin{aligned} \tilde{x}'(t) &= a(t)\tilde{x}(t) - b(t)\tilde{x}^2(t) - \frac{c(t)\tilde{x}(t)\tilde{y}(t)}{1+m(t)\tilde{x}(t)} \\ &\leq a(t)\tilde{x}(t) - b(t)\tilde{x}^2(t). \end{aligned} \tag{38}$$

About the solution of prey, the result $\exp(x(t)) = \tilde{x}(t) \leq x^*(t)$ is obtained by the comparison theorem for ODEs.

Since $\int_0^w -d(t)+f(t)x^*(t)/(1+m(t)x^*(t)) \leq 0$, then $y(t) \leq 0$, which means predator goes to extinction. Hence the proof follows. □

Lemma 5. *In system (3.13) from [1] take $\mathbb{T} = \mathbb{R}$; then this system has at least one w-periodic solution if and only if*

$$\int_0^w -d(t) + \frac{f(t)\,x^*(t)}{1+m(t)\,x^*(t)} > 0, \tag{39}$$

where $x^*(t) = (1-\exp(-\int_0^w a(s)ds))/\int_0^w b(t-s)\exp(-\int_0^s a(t-\tau)d\tau)ds$.

Proof. By using Lemma 4 and Remarks 2 and 3, one can prove Lemma 5. □

The permanence definition is taken from [19].

Corollary 6. *For system (3.13) from [1] for continuous case, the solution is permanent if and only if (39) is satisfied.*

Proof. Let us take (39) as satisfied. Therefore, from Lemma 4, predator does not become extinct and from Remark 3, prey does not become extinct also which means that both $\tilde{x}(t)$ and $\tilde{y}(t)$ are bounded from below with a positive number. From [22] (Proposition 3.1), the solution of the prey $\tilde{x}(t)$ is bounded from above. Assume that (39) is satisfied and $\tilde{y}(t)$ is not bounded from above which means that as t tends to infinity $\tilde{y}(t)$ also tends to infinity. Consider the first equation of transformed version of the system with Holling type functional response, which is

$$\begin{aligned} x'(t) &= a(t) - b(t)\exp(x(t)) \\ &\quad - \frac{c(t)\exp(y(t))}{1+m(t)\exp(x(t))}. \end{aligned} \tag{40}$$

Because of the assumption, as t tends to infinity $\tilde{y}(t) = \exp(y(t))$ also tends to infinity; then also $y(t)$ tends to infinity. Since we have found above that $\tilde{x}(t) = \exp(x(t))$ is bounded from above and below with positive constants, then $x(t)$ is bounded from above and below with constants from real numbers. Hence, for sufficiently large T, $x'(t)$ tends to $-\infty$ as t tends to infinity. Therefore, $\tilde{x}(t)$ tends to 0 as t tends to infinity. This is a contradiction with the boundedness of $\tilde{x}(t)$ from below with a positive constant. Therefore, $\tilde{y}(t)$ is bounded from above. Hence the solution of the considered system is permanent.

For the converse, let us assume that the system is permanent. Then prey and predator do not go to extinction; then by Remark 2 the considered system has at least one w-periodic solution. Thus, by using Lemma 5 obviously, the system satisfies (39). □

Theorem 7. *Assume that (39) holds true. Then, the w-periodic solution of system (3.13) from [1] for continuous case is globally attractive or globally asymptotically stable.*

Proof. Proof is similar with the proof of Lemma 5 in [4]. □

Corollary 8. *Consider the same conditions for the coefficient functions of system (3.13) from [1]. There exists w-periodic globally attractive solution for this system if and only if (39) is satisfied.*

Proof. The result is obtained from Theorem 7 and Lemma 5. □

3.1.2. Predator-Prey System with Holling Type II Functional Response. As a second case, consider the following predator-prey system with Holling type II functional response which is the same as system (3.14) from [1] for continuous case:

$$\begin{aligned} \tilde{x}'(t) &= a(t)\,\tilde{x}(t) - b(t)\,\tilde{x}^2(t) - \frac{c(t)\,\tilde{x}^2(t)\,\tilde{y}(t)}{1+m(t)\,\tilde{x}^2(t)}, \\ \tilde{y}'(t) &= -d(t)\,\tilde{y}(t) + \frac{f(t)\,\tilde{y}(t)\,\tilde{x}^2(t)}{1+m(t)\,\tilde{x}^2(t)}. \end{aligned} \tag{41}$$

When $\tilde{y}(t) = \exp(y(t))$ and $\tilde{x}(t) = \exp(x(t))$, the transformed form of this system is gotten as

$$\begin{aligned} x'(t) &= a(t) - b(t)\exp(x(t)) \\ &\quad - \frac{c(t)\exp(y(t)+x(t))}{1+m(t)\exp(2x(t))}, \\ y'(t) &= -d(t) + \frac{f(t)\exp(2x(t))}{1+m(t)\exp(2x(t))}. \end{aligned} \tag{42}$$

Assume that all the coefficient functions are positive for the above systems. Then

$$\begin{aligned} \Phi_1(t, x(t), y(t)) &= \frac{c(t)\,x^2(t)}{1+m(t)\,x^2(t)}, \\ \Phi_2(t, x(t), y(t)) &= \frac{f(t)\,x(t)\,y(t)}{1+m(t)\,x^2(t)}, \end{aligned} \tag{43}$$

$\Phi_1(t, x(t), y(t)) \le c(t)x^2(t)$ and $\Phi_2(t, x(t), y(t)) \le f(t)M_x y(t)$, where M_x is the maximum of the solution of system (41) for prey which can be found from Proposition 3.1 in [22]. Here if prey does not go to extinction, then there exists $k \in \mathbb{N}$ such that, for sufficiently large k, $\Phi_1(t, x(t), y(t)) \ge (1/k)c(t)x^2(t)$, since $c(t)x^2(t)/(1+m(t)x(t)) > 0$. Therefore, Theorem 1 has equal statement with the following remark for the systems with Holling type II functional response.

Remark 9. For continuous case of system (3.14) from [1] it has at least one w-periodic solution if and only if the prey does not go to extinction.

By using similar techniques of Lemma 3 from [4], it can be found that if predator does not go to extinction, then prey also does not go to extinction. Therefore, the statement of Remark 9 is the same as the following one: system (3.14) from [1] for continuous case has at least one w-periodic solution if and only if predator does not go to extinction.

Lemma 10. *For system (3.14) from [1], take $\mathbb{T} = \mathbb{R}$. Then predator becomes extinct if and only if*

$$\int_0^w -d(t) + \frac{f(t)\, x^{*2}(t)}{1 + m(t)\, x^{*2}(t)} \leq 0, \tag{44}$$

where $x^*(t) = (1 - \exp(-\int_0^w a(s)ds)) / \int_0^w b(t-s) \exp(-\int_0^s a(t-\tau)d\tau)ds$. *Here* $x^*(t)$ *is the unique globally attractive solution of the system*

$$x(t) = a(t)\, x(t) - b(t)\, x^2(t). \tag{45}$$

Proof. Proof is similar to the proof Lemma 4. □

Lemma 11. *In system (3.14) from [1] when it is taken as $\mathbb{T} = \mathbb{R}$, the system has at least one w-periodic solution if and only if*

$$\int_0^w -d(t) + \frac{f(t)\, x^{*2}(t)}{1 + m(t)\, x^{*2}(t)} > 0, \tag{46}$$

where $x^*(t) = (1 - \exp(-\int_0^w a(s)ds)) / \int_0^w b(t-s) \exp(-\int_0^s a(t-\tau)d\tau)ds$.

Proof. This lemma can be proven by Lemma 10 and Remark 9. □

Corollary 12. *In system (3.14) from [1] when it is taken as $\mathbb{T} = \mathbb{R}$, the solution of the considered system is permanent if and only if (46) is satisfied.*

Proof. The proof is similar to the proof of Corollary 6. □

Theorem 13. *Assume that (46) holds true. Hence, the w-periodic solution of system (3.14) from [1] for continuous case is globally attractive or globally asymptotically stable.*

Proof. This has a similar proof with the proof of Lemma 5 in [4]. □

Corollary 14. *Assume that the same conditions for the coefficient functions of system (3.14) from [1] hold. There exists w-periodic globally attractive solution for this system if and only if (46) is satisfied.*

Proof. By Theorem 13 and Lemma 11, one can get the desired result. □

3.1.3. Predator-Prey System with Beddington-DeAngelis Type Functional Response. Consider the following system which is taken from [4]:

$$\begin{aligned} \tilde{x}'(t) &= a(t)\, \tilde{x}(t) - b(t)\, \tilde{x}^2(t) \\ &\quad - \frac{c(t)\, \tilde{y}(t)\, \tilde{x}(t)}{\alpha(t) + \beta(t)\, \tilde{x}(t) + m(t)\, \tilde{y}(t)}, \\ \tilde{y}'(t) &= -d(t)\, \tilde{y}(t) + \frac{f(t)\, \tilde{x}(t)\, \tilde{y}(t)}{\alpha(t) + \beta(t)\, \tilde{x}(t) + m(t)\, \tilde{y}(t)}. \end{aligned} \tag{47}$$

If we take $\exp(x(t)) = \tilde{x}(t)$, then we have the following transformed system:

$$\begin{aligned} x'(t) &= a(t) - b(t) \exp(x(t)) \\ &\quad - \frac{c(t) \exp(y(t))}{\alpha(t) + \beta(t) \exp(x(t)) + m(t) \exp(y(t))}, \\ y'(t) &= -d(t) \\ &\quad + \frac{f(t) \exp(x(t))}{\alpha(t) + \beta(t) \exp(x(t)) + m(t) \exp(y(t))}. \end{aligned} \tag{48}$$

For the above systems, all the coefficient functions are positive. It is obvious that when we take

$$\begin{aligned} \Phi_1(t, x(t), y(t)) &= \frac{c(t)\, x(t)}{\alpha(t) + \beta(t)\, x(t) + m(t)\, y(t)}, \\ \Phi_2(t, x(t), y(t)) &= \frac{f(t)\, y(t)}{\alpha(t) + \beta(t)\, x(t) + m(t)\, y(t)}, \end{aligned} \tag{49}$$

$\Phi_1(t, x(t), y(t)) \leq (c(t)/\alpha(t))x(t)$ and $\Phi_2(t, x(t), y(t)) \leq (f(t)/\alpha(t))y(t)$. Here if prey does not go to extinction, then there exists $k \in \mathbb{N}$ such that, for sufficiently large k, $\Phi_1(t, x(t), y(t)) \geq (1/k)(c(t)/\alpha(t))x(t)$, since $c(t)x(t)/(\alpha(t) + \beta(t)x(t) + m(t)y(t)) > 0$. Hence, Theorem 1 is true for system (47). These findings are the same as the ones in [4].

4. Examples

Example 1.

$$\begin{aligned} x'(t) &= 3 - (\cos(t) + 2) \exp(x(t)) - \frac{2 \exp(y(t))}{\exp(x(t)) + 1}, \\ y'(t) &= -(0.1 \sin(t) + 0.5) + \frac{\exp(x(t))}{\exp(x(t)) + 1}. \end{aligned} \tag{50}$$

By doing some simple calculations, it can be seen that $x^* > 1$; therefore (39) is satisfied for Example 1, since

$$\int_0^{2\pi} -(0.1 \sin(t) + 0.5) + \frac{x^*}{x^* + 1} > 0. \tag{51}$$

Figure 1 supports this result.

Even if we have changed the initial conditions, after a while still we obtain the same solution which shows the global attractivity of the solutions. Figure 2 supports this result.

Example 2.

$$\begin{aligned} x'(t) &= 3 - (\cos(t) + 2) \exp(x(t)) - \frac{2 \exp(y(t))}{\exp(x(t)) + 1}, \\ y'(t) &= -(0.1 \sin(t) + 4) + \frac{\exp(x(t))}{\exp(x(t)) + 1}. \end{aligned} \tag{52}$$

Example 2 does not satisfy inequality (39). This is easily seen by doing some simple calculations. Hence predator goes to extinction which satisfies Lemma 4. This result is supported by Figure 3.

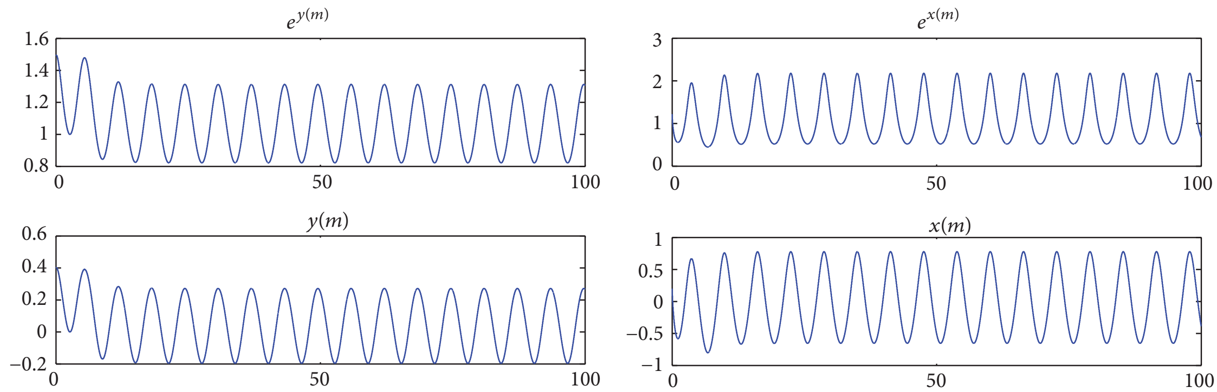

FIGURE 1: $x(0) = 0.2$ and $y(0) = 0.4$.

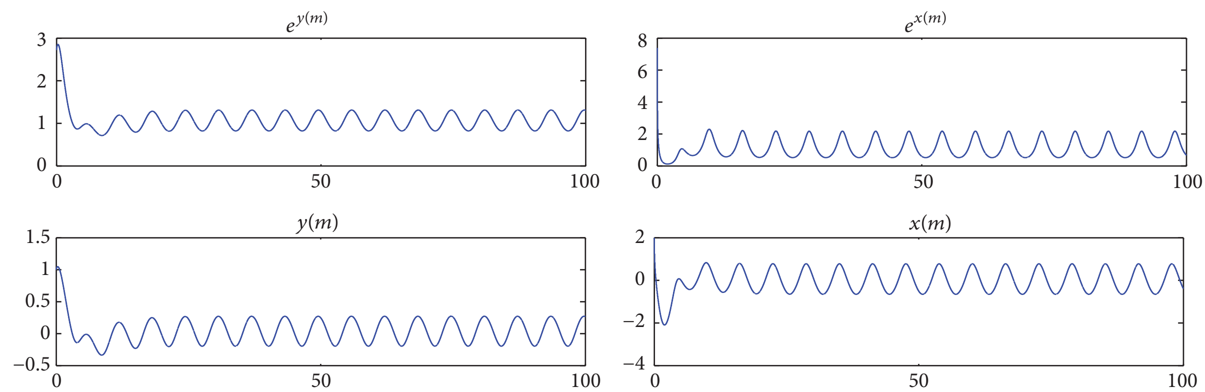

FIGURE 2: $x(0) = 2$ and $y(0) = 1$.

Example 3.

$$\begin{aligned} x'(t) &= 3 - (\cos(t) + 2)\exp(x(t)) \\ &\quad - \frac{2\exp\left(y(t) + x(t)\right)}{\exp(2x(t)) + 1}, \\ y'(t) &= -(0.1\sin(t) + 1) + \frac{3\exp(2x(t))}{\exp(2x(t)) + 1}. \end{aligned} \tag{53}$$

By doing some simple calculations it can be seen that $x^* > 1$; therefore (46) is satisfied by Example 3, since

$$\int_0^{2\pi} -(0.1\sin(t) + 1) + \frac{3\left(x^*\right)^2}{\left(x^*\right)^2 + 1} > 0.5. \tag{54}$$

This is supported by Figure 4.

Even if we have changed the initial conditions, after a while, we still obtain the same solution which shows the global attractivity of the solutions. Figure 5 supports this.

Example 4.

$$\begin{aligned} x'(t) &= 3 - (\cos(t) + 2)\exp(x(t)) \\ &\quad - \frac{2\exp\left(y(t) + x(t)\right)}{\exp(2x(t)) + 1}, \\ y'(t) &= -(0.1\sin(t) + 6) + \frac{3\exp(2x(t))}{\exp(2x(t)) + 1}. \end{aligned} \tag{55}$$

Example 4 does not satisfy inequality (46) and this can be easily seen by doing some simple calculations. Therefore, predator goes to extinction which satisfies Lemma 10 and this is supported by Figure 6.

For the examples of the system with Beddington-DeAngelis functional response, look at the study [4].

5. Discussion

In that study, two important analytical results are found. First, for the periodic solution of the system with generalized functional response, the if and only if condition is able to be found. Therefore, we are able to extend the study in [21] for continuous case. The second one is to be able to find an if and only if

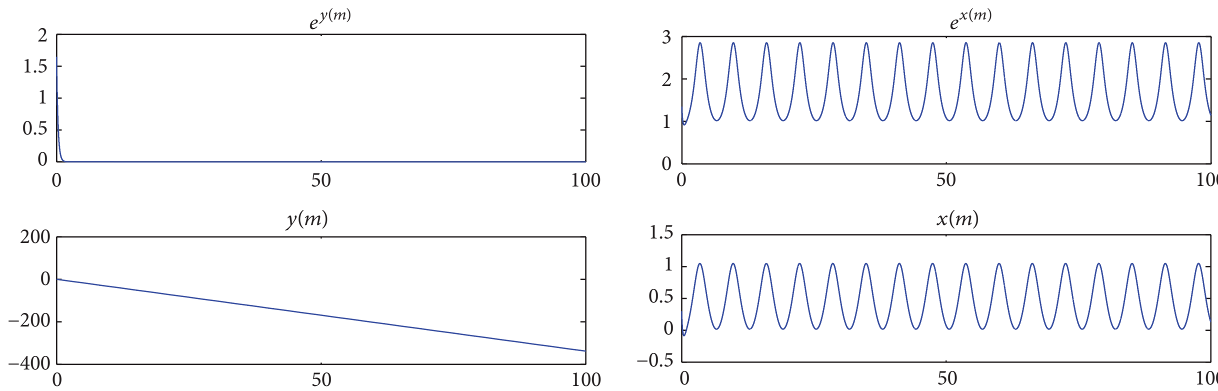

Figure 3: $x(0) = 0.3$ and $y(0) = 0.5$.

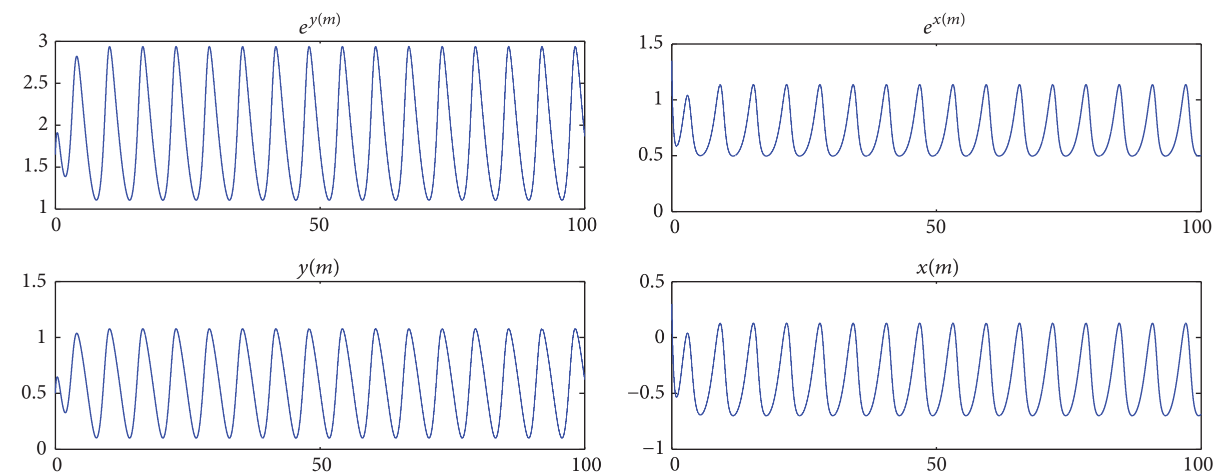

Figure 4: $x(0) = 0.3$ and $y(0) = 0.5$.

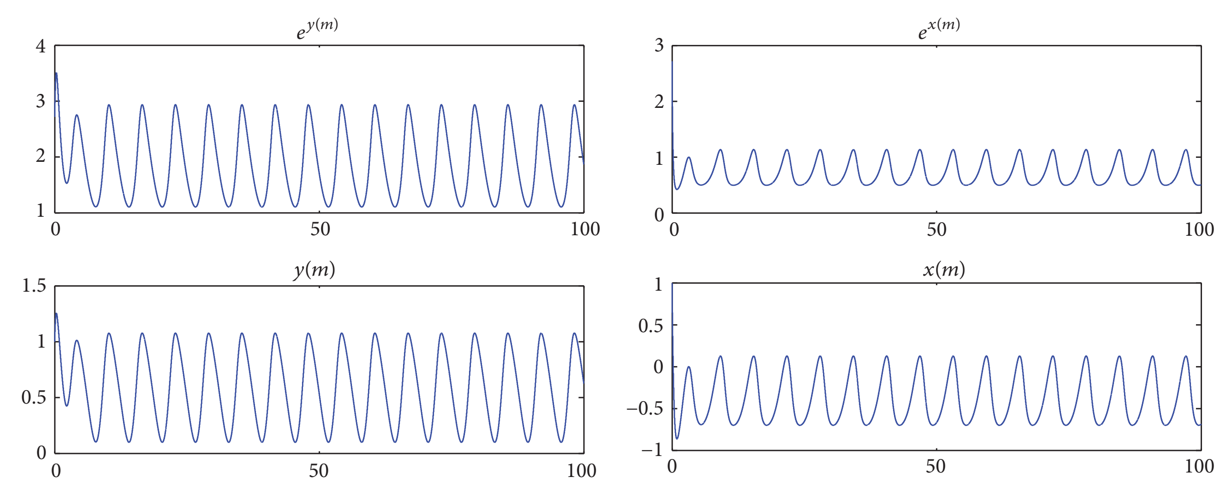

Figure 5: $x(0) = 1$ and $y(0) = 1$.

condition for the globally periodic solution of the predator-prey models with Holling type I and II functional responses. Additionally, the results of that paper is consistent with the findings of the paper [4] when the Beddington-DeAngelis type functional response is considered as an application.

Hence, the allover work in this study is important, since by using the results of that paper, the if and only if condition for at least one periodic solution and the globally attractive periodic solution can be found and these results can be generalized in many different types of functional responses.

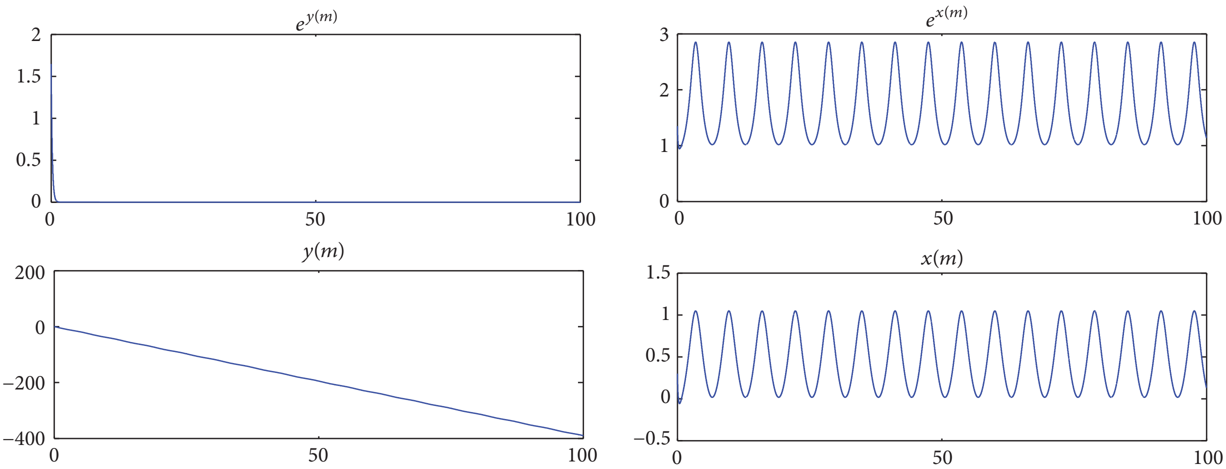

FIGURE 6: $x(0) = 0.3$ and $y(0) = 0.5$.

The suggested problem for the future works is to find the if and only if condition for the globally attractive periodic solution of the discrete predator-prey dynamic systems. Semigroup theory has been used in the analysis of the system when it has globally attractive periodic solution of the continuous system. For the discrete case, for further studies, the result that is related to the global attractivity of the system is another open problem.

Competing Interests

The author declares that there is no conflict of interests regarding the publication of this article.

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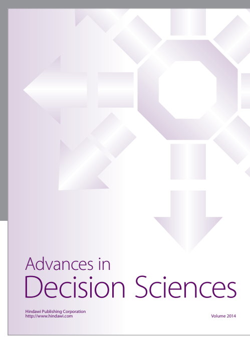

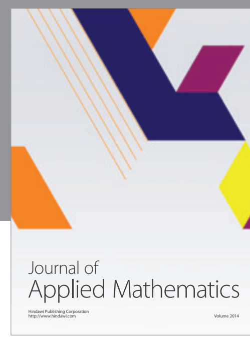

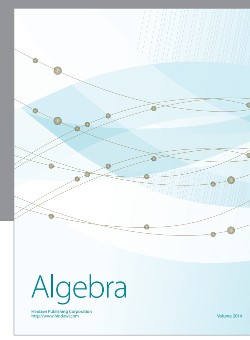

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