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Research Article

Dual Use of Auxiliary Information for Estimating the Finite Population Mean under the Stratified Random Sampling Scheme

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In this paper, we proposed an improved family of estimators for finite population mean under stratified random sampling, which needed a helping variable on the sample mean and rank of the auxiliary variable. The expression of the bias and mean square error of the proposed and existing estimators are computed up to the first-order approximation. The estimators proposed in different situations were investigated and provided a minimum mean square error relative to all other estimators considered. Four actual data sets and simulation studies are carried out to observe the performance of the estimators. For simulation study, R software is used. The mean square errors of all four data sets are minimum and percent relative efficiencies are more than a hundred percent higher than the other existing estimators, which indicated the importance of the newly proposed family of estimators. From the simulation study, it is concluded that the suggested family of estimators achieved better results. We demonstrate theoretically and numerically that the proposed estimator produces efficient results compared to all other contend estimators in entire situations. Overall, we conclude that the performance of the family of suggested estimators is better than all existing estimators.

1. Introduction

In the literature of survey sampling, the use of auxiliary variables was discussed by many researchers to improve the efficiency of their developed estimators, for estimating some usual parameters, such as mean, median, variance, and standard deviation. In such situations, traditional ratio, product, and regression estimators give efficient outcomes of the unspecified parameters. There is a situation where the strength of linear association between the study and auxiliary variable is positive; then the rank of the auxiliary variable is also positively correlated with the study variable. In this article, we developed a family of the estimator for estimating mean that utilizes supplementary variable, not only in the shape of auxiliary variable but also on the rank of auxiliary variable.

If the population of interest is homogeneous, then simple random sampling performs reasonably well. However, when the population of interest is heterogeneous, in such a situation it is advisable to use stratified random sampling instead of simple random sampling. In stratified random sampling, we split the whole aggregate into several non-overlapping groups or subgroups called strata. These groups are homogeneous entirely and a sample is drawn independently from each stratum separately. To obtain the maximum benefit from stratification, the values of the N_h must be known. When the strata have been determined, a sample is drawn from each stratum and the drawings are being made independently. If a simple random sample is taken from each stratum, then the entire procedure is described as stratified random sampling. Different researchers used different sample allocation techniques to distribute the sample in different strata. The use of a separate ratio estimate in each stratum is more precise if the sample in each stratum is large enough, so in this article, we use the proportional allocation method. More efforts have been pointed out about the population mean under stratified random sampling. Stratification improves efficiency when the variance between strata is much larger than the variances within strata as stated by Zaman and Kadilar [1]. More studies regarding population mean using auxiliary variables are Hussain et al. [2], Aladag and Cingi [3], Grover and Kaur [4], Shabbir and Gupta [5], Rao [6], Singh et al. [7], Kadilar and Cingi [8], Khalid [9, 10], Zaman and Bulut [11], Al-Marzouki [12], and Aamir et al. [13, 14].

In this paper, we propose a new family of estimators for estimating the mean using the information on the mean and ranks of the auxiliary variable based on stratified random sampling. The remaining document is set as follows.

In Section 2, notations and symbols are described. The literature review and proposed estimator are given in Sections 3 and 4. In Sections 5 and 6, theoretical and numerical identification are made, while Section 7 consists of the simulation study. In Section 8, discussion of the numerical results is provided. Concluding remarks of the article are given in Section 9.

2. Notations and Symbols

Suppose $T = \{1, 2, \ldots, N\}$ is the population of N definite units, which is partitions into L strata, where the size of h^{th} stratum is N_h, for $h = 1, 2, \ldots, L$ such that $\sum_{h=1}^{L} N_h = N$. Let Y and X be the study and auxiliary variables which take values y_h and x_h, respectively, where $i = 1, 2, \ldots, N_h$ and $h = 1, 2, \ldots, L$; for estimating mean, suppose a sample of size n_h is taken from the h^{th} stratum using SRSWOR. $\sum_{h=1}^{L} n_h = n$, where n is the sample size. Consider Y the study variable, X and Z the auxiliary and rank of the auxiliary variable, $\tilde{Y}_h = \sum_{i=1}^{N_h} Y_{ih}/N_h$, $\tilde{X}_h = \sum_{i=1}^{N_h} X_{ih}/N_h$, and $\tilde{Z}_h = \sum_{i=1}^{N_h} Z_{ih}/N_h$ the population mean of Y, X, and Z for the hth stratum,$\tilde{Y}_{st} = \tilde{Y} = \sum_{h=1}^{L} W_h \tilde{X}_h$, $\tilde{X}_{st} = \tilde{X} = \sum_{h=1}^{L} W_h \tilde{X}_h$, and $\tilde{Z}_{st} = \tilde{Z} = \sum_{h=1}^{L} W_h \tilde{Z}_h$ the population mean of Y, X, and Z based on stratified random sampling, $\hat{\tilde{Y}}_h = \sum_{i=1}^{n_h} Y_{ih}/n_h$, $\hat{\tilde{X}}_h = \sum_{i=1}^{n_h} X_{ih}/n_h$, and $\hat{\tilde{Z}}_h = \sum_{i=1}^{n_h} Z_{ih}/n_h$ the sample mean of Y, X, and Z for the hth stratum, $\hat{\tilde{Y}}_{st} = \hat{\tilde{Y}} = \sum_{h=1}^{L} W_h \hat{\tilde{Y}}_h$,$\hat{\tilde{X}}_{st} = \hat{\tilde{X}} = \sum_{h=1}^{L} W_h \hat{\tilde{X}}_h$, and $\hat{\tilde{Z}}_{st} = \hat{\tilde{Z}} = \sum_{h=1}^{L} W_h \hat{\tilde{Z}}_h$ the sample mean of Y, X, and Z based on stratified random sampling, $S_{y_h}^2 = \sum_{i=1}^{N_h} (Y_{ih} - \tilde{Y}_h)^2/(N_h - 1)$, $S_{x_h}^2 = \sum_{i=1}^{N_h} (X_{ih} - \tilde{X}_h)^2/(N_h - 1)$, and $S_{z_h}^2 = \sum_{i=1}^{N_h} (Z_{ih} - \tilde{Z}_h)^2/(N_h - 1)$ the population variance of Y, X, and Z for the hth stratum, $C_{yh} = S_{yh}/\tilde{Y}_h$, $C_{xh} = S_{xh}/\tilde{X}_h$, and $C_{zh} = S_{zh}/\tilde{Z}_h$ the population coefficient of variation of Y, X, and Z for the hth stratum, $S_{yxh} = \sum_{i=1}^{N_h} \{(Y_{ih} - \tilde{Y}_h)(X_{ih} - \tilde{X}_h)\}/(N_h - 1)$, $S_{yzh} = \sum_{i=1}^{N_h} \{(Y_{ih} - \tilde{Y}_h)(Z_{ih} - \tilde{Z}_{h\,h})\}/(N_h - 1)$, and $S_{xzh} = \sum_{i=1}^{N_h} \{(X_{ih} - X_h)(Z_{ih} - \tilde{Z}_h)\}/(N_h - 1)$ the covariance between (Y, X), (Y, Z), and (X, Z) for the hth stratum, $R_{yxh} = S_{yxh}/(S_{yh} S_{xh})$, $R_{yzh} = S_{yzh}/(S_{yh} S_{zh})$, and $R_{xzh} = S_{xzh}/(S_{xh} S_{zh})$ the population correlation between (Y, X), (Y, Z), and (X, Z) for the hth stratum, $R_{yx} = \sum_{h=1}^{L} W_h^2 \lambda_h R_{yxh} S_{yh} S_{xh}/(\sqrt{\sum_{h=1}^{L} W_h^2 \lambda_h S_{yh}^2} \sqrt{\sum_{h=1}^{L} W_h^2 \lambda_h S_{xh}^2})$ the population correlation coefficient between Y and X based on stratified random sampling, $R_{yz} = \sum_{h=1}^{L} W_h^2 \lambda_h R_{yzh} S_{yh} S_{zh}/(\sqrt{\sum_{h=1}^{L} W_h^2 \lambda_h S_{yh}^2} \sqrt{\sum_{h=1}^{L} W_h^2 \lambda_h S_{zh}^2})$ the correlation coefficient between Y and Z based on stratified random sampling, $R_{xz} = \sum_{h=1}^{L} W_h^2 \lambda_h R_{xzh} S_{2h} S_{xh}/(\sqrt{\sum_{h=1}^{L} W_h^2 \lambda_h S_{xh}^2} \sqrt{\sum_{h=1}^{L} W_h^2 \lambda_h S_{zh}^2})$ the correlation coefficient between X and Z based on stratified random sampling, and $R_{y.xz}^2 = (R_{yx}^2 + R_{yz}^2 - 2R_{yx} R_{yz} R_{xz})/(1 - R_{xz}^2)$ the strength of the linear relationship of Y on X and Z.

To obtain the properties of the existing and proposed estimators of $\tilde{Y}$, we consider the following relative error terms based on stratified random sampling.

Let $e_1 = \hat{\tilde{Y}} - \tilde{Y}/\tilde{Y}$, $e_2 = \hat{\tilde{X}} - \tilde{X}/\tilde{X}$, and $e_3 = \hat{\tilde{Z}} - \tilde{Z}/\tilde{Z}$, such that $E(e_i) = 0$ for $i = 1, 2, 3$.

$$
\begin{aligned}
E\left(e_1^2\right) &= \sum_{h=1}^{L} W_h^2 \lambda_h^2 C_{yh}^2 = V_{200},\\
E\left(e_2^2\right) &= \sum_{h=1}^{L} W_h^2 \lambda_h^2 C_{xh}^2 = V_{020},\\
E\left(e_3^2\right) &= \sum_{h=1}^{L} W_h^2 \lambda_h^2 C_{zh}^2 = V_{002},\\
E\left(e_1 e_2\right) &= \sum_{h=1}^{L} W_h^2 \lambda_h^2 R_{yxh} C_{yh} C_{xh} = V_{110},\\
E\left(e_1 e_3\right) &= \sum_{h=1}^{L} W_h^2 \lambda_h^2 R_{yzh} C_{yh} C_{xh} = V_{101},\\
E\left(e_2 e_3\right) &= \sum_{h=1}^{L} W_h^2 \lambda_h^2 R_{xzh} C_{xh} C_{zh} = V_{011}.
\end{aligned}
\tag{1}
$$

3. Existing Estimators

In this portion, we consider some estimators of population mean based on stratified sampling schemes that are available in the literature. The properties of these existing estimators are obtained up to the first degree of approximation. Different estimators with their error terms and mean square errors are as follows:

(1) The usual estimator of $\tilde{Y}$ is given by

$$\hat{\tilde{Y}}_{SRS_{st}} = \frac{1}{n} \sum_{i=1}^{n} Y_i = \hat{\tilde{Y}}. \tag{2}$$

The variance of $\hat{\tilde{Y}}_{\mathrm{SRS}_{st}}$ is given by

$$Var\left(\hat{\tilde{Y}}_{SRS_{st}}\right) = \tilde{Y}^2 V_{200}. \tag{3}$$

(2) The existing ratio estimator of $\tilde{Y}$ is given by Cochran [15].

$$\hat{\bar{Y}}_{R_{st}} = \hat{\bar{Y}}\left(\frac{\bar{X}}{\hat{\bar{X}}_{st}}\right). \tag{4}$$

The properties of $\hat{\bar{Y}}_{R_{st}}$ are given by

$$\begin{aligned}\text{Bias}\left(\hat{\bar{Y}}_{R_{st}}\right) &\cong \bar{Y}\left(V_{020} - V_{110}\right),\\ \text{MSE}\left(\hat{\bar{Y}}_{R_{st}}\right) &\cong \bar{Y}^2\left(V_{200} + V_{020} - 2V_{110}\right).\end{aligned} \tag{5}$$

(3) The existing product estimator of $\bar{Y}$ is given by Murthy [16].

$$\hat{\bar{Y}}_{P_{st}} = \hat{\bar{Y}}\left(\frac{\hat{\bar{X}}_{st}}{\bar{X}}\right). \tag{6}$$

The properties of $\hat{\bar{Y}}_{P_{st}}$ given as

$$\begin{aligned}B\left(\hat{\bar{Y}}_{P_{st}}\right) &= \bar{Y}V_{110},\\ \text{MSE}\left(\hat{\bar{Y}}_{P_{st}}\right) &\cong \bar{Y}^2\left(V_{200} + V_{020} + 2V_{110}\right).\end{aligned} \tag{7}$$

(4) The existing regression estimator of $\bar{Y}$ is given by

$$\hat{\bar{Y}}_{\text{Reg}_{st}} = \hat{\bar{Y}}_{st} + Q\left(\bar{X} - \hat{\bar{X}}_{st}\right). \tag{8}$$

The minimum variance of $\hat{\bar{X}}_{\text{Reg}_{st}}$ at the optimal value Q is given by

$$\text{Var}_{\min}\left(\bar{Y}_{\text{Reg}_{st}}\right) = \frac{\bar{Y}^2\left(V_{200}V_{020} - V_{110}^2\right)}{V_{020}}. \tag{9}$$

Equation (9) will become

$$\text{Var}_{\min}\left(\hat{\bar{Y}}_{\text{Reg}_{st}}\right) = \bar{Y}^2 V_{200}\left(1 - R_{yx}^2\right). \tag{10}$$

(5) The existing difference-type estimator of $\bar{Y}$ is given by

$$\hat{\bar{Y}}_{R,D_{st}} = Q_1\hat{\bar{Y}}_{st} + Q_2\left(\bar{X} - \hat{\bar{X}}_{st}\right), \tag{11}$$

where Q_1 and Q_2 are unknown constants. The bias and MSE of $\hat{\bar{Y}}_{R,D_{st}}$ are given as

$$\begin{aligned}B\left(\hat{\bar{Y}}_{R,D_{st}}\right) &= \bar{Y}\left(Q_1 - 1\right),\\ \text{MSE}\left(\hat{\bar{Y}}_{R,D_{st}}\right) &\cong \bar{Y}^2 - 2Q_1\bar{Y}^2 + Q_1^2\bar{Y}^2 + Q_1^2\bar{Y}^2V_{200}\\ &\quad - 2Q_1Q_2\bar{Y}\bar{X}V_{110} + Q_2^2\bar{X}^2V_{020}.\end{aligned} \tag{12}$$

The optimal values of Q_1 and Q_2 are given as

$$\begin{aligned}Q_{1\,(\text{opt})} &= \frac{V_{020}}{\left(V_{020}V_{200} - V_{110}^2 + V_{020}\right)},\\ Q_{2\,(\text{opt})} &= \frac{\bar{Y}V_{110}}{\bar{X}\left(V_{200}V_{020} - V_{110}^2 + V_{020}\right)}.\end{aligned} \tag{13}$$

The minimum MSE of $\hat{\bar{Y}}_{R,D_{st}}$ at the optimal values of Q_1, Q_2 is written as

$$\text{MSE}_{\min}\left(\hat{\bar{Y}}_{R,D_{st}}\right) = \frac{\bar{Y}^2\left(V_{200}V_{020} - V_{110}^2\right)}{\left(V_{200}V_{020} - V_{110}^2 + V_{020}\right)}. \tag{14}$$

Equation (14) can also be written as

$$\text{MSE}_{\min}\left(\hat{\bar{Y}}_{R,D_{st}}\right) = \frac{\bar{Y}^2V_{200}\left(1 - R_{yx}^2\right)}{1 + V_{200}\left(1 - R_{yx}^2\right)}. \tag{15}$$

(6) Exponential estimator [13] is given by

$$\hat{\bar{Y}}_{S_{st}} = \hat{\bar{Y}}_{st}\exp\left(\frac{a\left(\bar{X} - \hat{\bar{X}}_{st}\right)}{a\left(\bar{X} + \hat{\bar{X}}_{st}\right) + 2b}\right). \tag{16}$$

The properties of $\hat{\bar{Y}}_{S_{st}}$ is given by

$$\begin{aligned}B\left(\hat{\bar{Y}}_{S_{st}}\right) &\cong \bar{Y}\left(\frac{3}{8}\theta^2V_{020} - \frac{1}{2}\theta V_{110}\right),\\ \text{MSE}\left(\hat{\bar{Y}}_{S_{st}}\right) &\cong \frac{\bar{Y}^2}{4}\left(4V_{200} + \theta^2V_{020} - 4\theta V_{110}\right),\end{aligned} \tag{17}$$

where $\theta = a\bar{X}/(a\bar{X} + b)$.

(7) Grover and Kaur [17] are given by

$$\hat{\bar{Y}}_{gr,kr_{st}} = \left\{Q_3\hat{\bar{Y}}_{st} + Q_4\left(\bar{X} - \hat{\bar{X}}_{st}\right)\right\}\exp\left(\frac{a\left(\bar{X} - \hat{\bar{X}}_{st}\right)}{a\left(\bar{X} + \hat{\bar{X}}_{st}\right) + 2b}\right). \tag{18}$$

The properties of $\hat{\bar{Y}}_{gr,kr_{st}}$ are given by

$$\begin{aligned}B\left(\hat{\bar{Y}}_{gr,kr_{st}}\right) &\cong \bar{Y}\left(Q_3 - 1\right) + \frac{3}{8}\theta^2Q_3\bar{Y} + \frac{1}{2}\theta Q_4\bar{X}V_{020} - \frac{1}{2}\theta\bar{Y}V_{110},\\ \text{MSE}\left(\hat{\bar{Y}}_{gr,kr_{st}}\right) &\cong Q_4^2\bar{X}^2V_{020} + Q_3^2\bar{Y}^2V_{200} + 2\theta Q_3Q_4\bar{Y}\bar{X}V_{020}\\ &\quad - 2Q_3Q_4\bar{Y}\bar{X}V_{110} + \bar{Y}^2 - 2Q_3\bar{Y}^2 + \theta Q_3^2\bar{Y}^2\\ &\quad + Q_3\bar{Y}^2V_{110} - \theta Q_4\bar{Y}\bar{X}V_{020} - 2\theta Q_3^2\bar{Y}^2V_{110}\\ &\quad - \frac{3}{4}\theta^2Q_3\bar{Y}^2V_{020} + \theta^2Q_3^2\bar{Y}^2V_{020}.\end{aligned} \tag{19}$$

From (19), the optimal values are given by

$$Q_{3(\mathrm{opt})} = \frac{V_{020}\left(\theta^2 V_{020} - 8\right)}{8\left(-V_{200}V_{020} + V_{110}^2 - V_{020}\right)},$$
$$Q_{4(\mathrm{opt})} = \frac{\tilde{Y}\left(\theta^3 V_{020}^2 - \theta^2 V_{020}V_{110} + 4\theta V_{200}V_{020} - 4\theta V_{110}^2 - 4\theta V_{020} + 8V_{110}\right)}{8\tilde{X}\left(V_{200}V_{020} - V_{110}^2 + V_{020}\right)}. \quad (20)$$

The minimum MSE of $\hat{\hat{\tilde{Y}}}_{gr,kr_{st}}$ at the optimal values is given by

$$\mathrm{MSE}_{\min}\left(\hat{\hat{\tilde{Y}}}_{gr,kr_{st}}\right) \cong \frac{\tilde{Y}^2}{64}\left(64 - 16\theta^2 V_{020} - \frac{V_{020}\left(-8 + \theta^2 V_{020}\right)^2}{V_{020}\left(1 + V_{200}\right) - V_{1100}^2}\right), \quad (21)$$

or

$$MSE_{\min}\left(\hat{\hat{\tilde{Y}}}_{gr,kr_{st}}\right) \cong Var_{\min}\left(\hat{\hat{\tilde{Y}}}_{\mathrm{Reg}_{st}}\right) - \frac{\tilde{Y}^2\left(\theta^2 V_{020}^2 - 8V_{110}^2 + 8V_{020}V_{200}\right)^2}{64V_{020}^2\left\{1 + V_{200}\left(1 - R_{yx}^2\right)\right\}}, \quad (22)$$

and $\hat{\hat{\tilde{Y}}}_{gr,kr_{st}}$ is more efficient as compared to $\hat{\hat{\tilde{Y}}}_{\mathrm{Reg}_{st}}$.

4. Proposed Estimator

The use of auxiliary variables may improve the efficiency of an estimator in the design or estimation stages. When there is a correlation between the study and the auxiliary variable, the rank of the auxiliary variable is correlated with the study variables as well. Therefore, the rank of the auxiliary variable can be treated as a new auxiliary variable, and it is useful to increase the effectiveness of an estimator. By taking motivation from Hussain et al. [2], we developed a new family of estimators of $\tilde{Y}$ which needed a helping variable on the sample mean and rank of the auxiliary variable. Moreover, some members of our proposed and existing estimates are also presented in Table 1.

$$\hat{\hat{\tilde{Y}}}_{\mathrm{Pr}_{st}} = \left\{Q_5\hat{\hat{\tilde{Y}}}_{st} + Q_6\left(\frac{\tilde{X} - \hat{\hat{\tilde{X}}}_{st}}{\tilde{X}}\right) + Q_7\left(\frac{\tilde{Z} - \hat{\hat{\tilde{Z}}}_{st}}{\tilde{Z}}\right)\right\}\exp\left(\frac{a\left(\tilde{X} - \hat{\hat{\tilde{X}}}_{st}\right)}{a\left(\tilde{X} + \hat{\hat{\tilde{X}}}_{st}\right) + 2b}\right), \quad (23)$$

where Q_5, Q_6, and Q_7 are unknown values.

Equation (23) can be expressed as

$$\hat{\hat{\tilde{Y}}}_{\mathrm{Pr}_{st}} = \left\{Q_5\tilde{Y}\left(1 + e_1\right) - Q_6 e_2 - Q_7 e_3\right\}\left(1 - \frac{1}{2}\theta e_2 + \frac{3}{8}\theta^2 e_2^2 + \cdots\right). \quad (24)$$

After simplification (24), we have

$$\left(\hat{\hat{\tilde{Y}}}_{\mathrm{Pr}_{st}} - \tilde{Y}\right) = -\tilde{Y} + Q_5\tilde{Y} + Q_5\tilde{Y}e_1 - \frac{1}{2}\theta Q_5\tilde{Y}e_2 - Q_6 e_2 - Q_7 e_3 + \frac{3}{8}\theta^2 Q_5\tilde{Y}e_1^2 + \frac{1}{2}\theta Q_6 e_1^2 - \frac{1}{2}\theta Q_5\tilde{Y}e_1e_2 + \frac{1}{2}\theta Q_7 e_2 e_3. \quad (25)$$

The properties of $\hat{\hat{\tilde{Y}}}_{\mathrm{Pr}_{st}}$ are shown as

$$B\left(\hat{\hat{\tilde{Y}}}_{\mathrm{Pr}_{st}}\right) \cong \tilde{Y}\left(Q_5 - 1\right) + \frac{3}{8}\theta^2 Q_5\tilde{Y}V_{020} + \frac{1}{2}\theta Q_6 V_{020} - \frac{1}{2}\theta Q_5\tilde{Y}V_{110} + \frac{1}{2}\theta V_{011}$$
$$\mathrm{MSE}\left(\hat{\hat{\tilde{Y}}}_{\mathrm{Pr}_{st}}\right) \cong \tilde{Y}^2\left(Q_5 - 1\right)^2 + Q_5^2\tilde{Y}^2 V_{200} + Q_6 V_{020} + Q_7 V_{002} + \theta^2 Q_5^2\tilde{Y}^2 V_{020} - \theta Q_6\tilde{Y}V_{020} + 2\theta\tilde{Y}V_{020} - \frac{3}{4}\theta^2 Q_5\tilde{Y}^2 V_{020} + \theta Q_5\tilde{Y}^2 V_{110} - 2\theta Q_5^2\tilde{Y}^2 V_{110} - 2Q_5Q_6\tilde{Y}V_{110} - 2Q_5Q_7\tilde{Y}V_{101} - \theta Q_7\tilde{Y}V_{011} + 2\theta Q_5Q_7\tilde{Y}V_{011} - 2Q_6Q_7V_{011}. \quad (26)$$

Table 1: Members of our proposed and existing estimator.

a	**b**	$\widehat{\widetilde{Y}}_{\mathbf{S_{st}}}$	$\widehat{\widetilde{Y}}_{\mathbf{gr,kr_{st}}}$	$\widehat{\widetilde{Y}}_{\mathbf{Pr_{st}}}$
1	$C_{x_{st}}$	$\widehat{\widetilde{Y}}_{S_{st}}^{(1)}$	$\widehat{\widetilde{Y}}_{gr,kr_{st}}^{(1)}$	$\widehat{\widetilde{Y}}_{\mathrm{Pr}_{st}}^{(1)}$
1	$\beta_{2(st)}$	$\widehat{\widetilde{Y}}_{S_{st}}^{(2)}$	$\widehat{\widetilde{Y}}_{gr,kr_{st}}^{(2)}$	$\widehat{\widetilde{Y}}_{\mathrm{Pr}_{st}}^{(2)}$
$\beta_{2(st)}$	$C_{x_{st}}$	$\widehat{\widetilde{Y}}_{S_{st}}^{(3)}$	$\widehat{\widetilde{Y}}_{gr,kr_{st}}^{(3)}$	$\widehat{\widetilde{Y}}_{\mathrm{Pr}_{st}}^{(3)}$
$C_{x_{st}}$	$\beta_{2(st)}$	$\widehat{\widetilde{Y}}_{S_{st}}^{(4)}$	$\widehat{\widetilde{Y}}_{gr,kr_{st}}^{(4)}$	$\widehat{\widetilde{Y}}_{\mathrm{Pr}_{st}}^{(4)}$
1	R_{yx}	$\widehat{\widetilde{Y}}_{S_{st}}^{(5)}$	$\widehat{\widetilde{Y}}_{gr,kr_{st}}^{(5)}$	$\widehat{\widetilde{Y}}_{\mathrm{Pr}_{st}}^{(5)}$
$C_{x_{st}}$	R_{yx}	$\widehat{\widetilde{Y}}_{S_{st}}^{(6)}$	$\widehat{\widetilde{Y}}_{gr,kr_{st}}^{(6)}$	$\widehat{\widetilde{Y}}_{\mathrm{Pr}_{st}}^{(6)}$
R_{yx}	$C_{x_{st}}$	$\widehat{\widetilde{Y}}_{S_{st}}^{(7)}$	$\widehat{\widetilde{Y}}_{gr,kr_{st}}^{(7)}$	$\widehat{\widetilde{Y}}_{\mathrm{Pr}_{st}}^{(7)}$
$\beta_{2(st)}$	R_{yx}	$\widehat{\widetilde{Y}}_{S_{st}}^{(8)}$	$\widehat{\widetilde{Y}}_{gr,kr_{st}}^{(8)}$	$\widehat{\widetilde{Y}}_{\mathrm{Pr}_{st}}^{(8)}$
R_{yx}	$\beta_{2(st)}$	$\widehat{\widetilde{Y}}_{S_{st}}^{(9)}$	$\widehat{\widetilde{Y}}_{gr,kr_{st}}^{(9)}$	$\widehat{\widetilde{Y}}_{\mathrm{Pr}_{st}}^{(9)}$
1	$N\widetilde{X}$	$\widehat{\widetilde{Y}}_{S_{st}}^{(10)}$	$\widehat{\widetilde{Y}}_{gr,kr_{st}}^{(10)}$	$\widehat{\widetilde{Y}}_{\mathrm{Pr}_{st}}^{(10)}$

The optimal values of Q_5, Q_6, and Q_7, determined by reducing (26), are given by

$$
\begin{aligned}
Q_{5(\mathrm{opt})} &= \frac{8-\theta^2 V_{020}}{8\left\{1+V_{200}\left(1-R_{y.xz}^2\right)\right\}},\\
Q_{6(\mathrm{opt})} &= \frac{\widetilde{Y}\left[\begin{array}{c}\theta^3 V_{020}^{3/2}\left(R_{xz}^2-1\right)+V_{200}^{1/2}\left(-8+\theta^2 V_{020}\right)\left(R_{yx}-R_{xz}R_{yz}\right)\\ +4\theta V_{020}^{1/2}\left(R_{xz}^2-1\right)\left\{-1+V_{200}\left(1-R_{y.xz}^2\right)\right\}\end{array}\right]}{8V_{020}^{1/2}\left(R_{xz}^2-1\right)\left\{-1+V_{200}\left(1-R_{y.xz}^2\right)\right\}},\\
Q_{7(\mathrm{opt})} &= \frac{\widetilde{Y}V_{200}^{1/2}\left(8-\theta^2 V_{020}\right)\left(R_{yx}-R_{xz}R_{yz}\right)}{8V_{020}^{1/2}\left(R_{xz}^2-1\right)\left\{-1+V_{200}\left(1-R_{y.xz}^2\right)\right\}}.
\end{aligned}
\tag{27}
$$

The simplified least mean square error of $\widehat{\widetilde{Y}}_{\mathrm{Pr}_{st}}$ at optimal values of Q_5, Q_6, and Q_7 is as follows:

$$
\mathrm{MSE}_{\min}\left(\widehat{\widetilde{Y}}_{\mathrm{Pr}_{st}}\right) \cong \frac{\widetilde{Y}^2\left\{64V_{200}\left(1-R_{y.xz}^2\right)-\theta^4 V_{020}^2-16\theta^2 V_{020}V_{200}\left(1-R_{y.xz}^2\right)\right\}}{64\left\{1+V_{200}\left(1-R_{y.xz}^2\right)\right\}},
\tag{28}
$$

where $R^2_{1.23} = (V^2_{110}V_{002} + V^2_{101}V_{020} - 2V_{101}V_{110}V_{011}/V_{200}(V_{020}V_{002} - V^2_{011}))$.

Here, (28) may be written as

$$\mathrm{MSE}_{\min}\left(\hat{\tilde{Y}}_{\mathrm{Pr}_{st}}\right) \cong \mathrm{Var}_{\min}\left(\hat{\tilde{Y}}_{\mathrm{Reg}_{st}}\right) - T_1 - T_2, \tag{29}$$

where

$$\begin{aligned} T_1 &= \frac{\tilde{Y}^2\left(\theta^2 V^2_{020} - 8V^2_{110} + 8V_{020}V_{200}\right)^2}{64V^2_{020}\left\{1 + V_{200}\left(1 - R^2_{yx}\right)\right\}}, \\ T_2 &= \frac{\tilde{Y}^2\left(\theta^2 V_{020} - 8\right)^2\left(V_{020}V_{101} - V_{011}V_{110}\right)^2}{64V^2_{020}V_{002}\left(1 - R^2_{xz}\right)\left\{1 + V_{200}\left(1 - R^2_{yx}\right)\right\}\left\{1 + V_{200}\left(1 - R^2_{1.23}\right)\right\}}. \end{aligned} \tag{30}$$

5. Efficiency Comparisons

In the following portion, we made an efficiency comparison of all estimators.

(i) By taking (3) and (29),

$$\begin{aligned} &\mathrm{MSE}_{\min}\left(\hat{\tilde{Y}}_{\mathrm{Pr}_{st}}\right) < \mathrm{Var}\left(\hat{\tilde{Y}}_{\mathrm{SRS}_{st}}\right)\text{if} \\ &\tilde{Y}^2 V^2_{200} R^2_{yx} + T_1 + T_2 > 0. \end{aligned} \tag{31}$$

(ii) By taking (5) and (29),

$$\begin{aligned} &\mathrm{MSE}_{\min}\left(\hat{\tilde{Y}}_{\mathrm{Pr}_{st}}\right) < \mathrm{MSE}\left(\hat{\tilde{Y}}_{R_{st}}\right)\text{if} \\ &\frac{1}{V_{020}}\left(V_{020} - V_{110}\right)^2 + T_1 + T_2 > 0. \end{aligned} \tag{32}$$

(iii) By taking (7) and (29),

$$\begin{aligned} &\mathrm{MSE}_{\min}\left(\hat{\tilde{Y}}_{\mathrm{Pr}_{st}}\right) < \mathrm{MSE}\left(\hat{\tilde{Y}}_{P_{st}}\right)\text{if} \\ &\frac{1}{V_{020}}\left(V_{020} + V_{110}\right)^2 + T_1 + T_2 > 0. \end{aligned} \tag{33}$$

(iv) By taking (10) and (29),

$$\begin{aligned} &\mathrm{MSE}_{\min}\left(\hat{\tilde{Y}}_{\mathrm{Pr}_{st}}\right) < \mathrm{MSE}_{\min}\left(\hat{\tilde{Y}}_{\mathrm{Reg}_{st}}\right)\text{if} \\ &T_1 + T_2 > 0. \end{aligned} \tag{34}$$

(v) By taking (15) and (29),

$$\begin{aligned} &\mathrm{MSE}_{\min}\left(\hat{\tilde{Y}}_{\mathrm{Pr}_{st}}\right) < \mathrm{MSE}_{\min}\left(\hat{\tilde{Y}}_{R,D_{st}}\right)\text{if} \\ &\frac{\tilde{Y}^2\theta^2 V_{020}\left\{\theta^2 V_{020} + 16V_{200}\left(1 - R^2_{yx}\right)\right\}}{64\left\{1 + \left(1 - R^2_{yx}\right)\right\}} + T_2 > 0. \end{aligned} \tag{35}$$

(vi) By taking (17) and (29),

$$\begin{aligned} &\mathrm{MSE}_{\min}\left(\hat{\tilde{Y}}_{Pr_{st}}\right) < \mathrm{MSE}\left(\hat{\tilde{Y}}_{S_{st}}\right)\text{if} \\ &\frac{1}{V_{020}}\left(\frac{\theta V_{020}}{2} - V_{110}\right)^2 + T_1 + T_2 > 0. \end{aligned} \tag{36}$$

(vii) By taking (22) and (29),

$$\begin{aligned} &\mathrm{MSE}_{\min}\left(\hat{\tilde{Y}}_{\mathrm{Pr}_{st}}\right) < \mathrm{MSE}_{\min}\left(\hat{\tilde{Y}}_{gr,kr_{st}}\right)\text{if} \\ &T_2 > 0. \end{aligned} \tag{37}$$

6. Numerical Investigation

In this portion, the mathematical outcome is illustrated to check the efficiency of all estimators. Four data sets are contemplated. The finding outcomes of these data sets are listed in Tables 2–5. The percent relative efficiency of the estimator $\hat{\tilde{Y}}_i$ w.r.t. $\hat{\tilde{Y}}_{\mathrm{SRS}_{st}}$ is as follows:

$$\mathrm{PRE}\left(\hat{\tilde{Y}}_i, \hat{\tilde{Y}}_{\mathrm{SRS}_{st}}\right) = \frac{\mathrm{Var}\left(\hat{\tilde{Y}}_{\mathrm{SRS}_{st}}\right)}{\mathrm{MSE}_{\min}\left(\hat{\tilde{Y}}_i\right)} \times 100, \tag{38}$$

where $i = R_{st}, P_{st}, \ldots, \mathrm{Pr}_{st}$.

The percent relative efficiency of the finite population means, measured from four data sets, is listed in Tables 6–9.

Population 1 (source: Koyuncu and Kadilar [18]): Y is the number of instructors and X is the number of the trainees in 2007 for 923 districts in six regions.

Population 2 (source: Kadilar and Cingi [19]): Y is yield of apples in 1999 and X is yield of apples in 1998.

7. Simulation Study

A simulation study is performed to determine the efficiency of the estimators suggested for stratified sampling technique with the help of information of single auxiliary variable X and adapted rank set sampling procedure on auxiliary variable X and then made variable Z. In this section, the

Table 2: Data description of population 1 (scenario I).

h	N_h	n_h	W_h	λ_h	$\tilde{Y}_h$	$\tilde{X}_h$	$\tilde{Z}_h$	S_{yh}	S_{xh}	S_{zh}	R_{yxh}	R_{yzh}	R_{xzh}
1	127	31	0.1375	0.0244	704	20805	64	883.83	486.75	6.80	0.9366	0.8239	0.7834
2	117	21	0.1267	0.0390	413	9212	59	644.92	5180.77	3.92	0.9956	0.6584	0.6517
3	103	29	0.1115	0.0248	74	14309	52	1033.46	27549.7	9.87	0.9937	0.6337	0.6237
4	170	38	0.1841	0.0204	425	9479	86	810.58	8218.93	49.21	0.9834	0.6360	0.6442
5	205	22	0.2221	0.0406	267	5570	103	403.65	8497.77	59.32	0.9893	0.6595	0.6655
6	201	39	0.2177	0.0207	394	12998	101	711.72	3094.14	58.16	0.9651	0.5863	0.6162

Table 3: Data description of population 1 (scenario II).

h	N_h	n_h	W_h	λ_h	$\tilde{Y}_h$	$\tilde{X}_h$	$\tilde{Z}_h$	S_{yh}	S_{xh}	S_{zh}	R_{yxh}	R_{yzh}	R_{xzh}
1	127	31	0.1375	0.0244	704	498	64	883.83	55.58	36.80	0.9366	0.8239	0.7834
2	117	21	0.1267	0.0391	413	318	59	644.92	65.45	33.92	0.9956	0.6584	0.6517
3	103	29	0.1115	0.0248	574	431	52	1033.46	12.95	29.87	0.9937	0.6337	0.6237
4	170	38	0.1841	0.0204	425	311	86	810.58	458.02	49.22	0.9834	0.6360	0.6442
5	205	22	0.2221	0.0406	267	227	103	410.65	60.85	59.32	0.9893	0.6595	0.6655
6	201	39	0.2177	0.0207	394	314	101	711.72	97.05	58.16	0.9651	0.5863	0.6162

Table 4: Data description of population 2 (scenario I).

H	N_h	n_h	W_h	λ_h	$\tilde{Y}_h$	$\tilde{X}_h$	$\tilde{Z}_h$	S_{yh}	S_{xh}	S_{zh}	R_{yxh}	R_{yzh}	R_{xzh}
1	106	9	0.1241	0.1017	1537	24376	54	6425.08	49189.08	30.74	0.8156	0.3349	0.5930
2	106	17	0.1241	0.0494	2213	27422	54	11551.53	57460.61	30.74	0.8559	0.2816	0.6031
3	94	38	0.1100	0.0157	9384	72410	48	29907.48	160757.31	27.28	0.9011	0.4637	0.5873
4	171	67	0.2002	0.0090	5588	74365	87	28643.42	285603.12	49.51	0.9858	0.2981	0.3654
5	204	7	0.2389	0.4657	967	26442	103	2389.77	45402.78	45.41	0.7130	0.4547	0.6206
6	173	2	0.2026	0.4942	404	9844	87	945.74	18793.96	50.08	0.8935	0.5435	0.6262

Table 5: Data description of population 2 (scenario II).

h	N_h	n_h	W_h	λ_h	$\tilde{Y}_h$	$\tilde{X}_h$	$\tilde{Z}_h$	S_{yh}	S_{xh}	S_{zh}	R_{yxh}	R_{yzh}	R_{xzh}
1	106	9	0.1241	0.1017	1537	24712	54	6425.08	49134.76	30.74	0.8156	0.3346	0.5956
2	106	17	0.1241	0.0494	2213	26840	54	11551.53	53978.71	30.74	0.8359	0.2814	0.6246
3	94	38	0.1100	0.0157	9384	72722	48	29907.48	161109.50	27.27	0.8971	0.4626	0.5893
4	171	67	0.2002	0.0090	5588	73191	87	28643.42	262495.61	49.50	0.9814	0.2979	0.3885
5	204	7	0.2389	0.1379	967	26834	103	2389.77	45174.26	59.03	0.7107	0.4541	0.6317
6	173	2	0.2026	0.4942	404	9903	87	945.74	18977.28	50.08	0.8697	0.5366	0.6283

Table 6: MSEs using population 1 (scenario I).

Estimator	MSE	Estimator	MSE	Estimator	MSE	Estimator	MSE
$\hat{\tilde{\tilde{Y}}}_{\mathrm{SRS}_{st}}$	2229.266	$\hat{\tilde{\tilde{Y}}}^{(1)}_{S_{st}}$	5096.365	$\hat{\tilde{\tilde{Y}}}^{(1)}_{gr,kr_{st}}$	192.9490	$\hat{\tilde{\tilde{Y}}}^{(1)}_{\mathrm{Pr}_{st}}$	185.1848
$\hat{\tilde{\tilde{Y}}}_{R_{st}}$	9205.298	$\hat{\tilde{\tilde{Y}}}^{(2)}_{S_{st}}$	604.1097	$\hat{\tilde{\tilde{Y}}}^{(2)}_{gr,kr_{st}}$	192.9539	$\hat{\tilde{\tilde{Y}}}^{(2)}_{\mathrm{Pr}_{st}}$	185.1897
$\hat{\tilde{\tilde{Y}}}_{P_{st}}$	216.4183	$\hat{\tilde{\tilde{Y}}}^{(3)}_{S_{st}}$	602.4522	$\hat{\tilde{\tilde{Y}}}^{(3)}_{gr,kr_{st}}$	192.9485	$\hat{\tilde{\tilde{Y}}}^{(3)}_{\mathrm{Pr}_{st}}$	185.1844
$\hat{\tilde{\tilde{Y}}}_{\mathrm{Reg}_{st}}$	194.2832	$\hat{\tilde{\tilde{Y}}}^{(4)}_{S_{st}}$	603.4212	$\hat{\tilde{\tilde{Y}}}^{(4)}_{gr,kr_{st}}$	192.9516	$\hat{\tilde{\tilde{Y}}}^{(4)}_{\mathrm{Pr}_{st}}$	185.1875
$\hat{\tilde{\tilde{Y}}}_{R,D_{st}}$	194.0853	$\hat{\tilde{\tilde{Y}}}^{(5)}_{S_{st}}$	602.5302	$\hat{\tilde{\tilde{Y}}}^{(5)}_{gr,kr_{st}}$	192.9487	$\hat{\tilde{\tilde{Y}}}^{(5)}_{\mathrm{Pr}_{st}}$	185.1846
		$\hat{\tilde{\tilde{Y}}}^{(6)}_{S_{st}}$	602.4947	$\hat{\tilde{\tilde{Y}}}^{(6)}_{gr,kr_{st}}$	192.9486	$\hat{\tilde{\tilde{Y}}}^{(6)}_{\mathrm{Pr}_{st}}$	185.1845
		$\hat{\tilde{\tilde{Y}}}^{(7)}_{S_{st}}$	602.5978	$\hat{\tilde{\tilde{Y}}}^{(7)}_{gr,kr_{st}}$	192.9490	$\hat{\tilde{\tilde{Y}}}^{(7)}_{\mathrm{Pr}_{st}}$	185.1849
		$\hat{\tilde{\tilde{Y}}}^{(8)}_{S_{st}}$	602.4488	$\hat{\tilde{\tilde{Y}}}^{(8)}_{gr,kr_{st}}$	192.9485	$\hat{\tilde{\tilde{Y}}}^{(8)}_{\mathrm{Pr}_{st}}$	185.1844
		$\hat{\tilde{\tilde{Y}}}^{(9)}_{S_{st}}$	604.1488	$\hat{\tilde{\tilde{Y}}}^{(9)}_{gr,kr_{st}}$	192.9540	$\hat{\tilde{\tilde{Y}}}^{(9)}_{\mathrm{Pr}_{st}}$	185.1898
		$\hat{\tilde{\tilde{Y}}}^{(10)}_{S_{st}}$	2226.835	$\hat{\tilde{\tilde{Y}}}^{(10)}_{gr,kr_{st}}$	194.0853	$\hat{\tilde{\tilde{Y}}}^{(10)}_{\mathrm{Pr}_{st}}$	186.2958

Table 7: MSEs using population 1 (scenario II).

Estimator	MSE	Estimator	MSE	Estimator	MSE	Estimator	MSE
$\hat{\hat{Y}}_{\mathrm{SRS}_{st}}$	2229.266	$\hat{\hat{Y}}^{(1)}_{S_{st}}$	4240.170	$\hat{\hat{Y}}^{(1)}_{gr,kr_{st}}$	101.1275	$\hat{\hat{Y}}^{(1)}_{\mathrm{Pr}_{st}}$	79.35476
$\hat{\hat{Y}}_{R_{st}}$	6936.636	$\hat{\hat{Y}}^{(2)}_{S_{st}}$	914.5469	$\hat{\hat{Y}}^{(2)}_{gr,kr_{st}}$	101.1566	$\hat{\hat{Y}}^{(2)}_{\mathrm{Pr}_{st}}$	79.38143
$\hat{\hat{Y}}_{P_{st}}$	193.2885	$\hat{\hat{Y}}^{(3)}_{S_{st}}$	877.6616	$\hat{\hat{Y}}^{(3)}_{gr,kr_{st}}$	101.1242	$\hat{\hat{Y}}^{(3)}_{\mathrm{Pr}_{st}}$	79.35178
$\hat{\hat{Y}}_{\mathrm{Reg}_{st}}$	101.5021	$\hat{\hat{Y}}^{(4)}_{S_{st}}$	907.0444	$\hat{\hat{Y}}^{(4)}_{gr,kr_{st}}$	101.1503	$\hat{\hat{Y}}^{(4)}_{\mathrm{Pr}_{st}}$	79.37566
$\hat{\hat{Y}}_{R,D_{st}}$	101.4481	$\hat{\hat{Y}}^{(5)}_{S_{st}}$	880.3341	$\hat{\hat{Y}}^{(5)}_{gr,kr_{st}}$	101.1267	$\hat{\hat{Y}}^{(5)}_{\mathrm{Pr}_{st}}$	79.35404
		$\hat{\hat{Y}}^{(6)}_{S_{st}}$	879.7216	$\hat{\hat{Y}}^{(6)}_{gr,kr_{st}}$	101.1261	$\hat{\hat{Y}}^{(6)}_{\mathrm{Pr}_{st}}$	79.35352
		$\hat{\hat{Y}}^{(7)}_{S_{st}}$	881.2793	$\hat{\hat{Y}}^{(7)}_{gr,kr_{st}}$	101.1276	$\hat{\hat{Y}}^{(7)}_{\mathrm{Pr}_{st}}$	79.35483
		$\hat{\hat{Y}}^{(8)}_{S_{st}}$	877.5927	$\hat{\hat{Y}}^{(8)}_{gr,kr_{st}}$	101.1242	$\hat{\hat{Y}}^{(8)}_{\mathrm{Pr}_{st}}$	79.35172
		$\hat{\hat{Y}}^{(9)}_{S_{st}}$	915.3844	$\hat{\hat{Y}}^{(9)}_{gr,kr_{st}}$	101.1573	$\hat{\hat{Y}}^{(9)}_{\mathrm{Pr}_{st}}$	79.38207
		$\hat{\hat{Y}}^{(10)}_{S_{st}}$	2227.442	$\hat{\hat{Y}}^{(10)}_{gr,kr_{st}}$	101.4481	$\hat{\hat{Y}}^{(10)}_{\mathrm{Pr}_{st}}$	79.63742

Table 8: MSEs using population 2 (scenario I).

Estimator	MSE	Estimator	MSE	Estimator	MSE	Estimator	MSE
$\hat{\hat{Y}}_{\mathrm{SRS}_{st}}$	697789.6	$\hat{\hat{Y}}^{(1)}_{S_{st}}$	1225952	$\hat{\hat{Y}}^{(1)}_{gr,kr_{st}}$	214502.1	$\hat{\hat{Y}}^{(1)}_{\mathrm{Pr}_{st}}$	197279.1
$\hat{\hat{Y}}_{R_{st}}$	1949400	$\hat{\hat{Y}}^{(2)}_{S_{st}}$	365071.7	$\hat{\hat{Y}}^{(2)}_{gr,kr_{st}}$	214508.7	$\hat{\hat{Y}}^{(2)}_{\mathrm{Pr}_{st}}$	197285.3
$\hat{\hat{Y}}_{P_{st}}$	227015.9	$\hat{\hat{Y}}^{(3)}_{S_{st}}$	364798.4	$\hat{\hat{Y}}^{(3)}_{gr,kr_{st}}$	214501.7	$\hat{\hat{Y}}^{(3)}_{\mathrm{Pr}_{st}}$	197278.7
$\hat{\hat{Y}}_{\mathrm{Reg}_{st}}$	222881.3	$\hat{\hat{Y}}^{(4)}_{S_{st}}$	364915.7	$\hat{\hat{Y}}^{(4)}_{gr,kr_{st}}$	214504.7	$\hat{\hat{Y}}^{(4)}_{\mathrm{Pr}_{st}}$	197281.5
$\hat{\hat{Y}}_{R,D_{st}}$	217241.8	$\hat{\hat{Y}}^{(5)}_{S_{st}}$	364803.5	$\hat{\hat{Y}}^{(5)}_{gr,kr_{st}}$	214501.8	$\hat{\hat{Y}}^{(5)}_{\mathrm{Pr}_{st}}$	197278.9
		$\hat{\hat{Y}}^{(6)}_{S_{st}}$	364800.4	$\hat{\hat{Y}}^{(6)}_{gr,kr_{st}}$	214501.8	$\hat{\hat{Y}}^{(6)}_{\mathrm{Pr}_{st}}$	197278.8
		$\hat{\hat{Y}}^{(7)}_{S_{st}}$	364815.1	$\hat{\hat{Y}}^{(7)}_{gr,kr_{st}}$	214502.1	$\hat{\hat{Y}}^{(7)}_{\mathrm{Pr}_{st}}$	197279.1
		$\hat{\hat{Y}}^{(8)}_{S_{st}}$	364798.2	$\hat{\hat{Y}}^{(8)}_{gr,kr_{st}}$	214501.7	$\hat{\hat{Y}}^{(8)}_{\mathrm{Pr}_{st}}$	197278.7
		$\hat{\hat{Y}}^{(9)}_{S_{st}}$	365117.8	$\hat{\hat{Y}}^{(9)}_{gr,kr_{st}}$	214509.9	$\hat{\hat{Y}}^{(9)}_{\mathrm{Pr}_{st}}$	197286.4
		$\hat{\hat{Y}}^{(10)}_{S_{st}}$	697286.1	$\hat{\hat{Y}}^{(10)}_{gr,kr_{st}}$	217241.8	$\hat{\hat{Y}}^{(10)}_{\mathrm{Pr}_{st}}$	199821.3

Table 9: MSEs using population 2 (scenario II).

Estimator	MSE	Estimator	MSE	Estimator	MSE	Estimator	MSE
$\hat{\hat{Y}}_{\mathrm{SRS}_{st}}$	697789.6	$\hat{\hat{Y}}^{(1)}_{S_{st}}$	1225952	$\hat{\hat{Y}}^{(1)}_{gr,kr_{st}}$	228480.8	$\hat{\hat{Y}}^{(1)}_{\mathrm{Pr}_{st}}$	207876.6
$\hat{\hat{Y}}_{R_{st}}$	1878241	$\hat{\hat{Y}}^{(2)}_{S_{st}}$	365071.7	$\hat{\hat{Y}}^{(2)}_{gr,kr_{st}}$	228486.8	$\hat{\hat{Y}}^{(2)}_{\mathrm{Pr}_{st}}$	207882.2
$\hat{\hat{Y}}_{P_{st}}$	243316.9	$\hat{\hat{Y}}^{(3)}_{S_{st}}$	364798.4	$\hat{\hat{Y}}^{(3)}_{gr,kr_{st}}$	228480.4	$\hat{\hat{Y}}^{(3)}_{\mathrm{Pr}_{st}}$	207876.3
$\hat{\hat{Y}}_{\mathrm{Reg}_{st}}$	237552.8	$\hat{\hat{Y}}^{(4)}_{S_{st}}$	364915.7	$\hat{\hat{Y}}^{(4)}_{gr,kr_{st}}$	228483.3	$\hat{\hat{Y}}^{(4)}_{\mathrm{Pr}_{st}}$	207878.9
$\hat{\hat{Y}}_{R,D_{st}}$	231157.0	$\hat{\hat{Y}}^{(5)}_{S_{st}}$	364803.5	$\hat{\hat{Y}}^{(5)}_{gr,kr_{st}}$	228480.6	$\hat{\hat{Y}}^{(5)}_{\mathrm{Pr}_{st}}$	207876.4
		$\hat{\hat{Y}}^{(6)}_{S_{st}}$	364800.4	$\hat{\hat{Y}}^{(6)}_{gr,kr_{st}}$	228480.5	$\hat{\hat{Y}}^{(6)}_{\mathrm{Pr}_{st}}$	207876.3
		$\hat{\hat{Y}}^{(7)}_{S_{st}}$	364815.1	$\hat{\hat{Y}}^{(7)}_{gr,kr_{st}}$	228480.8	$\hat{\hat{Y}}^{(7)}_{\mathrm{Pr}_{st}}$	207876.7
		$\hat{\hat{Y}}^{(8)}_{S_{st}}$	364798.2	$\hat{\hat{Y}}^{(8)}_{gr,kr_{st}}$	228480.4	$\hat{\hat{Y}}^{(8)}_{\mathrm{Pr}_{st}}$	207876.3
		$\hat{\hat{Y}}^{(9)}_{S_{st}}$	365117.8	$\hat{\hat{Y}}^{(9)}_{gr,kr_{st}}$	228488.0	$\hat{\hat{Y}}^{(9)}_{\mathrm{Pr}_{st}}$	207883.3
		$\hat{\hat{Y}}^{(10)}_{S_{st}}$	697286.1	$\hat{\hat{Y}}^{(10)}_{gr,kr_{st}}$	231157.0	$\hat{\hat{Y}}^{(10)}_{\mathrm{Pr}_{st}}$	210333.4

Table 10: Simulation result for the percent relative efficiencies of the suggested family of exponential cum ratio type estimators w.r.t. existing estimators for different stratum.

Estimator	PRE	Estimator	PRE	Estimator	PRE	Estimator	PRE
$\hat{\hat{Y}}_{\mathrm{SRS}_{\mathrm{st}}}$	100.00	$\hat{\hat{Y}}_{S_{st}}^{(1)}$	145.79	$\hat{\hat{Y}}_{gr,kr_{st}}^{(1)}$	186.6	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(1)}$	1518.9
$\hat{\hat{Y}}_{R_{\mathrm{st}}}$	156.74	$\hat{\hat{Y}}_{S_{st}}^{(2)}$	100.21	$\hat{\hat{Y}}_{gr,kr_{st}}^{(2)}$	208.9	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(2)}$	1718.9
$\hat{\hat{Y}}_{P_{\mathrm{st}}}$	101.98	$\hat{\hat{Y}}_{S_{st}}^{(3)}$	170.96	$\hat{\hat{Y}}_{gr,kr_{st}}^{(3)}$	754.9	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(3)}$	1280.78
$\hat{\hat{Y}}_{\mathrm{Reg}_{\mathrm{st}}}$	145.62	$\hat{\hat{Y}}_{S_{st}}^{(4)}$	149.49	$\hat{\hat{Y}}_{gr,kr_{st}}^{(4)}$	416.10	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(4)}$	1281.55
$\hat{\hat{Y}}_{R,D_{st}}$	137.62	$\hat{\hat{Y}}_{S_{st}}^{(5)}$	110.21	$\hat{\hat{Y}}_{gr,kr_{st}}^{(5)}$	209.19	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(5)}$	1291.51
		$\hat{\hat{Y}}_{S_{st}}^{(6)}$	100.11	$\hat{\hat{Y}}_{gr,kr_{st}}^{(6)}$	210.19	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(6)}$	1254.59
		$\hat{\hat{Y}}_{S_{st}}^{(7)}$	179.00	$\hat{\hat{Y}}_{gr,kr_{st}}^{(7)}$	861.9	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(7)}$	1921.51
		$\hat{\hat{Y}}_{S_{st}}^{(8)}$	100.12	$\hat{\hat{Y}}_{gr,kr_{st}}^{(8)}$	254.9	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(8)}$	1890.59
		$\hat{\hat{Y}}_{S_{st}}^{(9)}$	162.35	$\hat{\hat{Y}}_{gr,kr_{st}}^{(9)}$	233.78	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(9)}$	1489.01
		$\hat{\hat{Y}}_{S_{st}}^{(10)}$	172.54	$\hat{\hat{Y}}_{gr,kr_{st}}^{(10)}$	243.58	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(10)}$	1220.45

Table 11: PREs of finite population mean estimators using population 1 (scenario I).

Estimator	PRE	Estimator	PRE	Estimator	PRE	Estimator	PRE
$\hat{\hat{Y}}_{\mathrm{SRS}_{st}}$	100.00	$\hat{\hat{Y}}_{S_{st}}^{(1)}$	43.74	$\hat{\hat{Y}}_{gr,kr_{st}}^{(1)}$	1155.37	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(1)}$	1203.81
$\hat{\hat{Y}}_{R_{st}}$	24.22	$\hat{\hat{Y}}_{S_{st}}^{(2)}$	369.02	$\hat{\hat{Y}}_{gr,kr_{st}}^{(2)}$	1155.34	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(2)}$	1203.77
$\hat{\hat{Y}}_{P_{st}}$	1030.07	$\hat{\hat{Y}}_{S_{st}}^{(3)}$	370.03	$\hat{\hat{Y}}_{gr,kr_{st}}^{(3)}$	1155.37	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(3)}$	1203.81
$\hat{\hat{Y}}_{\mathrm{Reg}_{st}}$	1147.43	$\hat{\hat{Y}}_{S_{st}}^{(4)}$	369.44	$\hat{\hat{Y}}_{gr,kr_{st}}^{(4)}$	1155.35	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(4)}$	1203.79
$\hat{\hat{Y}}_{R,D_{st}}$	1148.60	$\hat{\hat{Y}}_{S_{st}}^{(5)}$	369.98	$\hat{\hat{Y}}_{gr,kr_{st}}^{(5)}$	1155.37	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(5)}$	1203.81
		$\hat{\hat{Y}}_{S_{st}}^{(6)}$	370.01	$\hat{\hat{Y}}_{gr,kr_{st}}^{(6)}$	1155.37	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(6)}$	1203.81
		$\hat{\hat{Y}}_{S_{st}}^{(7)}$	369.94	$\hat{\hat{Y}}_{gr,kr_{st}}^{(7)}$	1155.37	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(7)}$	1203.81
		$\hat{\hat{Y}}_{S_{st}}^{(8)}$	370.03	$\hat{\hat{Y}}_{gr,kr_{st}}^{(8)}$	1155.37	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(8)}$	1203.81
		$\hat{\hat{Y}}_{S_{st}}^{(9)}$	368.99	$\hat{\hat{Y}}_{gr,kr_{st}}^{(9)}$	1155.34	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(9)}$	1203.77
		$\hat{\hat{Y}}_{S_{st}}^{(10)}$	100.11	$\hat{\hat{Y}}_{gr,kr_{st}}^{(10)}$	1148.60	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(10)}$	1196.63

Table 12: PREs of finite population mean estimators using population 1 (scenario II).

Estimator	PRE	Estimator	PRE	Estimator	PRE	Estimator	PRE
$\hat{\hat{Y}}_{\mathrm{SRS}_{st}}$	100.00	$\hat{\hat{Y}}_{S_{st}}^{(1)}$	52.57	$\hat{\hat{Y}}_{gr,kr_{st}}^{(1)}$	2204.41	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(1)}$	2809.24
$\hat{\hat{Y}}_{R_{st}}$	32.14	$\hat{\hat{Y}}_{S_{st}}^{(2)}$	243.76	$\hat{\hat{Y}}_{gr,kr_{st}}^{(2)}$	2203.78	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(2)}$	2808.30
$\hat{\hat{Y}}_{P_{st}}$	1153.34	$\hat{\hat{Y}}_{S_{st}}^{(3)}$	254.00	$\hat{\hat{Y}}_{gr,kr_{st}}^{(3)}$	2204.48	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(3)}$	2809.35
$\hat{\hat{Y}}_{\mathrm{Reg}_{st}}$	2196.27	$\hat{\hat{Y}}_{S_{st}}^{(4)}$	245.77	$\hat{\hat{Y}}_{gr,kr_{st}}^{(4)}$	2203.92	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(4)}$	2808.50
$\hat{\hat{Y}}_{R,D_{st}}$	2197.45	$\hat{\hat{Y}}_{S_{st}}^{(5)}$	253.23	$\hat{\hat{Y}}_{gr,kr_{st}}^{(5)}$	2204.43	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(5)}$	2809.27
		$\hat{\hat{Y}}_{S_{st}}^{(6)}$	253.41	$\hat{\hat{Y}}_{gr,kr_{st}}^{(6)}$	2204.44	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(6)}$	2809.28
		$\hat{\hat{Y}}_{S_{st}}^{(7)}$	252.96	$\hat{\hat{Y}}_{gr,kr_{st}}^{(7)}$	2204.41	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(7)}$	2809.24
		$\hat{\hat{Y}}_{S_{st}}^{(8)}$	254.02	$\hat{\hat{Y}}_{gr,kr_{st}}^{(8)}$	2204.48	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(8)}$	2809.35
		$\hat{\hat{Y}}_{S_{st}}^{(9)}$	243.53	$\hat{\hat{Y}}_{gr,kr_{st}}^{(9)}$	2203.76	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(9)}$	2808.27
		$\hat{\hat{Y}}_{S_{st}}^{(10)}$	100.08	$\hat{\hat{Y}}_{gr,kr_{st}}^{(10)}$	2197.45	$\hat{\hat{Y}}_{\mathrm{Pr}_{st}}^{(10)}$	2799.27

Table 13: PREs of finite population mean estimators using population 2 (scenario I).

Estimator	PRE	Estimator	PRE	Estimator	PRE	Estimator	PRE
$\hat{\bar{Y}}_{\mathrm{SRS}_{st}}$	100.00	$\hat{\bar{Y}}_{S_{st}}^{(1)}$	56.92	$\hat{\bar{Y}}_{gr,kr_{st}}^{(1)}$	325.31	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(1)}$	353.71
$\hat{\bar{Y}}_{R_{st}}$	35.80	$\hat{\bar{Y}}_{S_{st}}^{(2)}$	191.14	$\hat{\bar{Y}}_{gr,kr_{st}}^{(2)}$	325.30	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(2)}$	353.70
$\hat{\bar{Y}}_{P_{st}}$	307.37	$\hat{\bar{Y}}_{S_{st}}^{(3)}$	191.28	$\hat{\bar{Y}}_{gr,kr_{st}}^{(3)}$	325.31	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(3)}$	353.71
$\hat{\bar{Y}}_{\mathrm{Reg}_{st}}$	313.08	$\hat{\bar{Y}}_{S_{st}}^{(4)}$	191.22	$\hat{\bar{Y}}_{gr,kr_{st}}^{(4)}$	325.30	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(4)}$	353.70
$\hat{\bar{Y}}_{R,D_{st}}$	321.20	$\hat{\bar{Y}}_{S_{st}}^{(5)}$	191.28	$\hat{\bar{Y}}_{gr,kr_{st}}^{(5)}$	325.31	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(5)}$	353.71
		$\hat{\bar{Y}}_{S_{st}}^{(6)}$	191.28	$\hat{\bar{Y}}_{gr,kr_{st}}^{(6)}$	325.31	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(6)}$	353.71
		$\hat{\bar{Y}}_{S_{st}}^{(7)}$	191.27	$\hat{\bar{Y}}_{gr,kr_{st}}^{(7)}$	325.31	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(7)}$	353.71
		$\hat{\bar{Y}}_{S_{st}}^{(8)}$	191.28	$\hat{\bar{Y}}_{gr,kr_{st}}^{(8)}$	325.31	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(8)}$	353.71
		$\hat{\bar{Y}}_{S_{st}}^{(9)}$	191.11	$\hat{\bar{Y}}_{gr,kr_{st}}^{(9)}$	325.29	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(9)}$	353.69
		$\hat{\bar{Y}}_{S_{st}}^{(10)}$	100.07	$\hat{\bar{Y}}_{gr,kr_{st}}^{(10)}$	321.20	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(10)}$	349.21

Table 14: PREs of finite population mean estimators using population 2 (scenario II).

Estimator	PRE	Estimator	PRE	Estimator	PRE	Estimator	PRE
$\hat{\bar{Y}}_{\mathrm{SRS}_{st}}$	100.00	$\hat{\bar{Y}}_{S_{st}}^{(1)}$	58.28	$\hat{\bar{Y}}_{gr,kr_{st}}^{(1)}$	305.40	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(1)}$	335.67
$\hat{\bar{Y}}_{R_{st}}$	37.15	$\hat{\bar{Y}}_{S_{st}}^{(2)}$	183.60	$\hat{\bar{Y}}_{gr,kr_{st}}^{(2)}$	305.40	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(2)}$	335.67
$\hat{\bar{Y}}_{P_{st}}$	286.78	$\hat{\bar{Y}}_{S_{st}}^{(3)}$	183.72	$\hat{\bar{Y}}_{gr,kr_{st}}^{(3)}$	305.40	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(3)}$	335.68
$\hat{\bar{Y}}_{\mathrm{Reg}_{st}}$	293.74	$\hat{\bar{Y}}_{S_{st}}^{(4)}$	183.67	$\hat{\bar{Y}}_{gr,kr_{st}}^{(4)}$	305.40	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(4)}$	335.67
$\hat{\bar{Y}}_{R,D_{st}}$	301.87	$\hat{\bar{Y}}_{S_{st}}^{(5)}$	183.72	$\hat{\bar{Y}}_{gr,kr_{st}}^{(5)}$	305.40	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(5)}$	335.68
		$\hat{\bar{Y}}_{S_{st}}^{(6)}$	183.72	$\hat{\bar{Y}}_{gr,kr_{st}}^{(6)}$	305.40	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(6)}$	335.68
		$\hat{\bar{Y}}_{S_{st}}^{(7)}$	183.71	$\hat{\bar{Y}}_{gr,kr_{st}}^{(7)}$	305.40	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(7)}$	335.67
		$\hat{\bar{Y}}_{S_{st}}^{(8)}$	183.72	$\hat{\bar{Y}}_{gr,kr_{st}}^{(8)}$	305.40	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(8)}$	335.68
		$\hat{\bar{Y}}_{S_{st}}^{(9)}$	183.58	$\hat{\bar{Y}}_{gr,kr_{st}}^{(9)}$	305.39	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(9)}$	335.66
		$\hat{\bar{Y}}_{S_{st}}^{(10)}$	100.07	$\hat{\bar{Y}}_{gr,kr_{st}}^{(10)}$	301.87	$\hat{\bar{Y}}_{\mathrm{Pr}_{st}}^{(10)}$	331.75

performance of the suggested exponential cum ratio type family of estimators is appraised for the artificially generated population of different stratum and sample sizes. The population elements are randomly generated from $N_1 \sim (2,6)$ and $N_1 \sim (3,9)$ compared to that from the values of x_1 and x_2 generated y_1 and y_2. Applying the procedure of ranked set sampling, we obtained Z variable. The comparison is offered in the form of percentage relative efficiency (PRE). The relative efficiency in Table 10 is computed using the following formula:

$$\mathrm{PRE}\left(\hat{\bar{Y}}_i, \hat{\bar{Y}}_{\mathrm{SRS}_{st}}\right) = \frac{\mathrm{Var}\left(\hat{\bar{Y}}_{\mathrm{SRS}_{st}}\right)}{\mathrm{MSE}_{\min}\left(\hat{\bar{Y}}_i\right)} \times 100, \tag{39}$$

where $i = R_{st}, P_{st}, \ldots, \mathrm{Pr}_{st}$.

In this study, consider the generated population for summarizing the simulation procedures. The process is used to find out the efficiency of the family of suggested estimators over all the existing estimators.

From the described generated population of the normal population, the sample size is considered as $n_1 = 25$ and $n_2 = 25$. We take $r = 10$ and $m = 50$ for the ranks of the auxiliary variable x. The procedure has been repeated 10000 times and the whole aggregate is split into two strata, where $N = 1000$ and $N_1 = 500$, $N_2 = 500$ evaluate several available families of existing estimators under stratified random sampling technique. Using the sample, simulate 10,000 times estimators values of existing estimators class $\hat{\bar{Y}}_{S_{st}}^{(1)}$, $\hat{\bar{Y}}_{S_{st}}^{(2)}$, $\hat{\bar{Y}}_{S_{st}}^{(3)}$, $\hat{\bar{Y}}_{S_{st}}^{(4)}$, $\hat{\bar{Y}}_{S_{st}}^{(5)}$, $\hat{\bar{Y}}_{S_{st}}^{(6)}$, $\hat{\bar{Y}}_{S_{st}}^{(7)}$, $\hat{\bar{Y}}_{S_{st}}^{(8)}$, $\hat{\bar{Y}}_{S_{st}}^{(9)}$, $\hat{\bar{Y}}_{S_{st}}^{(10)}$, and $\hat{\bar{Y}}_{gr,kr_{st}}^{(1)}$, $\hat{\bar{Y}}_{gr,kr_{st}}^{(2)}$, $\hat{\bar{Y}}_{gr,kr_{st}}^{(3)}$, $\hat{\bar{Y}}_{gr,kr_{st}}^{(4)}$, $\hat{\bar{Y}}_{gr,kr_{st}}^{(5)}$, $\hat{\bar{Y}}_{gr,kr_{st}}^{(6)}$, $\hat{\bar{Y}}_{gr,kr_{st}}^{(7)}$, $\hat{\bar{Y}}_{gr,kr_{st}}^{(8)}$, $\hat{\bar{Y}}_{gr,kr_{st}}^{(9)}$, $\hat{\bar{Y}}_{gr,kr_{st}}^{(10)}$, and $\hat{\bar{Y}}_{S_{st}}^{(1)}$, $\hat{\bar{Y}}_{S_{st}}^{(2)}$, $\hat{\bar{Y}}_{S_{st}}^{(3)}$, $\hat{\bar{Y}}_{S_{st}}^{(4)}$, $\hat{\bar{Y}}_{S_{st}}^{(5)}$, $\hat{\bar{Y}}_{S_{st}}^{(6)}$, $\hat{\bar{Y}}_{S_{st}}^{(7)}$, $\hat{\bar{Y}}_{S_{st}}^{(8)}$, $\hat{\bar{Y}}_{S_{st}}^{(9)}$, $\hat{\bar{Y}}_{S_{st}}^{(10)}$, separately are obtained for the formula described in

(39), respectively. This simulation study is evaluating the percent relative efficiencies of the suggested exponential cum ratio type family of estimators and available estimators for sample sizes in two strata. From the critical comparison, it is concluded that the suggested family of estimators perform better result. Overall, we can conclude the performance of the family of suggested estimators is better than all existing estimators.

8. Discussion

As mentioned above, we used two real data sets and a simulation study to determine the efficiency of the estimators suggested for stratified sampling technique with the help of information of single auxiliary variable X and rank of the auxiliary variable. We also considered different sample sizes from the populations for scenario I and scenario II. The proposed family of estimators and the adapted stratified estimators were compared to each other with respect to their mean square error and percentage relative efficiency. In Tables 2–5, we present the data description. The results of mean square error for the real data sets are presented in Tables 6–9. And the percentage relative efficiency is shown in Tables 11–14.

From the numerical findings, which are put in Tables 6–9 and Tables 10–14, it is observed that the MSEs and PREs of our proposed family of estimators change with the choices of a and b. It is also observed that the proposed family of estimators are more precise than the existing estimators on both real data sets and simulation study, in terms of MSEs and PREs.

9. Concluding Remarks

In this manuscript, we proposed a new family of estimators for the finite population mean under the stratified sampling scheme, which needed a helping variable on the sample mean and rank of the auxiliary variable. Based on the real data sets, it is observed that the proposed class of estimators perform better theoretically and numerically compared to their existing counterparts and should be preferable over the existing estimators available in the literature. To check the robustness and generalizability of the proposed family of estimators, a simulation study is also conducted. The results of the simulation study also support the usefulness of the proposed family of estimators. Hence, it is recommended that the new suggested family of estimators should be used for better results when estimating population mean under stratified random sampling scheme.

Data Availability

All the data used for this study can be found within the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] T. Zaman, C. Kadilar, and C. Kadilar, "Exponential ratio and product type estimators of the mean in stratified two-phase sampling," *AIMS Mathematics*, vol. 6, no. 5, pp. 4265–4279, 2021.

[2] S. Hussain, S. Ahmad, M. Saleem, and S. Akhtar, "Finite population distribution function estimation with dual use of auxiliary information under simple and stratified random sampling," *Plos one*, vol. 15, no. 9, Article ID e0239098, 2020.

[3] S. Aladag and H. Cingi, "Improvement in estimating the population median in simple random sampling and stratified random sampling using auxiliary information," *Communications in Statistics - Theory and Methods*, vol. 44, no. 5, pp. 1013–1032, 2015.

[4] L. K. Grover and P. Kaur, "A generalized class of ratio type exponential estimators of population mean under linear transformation of auxiliary variable," *Communications in Statistics - Simulation and Computation*, vol. 43, no. 7, pp. 1552–1574, 2014.

[5] J. Shabbir and S. Gupta, "Estimation of finite population mean in simple and stratified random sampling using two auxiliary variables," *Communications in Statistics - Theory and Methods*, vol. 46, no. 20, pp. 10135–10148, 2017.

[6] T. J. Rao, "On certail methods of improving ration and regression estimators," *Communications in Statistics - Theory and Methods*, vol. 20, no. 10, pp. 3325–3340, 1991.

[7] R. Singh, P. Chauhan, N. Sawan, and F. Smarandache, "Improvement in estimating the population mean using exponential estimator in simple random sampling," *International Journal of Statistics & Economics*, vol. 3, no. 9, pp. 13–18, 2009.

[8] C. Kadilar and H. Cingi, "Ratio estimators in stratified random sampling," *Biometrical Journal*, vol. 45, no. 2, pp. 218–225, 2003.

[9] G. N. Singh and M. Khalid, "Exponential chain dual to ratio and regression type estimators of population mean in two-phase sampling," *Statistica*, vol. 75, no. 4, pp. 379–389, 2015.

[10] G. N. Singh and M. Khalid, "Efficient class of estimators for finite population mean using auxiliary information in two-occasion successive sampling," *Journal of Modern Applied Statistical Methods*, vol. 17, no. 2, pp. 1–43, 2019.

[11] T. Zaman and H. Bulut, "An efficient family of robust-type estimators for the population variance in simple and stratified random sampling," *Communications in Statistics - Theory and Methods*, vol. 26, pp. 1–15, 2021.

[12] S. Al-Marzouki, C. Chesneau, C. Chesneau et al., "Estimation of finite population mean under PPS in presence of maximum and minimum values," *AIMS Mathematics*, vol. 6, no. 5, pp. 5397–5409, 2021.

[13] M. Aamir, Z. U. Haq, M. Tariq, S. Jan, and M. Jahangir, "Improvement on estimating of median in two-phase sampling using two auxiliary variables," *Life Science Journal*, vol. 11, no. 4s, pp. 98–104, 2014.

[14] M. Aamir, A. Shabri, and M. Ishaq, "Improvement on estimating median for finite population using auxiliary variables in double-sampling," *Jurnal Teknologi*, vol. 80, no. 5, pp. 135–143, 2018.

[15] W. G. Cochran, "The estimation of the yields of cereal experiments by sampling for the ratio of grain to total produce," *The Journal of Agricultural Science*, vol. 30, no. 2, pp. 262–275, 1940.

[16] M. N. Murthy, "Product method of estimation," *Sankhya: The Indian Journal of Statistics, Series A*, vol. 26, no. 1, pp. 69–74, 1964.

[17] H. Kaur, S. S. Brar, and M. Sharma, "Efficent ratio type estimatiors of population variance through linear transformation in simple and statified random sampling," *International Journal of Statistics and Reliability Engineering*, vol. 4, no. 2, pp. 144–153, 2018.

[18] N. Koyuncu and C. Kadilar, "Ratio and Product estimators in stratified random sampling," *Journal of Statistical Planning and Inference*, vol. 139, no. 8, pp. 2552–2558, 2009.

[19] C. Kadilar and H. Cingi, "A new ratio estimator in stratified random sampling," *Communications in Statistics - Theory and Methods*, vol. 34, no. 3, pp. 597–602, 2005.