Hindawi
Journal of Mathematics
Volume 2020, Article ID 8872446, 25 pages
https://doi.org/10.1155/2020/8872446

*Review Article*

# Decision-Making Approach with Fuzzy Type-2 Soft Graphs

**Sundas Shahzadi,[1] Musavarah Sarwar ,[2] and Muhammad Akram [3]**

[1]*Division of Science and Technology, University of Education, Lahore 54000, Pakistan*
[2]*Department of Mathematics, Government College Women University, Sialkot, Pakistan*
[3]*Department of Mathematics, University of the Punjab, New Campus, Lahore, Pakistan*

Correspondence should be addressed to Muhammad Akram; m.akram@pucit.edu.pk

Received 30 August 2020; Revised 21 September 2020; Accepted 19 October 2020; Published 17 November 2020

Academic Editor: Lemnaouar Zedam



Molodtsov's theory of soft sets is free from the parameterizations insufficiency of fuzzy set theory. Type-2 soft set as an extension of a soft set has an essential mathematical structure to deal with parametrizations and their primary relationship. Fuzzy type-2 soft models play a key role to study the partial membership and uncertainty of objects along with underlying and primary set of parameters. In this research article, we introduce the concept of fuzzy type-2 soft set by integrating fuzzy set theory and type-2 soft set theory. We also introduce the notions of fuzzy type-2 soft graphs, regular fuzzy type-2 soft graphs, irregular fuzzy type-2 soft graphs, fuzzy type-2 soft trees, and fuzzy type-2 soft cycles. We construct some operations such as union, intersection, AND, and OR on fuzzy type-2 soft graphs and discuss these concepts with numerical examples. The fuzzy type-2 soft graph is an efficient model for dealing with uncertainty occurring in vertex-neighbors structure and is applicable in computational analysis, applied intelligence, and decision-making problems. We study the importance of fuzzy type-2 soft graphs in chemical digestion and national engineering services.

## 1. Introduction

Fuzzy set theory has its remarkable origin to the work of Zadeh [1] in 1965 to interact with vagueness and imprecision between absolute true and absolute false. The range of the values in a fuzzy set lies in [0, 1]. This remarkable discovery of fuzzy set theory paved a different way for dealing with uncertainties in various domains of science and technology.

Graph theory is moving quickly into the mainstream of mathematics, primarily due to its applications in engineering, communication networks, computer science, and artificial intelligence. In 1973, Kauffmann [2] introduced the notion of fuzzy graph, which is based on Zadeh's fuzzy relation [3]. Another elaborated definition of fuzzy graph was introduced by Rosenfeld [4]. Bhattacharya [5] subsequently gave some helpful results on fuzzy graphs and some operations on fuzzy graph theory were explored by Mordeson and Nair [6]. Many researchers studied fuzzy graphs in recent decades [7–9].

However, the theory of fuzzy sets has inadequacy to deal with parametrization tool. Soft set theory proposed by Molodtsov [10] has the ability to cope with this difficulty and is defined as a pair $(\xi, M)$, where $\xi$ is a mapping given by $\xi\colon M \longrightarrow P(E)$. Soft sets have been generalized to numerous directions beginning with Maji et al. [11, 12] who introduced fuzzy soft sets and Ahmad and Kharal [13] discussed some properties of fuzzy soft sets. In algebraic structures, soft sets and their hybrid models based on fuzzy soft sets, generalized fuzzy soft sets, rough soft sets, and soft rough sets have been implemented effectively [14–19]. Sarwar [20] elaborated the notion of rough graph and discussed decision-making approaches based on rough numbers and rough graphs. Akram and Nawaz [21] introduced the concepts of fuzzy soft graphs (named as fuzzy type-1 soft graph), vertex-induced soft graphs, and edge-induced soft graphs and also discussed some operations on soft graphs. Akram and Zafar [22] introduced various hybrid models based on fuzzy sets, soft sets, and rough sets. Further, Akram in cooperation with other researchers [23–26] discussed various applications and extensions of graph theory to study different types of uncertainties in real-world problems. Nowadays, researchers are actively working on interval type-2 fuzzy arc lengths [27], trapezoidal interval type-2 fuzzy soft sets [28], total uniformity of graph under fuzzy soft information [29], fuzzy soft cycles [30], and fuzzy soft $\beta$−coverings.

All these existing models have the same restriction that one cannot freely select the parameters. That is, if a correspondence or association occurs between parameters, then none of these models can solve the problems completely. Chatterjee et al. [31] proposed the concept of type-2 soft sets to deal with the correspondence between parameters, which is a generalization of Molodtsov's soft sets (called type-1 soft sets). Type-2 soft sets reparameterize the already parameterized crisp sets and thus have more freedom and effectiveness in dealing with imprecision as compared to type-1 soft sets. Hayat et al. [32–34] introduced vertex-neighbors-based type-2 soft sets, type-2 soft graphs, and irregular type-2 soft graphs and presented certain types of type-2 soft graphs.

The motives of this study are as follows:

(1) Soft sets and their hybrid models are used to deal with uncertainty based on parametrization tool. The correspondence, association, or relation occurring among parameters cannot be discussed with existing approaches. Type-2 soft models tackle this difficulty and present a mathematical approach to reparameterize the existing soft models. To deal with partial membership of objects, the main focus of this study is to introduce a hybrid model by combining fuzzy set theory with type-2 soft sets.

(2) Graph theory is an essential approach to study relations among objects using a figure consisting of vertices and lines joining these vertices. But there is an information loss in graphical models whether the objects are fully related or partially related, that is, uncertain and parameterized relations among objects. To handle this information loss, there is a need to represent the graphical models under fuzzy type-2 soft environment.

The main contribution of this study is as follows:

(1) The present study introduces the mathematical approaches of vertex-neighbors-based type-2 soft set and vertex-neighbors-based type-2 soft graphs under fuzzy environment. The notions of fuzzy type-2 soft graphs, regular fuzzy type-2 soft graphs, irregular fuzzy type-2 soft graphs, fuzzy type-2 soft trees, and fuzzy type-2 soft cycles are discussed with certain operations and numerical examples.

(2) The importance of presented concepts is studied with an application in chemical digestion and national engineering services.

## 2. Preliminaries

The term crisp graph on a nonvoid universe (named as set of vertices) $J$ is defined as a pair $G = (J, K)$, where $K \subseteq J \times J$ is named as set of edges. Crisp graph $(J, K)$ is a special case of the fuzzy graph with each vertex and edge of $(J, K)$ having degree of membership 1. A soft graph corresponding to a crisp graph $G$ is a parameterized family of subgraphs of $G$. A soft graph on a nonempty set $J$ is a 3−tuple $(J, K, A)$ such that, for each $e \in A$, $(J(e), K(e))$ is a graph, where $J(e) \subseteq J$ and $K(e) \subseteq J(e) \times J(e)$.

*Definition 1* (see [31]). Let $(E, M)$ be a soft universe and let $\eta(E)$ be the set of all T1SSs over $(E, M)$. Then a mapping $W\colon B \longrightarrow \eta(E)$, $B \subseteq M$ is called a type-2 soft set (T2SS) over $(E, M)$ and it is denoted by $[W^*, B]$. For all $\delta \in B, W^*(\delta)$ is a T1SS $(W_{(\delta)}, F_{(\delta)})$ such that $W^*(\delta) = (W_{(\delta)}, F_{(\delta)})$, where $W_{(\delta)}\colon F_{(\delta)} \longrightarrow P(E)$ and $F_{(\delta)} \subset M$. We refer to the parameter set $B$ as the "primary set of parameters" although the collection of parameters denoted by $\cup F_{(\delta)}$ is called "underlying set of parameters."

*Definition 2* (see [32]). Suppose that $G = (J, K)$ is a simple graph. Suppose that $B \subset J$ and $\Gamma(J)$ is the set of all T1SSs over $J$. Suppose that $[\xi^*, B]$ is a T2SS over $J$. Then a mapping $\xi^*\colon B \longrightarrow \Gamma(J)$ is said to be a T2SS over $J$ and is denoted as $[\xi^*, B]$. For every vertex $x \in B$, $[\xi^*, B]$ is a T1SS, where $\xi^*(x) = (\xi_{(x)}, \mathcal{N}\mathcal{B}_x)$ and $\xi_{(x)}\colon \mathcal{N}\mathcal{B}_x \longrightarrow P(J)$ can be explained as $\xi_x(u) = \{v \in J | uRv\}\ \forall u \in \mathcal{N}\mathcal{B}_x \subseteq J$, This T2SS is said to be a vertex-neighbors type-2 soft set (VN-T2SS) over $J$.

*Definition 3* (see [32]). Suppose that $G = (J, K)$ is a simple graph. Suppose that $B \subset J$ and $\Gamma(K)$ is the set of all T1SSs over $K$. Suppose that $[\xi^*, B]$ is a VN-T2SS over $J$. Then a mapping $\psi^*\colon B \longrightarrow \Gamma(K)$ is said to be a T2SS over $K$ and is denoted as $[\psi^*, B]$. For every vertex $x \in B$, $[\psi^*, B]$ is a T1SS, where $\psi^*(x) = (\psi_{(x)}, \mathcal{N}B_x)$ and $\psi_{(x)}\colon \mathcal{N}\mathcal{B}_x \longrightarrow P(K)$ can be explained as $\psi_x(u) = \{vw \in K | v, w \subseteq \xi_x(u)\}\quad \forall x \in \mathcal{N}\mathcal{B}_x \subseteq J$. This T2SS is said to be a VN-T2SS over $K$.

We present the notations that are used in this research article in Table 1.

## 3. Fuzzy Type-2 Soft Graphs

We refer to Maji's [11] fuzzy soft set as fuzzy type-1 soft set (FT1SS). Consider $B$ as a set of parameters that have a random nature (characterization of object, some functions, numeric values, etc.). Consider $E$ as a universal set and the class of all FT1SSs over $E$ will be indicated by $P(E)$. Recently, researchers have shown attraction to the application of fuzzy soft sets in science, advance technology, and decision problems. Fuzzy type-2 soft sets are considered as a generalized form of fuzzy soft set. Consider $E$ as a universal set and $M$ as the set of parameters. Fuzzy type-2 soft set is defined as follows.

*Definition 4.* Let $(E, M)$ be a fuzzy soft universe and let $P(E)$ be the collection of all FT1SSs over $(E, M)$. Then a mapping $S^*\colon B \longrightarrow P(E)$, $B \subset M$, is called a fuzzy type-2 soft set (FT2SS) over $(E, M)$ and it is denoted by $[S^*, B]$. In this case, corresponding to each parameter $e \in B$, $S^*(e)$ is

Table 1: List of abbreviations.

| Abbreviation | Description |
|---|---|
| T1SS | Type-1 soft set |
| T2SS | Type-2 soft set |
| VN-T2SS | Vertex-neighbors type-2 soft set |
| FT1SS | Fuzzy type-1 soft set |
| FT2SS | Fuzzy type-2 soft set |
| VN-FT2SS | Vertex-neighbors fuzzy type-2 soft set |
| FT1SG | Fuzzy type-1 soft graph |
| FT2SG | Fuzzy type-2 soft graph |
| FT1ST | Fuzzy type-1 soft tree |
| FT2ST | Fuzzy type-2 soft tree |
| FT2SST | Fuzzy type-1 soft subtree |
| FT2SST | Fuzzy type-2 soft subtree |
| FT1SC | Fuzzy type-1 soft cycle |
| FT2SC | Fuzzy type-2 soft cycle |

FT1SS. Thus, for each $e \in B$, there exists a FT1SS $(S_e, L_e)$ such that $S^*(e) = (S_e, L_e)$, where $S_e\colon L_e \longrightarrow P(E)$ and $L_e \subset M$. In this case, we refer to the parameter set $B$ as the "primary set of parameters," while the set of parameters $\cup L_e$ is known as the "underlying set of parameters."

*Definition 5.* Let $\mathscr{G} = (\mathscr{J}, \mathscr{K})$ be a fuzzy graph. The set of neighbors of an element $(j, \mu(j))$ is denoted by $\mathscr{N}\mathscr{B}_j$ and defined by $\mathscr{N}\mathscr{B}_j = ((i, \mu(i) | ij \in \mathscr{K}))$. Then $\mathscr{N}\mathscr{B}_B = \cup_{j\in B} \mathscr{N}\mathscr{B}_j$.

*Definition 6.* Let $\mathscr{G} = (\mathscr{J}, \mathscr{K})$ be a fuzzy graph. Suppose that $B \subset \mathscr{J}$ and $\Gamma(\mathscr{J})$ is the set of all FT1SSs over $\mathscr{J}$. Suppose that $[\xi, B]$ is a FT2SS over $\mathscr{J}$. Then a mapping $\xi\colon B \longrightarrow \Gamma(\mathscr{J})$ is said to be a FT2SS over $\mathscr{J}$ and is denoted as $[\xi, B]$. For every vertex $j \in B$, $\xi(j) = (\xi_{(j)}, \mathscr{N}\mathscr{B}_j)$ is a FT1SS and $\xi_{(j)}\colon \mathscr{N}\mathscr{B}_j \longrightarrow P(\mathscr{J})$ can be explained as $\xi_j(u) = \{v \in \mathscr{J} | uRv\}\ \forall u \in \mathscr{N}\mathscr{B}_j \subseteq \mathscr{J}$. This FT2SS is said to be a vertex-neighbors fuzzy type-2 soft set (VN-FT2SS) over $\mathscr{J}$.

*Definition 7.* Let $\mathscr{G} = (\mathscr{J}, \mathscr{K})$ be a fuzzy graph. Suppose that $B \subset \mathscr{J}$ and $\Gamma(K)$ is the set of all T1SSs over $\mathscr{K}$. Suppose that $[\psi, B]$ is a FT2SS over $\mathscr{K}$. Then a mapping $\psi\colon B \longrightarrow \Gamma(K)$ is said to be a FT2SS over $\mathscr{K}$ and is denoted as $[\psi, B]$. For every vertex $j \in B$, $\psi(j) = (\psi_{(j)}, \mathscr{N}\mathscr{B}_j)$ is a FT1SS and $\psi_{(j)}\colon \mathscr{N}\mathscr{B}_j \longrightarrow P(\mathscr{K})$ can be explained as $\psi_j(u) = \{uv \in \mathscr{K} | \{u, v\} \subseteq \xi_j(u)\}\ \forall u \in \mathscr{N}\mathscr{B}_j \subseteq \mathscr{J}$. This FT2SS is said to be a VN-FT2SS over $\mathscr{K}$.

$\xi(j) = (\xi_{(j)}, \mathscr{N}\mathscr{B}_j)$ and $\psi(j) = (\psi_{(j)}, \mathscr{N}\mathscr{B}_j)\ \forall j \in B$ are FT1SS over $\mathscr{J}$ and $\mathscr{K}$, respectively. If $(\xi_j(u), \psi_j(u))\ \forall u \in \mathscr{N}\mathscr{B}_j$ represent a fuzzy graph in fuzzy type-2 soft graph $\mathbb{G}$, then $(\xi(j), \psi(j))\ \forall j \in B$ is called FT1SG.

*Definition 8.* A 5-tuple $\mathbb{G} = (\mathscr{G}, \xi, \psi, B, \mathscr{N}\mathscr{B}_b)$ is called a fuzzy type-2 soft graph (FT2SG) if it satisfies the following conditions:

(a) $\mathscr{G} = (\mathscr{J}, \mathscr{K} \subseteq \mathscr{J} \times \mathscr{J})$ is a fuzzy graph.
(b) $B$ is a nonempty set of parameters.
(c) $[\xi, B]$ is a VN-FT2SS over $\mathscr{J}$.
(d) $[\psi, B]$ is a VN-FT2SS over $\mathscr{K}$.
(e) FT1SS corresponding to $(\xi(j), \psi(j))\ \forall j \in B$ represents a VN-fuzzy type-1 soft graph (FT1SG).

A FT2SG can also be defined by $\mathbb{G} = <\xi, \psi, B> = \{Z(j) | j \in B\}$, where $Z(j) = (Z_j, \mathscr{N}\mathscr{B}_j)$ such that $Z_j(u) = (\xi_j(u), \psi_j(u))$ for all $u \in \mathscr{N}\mathscr{B}_j$.

*Example 1.* Let $\mathscr{G} = (\mathscr{J}, \mathscr{K})$ be a fuzzy graph as shown in Figure 1. Let $B = \{(e_3, 0.5)\}$, $\mathscr{N}\mathscr{B}_{e_3} = \{(e_1, 0.8), (e_4, 0.4)\}$. Suppose that $[\xi, B]$ and $[\psi, B]$ are two FT2SSs over $\mathscr{J}$ and $\mathscr{K}$, respectively. We have

$$\begin{aligned} \xi(j) &= \left(\xi_j, \mathscr{N}\mathscr{B}_j\right), \\ \psi(j) &= \left(\psi_j, \mathscr{N}\mathscr{B}_j\right), \quad \text{for all } j \in B. \end{aligned} \tag{1}$$

Define $\xi_{e_3}(u) = \{v \in \mathscr{J} | u\mathscr{R}v \Longleftrightarrow S^{\infty}(P) = 0.3\}$ and $\psi_{e_3}(u) = \left\{vw \in \mathscr{K} | \{v, w\} \subseteq \xi_{e_3}(u)\right\}\ \forall u \in \mathscr{N}\mathscr{B}_{e_3} \subseteq \mathscr{J}$. Then FT2SSs $[\xi, B]$ and $[\psi, B]$ are defined as follows:

$$\begin{aligned} \xi_{e_3} &= \{\{(e_1, 0.8), \{(e_4, 0.4)\}\}, \{(e_4, 0.4), \{(e_1, 0.8), (e_2, 0.7), (e_3, 0.5), (e_5, 0.9)\}\}\}, \\ \psi_{e_3} &= \{\{(e_1, 0.8), \varnothing\}, \{(e_4, 0.4), \{(e_1e_3, 0.4), (e_1e_2, 0.6)\}\}\}. \end{aligned} \tag{2}$$

Fuzzy type-2 soft graph $\mathbb{G} = (Z(e_3))$ is shown in Figure 2.

*Definition 9.* Let $\mathbb{G} = (\mathscr{G}, \xi, \psi, B, \mathscr{N}\mathscr{B}_B)$ be a fuzzy type-2 soft graph; the complement of $\mathbb{G}$ is denoted by $\mathbb{G}^c$ and defined by $\mathbb{G}^c = (Z^c(z_1), Z^c(z_2), \ldots, Z^c(z_n))$ for all $z_1, z_2, \ldots, z_n \in B$, where $Z^c(z_i) = (\xi^c(z_i), \psi^c(z_i))$ is the complement of FT1SG corresponding to $Z(z_i) = (\xi(z_i), \psi(z_i))$ for all $z_i \in B, i = 1, 2, \ldots, n$.

*Example 2.* Let $\mathscr{G} = (\mathscr{J}, \mathscr{K})$ be a fuzzy graph as shown in Figure 3.

Let $B = \{(e_1, 0.7), (e_5, 0.5)\}$, $\mathscr{N}\mathscr{B}_{e_1} = \{(e_2, 0.8), (e_3, 0.5), (e_6, 0.6)\}$, and $\mathscr{N}\mathscr{B}_{e_5} = \{(e_4, 0.8), (e_6, 0.6)\}$.

Let $[\xi, B]$ and $[\psi, B]$ be two FT2SSs over $\mathscr{J}$ and $\mathscr{K}$, respectively. We have

$$\begin{aligned} \xi(j) &= \left(\xi_j, \mathscr{N}\mathscr{B}_j\right), \\ \psi(j) &= \left(\psi_j, \mathscr{N}\mathscr{B}_j\right), \quad \text{for all } j \in B. \end{aligned} \tag{3}$$

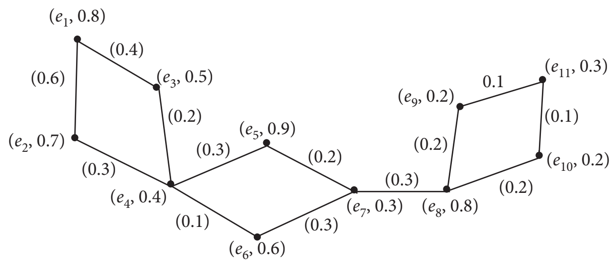


Figure 1: Fuzzy graph $\mathscr{G} = (\mathscr{J}, \mathscr{K})$.

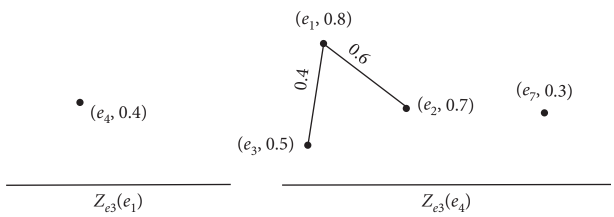


Figure 2: Fuzzy type-2 soft graph $\mathbb{G} = (Z(e_3))$.

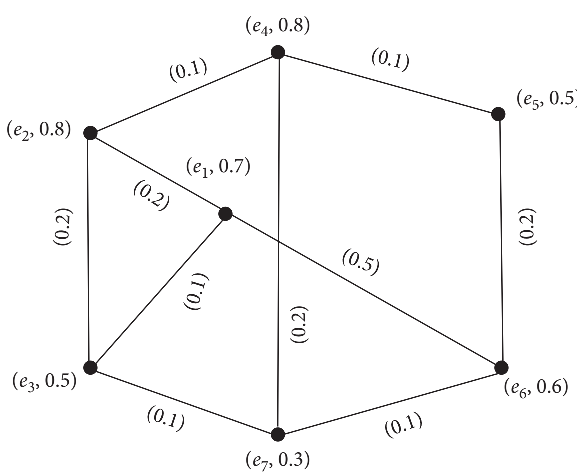


Figure 3: Fuzzy graph $\mathscr{G} = (\mathscr{J}, \mathscr{K})$.

Define

$$
\begin{aligned}
\xi_{e_1}(u) &= \{v \in \mathscr{J} \mid u\mathscr{R}v \Longleftrightarrow \mathrm{d}(u, v) \leq 0.2\},\\
\psi_{e_1}(u) &= \left\{vw \in \mathscr{K} \mid \{v, w\} \subseteq \xi_{e_1}(u)\right\}, \quad \text{for all } u \in \mathscr{N}\mathscr{B}_{e_1} \subseteq \mathscr{J},\\
\xi_{e_5}(u) &= \{v \in \mathscr{J} \mid u\mathscr{R}v \Longleftrightarrow \mathrm{d}(u, v) \leq 0.2\},\\
\psi_{e_5}(u) &= \left\{vw \in \mathscr{K} \mid \{v, w\} \subseteq \xi_{e_5}(u)\right\}, \quad \text{for all } u \in \mathscr{N}\mathscr{B}_{e_5} \subseteq \mathscr{J}.
\end{aligned}
\tag{4}
$$

The FT2SSs $[\xi, B]$ and $[\psi, B]$ are defined as follows:

$$
\begin{aligned}
\xi_{e_1} &= \left\{\begin{array}{c}
\{(e_2, 0.8), \{(e_1, 0.7), (e_3, 0.5), (e_4, 0.8), (e_5, 0.5)\}\},\\
\{(e_3, 0.5), \{(e_1, 0.7), (e_2, 0.8), (e_6, 0.6), (e_7, 0.3)\}\},\\
\{(e_6, 0.6), \{(e_3, 0.5), (e_5, 0.5), (e_7, 0.3)\}\}
\end{array}\right\},\\
\psi_{e_1} &= \left\{\begin{array}{c}
\{(e_2, 0.8), \{(e_1e_3, 0.1), (e_4e_5, 0.1)\}\},\\
\{(e_3, 0.5), \{(e_1e_2, 0.2), (e_1e_6, 0.5), (e_6e_7, 0.1)\}\},\\
\{(e_6, 0.6), \{(e_3e_7, 0.1)\}\}
\end{array}\right\},\\
\xi_{e_5} &= \{\{(e_4, 0.8), \{(e_2, 0.8), (e_5, 0.5), (e_7, 0.3)\}\}, \{(e_6, 0.6),\\
&\quad \{(e_7, 0.3), (e_3, 0.5), (e_5, 0.5)\}\}\},\\
\psi_{e_5} &= \{\{(e_6, 0.6), \{(e_3e_7, 0.1)\}\}, \{(e_4, 0.8), \varnothing\}\}.
\end{aligned}
\tag{5}
$$

Then $[\xi^c, B]$ and $[\psi^c, B]$ are defined as follows:

$$
\begin{aligned}
\xi^c_{e_1} &= \left\{\begin{array}{c}
\{(e_2, 0.8), \{(e_4, 0.8), (e_3, 0.5), (e_1, 0.7), (e_5, 0.5)\}\},\\
\{(e_3, 0.5), \{(e_1, 0.7), (e_2, 0.8), (e_6, 0.6)(e_7, 0.3)\}\},\\
\{(e_6, 0.6), \{(e_5, 0.5), (e_3, 0.5), (e_7, 0.3)\}\}
\end{array}\right\},\\
\psi^c_{e_1} &= \left\{\begin{array}{c}
\{(e_2, 0.8), \{(e_1e_4, 0.7), (e_1e_3, 0.4), (e_1e_5, 0.5)(e_4e_3, 0.5), (e_3e_5, 0.5), (e_4e_5, 0.4)\}\},\\
\{(e_3, 0.5), \{(e_1e_2, 0.5), (e_1e_7, 0.3), (e_1e_6, 0.1), (e_2e_6, 0.6), (e_2e_7, 0.3), (e_6e_7, 0.2)\}\},\\
\{(e_6, 0.6), \{(e_5e_7, 0.3), (e_5e_3, 0.5), (e_7e_3, 0.2)\}\}
\end{array}\right\},\\
\xi^c_{e_5} &= \{\{(e_4, 0.8), \{(e_2, 0.8), (e_5, 0.5), (e_7, 0.3)\}\}, \{(e_6, 0.6), \{(e_7, 0.3), (e_3, 0.5), (e_5, 0.5)\}\}\},\\
\psi^c_{e_5} &= \{\{(e_4, 0.8), \{(e_2e_7, 0.3), (e_5e_7, 0.3), (e_2e_5, 0.5)\}\}, \{(e_6, 0.6), \{(e_7e_3, 0.2), (e_5e_3, 0.5), (e_5e_7, 0.3)\}\}\}.
\end{aligned}
\tag{6}
$$

The complement of $\mathbb{G} = (Z_{e_1}, Z_{e_5})$ is a FT2SG $\mathbb{G}^c = (Z^c_{e_1}, Z^c_{e_5})$ such that $Z^c_{e_1} = (\xi^c_{e_1}, \psi^c_{e_1})$ is the complement of FT1SG corresponding to $Z(e_1) = (\xi_{e_1}, \psi_{e_1})$ and $Z^c_{e_5} = (\xi^c_{e_5}, \psi^c_{e_5})$ is the complement of FT1SG corresponding to $Z(e_5) = (\xi_{e_5}, \psi_{e_5})$ as shown in Figure 4.

*Definition 10.* Let $\mathbb{G}$ be a FT2SG; $\mathbb{G}$ is said to be a regular FT2SG if FT1SG corresponding to $Z(\chi)$ is a regular FT1SG for all $\chi \in B$.

**Proposition 1.** *If $\mathbb{G}$ is a regular FT2SG, then $\mathbb{G}^c$ is a regular FT2SG.*

*Proof.* Let $\mathbb{G}$ be a regular FT2SG. Suppose that $(Z_\sigma, \mathcal{N}\mathcal{B}_\sigma)$ is a FT1SG corresponding to $Z(\sigma)$ for all $\sigma \in B$; then $Z_\sigma(j)$ for all $j \in \mathcal{N}\mathcal{B}_\sigma$ must be a regular fuzzy graph. As we know that complement of a regular graph is regular, $Z^c_\sigma(j)\,\forall j \in \mathcal{N}\mathcal{B}_\sigma$ is also a regular fuzzy graph. It provides FT1SG corresponding to a $Z^c(\sigma)$ for all $\sigma \in B$ being regular FT1SG. Thus, $\mathbb{G}^c$ is a regular FT2SG of $\mathbb{G}$. □

*Definition 11.* Let $\mathbb{G}$ be a FT2SG; $\mathbb{G}$ is said to be an irregular FT2SG if FT1SG corresponding to $Z(\chi)$ is an irregular FT1SG for all $\chi \in B$.

*Example 3.* Let $\mathcal{G} = (\mathcal{J}, \mathcal{K})$ be a fuzzy graph as shown in Figure 5. Let $B = \{(e_4, 0.4), (e_5, 0.5)\}$, $\mathcal{N}\mathcal{B}_{e_4} = \{(e_1, 0.1), (e_3, 0.3), (e_5, 0.5)\}$, and $\mathcal{N}\mathcal{B}_{e_5} = \{(e_4, 0.4), (e_6, 0.9)\}$.

Let $[\xi, B]$ and $[\psi, B]$ be two FT2SSs over $\mathcal{J}$ and $\mathcal{K}$, respectively. We have

$$\begin{aligned} \xi(j) &= \left(\xi_j, \mathcal{N}\mathcal{B}_j\right), \\ \psi(j) &= \left(\psi_j, \mathcal{N}\mathcal{B}_j\right), \quad \text{for all } j \in B. \end{aligned} \tag{7}$$

Define

$$\begin{aligned} &\xi_{e_4}(u) = \{v \in \mathcal{J} | u\mathcal{R}v \Longleftrightarrow \mathrm{d}(u, v) \le 0.2\}, \\ &\psi_{e_4}(u) = \left\{vw \in \mathcal{K} | v, w \subseteq \xi_{e_4}(u)\right\}, \quad \text{for all } u \in \mathcal{N}\mathcal{B}_{e_4} \subseteq \mathcal{J}, \\ &\xi_{e_5}(u) = \{v \in \mathcal{J} | u\mathcal{R}v \Longleftrightarrow \mathrm{d}(u, v) \le 0.2\}, \\ &\psi_{e_5}(u) = \left\{vw \in \mathcal{K} | v, w \subseteq \xi_{e_5}(u)\right\}, \quad \text{for all } u \in \mathcal{N}\mathcal{B}_{e_5} \subseteq \mathcal{J}. \end{aligned} \tag{8}$$

The FT2SSs $[\xi, B]$ and $[\psi, B]$ are defined as follows:

$$\begin{aligned} \xi_{e_4} &= \left\{ \begin{array}{c} \{(e_1, 0.1), \{(e_2, 0.2), (e_3, 0.3), (e_4, 0.4), (e_5, 0.5)\}\}, \\ \{(e_3, 0.3), \{(e_1, 0.1), (e_2, 0.2), (e_4, 0.4)\}\}, \\ \{(e_5, 0.5), \{(e_1, 0.1), (e_4, 0.4), (e_6, 0.9)\}\} \end{array} \right\}, \\ \psi_{e_4} &= \left\{ \begin{array}{c} \{(e_1, 0.1), \{(e_2e_3, 0.1), (e_3e_4, 0.2), (e_4e_5, 0.1)\}\}, \{(e_3, 0.3), \{(e_1e_4, 0.1), (e_1e_2, 0.1)\}\}, \\ \{(e_5, 0.5), \{(e_1e_4, 0.1)\}\} \end{array} \right\}, \\ \xi_{e_5} &= \{(e_4, 0.4), \{(e_1, 0.1), (e_2, 0.2), (e_3, 0.3), (e_5, 0.5), (e_6, 0.9)\}, \{(e_6, 0.9), \{(e_4, 0.4), (e_5, 0.5)\}\}\}, \\ \psi_{e_5} &= \{\{(e_4, 0.4), \{(e_1e_2, 0.1), (e_2e_3, 0.1), (e_5e_6, 0.1), (e_6e_3, 0.3)\}\}, \{(e_6, 0.9), \{(e_4e_5, 0.1)\}\}\}. \end{aligned} \tag{9}$$

Then $\mathbb{G} = (Z(e_4), Z(e_5))$ is an irregular FT2SG as shown in Figure 6.

**Proposition 2.** *If $\mathcal{G}$ is a regular fuzzy graph, then every FT2SG of $\mathcal{G}$ is not necessarily a regular FT2SG.*

*Definition 12.* Let $\mathbb{G}$ be a FT2SG; $\mathbb{G}$ is called a neighborly irregular FT2SG if FT1SGs corresponding to $Z(\chi)$ are neighborly irregular FT1SG for all $\chi \in B$.

*Example 4.* Let $\mathcal{G} = (\mathcal{J}, \mathcal{K})$ be a fuzzy graph as shown in Figure 7. Let $B = \{(e_3, 0.9), (e_5, 0.4)\}$, $\mathcal{N}\mathcal{B}_{e_3} = \{(e_1, 0.2), (e_2, 0.4), (e_4, 0.6)\}$, and $\mathcal{N}\mathcal{B}_{e_5} = \{(e_4, 0.6), (e_6, 0.1), (e_7, 0.3)\}$.

Let $[\xi, B]$ and $[\psi, B]$ be two FT2SSs over $\mathcal{J}$ and $\mathcal{K}$, respectively. We have

$$\begin{aligned} \xi(j) &= \left(\xi_j, \mathcal{N}\mathcal{B}_j\right), \\ \psi(j) &= \left(\psi_j, \mathcal{N}\mathcal{B}_j\right), \quad \text{for all } j \in B. \end{aligned} \tag{10}$$

Define

$$\begin{aligned} &\xi_{e_3}(u) = (v \in \mathcal{J} | u\mathcal{R}v \Longleftrightarrow \mathrm{d}(u, v) \le 0.3), \\ &\psi_{e_3}(u) = \left\{vw \in \mathcal{K} \,|\{v, w\} \subseteq \xi_{e_3}(u)\right\}, \quad \text{for all } u \in \mathcal{N}\mathcal{B}_{e_3} \subseteq \mathcal{J}, \\ &\xi_{e_5}(u) = \{v \in \mathcal{J} | u\mathcal{R}v \Longleftrightarrow \mathrm{d}(u, v) \le 0.4\}, \\ &\psi_{e_5}(u) = \left\{vw \in \mathcal{K} \,|\{v, w\} \subseteq \xi_{e_5}(u)\right\}, \quad \text{for all } u \in \mathcal{N}\mathcal{B}_{e_5} \subseteq \mathcal{J}. \end{aligned} \tag{11}$$

The FT2SSs $[\xi, B]$ and $[\psi, B]$ are defined as follows:

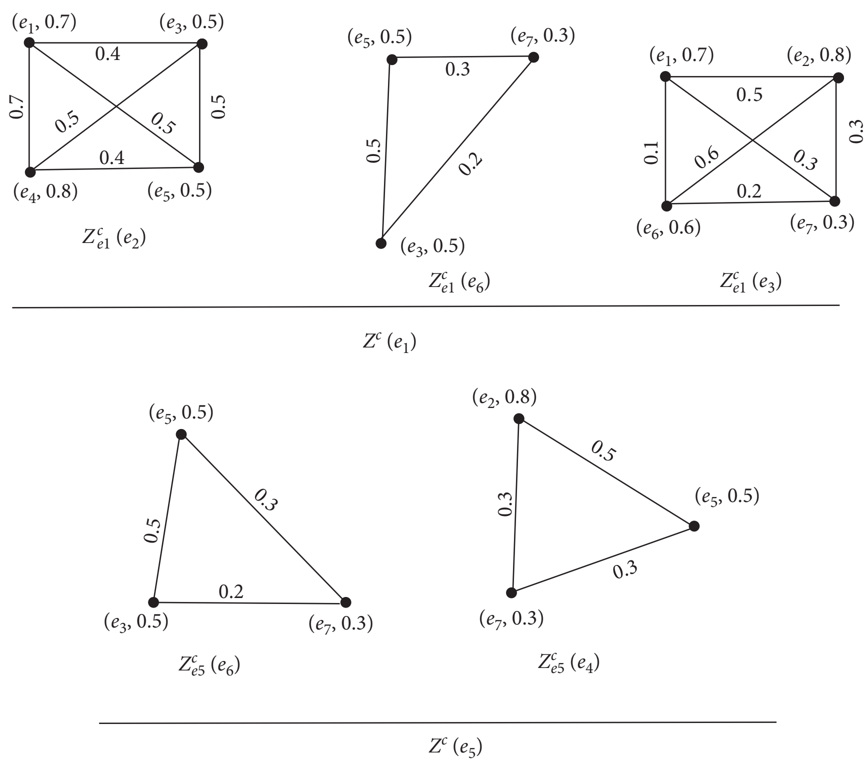


Figure 4: $\mathbb{G}^c = (Z^c(e_1), Z^c(e_5))$.

$$
\begin{aligned}
\xi_{e_3} &= \left\{ \begin{array}{c} \{(e_1, 0.2), \{(e_2, 0.4), (e_3, 0.9), (e_4, 0.6)\}\}, \{(e_2, 0.4), \{(e_1, 0.2), (e_3, 0.9), (e_4, 0.6)\}\}, \\ \{(e_4, 0.6), \{(e_1, 0.2), (e_2, 0.4), (e_3, 0.9), (e_5, 0.4)\}\} \end{array} \right\}, \\
\psi_{e_3} &= \left\{ \begin{array}{c} \{(e_1, 0.2), \{(e_2 e_3, 0.3), (e_3 e_4, 0.4)\}\}, \\ \{(e_2, 0.4), (e_1 e_3, 0.2), (e_3 e_4, 0.4), (e_1 e_4, 0.1)\}, \\ \{(e_4, 0.6), \{(e_1 e_2, 0.1), (e_3 e_2, 0.3), (e_1 e_3, 0.2)\}\} \end{array} \right\}, \\
\xi_{e_5} &= \left\{ \begin{array}{c} \{(e_4, 0.6), (e_1, 0.2), (e_2, 0.4), (e_3, 0.9), (e_5, 0.4)\}, \\ \{(e_6, 0.1), (\{e_5, 0.4\}, (e_8, 0.9), (e_7, 0.3))\}, \\ \{(e_7, 0.3), \{(e_6, 0.1), (e_8, 0.9), (e_5, 0.4)\}\} \end{array} \right\}, \\
\psi_{e_5} &= \left\{ \begin{array}{c} \{(e_4, 0.6), \{(e_1 e_2, 0.1), (e_2 e_3, 0.3), (e_1 e_3, 0.2)\}\}, \\ \{(e_6, 0.1), \{(e_5 e_7, 0.2), (e_7 e_8, 0.1)\}\}, \\ \{(e_7, 0.3), \{(e_6 e_5, 0.2), (e_6 e_8, 0.1)\}\} \end{array} \right\}.
\end{aligned}
\tag{12}
$$

Then $\mathbb{G} = (Z(e_3), Z(e_5))$ is a neighborly irregular FT2SG as shown in Figure 8.

*Definition 13.* Let $\mathbb{G}$ be a FT2SG and $Z(\chi) = (Z_\chi, \mathcal{NB}_\chi)$ is a FT1SG for all $\chi \in B$. An edge $uv$ in $\mathbb{G}$ is said to be a FT2S bridge if its removal disconnects $Z_\chi(u)$ for all $u \in \mathcal{NB}_\chi$.

*Definition 14.* Let $\mathbb{G}$ be a FT2SG and $Z(\chi) = (Z_\chi, \mathcal{NB}_\chi)$ is a FT1SG for all $\chi \in B$. A vertex $z$ in $\mathbb{G}$ is said to be a FT2S cut-vertex if its removal disconnects $Z_\chi(u)$ for all $u \in \mathcal{NB}_\chi$.

*Definition 15.* Let $\mathbb{G} = (\mathcal{G}, \xi, \psi, B, \mathcal{NB}_B)$ be a FT2SG; $\mathbb{G}$ is called a fuzzy type-2 soft tree (FT2ST) if FT1SGs corresponding to $Z(\chi)$ are FT1STs for all $\chi \in B$.

*Example 5.* Let $\mathcal{G} = (\mathcal{J}, \mathcal{K})$ be a fuzzy graph as shown in Figure 9. Let $B = \{(c, 0.6), (f, 0.2)\}, \mathcal{NB}_c = \{(b, 0.2), (d, 0.4)\}$, and $\mathcal{NB}_f = \{(e, 0.5), (g, 0.8)\}$.

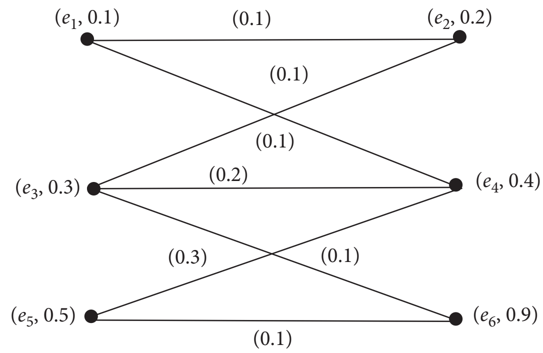


Figure 5: Fuzzy graph $\mathscr{G} = (\mathscr{J}, \mathscr{K})$.

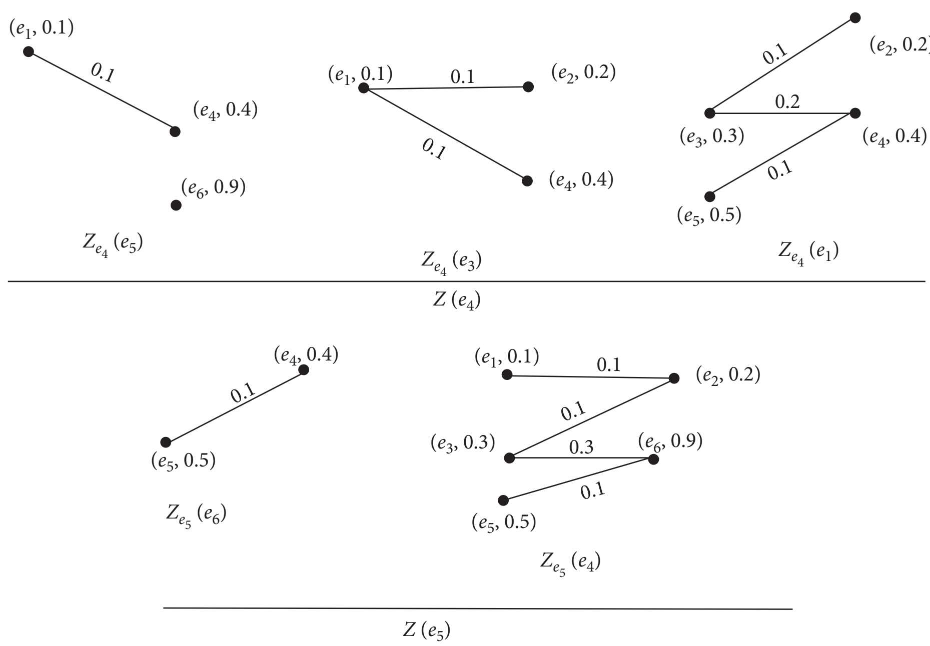


Figure 6: $\mathbb{G} = (Z(e_4), Z(e_5))$.

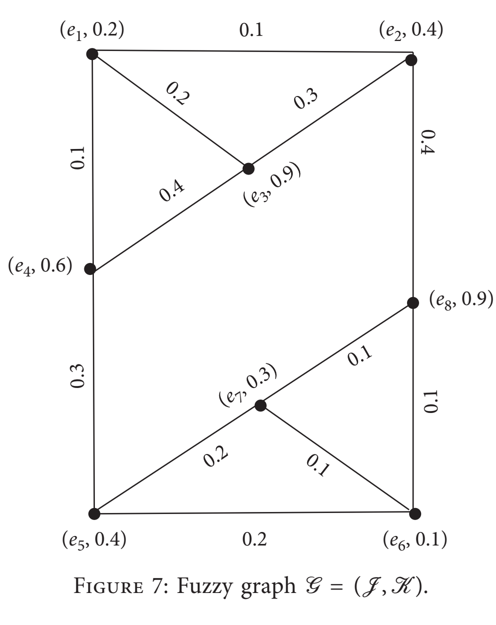


Figure 7: Fuzzy graph $\mathscr{G} = (\mathscr{J}, \mathscr{K})$.

Let $[\xi, B]$ and $[\psi, B]$ be two FT2SSs over $\mathscr{J}$ and $\mathscr{K}$, respectively. We have

$$\begin{aligned} \xi(j) &= \left(\xi_j, \mathscr{N}\mathscr{B}_j\right), \\ \psi(j) &= \left(\psi_j, \mathscr{N}\mathscr{B}_j\right), \quad \text{for all } j \in B. \end{aligned} \tag{13}$$

Define

$$\begin{aligned} \xi_c(u) &= \{v \in \mathscr{J} \mid u\mathscr{R}v \Longleftrightarrow \mathrm{d}(u, v) = \mathrm{rad}(G)\}, \\ \psi_c(u) &= \{vw \in \mathscr{K} \mid v, w \subseteq \xi_c(u)\}, \quad \text{for all } u \in \mathscr{N}\mathscr{B}_c \subseteq \mathscr{J}, \\ \xi_f(u) &= \{v \in \mathscr{J} \mid u\mathscr{R}v \Longleftrightarrow \mathrm{d}(u, v) = \mathrm{rad}(G)\}, \\ \psi_f(u) &= \left\{vw \in \mathscr{K} \mid v, w \subseteq \xi_f(u)\right\}, \quad \text{for all } u \in \mathscr{N}\mathscr{B}_f \subseteq \mathscr{J}. \end{aligned} \tag{14}$$

The FT2SSs $[\xi, B]$ and $[\psi, B]$ are defined as follows:

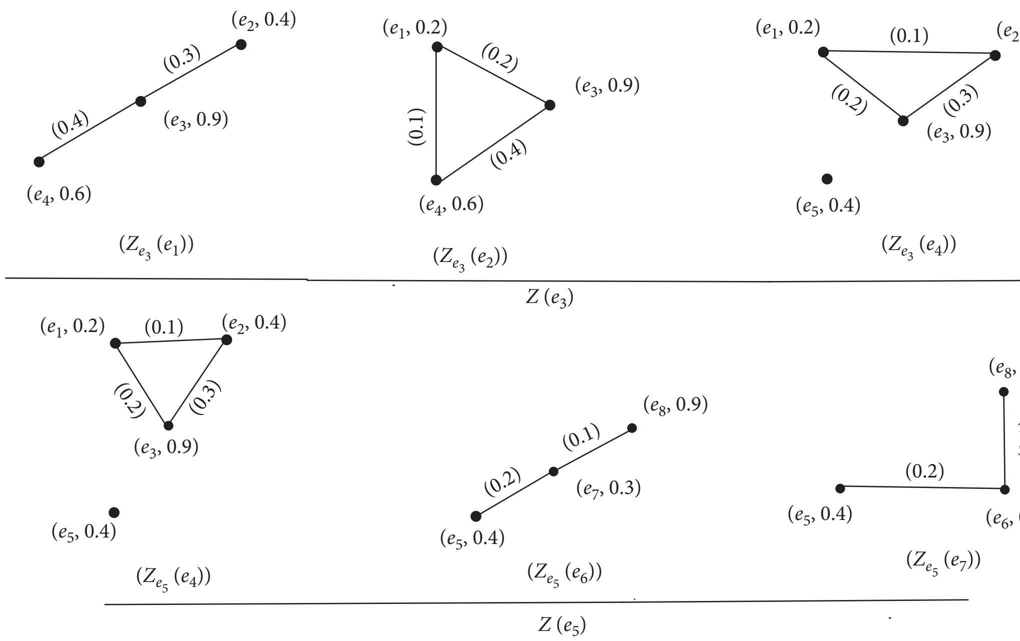


Figure 8: $\mathbb{G} = (Z(e_3), Z(e_5))$.

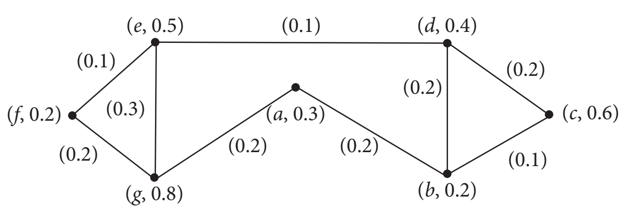


Figure 9: Fuzzy graph $\mathscr{G} = (\mathscr{J}, \mathscr{K})$.

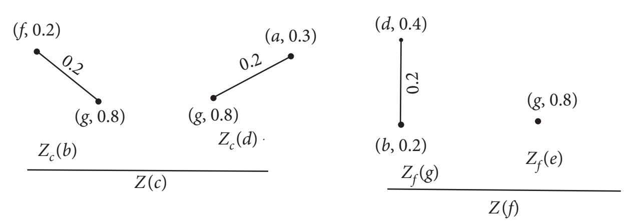


Figure 10: $\mathbb{G} = (Z(c), Z(f))$.

$$\begin{aligned}
\xi_c &= \{\{(b, 0.2), \{(g, 0.8), (f, 0.2)\}\}, \{(d, 0.4), \{(a, 0.3), (g, 0.8)\}\}\},\\
\psi_c &= \{\{(b, 0.2), \{(gf, 0.2)\}\}, \{(d, 0.4), \{(ag, 0.2)\}\}\},\\
\xi_f &= \{\{(e, 0.5), \varnothing\}, \{(g, 0.8), \{(b, 0.2), (d, 0.4)\}\}\},\\
\psi_f &= \{\{(e, 0.5), \varnothing\}, \{(g, 0.8), \{(bd, 0.2)\}\}\}.
\end{aligned} \tag{15}$$

Then $\mathbb{G} = (Z(c), Z(f))$ is a FT2ST as shown in Figure 10. It can also be defined as VN-type-2 soft tree.

**Theorem 1.** *Let $\mathbb{G}$ be a FT2SG and $Z(\chi) = (Z_\chi, \mathscr{N}\mathscr{B}_\chi)$ is a FT1SG for all $\chi \in B$. If $Z_\chi(j)\, \forall j \in \mathscr{N}\mathscr{B}_\chi$ is a FT1SG with $n \geq 3$ vertices, then $\mathbb{G}$ will not be a complete FT2SG.*

*Proof.* Let $\mathbb{G}$ be a FT2SG and $Z(\chi) = (Z_\chi, \mathscr{N}\mathscr{B}_\chi)$ is a FT1SG for all $\chi \in B$. On the contrary, assume that $\mathbb{G}$ is a complete FT2SG; then each $Z_\chi(j)\, \forall j \in \mathscr{N}\mathscr{B}_\chi$ will also be complete. Let $v, w$ be arbitrary nodes of $Z_\chi(j)$ joined by a line $vw$. Since $Z_\chi(j)$ having $n \geq 3$ vertices of $\mathscr{G}$ is a FT1SG, then a minimum one vertex $\eta$ which is connected to $v$ by an edge $v\eta$ and to $w$ by an edge $w\eta$ as $Z_\chi(j)$ be a complete fuzzy graph. Then there is a cycle $vw\eta v$. Therefore, $Z_\chi(j)\, \forall j \in \mathscr{N}\mathscr{B}_\chi$ cannot be a FT1ST, which is opposite to the fact that $Z_\chi(j)$ is a connected FT1SG of FT2SG. So, $\mathbb{G}$ is not a complete FT2SG. □

*Definition 16.* Let $\mathbb{G}$ be a FT2SG and $Z(\chi) = (Z_\chi, \mathscr{N}\mathscr{B}_\chi)$ is a FT1SG for all $\chi \in B$. Then $\mathbb{G}$ is called a fuzzy type-2 soft forest if $Z_\chi(j)$ consists of several disjoined fuzzy trees $\forall j \in \mathscr{N}\mathscr{B}_\chi$.

*Definition 17.* Let $\mathbb{G}$ be a FT2SG; $\mathbb{G}$ is said to be a FT2SC if FT1SG corresponding to $Z(\chi)$ is a fuzzy type-1 soft cycle, for all $\chi \in B$.

*Example 6.* Let $\mathscr{G} = (\mathscr{J}, \mathscr{K})$ be a fuzzy graph as shown in Figure 11, where

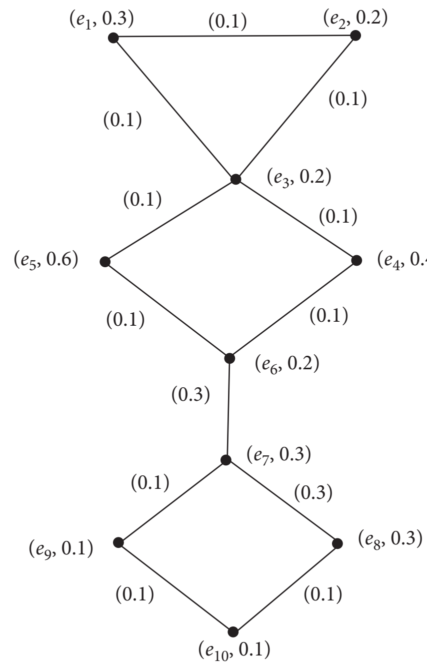


Figure 11: Fuzzy graph $\mathcal{G} = (\mathcal{J}, \mathcal{K})$.

$$\begin{aligned}
\mathcal{J} &= \{(e_1,0.3),(e_2,0.2),(e_3,0.2),(e_4,0.4),(e_5,0.6),(e_6,0.2),(e_7,0.3),(e_8,0.3),(e_9,0.1),(e_{10},0.1)\},\\
\mathcal{K} &= \left\{\begin{array}{c}(e_1e_2,0.1),(e_2e_3,0.1),(e_3e_1,0.1),(e_3e_4,0.1),(e_5e_3,0.1),\\(e_5e_6,0.1),(e_6e_4,0.1),(e_7e_6,0.3),(e_7e_9,0.1),(e_7e_8,0.3),(e_9e_{10},0.1),(e_8e_{10},0.1)\end{array}\right\}.
\end{aligned}\tag{16}$$

Let $B = \{(e_8, 0.3), (e_9, 0.1)\}$, $\mathcal{NB}_{e_9} = \{(e_7, 0.3), (e_{10}, 0.1)\}$, and $\mathcal{NB}_{e_8} = \{(e_7, 0.3), (e_{10}, 0.1)\}$.

Let $[\xi, B]$ and $[\psi, B]$ be two FT2SSs over $\mathcal{J}$ and $\mathcal{K}$, respectively. We have

$$\begin{aligned}
\xi(j) &= \left(\xi_j, \mathcal{NB}_j\right),\\
\psi(j) &= \left(\psi_j, \mathcal{NB}_j\right), \quad \text{for all } j \in B.
\end{aligned}\tag{17}$$

Define

$$\begin{aligned}
\xi_{e_9}(u) &= \{v \in \mathcal{J} \mid u\mathcal{R}v \Longleftrightarrow 0.5 \le \mathrm{d}(u,v) \le 0.7\},\\
\psi_{e_9}(u) &= \left\{vw \in \mathcal{K} \mid \{v,w\} \subseteq \xi_{e_9}(u)\right\}, \quad \text{for all } u \in \mathcal{NB}_{e_9} \subseteq \mathcal{J},\\
\xi_{e_8}(u) &= \{v \in \mathcal{J} \mid u\mathcal{R}v \Longleftrightarrow 0.5 \le \mathrm{d}(u,v) \le 0.7\},\\
\psi_{e_8}(u) &= \left\{vw \in \mathcal{K} \mid \{v,w\} \subseteq \xi_{e_8}(u)\right\}, \quad \text{for all } u \in \mathcal{NB}_{e_8} \subseteq \mathcal{J}.
\end{aligned}\tag{18}$$

The FT2SSs $[\xi, B]$ and $[\psi, B]$ are defined as follows:

$$\begin{aligned}
\xi_{e_9} &= \{\{(e_7,0.3),\{(e_1,0.3),(e_2,0.2),(e_3,0.2)\}\},\{(e_{10},0.1),\{(e_3,0.2),(e_4,0.4),(e_5,0.6),(e_6,0.2)\}\}\},\\
\psi_{e_9} &= \left\{\begin{array}{c}\{(e_7,0.3),\{(e_3e_2,0.1),(e_1e_3,0.1),(e_1e_2,0.1)\}\},\\\{(e_{10},0.1),\{(e_4e_3,0.1),(e_3e_5,0.1),(e_6e_5,0.1),(e_6e_4,0.1)\}\}\end{array}\right\},\\
\xi_{e_8} &= \{\{(e_7,0.3),\{(e_1,0.3),(e_2,0.2),(e_3,0.2)\}\},\{(e_{10},0.1),\{(e_3,0.2),(e_4,0.4),(e_5,0.6),(e_6,0.2)\}\}\},\\
\psi_{e_8} &= \left\{\begin{array}{c}\{(e_7,0.3),\{(e_3e_2,0.1),(e_1e_3,0.1),(e_1e_2,0.1)\}\},\\\{(e_{10},0.1),\{(e_4e_3,0.1),(e_3e_5,0.1),(e_6e_5,0.1),(e_6e_4,0.1)\}\}\end{array}\right\}.
\end{aligned}\tag{19}$$

We can check that $\mathbb{G} = (Z(e_9), Z(e_8))$ is a FT2SC as shown in Figure 12. It is also defined as a fuzzy VN-type-2 soft cycle.

*Example 7.* Let $\mathcal{G} = (\mathcal{J}, \mathcal{K})$ be a fuzzy graph as shown in Figure 13. Let $B = \{(a, 0.4), (b, 0.2)\} \subset \mathcal{J}$, $\mathcal{NB}_a = \{(b, 0.2), (c, 0.3), (d, 0.3)\}$, and $\mathcal{NB}_b = \{(a, 0.4), (c, 0.3), (d, 0.3)\}$. Let $[\xi, B]$ and $[\psi, B]$ be two FT2SSs over $\mathcal{J}$ and $\mathcal{K}$, respectively. We have

$$\begin{aligned}
\xi(j) &= \left(\xi_j, \mathcal{NB}_j\right),\\
\psi(j) &= \left(\psi_j, \mathcal{NB}_j\right), \quad \text{for all } j \in B.
\end{aligned}\tag{20}$$

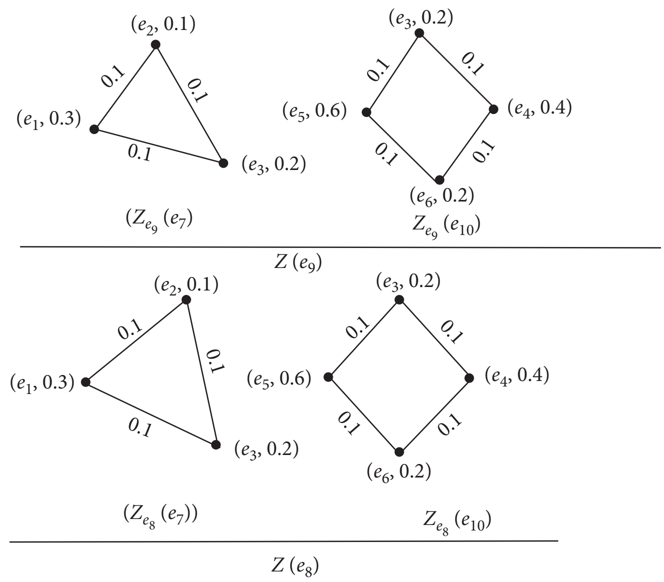


Figure 12: $\mathbb{G} = (Z(e_9), Z(e_8))$.

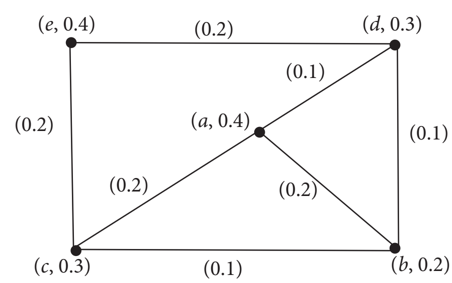


Figure 13: Fuzzy graph $\mathscr{G} = (\mathscr{J}, \mathscr{K})$.

Define $\xi_a(u) = \{v \in \mathscr{J} | u\mathscr{R}v \Longleftrightarrow \mathrm{d}(u,v) \le 0.2\}$, $\psi_a(u) = \{vw \in \mathscr{K} | v,\ w \subseteq \xi_a(u)\}\ \forall u \in \mathscr{N}\mathscr{B}_a \subseteq \mathscr{J}$, $\xi_b(u) = \{v \in \mathscr{J} | u\mathscr{R}v \Longleftrightarrow \mathrm{d}(u,v) \le 0.2\}$, $\psi_b(u) = \{vw \in \mathscr{K} | v, w \subseteq \xi_b(u)\} \forall u \in \mathscr{N}\mathscr{B}_b \subseteq \mathscr{J}$.

The FT2SSs $[\xi, B]$ and $[\psi, B]$ are defined as follows:

$$
\begin{aligned}
\xi_a &= \left\{ \begin{array}{c} \{(b,0.2), \{(a,0.4), (c,0.3), (d,0.3)\}\}, \{(c,0.3), \{(a,0.4), (b,0.2), (d,0.3), (e,0.4)\}\}, \\ \{(d,0.3), \{(a,0.4), (b,0.2), (c,0.3), (e,0.4)\}\} \end{array} \right\}, \\
\psi_a &= \left\{ \begin{array}{c} \{(b,0.2), \{(ad,0.1), (ac,0.2)\}\}, \{(c,0.3), \{(ab,0.2), (ad,0.1), (bd,0.1), (de,0.2)\}\}, \\ \{(d,0.3), \{(ab,0.2), (ac,0.2), (bc,0.1), (ce,0.2)\}\} \end{array} \right\}, \\
\xi_b &= \left\{ \begin{array}{c} \{(a,0.4), \{(c,0.3), (b,0.2), (d,0.3)\}\}, \{(c,0.3), \{(a,0.4), (b,0.2), (e,0.4), (d,0.3)\}\}, \\ \{(d,0.3), \{(a,0.4), (b,0.2), (c,0.3), (e,0.4)\}\} \end{array} \right\}, \\
\psi_b &= \left\{ \begin{array}{c} \{(a,0.4), \{(bc,0.1), (bd,0.1)\}\}, \{(c,0.3), \{(ad,0.1), (ed,0.2), (ab,0.2), (bd,0.1)\}\}, \\ \{(d,0.3), \{(ec,0.2), (ab,0.2), (ac,0.2), (bc,0.1)\}\} \end{array} \right\}.
\end{aligned}
\tag{21}
$$

$Z(a) = (\xi(a), \psi(a))$ and $Z(b) = (\xi(b), \psi(b))$ are FT1SGs as shown in Figure 14. We can see that $Z_a(b) = (\xi_a(b), \psi_a(b))$, $Z_a(c) = (\xi_a(c), \psi_a(c))$, $Z_a(d) = (\xi_a(d), \psi_a(d))$, $Z_b(c) = (\xi_b(c), \psi_b(c))$ and $Z_b(d) = (\xi_b(d), \psi_b(d))$ are all not trees. Hence $\mathbb{G} = (Z(a), Z(b))$ is not a FT2ST and $\mathbb{G}$ is also not a FT2SC.

**Proposition 3.** *Every fuzzy type-2 soft cycle is a regular fuzzy type-2 soft cycle.*

*Proof.* Let $\mathbb{G}$ be a FT2SC. Let $(Z_\chi, \mathscr{N}\mathscr{B}_\chi)$ be a T1FSC corresponding to $Z(\chi)$ for every $\chi \in B$. Then, $Z_\chi(j)$ is a cycle $\forall j \in \mathscr{N}\mathscr{B}_\chi$. We know that cycle is a path that is closed and

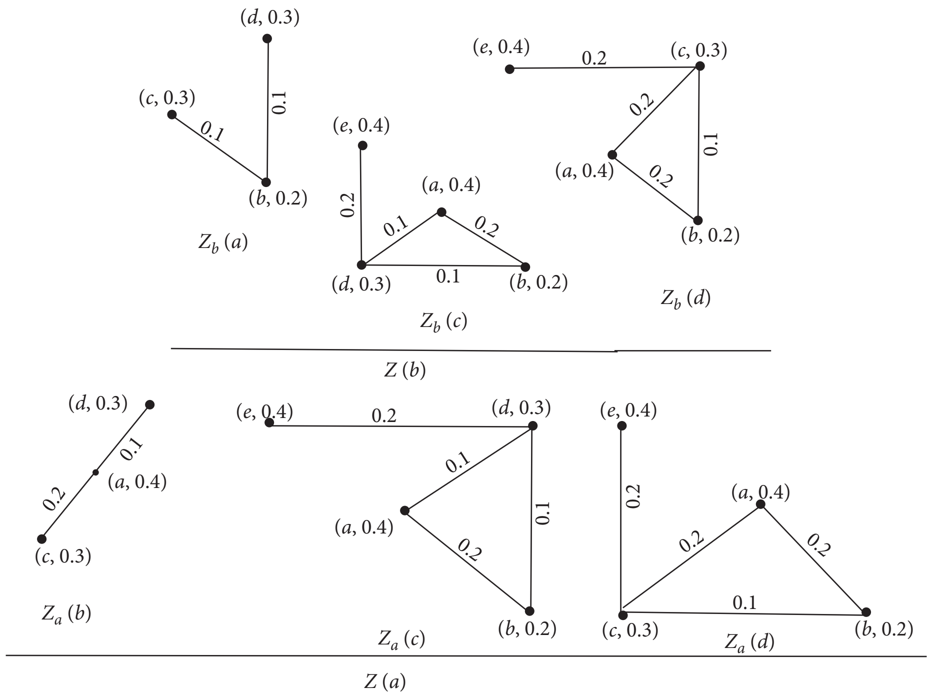


Figure 14: $\mathbb{G} = (Z(b), Z(a))$.

every vertex of cycle is of degree 2; this signifies that $Z_\chi(j)$ is a regular fuzzy graph for all $j \in \mathcal{NB}_\chi$. Therefore, $(Z_\chi, \mathcal{NB}_\chi)$ is a regular FT1SG, for all $\chi \in B$. Hence $\mathbb{G}$ is a regular FT2SG. □

*Definition 18.* Let $\mathbb{G}_1 = <\xi_1, \psi_1, B_1>$ and $\mathbb{G}_2 = <\xi_2, \psi_2, B_2>$ be two FT2STs. $\mathbb{G}_2$ is a fuzzy type-2 soft subtree (FT2SST) of $\mathbb{G}_1$ if

(i) $B_2 \subseteq B_1$

(ii) For each $j \in B_2$, FT1ST corresponding to $Z_{2(j)} = (\xi_{2(j)}, \psi_{2(j)})$ is a fuzzy type-1 soft subtree (FT1SST) of FT1ST corresponding to $Z_{1(j)} = (\xi_{1(j)}, \psi_{1(j)})$

*Example 8.* Let $\mathcal{G} = (\mathcal{J}, \mathcal{K})$ be a fuzzy graph as shown in Figure 15, where

$$\begin{aligned}\mathcal{J} &= \{(e_1, 0.3), (e_2, 0.2), (e_3, 0.2), (e_4, 0.4), (e_5, 0.6), (e_6, 0.2), (e_7, 0.3)\},\\ \mathcal{K} &= \{(e_1e_2, 0.1), (e_2e_3, 0.1), (e_3e_4, 0.2), (e_5e_4, 0.1), (e_5e_6, 0.1), (e_7e_6, 0.2), (e_7e_1, 0.1)\}.\end{aligned} \tag{22}$$

Let $B = \{(e_2, 0.2), (e_4, 0.4)\}$, $B^* = \{(e_2, 0.2), (e_4, 0.4)\}$, $\mathcal{NB}_{e_2} = \{(e_1, 0.3), (e_3, 0.2)\}$, $\mathcal{NB}_{e_4} = \{(e_3, 0.2), (e_5, 0.6)\}$. Let $[\xi, B]$ and $[\psi, B]$ be two FT2SSs over $\mathcal{J}$ and $\mathcal{K}$, respectively. We have

$$\xi(j) = \left(\xi_j, \mathcal{NB}_j\right) \psi(j) = \left(\psi_j, \mathcal{NB}_j\right), \quad \text{for all } j \in B. \tag{23}$$

Define $\xi_{e_2}(u) = \{v \in \mathcal{J} | u\mathcal{R}v \Longleftrightarrow \mathrm{d}(u, v) \le 0.2\}$, $\psi_{e_2}(u) = \left\{vw \in \mathcal{K} | v, w \subseteq \xi_{e_2}(u)\right\}$ $\forall u \in \mathcal{NB}_{e_2} \subseteq \mathcal{J}$ and $\xi_{e_4}(u) = \{v \in \mathcal{J} | u\mathcal{R}v \Longleftrightarrow \mathrm{d}(u, v) = 0.2\}$, $\psi_{e_4}(u) = \left\{vw \in \mathcal{K} | v, w \subseteq \xi_{e_4}(u)\right\}$ $\forall u \in \mathcal{NB}_{e_4} \subseteq \mathcal{J}$.

Then FT2SSs $[\xi, B]$ and $[\psi, B]$ are defined as follows:

$$\begin{aligned}\xi_{e_2} &= \{\{(e_1, 0.3), \{(e_2, 0.2), (e_3, 0.2), (e_7, 0.3)\}\}, \{(e_3, 0.2), \{(e_1, 0.3), (e_2, 0.2), (e_4, 0.4)\}\}\},\\ \psi_{e_2} &= \{\{(e_1, 0.3), \{(e_2e_3, 0.1)\}\}, \{(e_3, 0.2), \{(e_1e_2, 0.1)\}\}\},\\ \xi_{e_4} &= \{\{(e_3, 0.2), \{(e_1, 0.3), (e_4, 0.4)\}\}, \{(e_5, 0.6), \varnothing\}\},\\ \psi_{e_4} &= \{\{(e_3, 0.2), \varnothing\}, \{(e_5, 0.6), \varnothing\}\}.\end{aligned} \tag{24}$$

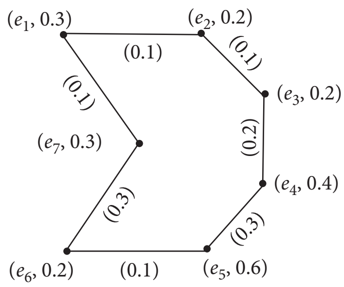


FIGURE 15: $\mathscr{G} = (\mathscr{J}, \mathscr{K})$ is a fuzzy graph.

$\mathbb{G} = (Z(e_2), Z(e_4))$ is a FT2ST as shown in Figure 16.

Let $[\xi', N]$ and $[\psi', N]$ be two FT2SSs over $\mathscr{J}$ and $\mathscr{K}$, respectively. We have

$$\begin{aligned} \xi'(j) &= \left(\xi_j, \mathcal{N}\mathcal{B}_j\right), \\ \psi'(j) &= \left(\psi_j, \mathcal{N}\mathcal{B}_j\right), \quad \text{for all } j \in N. \end{aligned} \tag{25}$$

Define $\xi'_{e_2}(u) = \{v \in \mathscr{J} | u\mathscr{R}v \Longleftrightarrow \mathrm{d}(u, v) \le 0.3\}$, $\psi'_{e_2}(u) = \left\{vw \in \mathscr{K} | v, w \subseteq \xi'_{e_2}(u)\right\}$ $\forall u \in \mathcal{N}\mathcal{B}_{e_2} \subseteq \mathscr{J}$ and $\xi'_{e_4}(u) = \{v \in \mathscr{J} | u\mathscr{R}v \Longleftrightarrow \mathrm{d}(u, v) \le 0.3\}$, $\psi'_{e_4}(u) = \left\{vw \in \mathscr{K} | v, w \subseteq \xi'_{e_4}(u)\right\}$ $\forall u \in \mathcal{N}\mathcal{B}_{e_4} \subseteq \mathscr{J}$.

FT2SSs $[\xi', N]$ and $[\psi', N]$ are defined as follows:

$$\begin{aligned} \xi'_{e_2} &= \{\{(e_1, 0.3), \{(e_2, 0.2), (e_3, 0.2), (e_7, 0.3)\}\}, \{(e_3, 0.2), \{(e_1, 0.3), (e_2, 0.2), (e_4, 0.4), (e_7, 0.3)\}\}\}, \\ \psi'_{e_2} &= \{\{(e_1, 0.3), \{(e_2e_3, 0.1)\}\}, \{(e_3, 0.2), \{(e_1e_7, 0.1), (e_1e_2, 0.1)\}\}\}, \\ \xi'_{e_4} &= \{\{(e_3, 0.2), \{(e_1, 0.3), (e_2, 0.2), (e_4, 0.4), (e_7, 0.3)\}\}, \{(e_5, 0.6), \{(e_4, 0.4), (e_6, 0.2)\}\}\}, \\ \psi'_{e_4} &= \{\{(e_3, 0.2), \{(e_2e_1, 0.1)(e_1e_7, 0.1)\}\}, \{(e_5, 0.6), \varnothing\}\}. \end{aligned} \tag{26}$$

$\mathbb{G}' = (Z'(e_2), Z'(e_4))$ is a FT2SST of $\mathbb{G}$ as shown in Figure 17. We can see that $B \subset B^*$ and $Z(e_2) \subseteq Z'(e_2), Z(e_4) \subseteq Z'(e_4)$. Hence, $\mathbb{G}$ is a FT2SST of $\mathbb{G}'$.

**Theorem 2.** *Let $\mathbb{G}_1 = <\xi_1, \psi_1, B_1>$ and $\mathbb{G}_2 = <\xi_2, \psi_2, B_2>$ be two FT2STs. Then $\mathbb{G}_2$ is said to be a FT2SST of $\mathbb{G}_1$ if and only if $\xi_2 \subseteq \xi_1$ and $\psi_2 \subseteq \psi_1$.*

*Proof.* Let $\mathbb{G}_2$ be a FT2SST of $\mathbb{G}_1$. Then, by using the definition of FT2SST,

(i) $B_2 \subseteq B_1$

(ii) For all $j \in B_2$, FT1ST corresponding to $Z_2(j) = (\xi_2(j), \psi_2(j))$ is a FT1SST of FT1ST corresponding to $Z_1(j) = (\xi_1(j), \psi_1(j))$

Since FT1ST corresponding to $Z_2(j)$ is a FT1SST of FT1ST corresponding to $Z_1(j)$ for all $j \in B_2$, we have $\xi_2 \subseteq \xi_1$ and $\psi_2 \subseteq \psi_1$ $\forall j \in B_2$. Conversely, we have $\xi_2(j) \subseteq \xi_1(j)$ and $\psi_2(j) \subseteq \psi_1(j) \forall j \in B_2$. As $\mathbb{G}_1$ is a fuzzy type-2 soft tree, fuzzy type-1 soft set corresponding to $Z_1(j)$ forms a FT1ST of $\mathbb{G}_2$ for all $j \in B_1$. Also, $\mathbb{G}_2$ is a fuzzy type-2 soft tree, and fuzzy type-1 soft set corresponding to $Z_2(j)$ forms a FT1ST of $\mathbb{G}_1$ for all $j \in B_2$. This implies that FT1ST corresponding to $Z_2(j)$ is a FT1SST of FT1ST corresponding to $Z_1(j)$ for all $j \in B_2$. Hence, $\mathbb{G}_2$ is a FT2SST of $\mathbb{G}_1$. □

*Definition 19.* Let $\mathbb{G}_1 = <\xi_1, \psi_1, B_1>$ and $\mathbb{G}_2 = <\xi_2, \psi_2, B_2>$ be two FT2STs. The union of $\mathbb{G}_1$ and $\mathbb{G}_2$ is denoted by $\mathbb{G}_1 \cup \mathbb{G}_2 = \mathbb{G} = <\xi, \psi, P>$, where $P = B_1 \cup B_2$, such that

$$\begin{aligned} \xi(v) &= \begin{cases} \xi_1(v), & \text{if } v \in B_1 - B_2, \\ \xi_2(v), & \text{if } v \in B_2 - B_1, \\ \xi_1(v) \cup \xi_2(v), & \text{if } v \in B_1 \cap B_2, \end{cases} \\ \psi(v) &= \begin{cases} \psi_1(v), & \text{if } v \in B_1 - B_2, \\ \psi_2(v), & \text{if } v \in B_2 - B_1, \\ \psi_1(v) \cup \psi_2(v), & \text{if } v \in B_2 \cap B_1, \end{cases} \end{aligned} \tag{27}$$

where $(\xi_1(v) \cup \xi_2(v), \psi_1(v) \cup \psi_2(v))$ for all $v \in B_1 \cap B_2$ relates to the fuzzy type-1 soft union between the relevant FT1STs corresponding to $(\xi_1(v), \psi_1(v))$ and $(\xi_2(v), \psi_2(v))$, respectively. It can be written as $\mathbb{G}_1 \cup \mathbb{G}_2 = \left\{Z(v) = (\xi(v), \psi(v)) | v \in P\right\}$.

**Theorem 3.** *Let $\mathbb{G}_1 = <\xi_1, \psi_1, B_1>$ and $\mathbb{G}_2 = <\xi_2, \psi_2, B_2>$ be two FT2STs with $B_1 \cap B_2 = \varnothing$. Then $\mathbb{G}_1 \cup \mathbb{G}_2$ is a FT2ST.*

*Proof.* Let $\mathbb{G}_1 = <\xi_1, \psi_1, B_1>$ and $\mathbb{G}_2 = <\xi_2, \psi_2, B_2>$ be two FT2STs. The union of $\mathbb{G}_1$ and $\mathbb{G}_2$ is denoted by $\mathbb{G}_1 \cup \mathbb{G}_2 = \mathbb{G} = <\xi, \psi, P>$, where $P = B_1 \cup B_2$ is defined $\forall v \in P$:

$$\begin{aligned} \xi(v) &= \begin{cases} \xi_1(v), & \text{if } v \in B_1 - B_2, \\ \xi_2(v), & \text{if } v \in B_2 - B_1, \\ \xi_1(v) \cup \psi_2(v), & \text{if } v \in B_1 \cap B_2, \end{cases} \\ \psi(v) &= \begin{cases} \psi_1(v), & \text{if } v \in B_1 - B_2, \\ \psi_2(v), & \text{if } v \in B_2 - B_1, \\ \psi_1(v) \cup \psi_2(v), & \text{if } v \in B_2 \cap B_1, \end{cases} \end{aligned} \tag{28}$$

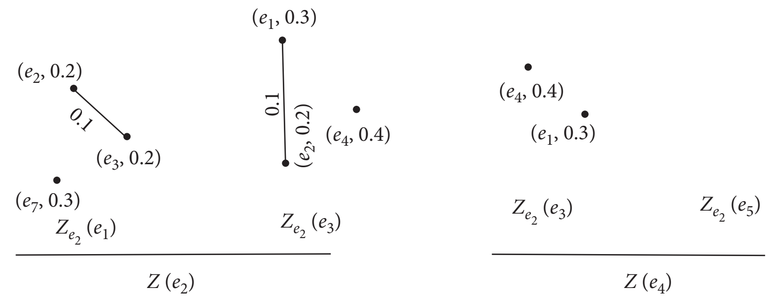


Figure 16: $\mathbb{G} = (Z(e_2), Z(e_4))$.

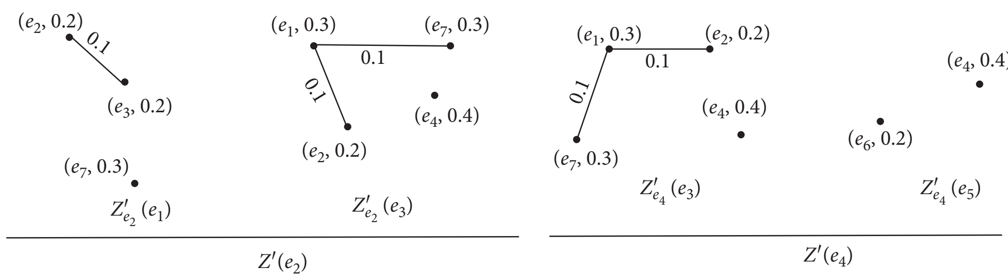


Figure 17: $\mathbb{G}' = (Z_{e_4}{}', Z_{e_2}{}')$.

where $\psi_1(v) \cup \psi_2(v)$ for all $v \in B_2 \cap B_1$ relates to the fuzzy type-1 soft extended union among the relevant FT1STs corresponding to $\psi_1(v)$ and $\psi_2(v)$, respectively, and $\xi_1(v) \cup \xi_2(v)$ for all $v \in B_1 \cap B_2$ relates to the fuzzy type-1 soft union between the relevant FT1STs corresponding to $\xi_1(v)$ and $\xi_2(v)$, respectively. Since $\mathbb{G}_1$ is a FT2ST, FT1ST corresponding to $(\xi_1(j)$ and $\psi_1(j))$ is a FT2ST for all $j \in B_1 - B_2$.

Since $\mathbb{G}_2$ is a FT2ST, FT1ST corresponding to $(\xi_2(j)$ and $\psi_2(j))$ is a FT2ST for all $j \in B_2 - B_1$. It is given that $B_1 \cap B_2 = \varnothing$. Thus, $\mathbb{G}_1 \cup \mathbb{G}_2 = \mathbb{G} = \; <\xi, \psi, B_1 \cap B_2>$ is a FT2ST. □

*Definition 20.* Let $\mathbb{G}_1 = \; <\xi_1, \psi_1, B_1>$ and $\mathbb{G}_2 = \; <\xi_2, \psi_2, B_2>$ be two FT2STs. The intersection of $\mathbb{G}_1$ and $\mathbb{G}_2$ is denoted by $\mathbb{G}_1 \cap \mathbb{G}_2 = \mathbb{G} = \; <\xi, \psi, P>$, where $P = B_1 \cup B_2$ such that

$$\begin{aligned}
\xi(v) &= \begin{cases} \xi_1(v), & \text{if } v \in B_1 - B_2, \\ \xi_2(v), & \text{if } v \in B_2 - B_1, \\ \xi_1(v) \cap \xi_2(v), & \text{if } v \in B_1 \cap B_2, \end{cases} \\
\psi(v) &= \begin{cases} \psi_1(v), & \text{if } v \in B_1 - B_2, \\ \psi_2(v), & \text{if } v \in B_2 - B_1, \\ \psi_1(v) \cap \psi_2(v), & \text{if } v \in B_2 \cap B_1, \end{cases}
\end{aligned} \tag{29}$$

where $(\xi_1(v) \cap \xi_2(v), \psi_1(v) \cap \psi_2(v))$ for all $v \in B_1 \cap B_2$ relates to the fuzzy type-1 soft intersection between the relevant FT1STs corresponding to $(\xi_1(v), \psi_1(v))$ and $(\xi_2(v), \psi_2(v))$, respectively.

It can be written as $\mathbb{G}_1 \cap \mathbb{G}_2 = \{Z(v) = \; (\xi(v), \psi(v)) \,|\, v \in P\}$.

*Example 9.* Let $\mathcal{G}$ be a fuzzy graph as shown in Figure 18. Let $B = \{(a, 0.9), (b, 0.1)\}, B^* = \{(a, 0.9), (e, 0.2)\}$. It can be written as $\mathcal{NB}_a = \{(v, 0.7), (b, 0.1)\}$, $\mathcal{NB}_b = \{(a, 0.9), (c, 0.1)\}$, $\mathcal{NB}_e = \{(d, 0.3), (f, 0.5)\}$. Let $[\xi, B]$ and $[\psi, B]$ be two FT2SSs over $\mathcal{J}$ and $\mathcal{K}$, respectively. We have

$$\begin{aligned}
\xi(j) &= \left(\xi_j, \mathcal{NB}_j\right), \\
\psi(j) &= \left(\psi_j, \mathcal{NB}_j\right), \quad \text{for all } j \in B.
\end{aligned} \tag{30}$$

Define $\xi_a(u) = \{v \in \mathcal{J} | u\mathcal{R}v \Longleftrightarrow \mathrm{d}(u, v) \leq 0.3\}$, $\psi_a(u) = \{vw \in \mathcal{K} | \{v, w\} \subseteq \xi_a(u)\} \; \forall u \in \mathcal{NB}_a$ and $\xi_b(u) = \{v \in \mathcal{J} | u\mathcal{R}v \Longleftrightarrow \mathrm{d}(u, v) \leq 0.3\}$, $\psi_b(u) = \{vw \in \mathcal{K} | \{v, w\} \subseteq \xi_b(u)\} \; \forall z \in \mathcal{NB}_b$.

The FT2SSs $[\xi, B]$ and $[\psi, B]$ are defined as follows:

$$\begin{aligned}
\xi_a &= \{\{(b, 0.1), \{(a, 0.9), (c, 0.1), (d, 0.3)\}\}, \{(v, 0.7), \{(d, 0.3), (g, 0.1), (f, 0.5), (h, 0.2)\}\}\}, \\
\psi_a &= \{\{(b, 0.1), \{(cd, 0.1)\}\}, \{(v, 0.7), \{(gh, 0.1), (gf, 0.1)\}\}\}, \\
\xi_b &= \{\{(c, 0.1), \{(a, 0.9), (b, 0.1), (d, 0.3), (e, 0.2)\}\}, \{(a, 0.9), \{(b, 0.1), (c, 0.1), (d, 0.3)\}\}\}, \\
\psi_b &= \{\{(c, 0.1), \{(ab, 0.1), (ed, 0.2)\}\}, \{(a, 0.9), \{(bc, 0.1), (cd, 0.1)\}\}\}.
\end{aligned} \tag{31}$$

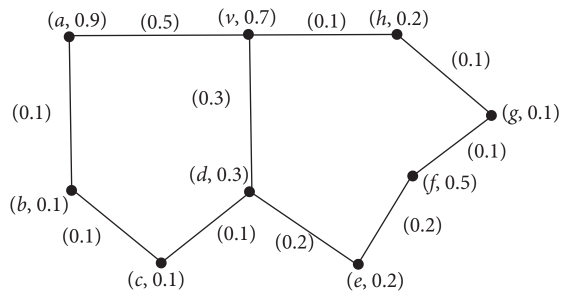


Figure 18: Fuzzy graph $\mathscr{G} = (\mathscr{J}, \mathscr{K})$.

Let $[\xi', B^*]$ and $[\psi', B^*]$ be two FT2SSs over $\mathscr{J}$ and $\mathscr{K}$, respectively. We have

$$\begin{aligned}\xi'(j) &= (\xi_j, \mathscr{N}\mathscr{B}_j),\\ \psi'(j) &= (\psi_j, \mathscr{N}\mathscr{B}_j), \quad \text{for all } j \in B^*,\\ \text{define } \xi'_a(u) &= \{v \in \mathscr{J} | u\mathscr{R}v \Longleftrightarrow \mathrm{d}(u,v) \le 0.2\},\\ \psi'_a(u) &= \{vw \in \mathscr{K} | \{v,w\} \subseteq \xi'_a(u)\}, \quad \forall u \in \mathscr{N}\mathscr{B}_a \subseteq \mathscr{J},\\ \xi'_e(u) &= \{v \in \mathscr{J} | u\mathscr{R}v \Longleftrightarrow \mathrm{d}(u,v) \le 0.3\},\\ \psi'_e(u) &= \{vw \in \mathscr{K} | \{v,w\} \subseteq \xi'_e(u)\}, \quad \forall u \in \mathscr{N}\mathscr{B}_e \subseteq \mathscr{J}.\end{aligned} \tag{32}$$

The FT2SSs $[\xi', B^*]$ and $[\psi', B^*]$ are defined as follows:

$$\begin{aligned}\xi'_a &= \{\{(b,0.1), \{(a,0.9), (c,0.1), (d,0.3)\}\}, \{(v,0.7), \{(g,0.1), (h,0.2)\}\}\},\\ \psi'_a &= \{\{(b,0.1), \{(cd,0.1)\}\}, \{(v,0.7), \{(gh,0.1)\}\}\},\\ \xi'_e &= \left\{\begin{matrix}\{(d,0.3), \{(a,0.9), (b,0.1), (c,0.1), (e,0.2), (v,0.7)\}\},\\ \{(f,0.5), \{(e,0.2), (g,0.1), (h,0.2), (v,0.7)\}\}\end{matrix}\right\},\\ \psi'_e &= \{\{(d,0.3), \{(bc,0.1), (av,0.5), (ba,0.1)\}\}, \{(f,0.5), \{(vh,0.1), (hg,0.1)\}\}\}.\end{aligned} \tag{33}$$

Then $\mathbb{G} = (Z(a), Z(b))$ and $\mathscr{G}' = (Z'(e), Z'(a))$ are FT2STs as shown in Figure 19. By the definition of intersection of FT2STs, $\xi(a) = \xi(a) \cap \xi'(a)$ and $\psi(a) = \psi(a) \cap \psi'(a)$ where $a \in B^* \cap B$.

Therefore, $\mathbb{G}_1 \cap \mathbb{G}_2 = \mathbb{G} = <\xi, \psi, B \cup B^*>$ is a FT2ST as shown in Figure 20.

*Definition 21.* Let $\mathbb{G}_1 = <\xi_1, \psi_1, M_1>$ and $\mathbb{G}_2 = <\xi_2, \psi_2, B_2>$ be two FT2STs. The AND operation of $\mathbb{G}_1$ and $\mathbb{G}_2$ is denoted by $\mathbb{G}_1 \wedge \mathbb{G}_2 = \mathbb{G} = <\xi, \psi, B_1 \times B_2>$ such that $\xi(\chi, \eta) = \xi_1(\chi) \wedge \xi_2(\eta)$, $\psi(\chi, \eta) = \psi_1(\chi) \wedge \psi_2(\eta)$ for all $(\chi, \eta) \in B_1 \times B_2$. $(\xi(\chi, \eta), \psi(\chi, \eta))$ for all $(\chi, \eta) \in B_1 \times B_2$ is the fuzzy type-1 soft AND operation between the relevant FT1SGs corresponding to $(\xi_1(\chi), \psi_1(\chi))$ and $(\xi_2(\eta), \psi_2(\eta))$, respectively.

*Example 10.* Let $\mathscr{G} = (\mathscr{J}, \mathscr{K})$ be the fuzzy graph as shown in Figure 21, where

$$\begin{aligned}\mathscr{J} &= \{(e_1,0.3), (e_2,0.1), (e_3,0.2), (e_4,0.4), (e_5,0.6),\\ &\quad (e_6,0.2), (e_7,0.3), (e_8,0.3)\},\\ \mathscr{K} &= \{(e_1e_2,0.1), (e_2e_3,0.1), (e_3e_4,0.2), (e_5e_4,0.3),\\ &\quad (e_5e_6,0.1), (e_7e_6,0.2), (e_7e_8,0.2), (e_8e_1,0.1)\}.\end{aligned} \tag{34}$$

Let $B = \{(e_3,0.2), (e_4,0.4)\}$, $B^* = \{(e_7,0.3)\}$, $\mathscr{N}\mathscr{B}_{e_3} = \{(e_2,0.1), (e_4,0.4)\}$, $\mathscr{N}\mathscr{B}_{e_4} = \{(e_3,0.2), (e_5, 0.6)\}$, $\mathscr{N}\mathscr{B}_{e_7} = \{(e_6,0.2), (e_8,0.3)\}$.

Let $[\xi, B]$ and $[\psi, B]$ be two FT2SSs over $\mathscr{J}$ and $\mathscr{K}$, respectively. We have

$$\begin{aligned}\xi(j) &= (\xi_j, \mathscr{N}\mathscr{B}_j),\\ \psi(j) &= (\psi_j, \mathscr{N}\mathscr{B}_j), \quad \text{for all } J \in B,\\ \text{define } \xi_{e_3}(u) &= \{v \in \mathscr{J} | u\mathscr{R}v \Longleftrightarrow \mathrm{d}(u,v) \le 0.3\},\\ \psi_{e_3}(u) &= \{vw \in \mathscr{K} | \{v,w\} \subseteq \xi_{e_3}(u)\}, \quad \forall u \in \mathscr{N}\mathscr{B}_{e_3},\\ \xi_{e_4}(u) &= \{v \in \mathscr{J} | u\mathscr{R}v \Longleftrightarrow \mathrm{d}(u,v) \le 0.4\},\\ \psi_{e_4}(u) &= \{vw \in \mathscr{K} | \{v,w\} \subseteq \xi_{e_4}(u)\}, \quad \forall z \in \mathscr{N}\mathscr{B}_{e_3}.\end{aligned} \tag{35}$$

The FT2SSs $[\xi, B]$ and $[\psi, B]$ are defined as follows:

$$\begin{aligned}\xi_{e_3} &= \{\{(e_4,0.4), \{(e_2,0.1), (e_3,0.2), (e_5,0.6)\}\}, \{(e_2,0.1), \{(e_1,0.3), (e_3,0.2), (e_4,0.4), (e_8,0.3)\}\}\},\\ \psi_{e_3} &= \{\{(e_4,0.4), \{(e_2e_3,0.1)\}\}, \{(e_2,0.1), \{(e_3e_4,0.2), (e_1e_8,0.1)\}\}\},\\ \xi_{e_4} &= \{\{(e_5,0.6), \{(e_4,0.4), (e_6,0.2), (e_7,0.3)\}\}, \{(e_3,0.2), \{(e_1,0.3), (e_2,0.1), (e_4,0.4), (e_8,0.3)\}\}\},\\ \psi_{e_4} &= \{\{(e_3,0.2), \{(e_8e_1,0.1), (e_1e_2,0.1)\}\}, \{(e_5,0.6), \{(e_6e_7,0.2)\}\}\}.\end{aligned} \tag{36}$$

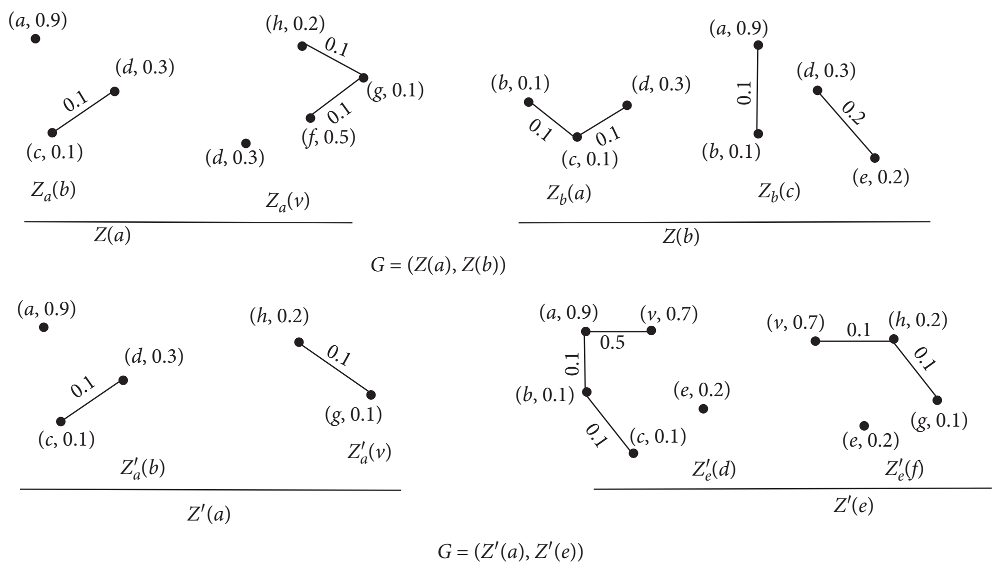


Figure 19: $\mathbb{G} = (Z(a), Z(b))$ and $\mathbb{G}' = (Z'(e), Z'(a))$.

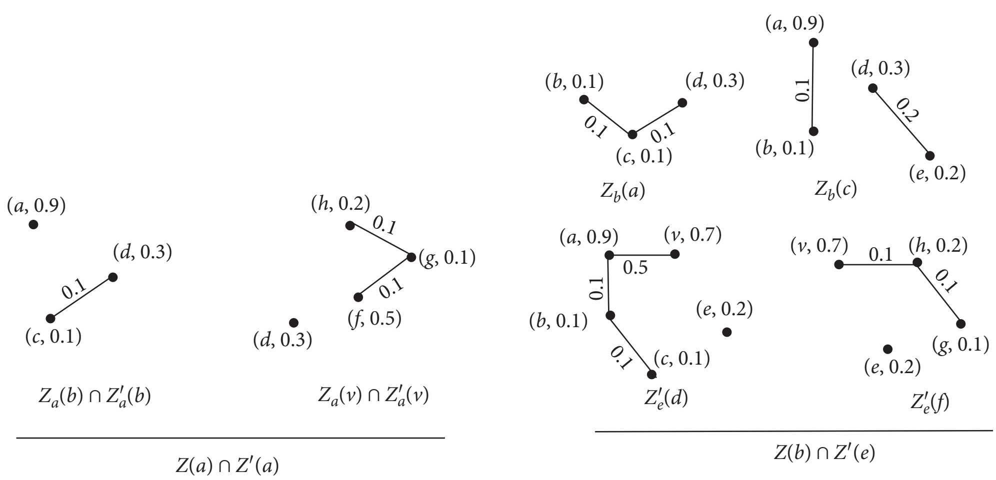


Figure 20: Intersection of $\mathbb{G}$ and $\mathbb{G}'$ is $\mathbb{G} \cap \mathbb{G}'$.

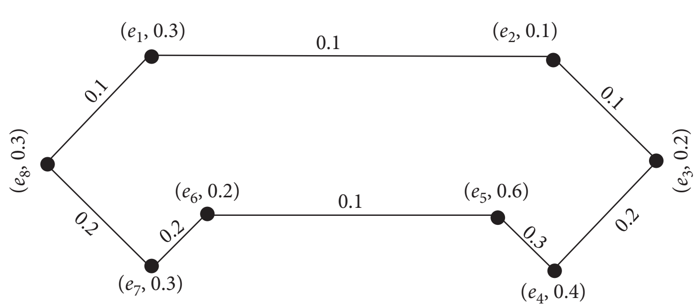


Figure 21: Fuzzy graph $\mathcal{G} = (\mathcal{J}, \mathcal{K})$.

Then $\mathbb{G} = (Z(e_3), Z(e_4))$ is a FT2ST. Let $[\xi', B^*]$ and $[\psi', B^*]$ be two FT2SSs over $\mathcal{J}$ and $\mathcal{K}$, respectively. We have

$$\begin{aligned} \xi'(j) &= \left(\xi_j, \mathcal{NB}_j\right), \\ \psi'(j) &= \left(\psi_j, \mathcal{NB}_j\right), \quad \text{for all } j \in B^*. \end{aligned} \tag{37}$$

Define $\xi'_{e_7}(u) = \{v \in \mathcal{J} | u\mathcal{R}v \Longleftrightarrow \mathrm{d}(u, v) \leq 0.4\}$, $\psi'_{e_7}(u) = \left\{vw \in \mathcal{K} | \{v, w\} \subseteq \xi'_{e_7}(u)\right\} \forall u \in \mathcal{NB}_{e_7} \subseteq \mathcal{J}$,

$$\begin{aligned} \xi'_{e_7} &= \left\{ \begin{array}{c} \{(e_8, 0.3), \{(e_1, 0.3), (e_2, 0.1), (e_3, 0.2), (e_6, 0.2), (e_7, 0.3)\}\}, \\ \{(e_6, 0.2), \{(e_4, 0.4), (e_5, 0.6), (e_7, 0.3), (e_8, 0.3)\}\} \end{array} \right\}, \\ \psi'_{e_7} &= \left\{ \begin{array}{c} \{(e_8, 0.3), \{(e_1e_2, 0.1), (e_2e_3, 0.1), (e_7e_6, 0.2)\}\}, \\ \{(e_6, 0.2), \{(e_4e_5, 0.3), (e_7e_8, 0.2)\}\} \end{array} \right\}. \end{aligned} \tag{38}$$

$\mathbb{G}' = Z'(e_7)$ is a FT2ST. The AND operation of $\mathbb{G}$ and $\mathbb{G}'$ is defined as follows:

$$\begin{aligned} \xi(e_3, e_7) &= \xi_{e_3} \wedge \xi'_{e_7} \left\{ \begin{array}{c} \{((e_4, 0.4), (e_8, 0.3)), \{(e_3, 0.2), (e_2, 0.1)\}\}, \\ \{((e_4, 0.4), (e_6, 0.2)), \{(e_5, 0.6)\}\}, \\ \{(e_2, 0.1), (e_6, 0.2), \{(e_4, 0.4), (e_8, 0.3)\}\}, \\ \{(e_8, 0.3), (e_2, 0.1), \{(e_1, 0.3), (e_3, 0.2)\}\} \end{array} \right\}, \\ \psi(e_3, e_7) &= \psi_{e_3} \wedge \psi'_{e_7} \left\{ \begin{array}{c} \{((e_4, 0.4), (e_8, 0.3)), \{(e_2e_3, 0.1)\}\}, \{((e_4, 0.4), (e_6, 0.2)), \varnothing\}, \\ \{((e_2, 0.1), (e_8, 0.3)), \varnothing\}, \{((e_2, 0.1), (e_6, 0.2)), \varnothing\} \end{array} \right\}, \\ \xi(e_4, e_7) &= \xi_{e_4} \wedge \xi'_{e_7} \left\{ \begin{array}{c} \{((e_5, 0.6), (e_8, 0.3)), \{(e_6, 0.2), (e_7, 0.3)\}\}, \\ \{((e_5, 0.6), (e_6, 0.2)), \{(e_7, 0.3), (e_4, 0.4)\}\}, \\ \{((e_3, 0.2), (e_8, 0.3)), \{(e_1, 0.3), (e_2, 0.1)\}\}, \\ \{((e_3, 0.2), (e_6, 0.2)), \{(e_8, 0.3), (e_4, 0.4)\}\} \end{array} \right\}, \\ \psi(e_4, e_7) &= \psi_{e_4} \wedge \psi'_{e_7} \left\{ \begin{array}{c} \{((e_5, 0.6), (e_8, 0.3)), \{(e_6e_7, 0.2)\}\}, \{((e_5, 0.6), (e_6, 0.2)), \varnothing\}, \\ \{\{((e_3, 0.2), (e_8, 0.3)), \{(e_2e_1, 0.1)\}\}, \{(e_3, 0.2), (e_6, 0.2), \varnothing\}\} \end{array} \right\}. \end{aligned} \tag{39}$$

The AND operation of $\mathbb{G}$ and $\mathbb{G}'$ is shown in Figure 22.

*Definition 22.* Let $\mathbb{G}_1 = <\xi_1, \psi_1, M_1>$ and $\mathbb{G}_2 = <\xi_2, \psi_2, B_2>$ be two FT2STs. The OR operation of $\mathbb{G}_1$ and $\mathbb{G}_2$ is denoted by $\mathbb{G}_1 \vee \mathbb{G}_2 = \mathbb{G} = <\xi, \psi, B_1 \times B_2>$ such that $\xi(\chi, \eta) = \xi_1(\chi) \vee \xi_2(\eta)$, $\psi(\chi, \eta) = \psi_1(\chi) \vee \psi_2(\eta)$ for all $(\chi, \eta) \in B_1 \times B_2$. $(\xi(\chi, \eta), \psi(\chi, \eta))$ for all $(\chi, \eta) \in B_1 \times B_2$ is the fuzzy type-1 soft OR operation between the relevant FT1SGs corresponding to $(\xi_1(\chi), \psi_1(\chi))$ and $(\xi_2(\eta), \psi_2(\eta))$, respectively.

## 4. Applications of Fuzzy Type-2 Soft Graphs

In this section, we apply the concept of fuzzy type-2 soft graphs to decision-making problems in chemical digestion and national engineering services. The selection of a suitable object problem can be considered as a decision-making problem, in which final identification of object is decided on a given set of information. A detailed description of the algorithm for the selection of most suitable object based on available set of parameters is given in Algorithm 1 and the flow chart shown in Figure 23; purposed algorithm can be used to find out the best correspondence relationship between the neighboring objects in the decision-making problem. This method can be applied in various domains for multicriteria selection of objects.

*4.1. Determination of Dominant Food Components in Chemical Digestion.* We present an application of FT2SG in chemical digestion and discuss how to apply FT2SG in

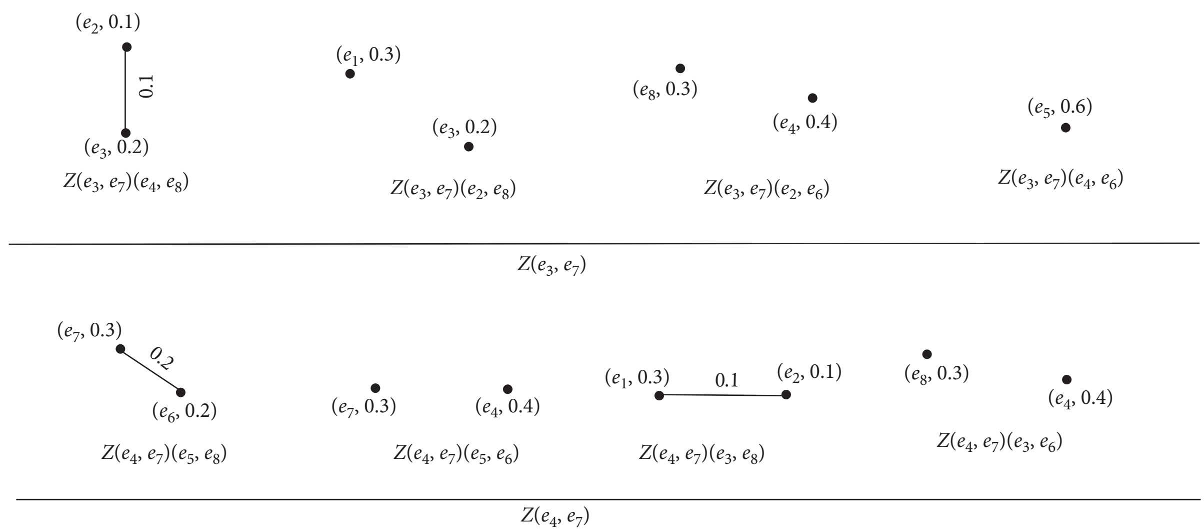


Figure 22: AND operation of $\mathbb{G}$ and $\mathbb{G}'$ is $\mathbb{G} \wedge \mathbb{G}' = (Z(e_3, e_7), Z(e_4, e_7))$.

(1) Input the fuzzy graph $\mathscr{G} = (\mathscr{J}, \mathscr{K})$.
(2) Input the choice parameter set $B = \{e_1, e_2, \ldots, e_n\}$ for suitable selection of the object.
(3) Input the VN-FT2SS $[\xi, B]$ over $\mathscr{J}$ and VN-FT2SS $[\psi, B]$ over $\mathscr{K}$.
(4) Construct a FT2SG $\mathbb{G} = <\xi, \psi, B> = \{Z(j) | j \in B\}$, where $Z(j) = (Z_{(j)}, \mathscr{N}\mathscr{B}_j)$ such that $Z_j(u) = (\xi_j(u), \psi_j(u)), \forall u \in \mathscr{N}\mathscr{B}_j$.
(5) Construct the resultant VN-fuzzy graph by taking the intersection of vertex-neighbors fuzzy graphs $Z^*(j) = \bigcap_u Z_j(u), \forall u \in \mathscr{N}\mathscr{B}_j$.
(6) Tabular representation of resultant VN-fuzzy graph $Z^*(j) \forall j \in B$ with the choice values $C_i^j$.
(7) The decision is $S_i$ if $S_i = \bigvee_i^n (\bigwedge_j C_i^j)$.
(8) If $i$ has more than one value, then any one of $S_i$ may be chosen.

Algorithm 1: Algorithm for the selection of most suitable objects.

chemical digestion of spinach. Spinach is generally composed of carbohydrates, protein, lipids, minerals, vitamins, and nucleic acids. We mainly focused on the digestion of carbohydrates, proteins, lipids, and nucleic acids, which is carried out by a variety of salivary enzymes and the enzymes present in other parts of digestive system; that is, amylase, pepsin, and trypsin are released as a result of involuntary signal generated by our body to digest the food. When 25 g of spinach is taken, it contains carbohydrates (0.9g), protein (0.7g), lipids (0.1g), nucleic acids (0.3g), involuntary signal (0.3), pepsin (0.2), amylase (0.2), and trypsin (0.1), represented as vertices donated by $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$, respectively. "Chemical digestion" is the enzyme-mediated, hydrolysis method that converts large macronutrients into smaller molecules.

(i) Carbohydrate mostly comprises amylose and glycogen. Long carbohydrates chains are broken down into disaccharides which are decomposed by amylase enzyme.
(ii) Proteins are usually broken down into amino acids by peptidase enzyme as well as trypsin and chymotrypsin.
(iii) Lipids are hydrolyzed by pancreatic lipase enzyme.
(iv) Nucleic acids, that is, DNA and RNA, are hydrolyzed by pancreatic nuclease.
(v) Involuntary signal is generated by the brain in order to carry out chemical digestion in the digestive system.

Protein digestion occurs in stomach and duodenum by the action of three primary enzymes.

(i) Pepsin, disguised by abdomen
(ii) Trypsin, disguised through pancreas
(iii) Amylase, disguised through saliva and pancreas

Note that the values of pepsin, trypsin, amylase, and involuntary signal are supposed as we cannot calculate the amounts of these products released as a result of con-

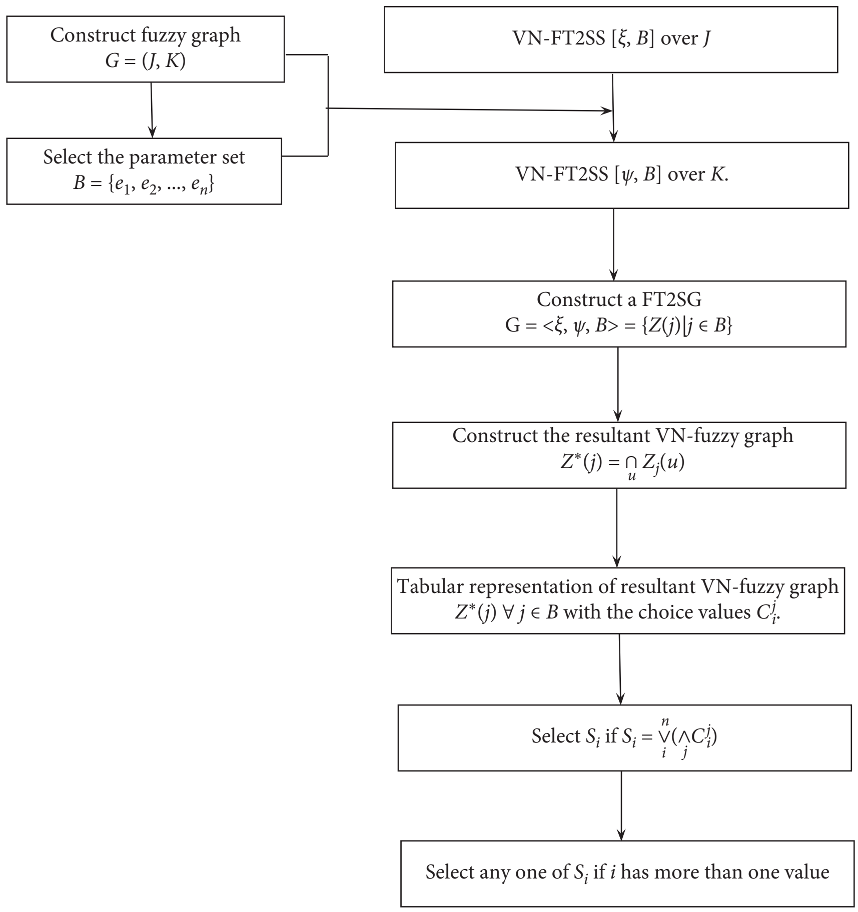


Figure 23: Flow chart for suitable selection of objects.

sumption of little amount of food through previous literature findings.

Consider

$$\mathcal{J} = \{(e_1, 0.9), (e_2, 0.7), (e_3, 0.1), (e_4, 0.3), (e_5, 0.3), (e_6, 0.2), (e_7, 0.2), (e_8, 0.1)\},$$
$$\mathcal{K} = \left\{ \begin{array}{c} (e_1e_2, 0.3), (e_1e_4, 0.3), (e_3e_2, 0.1), (e_3e_4, 0.1), (e_5e_4, 0.2), \\ (e_5e_6, 0.2), (e_5e_8, 0.1), (e_8e_7, 0.1), (e_6e_7, 0.1) \end{array} \right\}. \tag{40}$$

In fuzzy graph $(\mathcal{J}, \mathcal{K})$ as shown in Figure 24, edges represent the amount of energy utilized by the body in order to carry out the digestion process. Let $B = \{(e_1, 0.9), (e_2, 0.7)\}$ represent the amounts of carbohydrates and protein released when 25 g of spinach is consumed. We have $\mathcal{NB}_{e_1} = \{(e_2, 0.7), (e_4, 0.3)\}$, $\mathcal{NB}_{e_2} = \{(e_3, 0.1), (e_1, 0.9)\}$.

Let $[\xi, B]$ and $[\psi, B]$ be two FT2SSs over $\mathcal{J}$ and $\mathcal{K}$, respectively. We have

$$\begin{aligned} \xi(j) &= \left(\xi_j, \mathcal{NB}_j\right), \\ \psi(j) &= \left(\psi_j, \mathcal{NB}_j\right), \quad \text{for all } j \in B, \\ \text{define } \xi_{e_1}(u) &= \{v \in \mathcal{J} | u\mathcal{R}v \Longleftrightarrow \mathrm{d}(u, v) \leq 0.5\}, \\ \psi_{e_1}(u) &= \left\{vw \in \mathcal{K} | v, w \subseteq \xi_{e_1}(u)\right\}, \\ \xi_{e_2}(u) &= \{v \in \mathcal{J} | u\mathcal{R}v \Longleftrightarrow \mathrm{d}(u, v) \leq 0.5\}, \\ \psi_{e_2}(u) &= \left\{vw \in \mathcal{K} | v, w \subseteq \xi_{e_2}(u)\right\}. \end{aligned} \tag{41}$$

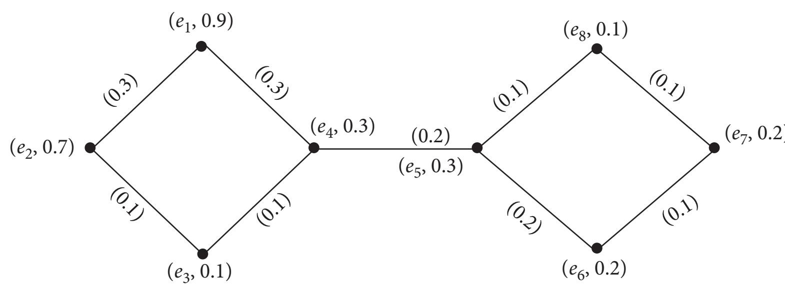


Figure 24: $\mathscr{G} = (\mathscr{J}, \mathscr{K})$.

The FT2SSs $[\xi, B]$ and $[\psi, B]$ are defined as follows:

$$
\begin{aligned}
\xi_{e_1} &= \left\{ \begin{array}{c} \{(e_2, 0.7), \{(e_1, 0.9), (e_3, 0.1), (e_4, 0.3), (e_5, 0.3)\}\}, \\ \{(e_4, 0.3), \{(e_1, 0.9), (e_2, 0.7), (e_3, 0.1), (e_5, 0.3), (e_6, 0.2), (e_7, 0.2), (e_8, 0.1)\}\} \end{array} \right\}, \\
\psi_{e_1} &= \left\{ \begin{array}{c} \{(e_2, 0.7), \{(e_3e_4, 0.1), (e_1e_4, 0.3), (e_5e_4, 0.2)\}\}, \\ \{(e_4, 0.5), (e_1e_2, 0.3), (e_3e_2, 0.1), (e_5e_6, 0.2), (e_5e_8, 0.1), (e_6e_7, 0.1), (e_7e_8, 0.1)\} \end{array} \right\}, \\
\xi_{e_2} &= \left\{ \begin{array}{c} \{(e_1, 0.9), \{(e_2, 0.7), (e_3, 0.1), (e_4, 0.3), (e_5, 0.3)\}\}, \\ \{(e_3, 0.1), \{(e_1, 0.9), (e_2, 0.7), (e_4, 0.3), (e_5, 0.3), (e_6, 0.2), (e_8, 0.1)\}\} \end{array} \right\}, \\
\psi_{e_2} &= \left\{ \begin{array}{c} \{(e_1, 0.9), \{(e_5e_4, 0.2), (e_3e_4, 0.2), (e_2e_3, 0.1)\}\}, \\ \{(e_3, 0.4), \{(e_1e_2, 0.3), (e_1e_4, 0.3), (e_4e_5, 0.2), (e_5e_6, 0.2), (e_5e_8, 0.1)\}\} \end{array} \right\}.
\end{aligned}
\tag{42}
$$

The fuzzy type-2 soft graph $\mathbb{G}$ is shown in Figure 25.

The tabular representations of resultant vertex-neighbors fuzzy graphs $Z^*(e_j)$ shown in Figure 26 corresponding to the parameter $e_j$, $j = 1, 2$ with the choice values $C_i^j = \sum_k S_{ik}$ for all $i, k$ are given in Tables 2 and 3.

The decision value is $S_i = \vee_i^7 (\wedge_j C_i^j) = \vee_{i=1}^7 \{0.6 \wedge 0.6, 0.4 \wedge 0.4, 0.2 \wedge 0.2, 0.6 \wedge 0.5, 0.3 \wedge 0.2, 0.2 \wedge 0, 0.3 \wedge 0.2\} = 0.6$ from the choice value $C_i^j$ of fuzzy type-2 soft graphs for $j = 1, 2$. The prominent food components are $e_1$ as carbohydrates and $e_2$ as lipids as carbohydrates are consumed as sugar and lipids are consumed as fats. Clearly, the dominant food components are $e_1$ or $e_4$.

*4.2. Water Supply for National Engineering Services.* We present the application of fuzzy type-2 soft graph in the National Engineering Services Pakistan (NESPAK). The National Engineering Services Pakistan is a Pakistani multinational state-owned corporation that provides construction, management, and consulting services globally. Every government project has something to do with NESPAK at some time of its planning or implementation. In fuzzy graph $\mathscr{G} = (\mathscr{J}, \mathscr{K})$ as shown in Figure 27, vertices represent some important projects.

$$
\mathscr{J} = \left\{ \begin{array}{c} (a = \text{Water Supply}, 0.9), (b = \text{Sewerage}, 0.7), (c = \text{Drainage}, 0.6), \\ (\text{d} = \text{Solid Waste Management}, 0.4), (e = \text{Plumbing}, 0.3), (f = \text{Industrial Wastes}, 0.2) \end{array} \right\}. \tag{43}
$$

NESPAK provides engineering services for these projects, the membership value of a vertex showing the working capability of the relevant project and values of edges represents the strength of the relationship between different projects to complete the tasks.

Now, we take two important projects Plumbing and Solid Waste Management named as $(e, 0.3)$, $(d, 0.4)$, respectively, and $B = \{(e, 0.3), (d, 0.4)\} \subset \mathscr{J}$. The vertex-neighbors of these selected projects are $\mathscr{N}\mathscr{B}_e = \{(b, 0.7), (c, 0.6), (d, 0.4), (f, 0.2)\}$ and $\mathscr{N}\mathscr{B}_d = \{(a, 0.9), (c, 0.6), (e, 0.3), (f, 0.2)\}$. Let $[\xi, B]$ and $[\psi, B]$ be two FT2SSs over $\mathscr{J}$ and $\mathscr{K}$, respectively. We have

$$
\begin{aligned}
\xi(j) &= \left(\xi_j, \mathscr{N}\mathscr{B}_j\right), \\
\psi(j) &= \left(\psi_j, \mathscr{N}\mathscr{B}_j\right), \quad \text{for all } j \in B, \\
\text{define } \xi_e(u) &= \{v \in \mathscr{J} \mid u\mathscr{R}v \Longleftrightarrow \text{d}(u, v) \leq 0.2\}, \\
\psi_e(u) &= \{vw \in \mathscr{K} \mid v, w \subseteq \xi_e(u)\}, \quad \forall u \in \mathscr{N}\mathscr{B}_e \subseteq \mathscr{J}, \\
\xi_d(u) &= \{v \in \mathscr{J} \mid u\mathscr{R}v \Longleftrightarrow 0.1 \leq \text{d}(u, v) \leq 0.2\}, \\
\psi_d(u) &= \{vw \in \mathscr{K} \mid v, w \subseteq \xi_d(u)\}, \quad \forall u \in \mathscr{N}\mathscr{B}_d \subseteq \mathscr{J}.
\end{aligned}
\tag{44}
$$

The FT2SSs $[\xi, B]$ and $[\psi, B]$ are defined as follows:

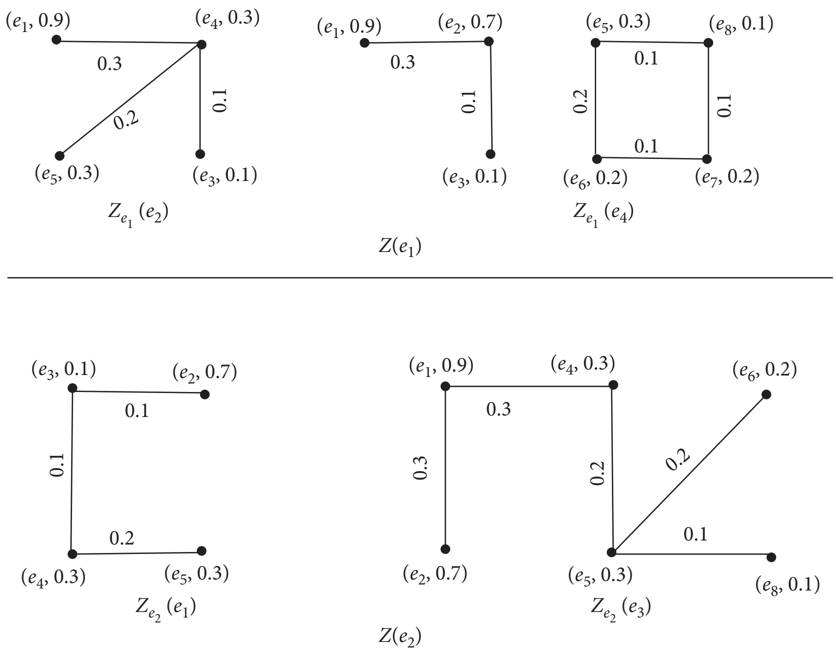


Figure 25: Fuzzy type-2 soft graph $\mathbb{G} = (Z(e_1), Z(e_2))$.

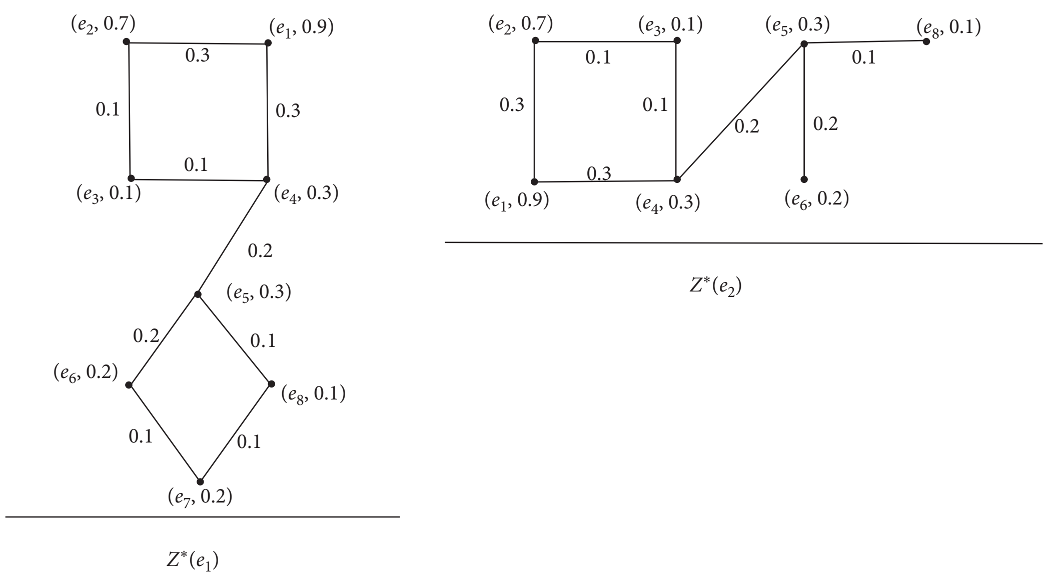


Figure 26: Resultant VN-fuzzy graphs $Z^*(e_1)$ and $Z^*(e_2)$.

Table 2: The tabular representation of $Z^*(e_1)$ with choice values.

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ | $e_7$ | $e_8$ | $C_i^1$ |
|---|---|---|---|---|---|---|---|---|---|
| $e_1$ | 0 | 0.3 | 0 | 0.3 | 0 | 0 | 0 | 0 | 0.6 |
| $e_2$ | 0.3 | 0 | 0.1 | 0 | 0 | 0 | 0 | 0 | 0.4 |
| $e_3$ | 0 | 0.1 | 0 | 0.1 | 0 | 0 | 0 | 0 | 0.2 |
| $e_4$ | 0.3 | 0 | 0.1 | 0 | 0.2 | 0 | 0 | 0 | 0.6 |
| $e_5$ | 0 | 0 | 0 | 0.2 | 0 | 0.2 | 0 | 0.1 | 0.5 |
| $e_6$ | 0 | 0 | 0 | 0 | 0.2 | 0 | 0.1 | 0 | 0.3 |
| $e_7$ | 0 | 0 | 0 | 0 | 0 | 0.1 | 0 | 0.1 | 0.2 |
| $e_8$ | 0 | 0 | 0 | 0 | 0.1 | 0 | 0.1 | 0 | 0.2 |

Table 3: The tabular representation of $Z^*(e_2)$ with choice values $C_i^2$.

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ | $e_7$ | $C_i^2$ |
|---|---|---|---|---|---|---|---|---|
| $e_1$ | 0 | 0.3 | 0 | 0.3 | 0 | 0 | 0 | 0.6 |
| $e_2$ | 0.3 | 0 | 0.1 | 0 | 0 | 0 | 0 | 0.4 |
| $e_3$ | 0 | 0.1 | 0 | 0.1 | 0 | 0 | 0 | 0.2 |
| $e_4$ | 0.3 | 0 | 0.1 | 0 | 0.2 | 0 | 0 | 0.6 |
| $e_5$ | 0 | 0 | 0 | 0.2 | 0 | 0.2 | 0.1 | 0.5 |
| $e_6$ | 0 | 0 | 0 | 0 | 0.2 | 0 | 0 | 0.2 |
| $e_8$ | 0 | 0 | 0 | 0 | 0.1 | 0 | 0 | 0.1 |

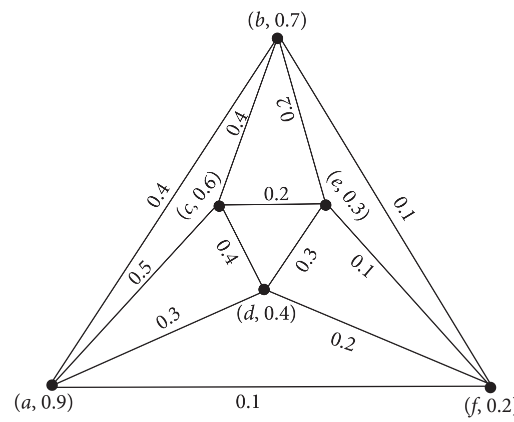


Figure 27: Fuzzy graph $\mathscr{G} = (\mathscr{J}, \mathscr{K})$.

$$
\begin{aligned}
\xi_e &= \left\{ \begin{array}{c} \{(f,0.2),\{(a,0.9),\,(b,0.7),\,(d,0.4),\,(e,0.3)\}\},\{(d,0.4),\{(f,0.2)\}\},\{(c,0.6),\{(e,0.3)\}\}, \\ \{(b,0.7),\{(e,0.3),\,(f,0.2)\}\} \end{array} \right\}, \\
\psi_e &= \left\{ \begin{array}{c} \{(f,0.2),\{(ad,0.3),\,(ab,0.4),\,(de,0.3),\,(eb,0.2)\}\}, \\ \{(d,0.4),\varnothing\},\{(c,0.6),\varnothing\},\{(b,0.7),\{(fe,0.1)\}\} \end{array} \right\}, \\
\xi_d &= \left\{ \begin{array}{c} \{(e,0.3),\{(b,0.7),\,(c,0.6),\,(f,0.2)\}\},\{(a,0.9),\{(f,0.2)\}\}, \\ \{(f,0.2),\{(a,0.9),\,(b,0.7),\,(d,0.4),\,(e,0.3)\}\},\{(c,0.6),\{(e,0.3)\}\} \end{array} \right\}, \\
\psi_d &= \left\{ \begin{array}{c} \{(e,0.3),\{(bc,0.4),\,(fb,0.1)\}\},\{(a,0.9),\varnothing\}, \\ \{(c,0.6),\varnothing\},\{(f,0.2),\{(a\,d,0.3),\,(ab,0.4),\,(de\,,0.3),\,(eb,0.2)\}\} \end{array} \right\}.
\end{aligned}
\tag{45}
$$

FT1SGs corresponding to $Z(e) = (\xi(e), \psi(e))$ and $Z(d) = (\xi(d), \psi(d))$, respectively, are shown in fuzzy type-2 soft graph 28. (Figure 28)

The tabular representations of resultant vertex-neighbors fuzzy graphs $Z^*(e)$ and $Z^*(d)$ shown in Figure 29 with the choice values $C_i^j = \sum_k S_{ik}$ for all $i, k$ are given in Tables 4 and 5.

The decision value is $S_i = \vee_i^6 (\wedge_j C_i^j) = \vee_{i=1}^6 \{0.7 \wedge 0.7, 0.6 \wedge 1.1, 0 \wedge 0.4, 0.6 \wedge 0.6, 0.6 \wedge 0.5, 0.1 \wedge 0.1\} = 0.7$, from the choice value $C_i^j$ of fuzzy type-2 soft graphs for $j = 1, 2$. The optimal project is "$a$ = water supply." So, NESPAK provides the best engineering services to the project of "water supply."

Advantages of the Proposed Method.

The advantages of the proposed method based on FT2SGs are as follows:

(1) The method can be effectively used to handle uncertainty and vagueness with correspondence, assertion, and relations among parameters.

(2) The proposed method incorporates parametrization tool with fuzzy information to effectively handle more uncertain conditions and errors in given data.

(3) The presented method considers vertex-neighbors coordination tool along with reparametrization to study the interrelationship and ambiguity among objects.

## 5. Comparison Analysis

In this section, we discuss the comparison of fuzzy type-2 soft graphs with fuzzy soft graphs and type-2 soft graphs.

*5.1. Comparison with Fuzzy Soft Graphs.* Fuzzy soft graph [21] is a parameterized family of fuzzy graphs, and it is an extension of a soft graph. The fuzzy type-2 soft graph is a parameterized family of VN-fuzzy soft graphs and an extension of type-2 soft graph. Fuzzy type-2 soft graphs show

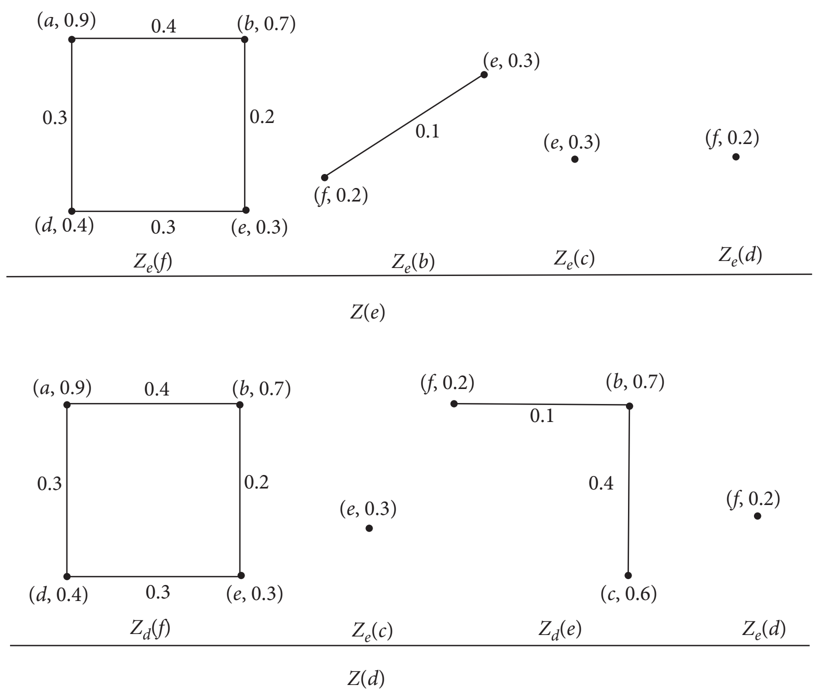


Figure 28: Fuzzy type-2 soft graph for national engineering services.

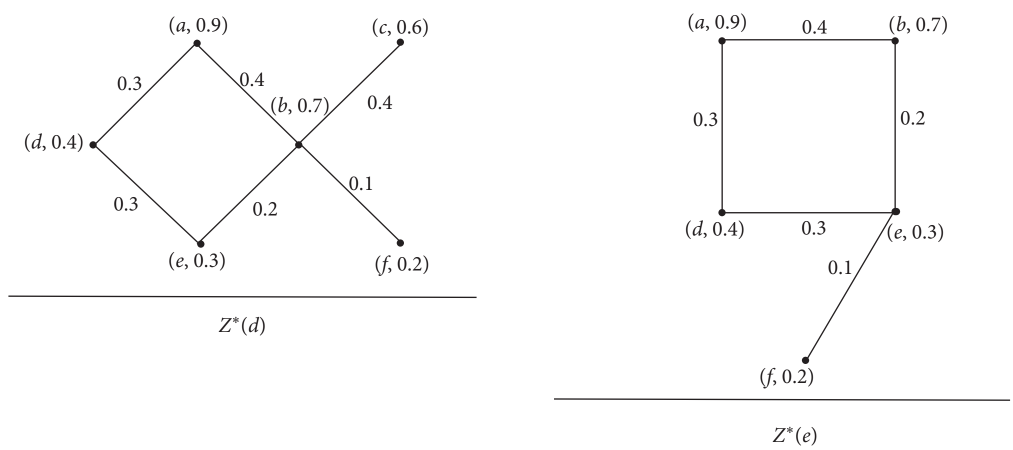


Figure 29: Resultant VN-fuzzy graphs $Z^*(e)$ and $Z^*(d)$.

vertex-neighbors coordination relation among objects in a parameterized VN-fuzzy graph. The proposed models take the set of parameters from a given fuzzy vertex set and, corresponding to each selected parameter, there exists a VN-fuzzy soft graph. As fuzzy soft graph is a parameterized family of fuzzy graphs and, corresponding to each parameter, there exists a fuzzy graph. For handling vagueness and ambiguity in decision-making problems, different fuzzy models were introduced. Fuzzy type-2 soft graph shows vertex-neighbors correspondence among objects as well as relations of parameters, while fuzzy soft graphs cannot study these correspondences and thus cannot give accurate and effective results. The decision-making problem discussed in Section 4.1 can be discussed using fuzzy soft graphs.

We consider a fuzzy soft graph $G = (\Phi, \Psi, M)$, where $(\Phi, M)$ is a fuzzy soft set over $V = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ which describes the membership of the objects based upon the given parameters $e_1$ and $e_2$; $(\Psi, M)$ is a fuzzy soft set over $E = \{e_1e_2, e_1e_4, e_2e_3, e_3e_4, e_4e_5, e_5e_6, e_5e_6, e_5e_8, e_6e_7, e_8e_7\} \subseteq V \times V$ describing the membership between two objects corresponding to the given parameters $e_1$ and $e_2$. A fuzzy soft graph $G = \{H(e_1), H(e_2)\}$ is given in Tables 6 and 7.

The fuzzy graphs $H(e_1)$ and $H(e_2)$ of fuzzy soft graph $G = \{H(e_1), H(e_2)\}$ corresponding to the parameters "carbohydrates" and "protein" are shown in Figure 30.

The fuzzy graphs $H(e_1)$ and $H(e_2)$ and the choice values $C_i^k = \sum_j S_{ij}$ for all $i, j, k = 1, 2$ are given in Tables 8 and 9, respectively.

The decision value is $S_i = \vee_i^8 (\wedge_k C_i^k) = \vee_{i=1}^8 \{0.5 \wedge 0.6, 0.6 \wedge 0.4, 0.4 \wedge 0.3, 0.4 \wedge 0.5, 0.4 \wedge 0.3, 0.4 \wedge 0.1, 0.1 \wedge 0.2, 0.3 \wedge 0.2\} = 0.5$ from the choice value $C_i^k$ of fuzzy graph $H(e_k)$ for $k = 1, 2$. Clearly, the dominant object is $e_1$ or $e_4$. The

Table 4: The tabular representation of $Z^*(e)$ with choice values.

| | $a$ | $b$ | $d$ | $e$ | $f$ | $C_i^1$ |
|---|---|---|---|---|---|---|
| $a$ | 0 | 0.4 | 0.3 | 0 | 0 | 0.7 |
| $b$ | 0.4 | 0 | 0 | 0.2 | 0 | 0.6 |
| $d$ | 0.3 | 0 | 0 | 0.3 | 0 | 0.6 |
| $e$ | 0 | 0.2 | 0.3 | 0 | 0.1 | 0.6 |
| $f$ | 0 | 0 | 0 | 0.1 | 0 | 0.1 |

Table 5: The tabular representation of $Z^*(d)$ with choice values.

| | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $C_i^2$ |
|---|---|---|---|---|---|---|---|
| $a$ | 0 | 0.4 | 0 | 0.3 | 0 | 0 | 0.7 |
| $b$ | 0.4 | 0 | 0.4 | 0 | 0.2 | 0.1 | 1.1 |
| $c$ | 0 | 0.4 | 0 | 0 | 0 | 0 | 0.4 |
| $d$ | 0.3 | 0 | 0 | 0 | 0.3 | 0 | 0.6 |
| $e$ | 0 | 0.2 | 0 | 0.3 | 0 | 0 | 0.5 |
| $f$ | 0 | 0.1 | 0 | 0 | 0 | 0 | 0.1 |

Table 6: Tabular representation of a fuzzy soft vertex set.

| $\Phi$ | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ | $e_7$ | $e_8$ |
|---|---|---|---|---|---|---|---|---|
| $e_1$ | 0.8 | 0.7 | 0.2 | 0.4 | 0.3 | 0.3 | 0.2 | 0.2 |
| $e_2$ | 0.8 | 0.5 | 0.5 | 0.4 | 0.4 | 0.5 | 0.6 | 0.5 |

Table 7: Tabular representation of a fuzzy soft edge set.

| $\Psi$ | $e_1e_2$ | $e_1e_6$ | $e_1e_4$ | $e_1e_7$ | $e_2e_3$ | $e_2e_4$ | $e_2e_8$ | $e_3e_4$ | $e_3e_7$ | $e_4e_5$ | $e_5e_6$ | $e_5e_8$ | $e_7e_8$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $e_1$ | 0.3 | 0.2 | 0.0 | 0.0 | 0.2 | 0.1 | 0.0 | 0.1 | 0.1 | 0.2 | 0.1 | 0.1 | 0.1 |
| $e_2$ | 0.3 | 0.0 | 0.2 | 0.1 | 0.0 | 0.0 | 0.1 | 0.2 | 0.1 | 0.2 | 0.1 | 0.1 | 0.0 |

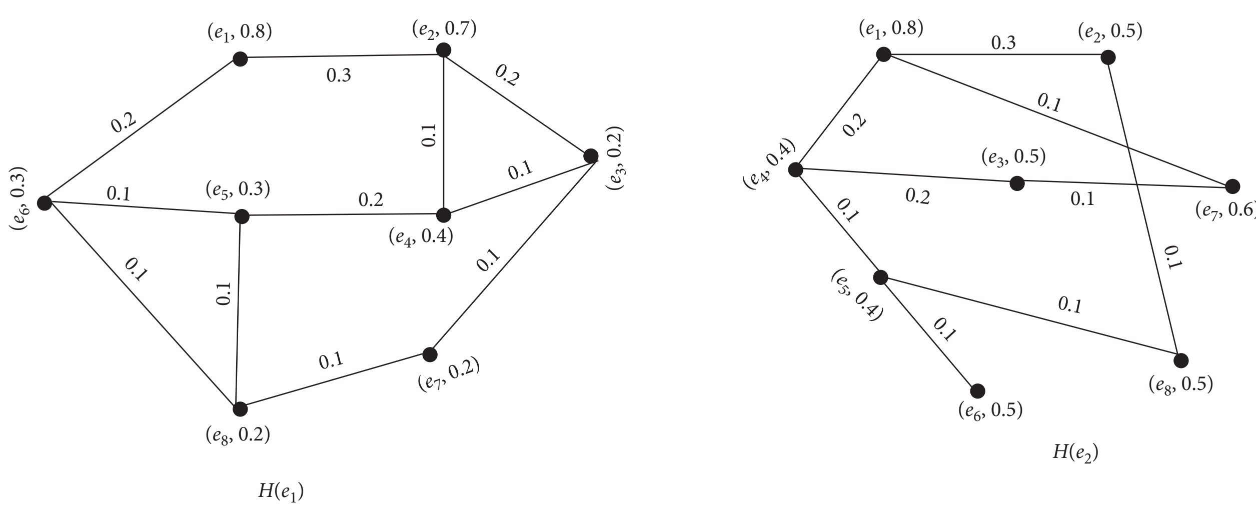


Figure 30: Fuzzy soft graph G = $\{H(e_1), H(e_2)\}$.

suitable object determined by fuzzy soft graph as above and fuzzy type-2 soft graph in Section 4.1 is dependent on information determined by selected set of parameters and fuzzy values in VN-fuzzy graphs, respectively. As the coordination among objects varies, the solution changes accordingly. So, in this case, when the objects show close vertex-neighbors coordination according to observed data, fuzzy type-2 soft graph model can be used and in the case when fuzzy relations are given along with different parameters, fuzzy soft graph model can be used.

Table 8: Tabular representation of $H(e_1)$ with choice values.

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ | $e_7$ | $e_8$ | $C_i^1$ |
|---|---|---|---|---|---|---|---|---|---|
| $e_1$ | 0.0 | 0.3 | 0.0 | 0.0 | 0.0 | 0.2 | 0.0 | 0.0 | 0.5 |
| $e_2$ | 0.3 | 0.0 | 0.2 | 0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.6 |
| $e_3$ | 0.0 | 0.2 | 0.0 | 0.1 | 0.0 | 0.0 | 0.1 | 0.0 | 0.4 |
| $e_4$ | 0.0 | 0.1 | 0.1 | 0.0 | 0.2 | 0.0 | 0.0 | 0.0 | 0.4 |
| $e_5$ | 0.0 | 0.0 | 0.0 | 0.2 | 0.0 | 0.1 | 0.0 | 0.1 | 0.4 |
| $e_6$ | 0.2 | 0.0 | 0.0 | 0.0 | 0.1 | 0.0 | 0.0 | 0.1 | 0.4 |
| $e_7$ | 0.0 | 0.0 | 0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.1 |
| $e_8$ | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.1 | 0.1 | 0.0 | 0.3 |

Table 9: Tabular representation of $H(e_2)$ with choice values.

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ | $e_7$ | $e_8$ | $C_i^2$ |
|---|---|---|---|---|---|---|---|---|---|
| $e_1$ | 0.0 | 0.3 | 0.0 | 0.2 | 0.0 | 0.0 | 0.1 | 0.0 | 0.6 |
| $e_2$ | 0.3 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.4 |
| $e_3$ | 0.0 | 0.0 | 0.0 | 0.2 | 0.0 | 0.0 | 0.1 | 0.0 | 0.3 |
| $e_4$ | 0.2 | 0.0 | 0.2 | 0.0 | 0.1 | 0.0 | 0.0 | 0.0 | 0.5 |
| $e_5$ | 0.0 | 0.0 | 0.0 | 0.1 | 0.0 | 0.1 | 0.0 | 0.1 | 0.3 |
| $e_6$ | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.0 | 0.0 | 0.0 | 0.1 |
| $e_7$ | 0.1 | 0.0 | 0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.2 |
| $e_8$ | 0.0 | 0.1 | 0.0 | 0.0 | 0.1 | 0.0 | 0.0 | 0.0 | 0.2 |

*5.2. Comparison with Type-2 Soft Graphs.* In structure of a graph, the vertex-neighbors correspondence has an important role. The type-2 soft graph [32] is based on the correspondence of initial parameters (vertex soft set) and underlying parameters. The type-2 soft graph is an efficient model for dealing with uncertainty occurring in vertex-neighbors' structure and is applicable in computational analysis, applied intelligence, and decision-making problems. The theory of fuzzy sets has played an important role to form useful models for handling partial membership of objects. To overcome the parameterized limitations of fuzzy set, the theory of fuzzy type-2 soft set was introduced. Fuzzy type-2 soft graph model is a more efficient model as compared to type-2 soft graph model to represent the parametric uncertainty in graphical networks. It is observed that, for the selection of dominant food components in chemical digestion using given type-2 soft information, we are not able to identify any object (dominating component). In this case, the simple type-2 soft information provides no solution. To determine the solution of the problem, it is necessary to have fuzzy information or define a fuzzy relation in order to attain a suitable approximation approach for selecting at least one object. So, fuzzy type-2 soft graph is more reliable in such decision-making problems.

## 6. Conclusions and Future Directions

Molodtsov's soft set theory is an effective and rational approach to understand uncertainties in terms of parameters. Type-2 soft sets have been introduced by adding the primary relations among parameters in soft sets. We have introduced the notions of fuzzy type-2 soft sets and fuzzy type-2 soft graphs to study the partial membership and uncertainty of objects along with underlying and primary set of parameters. We have discussed certain properties of fuzzy type-2 soft graphs, regular fuzzy type-2 soft graphs, irregular fuzzy type-2 soft graphs, fuzzy type-2 soft trees, and fuzzy type-2 soft cycles. We have discussed different methods of construction of fuzzy type-2 soft graphs with certain operations and elaborated these concepts with numerical examples. We have studied the importance of fuzzy type-2 soft graphs in chemical digestion and national engineering services. The present study can be extended to various directions including (1) Pythagorean fuzzy type-2 soft graphs, (2) spherical fuzzy type-2 soft graphs, and (3) picture fuzzy type-2 soft trees.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## References

[1] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.
[2] A. Kaufmann, *Introduction àla théorie des sous-ensembles flous à l'usage des ingénieurs (Fuzzy sets theory)*, Masson, Paris, France, 1975.
[3] L. A. Zadeh, "Similarity relations and fuzzy orderings," *Information Sciences*, vol. 3, no. 2, pp. 177–200, 1971.
[4] A. Rosenfeld, "Fuzzy graphs," in *Fuzzy Sets and Their Applications*, L. A. Zadeh, K. S. Fu, and M. Shimura, Eds., pp. 77–95, Academic Press, New York, NY, USA, 1975.
[5] P. Bhattacharya, "Some remarks on fuzzy graphs," *Pattern Recognition Letters*, vol. 6, no. 5, pp. 297–302, 1987.
[6] J. N. Mordeson and P. S. Nair, *Fuzzy Graphs and Fuzzy Hypergraphs*, Physica Verlag, Heidelberg, MA, USA, 2012.

[7] L. Kóczy, "Fuzzy graphs in the evaluation and optimization of networks," *Fuzzy Sets and Systems*, vol. 46, no. 3, pp. 307–319, 1992.
[8] R. T. Yeh and S. Y. Bang, "Fuzzy relations, fuzzy graphs, and their applications to clustering analysis," *Fuzzy Sets and Their Applications to Cognitive and Decision Processes*, vol. 17, pp. 125–149, 1975.
[9] S. Mathew and M. S. Sunitha, "Types of arcs in a fuzzy graph," *Information Sciences*, vol. 179, no. 11, pp. 1760–1768, 2009.
[10] D. Molodtsov, "Soft set theory–first results," *Computers & Mathematics with Applications*, vol. 37, no. 4-5, pp. 19–31, 1999.
[11] P. K. Maji, R. K. Biswas, and A. Roy, "Fuzzy soft sets," *The Journal of Fuzzy Mathematics*, vol. 9, no. 3, pp. 589–602, 2001.
[12] A. R. Roy and P. K. Maji, "A fuzzy soft set theoretic approach to decision making problems," *Journal of Computational and Applied Mathematics*, vol. 203, no. 2, pp. 412–418, 2007.
[13] B. Ahmad and A. Kharal, "On fuzzy soft sets," *Advances in Fuzzy Systems*, vol. 6, 2009.
[14] J. C. R. Alcantud, "Some formal relationships among soft sets, fuzzy sets, and their extensions," *International Journal of Approximate Reasoning*, vol. 68, pp. 45–53, 2016.
[15] M. I. Ali, "A note on soft sets, rough soft sets and fuzzy soft sets," *Applied Soft Computing*, vol. 11, no. 4, pp. 3329–3332, 2011.
[16] F. Feng, C. Li, B. Davvaz, and M. I. Ali, "Soft sets combined with fuzzy sets and rough sets: a tentative approach," *Soft Computing*, vol. 14, no. 9, pp. 899–911, 2010.
[17] P. Majumdar and S. K. Samanta, "Generalised fuzzy soft sets," *Computers & Mathematics with Applications*, vol. 59, no. 4, pp. 1425–1432, 2010.
[18] L. Zhang and J. Zhan, "Fuzzy soft $$\beta$$ $\beta$ -covering based fuzzy rough sets and corresponding decision-making applications," *International Journal of Machine Learning and Cybernetics*, vol. 10, no. 6, pp. 1487–1502, 2019.
[19] Z. Zhang and S. Zhang, "Type-2 fuzzy soft sets and their applications in decision making," *Journal of Applied Mathematics*, vol. 35, 2012.
[20] M. Sarwar, "Decision-making approaches based on color spectrum and D-TOPSIS method under rough environment," *Computational and Applied Mathematics*, vol. 39, no. 4, 2020.
[21] M. Akram and S. Nawaz, "Fuzzy soft graphs with applications," *Journal of Intelligent & Fuzzy Systems*, vol. 30, no. 6, pp. 3619–3632, 2016.
[22] M. Akram and F. Zafar, "Hybrid soft computing models applied to graph theory," *Studies in Fuzziness and Soft Computing*, vol. 29, 2020.
[23] M. Akram and G. Shahzadi, "Decision-making approach based on Pythagorean Dombi fuzzy soft graphs," *Granular Computing*, vol. 29, 2020.
[24] M. Akram and S. Shahzadi, "Novel intuitionistic fuzzy soft multiple-attribute decision-making methods," *Neural Computing and Applications*, vol. 29, no. 7, pp. 435–447, 2018.
[25] K. V. Babitha and J. J. Sunil, "Soft set relations and functions," *Computers & Mathematics with Applications*, vol. 60, no. 7, pp. 1840–1849, 2010.
[26] S. Shahzadi and M. Akram, "Graphs in an intuitionistic fuzzy soft environment," *Axioms*, vol. 7, no. 2, p. 20, 2018.
[27] A. Dey, L. H. Son, A. Pal, and H. V. Long, "Fuzzy minimum spanning tree with interval type 2 fuzzy arc length: formulation and a new genetic algorithm," *Soft Computing*, vol. 24, no. 6, pp. 3963–3974, 2020.
[28] A. M. Khalil and N. Hassan, "A note on "A novel approach to multi attribute group decision making based on trapezoidal interval type-2 fuzzy soft sets," *Applied Mathematical Modelling*, vol. 41, pp. 684–690, 2017.
[29] M. A. Rashid, S. Ahmad, and M. K. Siddiqui, "On total uniform fuzzy soft graphs," *Journal of Intelligent & Fuzzy Systems*, vol. 39, no. 1, pp. 263–275, 2020.
[30] S. Shashikala and P. N. Anil, "Fuzzy soft cycles in Fuzzy soft graphs," *Journal of New Results in Science*, vol. 8, no. 1, pp. 26–35, 2019.
[31] R. Chatterjee, P. Majumdar, and S. Samanta, "Type-2 soft sets," *Journal of Intelligent & Fuzzy Systems*, vol. 29, no. 2, pp. 885–898, 2015.
[32] K. Hayat, M. I. Ali, B. Y. Cao, and X. P. Yang, "A new type-2 soft set: type-2 soft graphs and their applications," *Advances in Fuzzy Systems*, vol. 29, 2017.
[33] K. Hayat, M. I. Ali, B. Y. Cao, and F. Karaaslan, "New results on type-2 soft sets," *Hacettepe Journal of Mathematics and Statistics*, vol. 47, no. 4, pp. 855–876, 2018.
[34] K. Hayat, B. Y. Cao, M. I. Ali, F. Karaaslan, and Z. Qin, "Characterizations of certain types of type 2 soft graphs," *Discrete Dynamics in Nature and Society*, vol. 29, 2018.