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Research Article

Preservation of the Classical Meanness Property of Some Graphs Based on Line Graph Operation

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In the present paper, we introduce the classical mean labeling of graphs and investigate their related properties. Moreover, it is obtained that the line graph operation preserves the classical meanness property for some standard graphs.

1. Introduction and Preliminaries

All through this paper, by a graph we mean a simple, undirected, and finite graph. For documentations and wording, we follow [1–5]. For a point by point review on graph labeling, we refer [6]. The line graph $L(G)$ of a graph G is defined to have as its vertices the edges of G, with two being adjacent if the corresponding edges share a vertex in G. The graph $G^{\circ}S_m$ is obtained from G by attaching m pendant vertices to each vertex of G. Let u_α: $1 \leq \alpha \leq n$ and $v_\beta^{(\alpha)}$: $1 \leq \beta \leq m+1$ be the nodes of path P_n and α^{th} copy of the star graph S_m, respectively, then the graph $[P_n; S_m]$ is obtained from n copies of S_m and the path P_n by joining u_α with the central vertex $v_1^{(\alpha)}$ of the α^{th} copy of S_m by means of an edge, for $1 \leq \alpha \leq n$. A graph obtained by subdividing edge of G by a vertex is called subdivision graph $S(G)$ and a graph obtained from the path by replacing every edge of a path by a C_3 is called triangular snake graph T_n.

2. Literature Survey

The investigation of graceful labeling is characterized by Rosa in [7] and prime labeling is defined by Tout et al. in [8]. Somasundram and Ponraj introduced the mean labeling of graphs in [9]. Durai Baskar and Arockiaraj defined the F-harmonic mean labeling [10] and discussed its meanness for some standard graphs. The idea of F-geometric was presented by Durai Baskar et al. in [11] and F-root mean labeling was presented by Arockiaraj et al. in [12] and talked about its meanness of ladder graph in [13]. Vaidya and Barasara in [14] have discussed so many results on product cordial labeling. Vaidya and Lekha in [15] presented the idea of a bi-odd sequential labeling. The labeling of L (2, 1) in [16] is researched by Prajapati and Patel. Rajesh Kannan et al. discussed the FCM labeling of graphs and its line graphs in [17]. Propelled by and crafted by such a large number of creators in the territory of graph labeling, we present another labeling called classical mean labeling. A classical mean of two positive integers need not be an integer in general. For the classical mean is to be an integer, we may use either flooring or ceiling function. In this paper, we consider only the flooring function of our discussion and try to analyze that the line graph operation preserves the classical meanness property for some standard graphs. The labeling is one of the well studied area in Graph Theory. So, we are interested in defining new labeling called classical mean labeling. A classical mean labeling is for getting more accuracy of all the edge labeling by using the average of four different types of means of the vertex labeling of the given graph. Recently, Muhiuddin et al. studied various related concepts on graphs (see [18–22]).

Graph labeling assumes an essential job in different areas of the real world system. The concepts of classical mean labeling are utilized to demonstrate numerous kinds of processes and relations in biological, social, material physical, and data systems. It is a powerful tool that makes complicated patterns to be learned easily and conveniently in various fields. A static network can be represented as a specific kind of graph by connecting nodes in some topology, and labeling can be applied for automatic routing of data in a network. The graph can be cycle, path, circuit, walk, and connected which represent a fixed network. For each network, labeling is done with a constant which helps routing to automatically detect next node in the network. The classical mean labeling is used in fast communication in sensor networks for finding the more accuracy level of sensor units.

3. Methodology

A function χ is known as a classical mean labeling of a graph $G(V, E)$ with p nodes and q edges if $\chi\colon V(G) \longrightarrow \{1, 2, 3, \ldots, q+1\}$ is injective and the incited edge assignment function $\chi^*\colon E(G) \longrightarrow \{1, 2, 3, \ldots, q\}$ characterized as

$$\chi^*(uv) = \left\lfloor \frac{1}{4}\left(\frac{\chi(u)+\chi(v)}{2} + \sqrt{\chi(u)\chi(v)} + \frac{2\chi(u)\chi(v)}{\chi(u)+\chi(v)} + \sqrt{\frac{\chi(u)^2+\chi(v)^2}{2}}\right)\right\rfloor, \tag{1}$$

for all $uv \in E(G)$, is bijective. From Figure 1, a graph that concedes a classical mean labeling is said to be classical mean graph.

As q is the number of edges of the given graph, it cannot take a randomly large number so that such a labeling exists. However more than one classical mean labeling exists for the given graph. So, we show one among in the proof.

Here, it is found that the line graph operation preserves the classical meanness property for some standard graphs.

4. Classical Meanness of Some Standard Graphs and Its Line Graph

Theorem 1. *Every path P_n for $n \geq 1$ and its line graph $L(P_n)$ for $n \geq 2$ are classical mean graphs.*

Proof. Develop a mapping χ from the vertex set of path to $\{1, 2, 3, \ldots, n\}$ by $\chi(v_\alpha) = \alpha$, for $1 \leq \alpha \leq n$, where $\{v_\alpha\colon 1 \leq \alpha \leq n\}$ be the nodes of the path. Therefore, for $1 \leq \alpha \leq n-1$, $\chi^*(v_\alpha v_{\alpha+1}) = \alpha$. Since $L(P_n)$ is again a path, $L(P_n)$ is also a classical mean graph. Hence, every path P_n for $n \geq 1$ and its line graph $L(P_n)$ for $n \geq 2$ are classical mean graphs. □

Theorem 2. *Every cycle C_n and its line graph $L(C_n)$ are classical mean graphs, for $n \geq 3$.*

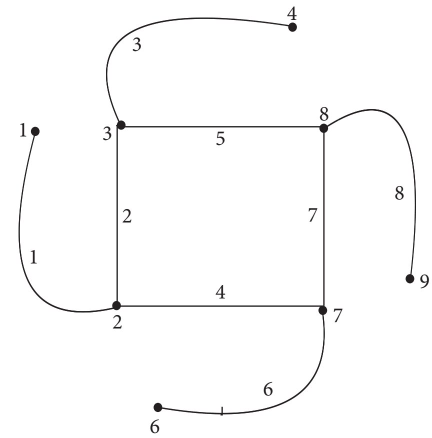

Figure 1: A classical mean labeling of $C_4{\circ}S_1$.

Proof. Develop a mapping χ from the vertex set of cycle to $\{1, 2, 3, \ldots, n+1\}$ by

$$\chi(u_\alpha) = \begin{cases} -1 + 2\alpha, & 1 \leq \alpha \leq \left\lfloor \dfrac{n}{2} \right\rfloor + 1, \\ 2n - 2\alpha + 4, & \left\lfloor \dfrac{n}{2} \right\rfloor + 2 \leq \alpha \leq n. \end{cases} \tag{2}$$

Therefore,

$$\chi^*(u_\alpha u_{1+\alpha}) = \begin{cases} 2\alpha - 1, & 1 \leq \alpha \leq \left\lfloor \dfrac{n}{2} \right\rfloor, \\ n, & \alpha = \left\lfloor \dfrac{n}{2} \right\rfloor + 1, \\ 2n - 2\alpha + 2, & \left\lfloor \dfrac{n}{2} \right\rfloor + 2 \leq \alpha \leq -1 + n \text{ and} \end{cases} \quad \chi^*(u_n u_1) = 2. \tag{3}$$

Also, the graph $L(C_n)$ is again a cycle, which is given by Figure 2. Hence, every cycle C_n and its line graph $L(C_n)$ are classical mean graphs, for $n \geq 3$. □

5. Classical Meanness of Graphs Obtained from Vertex Identification and Its Line Graph

Theorem 3. *. The graph $P_n{\circ}S_m$ and its line graph $L(P_n \circ S_m)$ are classical mean graphs, for $n \geq 1$ and $m \leq 2$.*

Proof. Let $\left\{u_\beta^\alpha\colon 1 \leq \alpha \leq n \text{ and } 1 \leq \beta \leq m\right\}$ be the pendant vertices at each v_α.

Case i. $m = 1$ in the graph $P_n{\circ}S_m$. Develop a mapping χ from the vertex set of $P_n{\circ}S_1$ to $\{1, 2, 3, \ldots, 2n\}$ by

$$\chi(v_\alpha) = \begin{cases} 1, & \alpha = 1, \\ 2\alpha, & 2 \leq \alpha \leq n \text{ and,} \end{cases}$$
$$\chi\left(u_1^{(\alpha)}\right) = \begin{cases} 2, & \alpha = 1, \\ 2\alpha - 1, & 2 \leq \alpha \leq n. \end{cases} \tag{4}$$

Therefore,

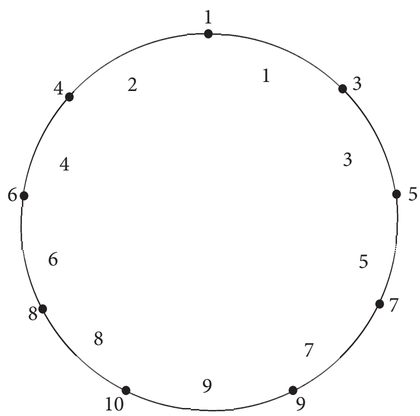

Figure 2: A classical mean labeling of C_9.

$$\chi^*(v_{\alpha+1}v_\alpha) = 2\alpha, \text{ for } 1 \le \alpha \le n-1 \text{ and,}$$
$$\chi^*\left(u_1^{(\alpha)v_\alpha}\right) = -1 + 2\alpha,\ 1 \le \alpha \le n. \tag{5}$$

Case ii. $m = 2$ in the graph $P_n \circ S_m$.

Develop a mapping χ from the vertex set of $P_n \circ S_2$ to $\{1, 2, 3, \ldots, 3n\}$ by

$$\chi(v_\alpha) = 3\alpha - 1, \text{ for } 1 \le \alpha \le n,$$
$$\chi\left(u_1^{(\alpha)}\right) = 3\alpha - 2, \text{ for } 1 \le \alpha \le n \text{ and,} \tag{6}$$
$$\chi\left(u_2^{(\alpha)}\right) = 3\alpha, \text{ for } 1 \le \alpha \le n.$$

Therefore,

$$\chi^*(v_\alpha v_{\alpha+1}) = 3\alpha, \text{ for } 1 \le \alpha \le n-1,$$
$$\chi^*\left(v_\alpha u_1^{(\alpha)}\right) = 3\alpha - 2, \text{ for } 1 \le \alpha \le n \text{ and,} \tag{7}$$
$$\chi^*\left(v_\alpha u_2^{(\alpha)}\right) = 3\alpha - 1, \text{ for } 1 \le \alpha \le n.$$

Hence, a classical mean labeling of $P_7 \circ S_1$ and $P_7 \circ S_2$ is given by Figure 3.

Let $V(L(P_n \circ S_1)) = \{v_1, v_2, v_3, \ldots, v_n, e_1, e_2, e_3, \ldots, e_{n-1}\}$ and $E(L(P_n \circ S_1)) = \{v_\alpha e_\alpha, e_\alpha v_{\alpha+1}\colon 1 \le \alpha \le n-1\} \cup \{e_\alpha e_{\alpha+1}\colon 1 \le \alpha \le n-2\}$.

Case iii. $m = 1$ in the graph $L(P_n \circ S_m)$.

Develop a mapping χ from the line graph of vertex set of $P_n \circ S_1$ to $\{1, 2, 3, \ldots, 3n-3\}$ by

$$\chi(v_\alpha) = \begin{cases} 2\alpha - 1, & 1 \le \alpha \le 3, \\ 3\alpha - 4, & 4 \le \alpha \le n \text{ and,} \end{cases}$$
$$\chi(e_\alpha) = \begin{cases} 2, & \alpha = 1, \\ 3\alpha, & 2 \le \alpha \le n-1. \end{cases} \tag{8}$$

Therefore,

$$\chi^*(v_\alpha e_\alpha) = \begin{cases} 3\alpha - 2, & 1 \le \alpha \le 2, \\ 3\alpha - 3, & 3 \le \alpha \le -1 + n, \end{cases}$$
$$\chi^*(e_\alpha v_{\alpha+1}) = 3\alpha - 1, \quad \text{for } 1 \le \alpha \le n-1 \text{ and,} \tag{9}$$
$$\chi^*(e_\alpha e_{\alpha+1}) = \begin{cases} 3, & \alpha = 1, \\ 3\alpha + 1, & 2 \le \alpha \le -2 + n. \end{cases}$$

Let $V(L(P_n \circ S_2)) = \{x_\alpha\colon 1 \le \alpha \le n-1\} \cup \{y_\alpha, z_\alpha\colon 1 \le \alpha \le n\}$ and $E(L(P_n \circ S_2)) = \{x_\alpha z_\alpha, x_\alpha y_{\alpha+1}, x_\alpha z_{\alpha+1}, x_\alpha y_\alpha\colon 1 \le \alpha \le n-1\} \cup \{x_\alpha x_{\alpha+1}\colon 1 \le \alpha \le n-2\} \cup \{y_\alpha z_\alpha\colon 1 \le \alpha \le n\}$.

Case iv. $m = 2$ in the graph $L(P_n \circ S_m)$. Develop a mapping χ from the line graph of vertex set of $P_n \circ S_2$ to $\{1, 2, 3, \ldots, 6n-5\}$ by

$$\chi(x_\alpha) = \begin{cases} 5, & \alpha = 1, \\ 6\alpha, & 2 \le \alpha \le n-1, \end{cases}$$
$$\chi(y_\alpha) = \begin{cases} 1, & \alpha = 1, \\ 6\alpha - 8, & 2 \le \alpha \le n \text{ and,} \end{cases} \tag{10}$$
$$\chi(z_\alpha) = \begin{cases} 6\alpha - 4, & 1 \le \alpha \le 2, \\ 6\alpha - 5, & 3 \le \alpha \le n. \end{cases}$$

Therefore,

$$\chi^*(x_\alpha x_{\alpha+1}) = 2 + 6\alpha, \quad \text{for } 1 \le \alpha \le n-2,$$
$$\chi^*(x_\alpha y_\alpha) = \begin{cases} 2, & \alpha = 1, \\ 6\alpha - 5, & 2 \le \alpha \le -1 + n, \end{cases}$$
$$\chi^*(x_\alpha z_\alpha) = 6\alpha - 3, \quad \text{for } 1 \le \alpha \le n-1,$$
$$\chi^*(x_\alpha z_{\alpha+1}) = 6\alpha, \quad \text{for } 1 \le \alpha \le -1 + n, \tag{11}$$
$$\chi^*(x_\alpha y_{\alpha+1}) = 6\alpha - 2, \quad \text{for } 1 \le \alpha \le -1 + n \text{ and,}$$
$$\chi^*(y_\alpha z_\alpha) = \begin{cases} 1, & \alpha = 1, \\ 6\alpha - 7, & 2 \le \alpha \le n. \end{cases}$$

Hence, from Figure 4, the graph $P_n \circ S_m$ and its line graph $L(P_n \circ S_m)$ are classical mean graphs, for $m \le 2$ and $n \ge 1$. □

Theorem 4. *The graph $[P_n; S_m]$ and its line graph $L([P_n; S_m])$ are classical mean graphs, for $m \le 2$ and $n \ge 1$.*

Proof.

Case i. $m = 1$ in the graph $[P_n; S_m]$. Develop a mapping χ from the vertex set of $[P_n; S_1]$ to $\{1, 2, 3, \ldots, 3n\}$ by

$$\chi(u_\alpha) = \begin{cases} 3\alpha, & \alpha \text{ is odd and } 1 \le \alpha \le n, \\ 3\alpha - 2, & \alpha \text{ is even and } 1 \le \alpha \le n, \end{cases}$$
$$\chi\left(v_1^{(\alpha)}\right) = 3\alpha - 1, \quad 1 \le \alpha \le n \text{ and,} \tag{12}$$
$$\chi\left(v_2^{(\alpha)}\right) = \begin{cases} -2 + 3\alpha, & \alpha \text{ is odd and } 1 \le \alpha \le n, \\ 3\alpha, & \alpha \text{ is even and } 1 \le \alpha \le n. \end{cases}$$

Therefore,

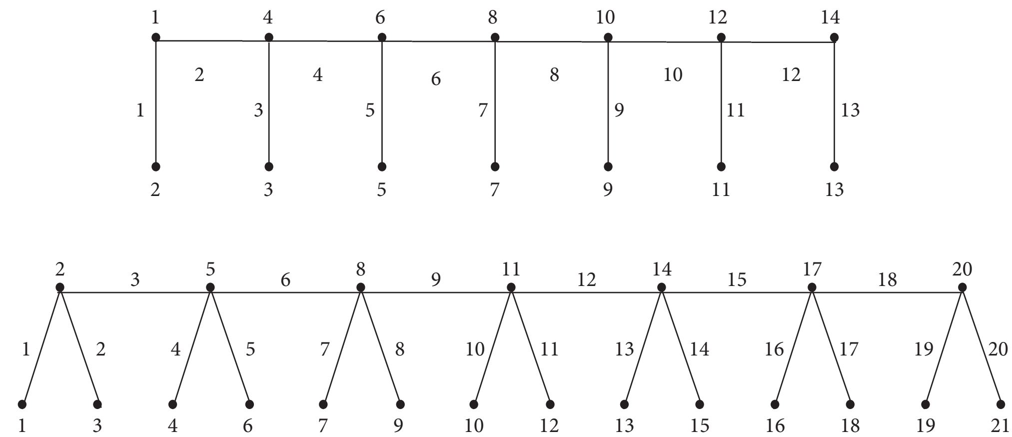

Figure 3: A classical mean labeling of $P_7 \circ S_1$ and $P_7 \circ S_2$.

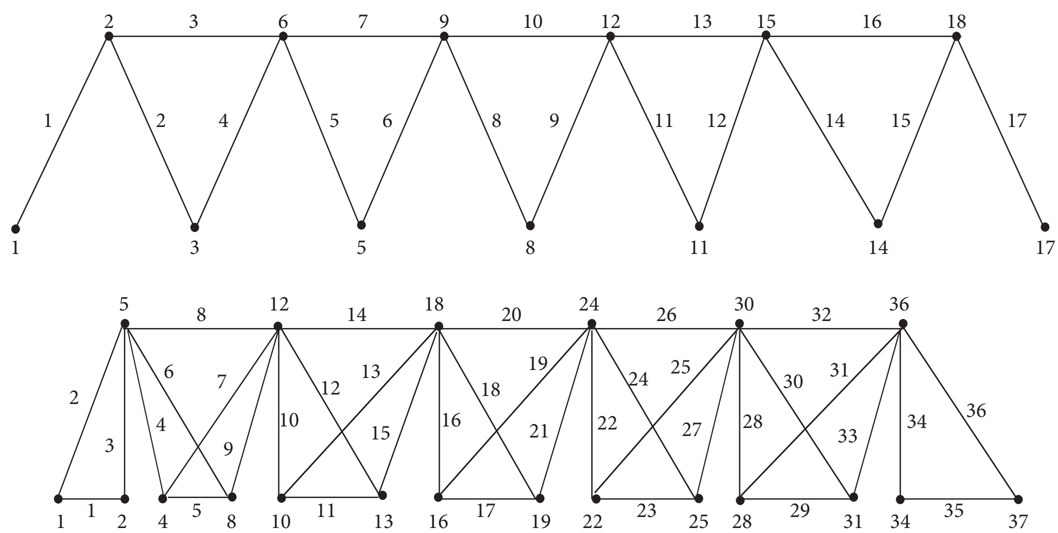

Figure 4: A classical mean labeling of $L(P_7 \circ S_1)$ and $L(P_7 \circ S_2)$.

$$\chi^*(u_\alpha u_{\alpha+1}) = 3\alpha, \text{ for } 1 \le \alpha \le n-1,$$

$$\chi^*\left(u_\alpha v_1^{(\alpha)}\right) = \begin{cases} -1 + 3\alpha, & \alpha \text{ is odd and } 1 \le \alpha \le n, \\ -2 + 3\alpha, & \alpha \text{ is even and } 1 \le \alpha \le n \text{ and} \end{cases}$$

$$\chi^*\left(v_1^{(\alpha)} v_2^{(\alpha)}\right) = \begin{cases} -2 + 3\alpha, & \alpha \text{ is odd and } 1 \le \alpha \le n, \\ -1 + 3\alpha, & \alpha \text{ is even and } 1 \le \alpha \le n. \end{cases} \tag{13}$$

Case ii. $m = 2$ in the graph $[P_n; S_m]$. Develop a mapping χ from the vertex set of $[P_n; S_2]$ to $\{1, 2, 3, \ldots, 4n\}$ by

$$\chi(u_\alpha) = \begin{cases} 4\alpha, & \alpha \text{ is odd and } 1 \le \alpha \le n, \\ -2 + 4\alpha, & \alpha \text{ is even and } 1 \le \alpha \le n, \end{cases}$$

$$\chi\left(v_1^{(\alpha)}\right) = 4\alpha - 1, \quad 1 \le \alpha \le n,$$

$$\chi\left(v_2^{(\alpha)}\right) = \begin{cases} 1, & \alpha = 1, \\ 4\alpha + 1, & 2 \le \alpha \le n \text{ and,} \end{cases} \tag{14}$$

$$\chi\left(v_3^{(\alpha)}\right) = \begin{cases} 4\alpha - 2, & \alpha \text{ is odd and } 1 \le \alpha \le n, \\ 4\alpha, & \alpha \text{ is even and } 1 \le \alpha \le n. \end{cases}$$

Therefore,

$$\chi^*(u_\alpha u_{\alpha+1}) = 4\alpha, \text{ for } 1 \le \alpha \le n-1,$$
$$\chi^*\left(u_\alpha v_1^{(\alpha)}\right) = \begin{cases} -1+4\alpha, & \alpha \text{ is odd and } 1 \le \alpha \le n, \\ -2+4\alpha, & \alpha \text{ is even and } 1 \le \alpha \le n, \end{cases}$$
$$\chi^*\left(v_1^{(\alpha)} v_2^{(\alpha)}\right) = -3+4\alpha, \quad \text{for } 1 \le \alpha \le n-1, \tag{15}$$
$$\chi^*\left(v_1^{(\alpha)} v_3^{(\alpha)}\right) = \begin{cases} -2+4\alpha, & \alpha \text{ is odd and } 1 \le \alpha \le n, \\ 4\alpha-1, & \alpha \text{ is even and } 1 \le \alpha \le n. \end{cases}$$

It is clearly seen that a classical mean labeling of $[P_7; S_1]$ and $[P_6; S_2]$ is given by Figure 5.

Case iii. $m = 1$ and $n \ge 2$ in the graph $L([P_n; S_m])$. Develop a mapping χ from the line graph of vertex set of $[P_n; S_1]$ to $\{1, 2, 3, \ldots, 4n-3\}$ by

$$\chi(u_\alpha) = 4\alpha, \quad \text{for } 1 \le \alpha \le n-1,$$
$$\chi\left(v_1^{(\alpha)}\right) = \begin{cases} 2, & \alpha = 1, \\ 4\alpha-3, & 2 \le \alpha \le n \text{ and}, \end{cases} \tag{16}$$
$$\chi\left(v_2^{(\alpha)}\right) = \begin{cases} 1, & \alpha = 1, \\ 4\alpha-5, & 2 \le \alpha \le n. \end{cases}$$

Therefore,

$$\chi^*(u_\alpha u_{\alpha+1}) = 4\alpha+1, \quad \text{for } 1 \le \alpha \le n-1,$$
$$\chi^*\left(u_\alpha v_1^{(\alpha+1)}\right) = 4\alpha, \quad \text{for } 1 \le \alpha \le n-1,$$
$$\chi^*\left(u_\alpha v_1^{(\alpha)}\right) = 4\alpha-2, \quad \text{for } 1 \le \alpha \le n-1 \text{ and}, \tag{17}$$
$$\chi^*\left(v_1^{(\alpha)} v_2^{(\alpha)}\right) = \begin{cases} 1, & \alpha = 1, \\ 4\alpha-5, & 2 \le \alpha \le n. \end{cases}$$

Case iv. $m = 1$ and $n = 1$ in the graph $L([P_n; S_m])$. For $n = 1$, the graph $L([P_n; S_1])$ is a path and by Theorem 1, the result follows.

Case v. $m = 2$ and $n \ge 2$ in the graph $L([P_n; S_m])$. Develop a mapping χ from the line graph of vertex set of $[P_n; S_2]$ to $\{1, 2, 3, \ldots, 5n-3\}$ by

$$\chi(u_\alpha) = \begin{cases} -4+8\alpha, & 1 \le \alpha \le 2, \\ 5\alpha, & \alpha \text{ is odd and } 3 \le \alpha \le n, \\ 1+5\alpha, & \alpha \text{ is even and } 1 \le \alpha \le n, \end{cases}$$
$$\chi\left(v_1^{(\alpha)}\right) = \begin{cases} 3\alpha, & 1 \le \alpha \le 2, \\ 11, & \alpha = 3, \\ 5\alpha-3, & 4 \le \alpha \le n \text{ and } \alpha \text{ is odd}, \\ 5\alpha-4, & 4 \le \alpha \le n \text{ and } \alpha \text{ is even}, \end{cases}$$
$$\chi\left(v_2^{(\alpha)}\right) = \begin{cases} 4\alpha-3, & 1 \le \alpha \le 2, \\ 5\alpha-7, & 3 \le \alpha \le n \text{ and } \alpha \text{ is odd}, \\ -6+5\alpha, & 3 \le \alpha \le n \text{ and } \alpha \text{ is even and}, \end{cases}$$
$$\chi\left(v_3^{(\alpha)}\right) = \begin{cases} 2, & \alpha = 1, \\ -5+5\alpha, & \alpha \text{ is odd and } 2 \le \alpha \le n, \\ -3+5\alpha, & \alpha \text{ is even and } 2 \le \alpha \le n. \end{cases} \tag{18}$$

Therefore,

$$\chi^*(u_\alpha u_{\alpha+1}) = \begin{cases} 6\alpha+1, & 1 \le \alpha \le 2, \\ 5\alpha+2, & 3 \le \alpha \le n-2, \end{cases}$$
$$\chi^*\left(u_\alpha v_1^{(\alpha)}\right) = \begin{cases} 3, & \alpha = 1, \\ 4\alpha, & 2 \le \alpha \le 3, \\ 5\alpha-2, & 4 \le \alpha \le n-1, \end{cases}$$
$$\chi^*\left(u_\alpha v_1^{(\alpha+1)}\right) = \begin{cases} 4, & \alpha = 1, \\ 5\alpha, & \alpha \text{ is odd and } 2 \le \alpha \le n, \\ 5\alpha+1, & \alpha \text{ is even and } 2 \le \alpha \le n, \end{cases}$$
$$\chi^*\left(v_1^{(\alpha)} v_2^{(\alpha)}\right) = \begin{cases} 4\alpha-3, & 1 \le \alpha \le 2, \\ 5\alpha-6, & 3 \le \alpha \le n \text{ and}, \end{cases}$$
$$\chi^*\left(v_1^{(\alpha)} v_3^{(\alpha)}\right) = \begin{cases} 2, & \alpha = 1, \\ 5\alpha-5, & \alpha \text{ is odd and } 2 \le \alpha \le n, \\ 5\alpha-4, & \alpha \text{ is even and } 2 \le \alpha \le n. \end{cases} \tag{19}$$

Case vi. $m = 2$ and $n = 1$ in the graph $L([P_n; S_m])$.

For $n = 1$, the graph $L([P_1; S_2])$ is C_3 and by Theorem 2, the result follows.

Hence, the graph $[P_n; S_m]$ for $m \le 2$ and $n \ge 1$ and its line graph $L([P_n; S_m])$ for $n \ge 1$ are classical mean graphs given by Figure 6. □

6. Classical Meanness of Graphs Obtained from Other Graph Operations and Its Line Graph

Theorem 5. *. For $n \ge 2$, $M(P_n)$ and its line graph $L(M(P_n))$ are classical mean graphs.*

Proof. Since $M(P_n)$ is a graph $L(P_n \circ S_1)$, for $n \ge 2$ and by Theorem 3, the result follows. Let $V(L(M(P_n))) = \{x_\alpha\colon 1 \le \alpha \le n+1\} \cup \{x'_\alpha\colon 1 \le \alpha \le n-3\} \cup \{y_\alpha\colon 1 \le \alpha \le n-2\}$ and $E(L(M(P_n))) = \{y_\alpha y_{\alpha+1}, x'_\alpha x_{\alpha+3}, x'_\alpha y_{\alpha+1}, y_\alpha x'_\alpha\colon 1 \le \alpha \le n-3\} \cup \{y_\alpha x_{\alpha+2}\colon 1 \le \alpha \le n-2\} \cup \{y_\alpha\ x_{\alpha+1}\colon 2 \le \alpha \le\ n-2\} \cup \{x_\alpha x_{\alpha+1}, x_\alpha y_1\colon 1 \le \alpha \le\ 2\} \cup \{y_{n-2} x_{n+1}, x_n x_{n+1}\}$.

Case i. $n \ge 3$ in the graph $L(M(P_n))$. Develop a mapping χ from the line graph of vertex set of $M(P_n)$ to $\{1, 2, 3, \ldots, 7n-13\}$ by

$$\chi(x_\alpha) = \begin{cases} \alpha, & 1 \le \alpha \le 2, \\ -14+7\alpha, & 3 \le \alpha \le n, \\ -13+7n, & \alpha = n+1, \end{cases} \tag{20}$$
$$\chi(x'_\alpha) = 2+7\alpha, \quad \text{for } 1 \le \alpha \le n-3 \text{ and},$$
$$\chi(y_\alpha) = -2+7\alpha, \quad \text{for } 1 \le \alpha \le n-2.$$

Therefore,

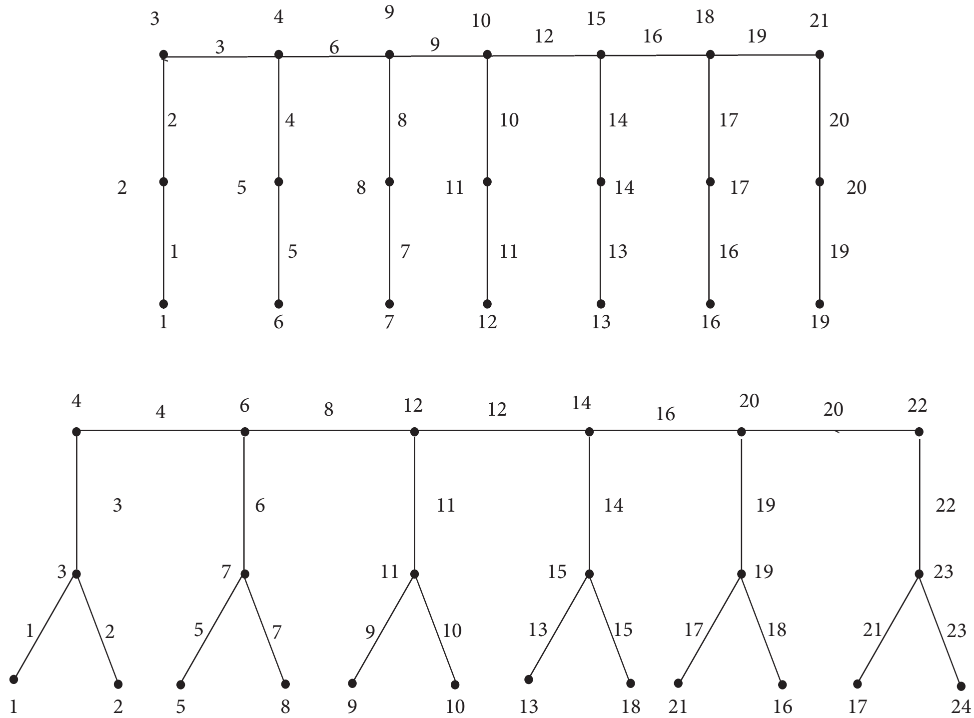

FIGURE 5: A classical mean labeling of $[P_7; S_1]$ and $[P_6; S_2]$.

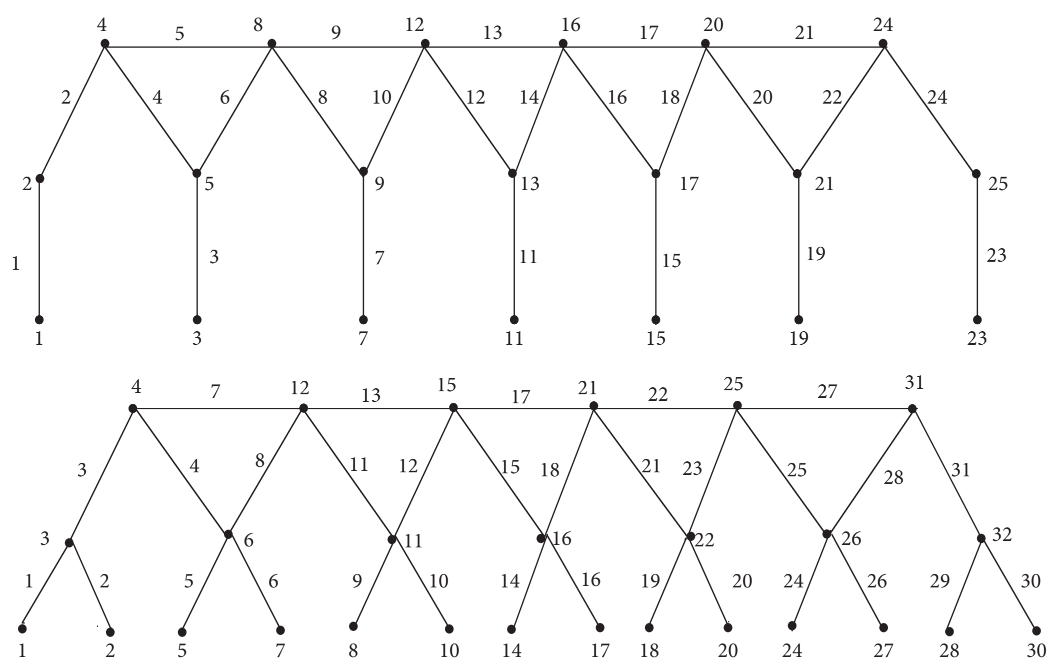

FIGURE 6: A classical mean labeling of $L([P_7; S_1])$ and $L([P_7; S_2])$.

$$\chi^*(y_\alpha y_{\alpha+1}) = 7\alpha + 1, \quad \text{for } 1 \le \alpha \le n-3,$$
$$\chi^*(x'_\alpha x_{\alpha+3}) = 7\alpha + 4, \quad \text{for } 1 \le \alpha \le n-3,$$
$$\chi^*(x'_\alpha y_{\alpha+1}) = 7\alpha + 3, \quad \text{for } 1 \le \alpha \le n-3,$$
$$\chi^*(y_\alpha x'_\alpha) = 7\alpha - 1, \quad \text{for } 1 \le \alpha \le n-3,$$
$$\chi^*(y_\alpha x_{\alpha+2}) = 7\alpha - 2, \quad \text{for } 1 \le \alpha \le n-2,$$
$$\chi^*(y_\alpha x_{\alpha+1}) = \begin{cases} 3, & \alpha = 1, \\ 7\alpha - 5, & 2 \le \alpha \le n-2, \end{cases} \tag{21}$$
$$\chi^*(x_\alpha x_{\alpha+1}) = 3\alpha - 2, \quad \text{for } 1 \le \alpha \le 2,$$
$$\chi^*(x_\alpha y_1) = \alpha + 1, \quad \text{for } 1 \le \alpha \le 2,$$
$$\chi^*(y_{n-2} x_{n+1}) = 7n - 15 \text{ and}$$
$$\chi^*(x_n x_{n+1}) = -14 + 7n.$$

Case ii. $n = 2$ in the graph $L(M(P_n))$. For $n = 2$, the graph $L(M(P_n))$ is P_2 and by Theorem 1, the result follows. Hence, from Figure 7, for $n \ge 2$, the graph $M(P_n)$ and its line graph $L(M(P_n))$ are classical mean graphs. □

Theorem 6. . *The graph $S(P_n \circ K_1)$ and its line graph $L(S(P_n \circ K_1))$ are classical mean graphs, for $n \ge 1$.*

Proof. Let $V(S(P_n \circ K_1)) = \{u_\alpha, v_\alpha, x_\alpha, y_\beta\colon 1 \le \alpha \le n, 1 \le \beta \le n-1\}$ and $E(S(P_n \circ K_1)) = \{u_\alpha x_\alpha, v_\alpha x_\alpha\colon 1 \le \alpha \le n\} \cup \{u_\alpha y_\alpha, y_\alpha u_{\alpha+1}\colon 1 \le \alpha \le n-1\}$. Develop a mapping χ from the vertex set of $S(P_n \circ K_1)$ to $\{1, 2, 3, \ldots, 4n-1\}$ by

$$\chi(u_\alpha) = \begin{cases} 3\alpha, & 1 \le \alpha \le 2, \\ 4\alpha - 1, & 3 \le \alpha \le n, \end{cases}$$
$$\chi(y_\alpha) = \begin{cases} 4, & \alpha = 1, \\ 4\alpha + 1, & 2 \le \alpha \le n-1, \end{cases}$$
$$\chi(x_\alpha) = \begin{cases} 5\alpha - 3, & 1 \le \alpha \le 2, \\ 4\alpha - 2, & 3 \le \alpha \le n \text{ and}, \end{cases} \tag{22}$$
$$\chi(v_\alpha) = \begin{cases} 4\alpha - 3, & 1 \le \alpha \le 2, \\ 4\alpha - 4, & 3 \le \alpha \le n. \end{cases}$$

Therefore,

$$\chi^*(u_\alpha y_\alpha) = -1 + 4\alpha, \quad 1 \le \alpha \le n-1,$$
$$\chi^*(y_\alpha u_{\alpha+1}) = \begin{cases} 4, & \alpha = 1, \\ 1 + 4\alpha, & 2 \le \alpha \le n-1, \end{cases}$$
$$\chi^*(u_\alpha x_\alpha) = \begin{cases} 2, & \alpha = 1, \\ 4\alpha - 2, & 2 \le \alpha \le n \text{ and}, \end{cases} \tag{23}$$
$$\chi^*(v_\alpha x_\alpha) = \begin{cases} 4\alpha - 3, & 1 \le \alpha \le 2, \\ 4\alpha - 4, & 3 \le \alpha \le n. \end{cases}$$

Let $V(L(S(P_n \circ K_1))) = \{u_\alpha, u'_\beta, v_\alpha, w_\alpha\colon 1 \le \alpha \le n, 1 \le \beta \le n-2\}$ and $E(L(S(P_n \circ K_1))) = \{u_\alpha v_\alpha, v_\alpha w_\alpha\colon 1 \le \alpha \le n\} \cup \{u'_\alpha v_{\alpha+1}\colon 1 \le \alpha \le n-2\} \cup \{u_\alpha u'_{\alpha-1}\colon 2 \le \alpha \le n-1\} \cup \{u'_\alpha u_{\alpha+2}\colon 1 \le \alpha \le n-2\} \cup u_1 u_2$.

Case i. $n \ge 3$ in the graph $L(S(P_n \circ K_1))$. Develop a mapping χ from the line graph of vertex set of $S(P_n \circ K_1)$ to $\{1, 2, 3, \ldots, 5n-4\}$ by

$$\chi(u_\alpha) = \begin{cases} 3, & \alpha = 1, \\ 5\alpha - 6, & 2 \le \alpha \le n-1, \\ 5n - 4, & \alpha = n, \end{cases}$$
$$\chi(u'_\alpha) = 5\alpha + 3, \quad \text{for } 1 \le \alpha \le n-2,$$
$$\chi(v_\alpha) = \begin{cases} 2, & \alpha = 1, \\ 5\alpha - 4, & 2 \le \alpha \le n-1, \\ 5\alpha - 5, & \alpha = n \text{ and}, \end{cases} \tag{24}$$
$$\chi(w_\alpha) = \begin{cases} 1, & \alpha = 1, \\ 5\alpha, & 2 \le \alpha \le n-2, \\ 5n - 6, & \alpha = n-1, \\ 5n - 8, & \alpha = n. \end{cases}$$

Therefore,

$$\chi^*(u_\alpha v_\alpha) = \begin{cases} 2, & \alpha = 1 \\ 5\alpha - 6, & 2 \le \alpha \le n-1, \\ 5n - 5, & \alpha = n, \end{cases}$$
$$\chi^*(v_\alpha w_\alpha) = \begin{cases} 1, & \alpha = 1 \\ 5\alpha - 3, & 2 \le \alpha \le n-1 \\ 5n - 7, & \alpha = n, \end{cases}$$
$$\chi^*(u'_\alpha v_{\alpha+1}) = \begin{cases} 6, & \alpha = 1 \\ 5\alpha + 1, & 2 \le \alpha \le n-2, \end{cases} \tag{25}$$
$$\chi^*(u_\alpha u'_{\alpha-1}) = 5\alpha, \quad \text{for } 2 \le \alpha \le n-1,$$
$$\chi^*(u'_\alpha u_{\alpha+2}) = \begin{cases} 5\alpha + 3, & 1 \le \alpha \le n-3 \\ 6\alpha - 1, & \alpha = n-2 \text{ and}, \end{cases}$$
$$\chi^*(u_1 u_2) = 3.$$

Case ii. $1 \le n \le 2$ in the graph $L(S(P_n \circ K_1))$. For $1 \le n \le 2$, the graph $L(S(P_n \circ K_1))$ is a path and by Theorem 1, the result follows. Hence, from Figure 9, the graph $S(P_n \circ K_1)$ and its line graph $L(S(P_n \circ K_1))$ are classical mean graphs, for $n \ge 1$. □

Theorem 7. *The triangular snake T_n and its line graph $L(T_n)$ are classical mean graphs, for $n \ge 2$.*

Proof. Let $V(T_n) = \{u_\alpha\colon 1 \le \alpha \le n-1\} \cup \{v'_\alpha\colon 1 \le \alpha \le n\}$ and $E(T_n) = \{u_\alpha v_\alpha, u_\alpha v_{\alpha+1}, v_\alpha v_{\alpha+1}\colon 1 \le \alpha \le n-1\}$. Develop a mapping χ from the vertex set of T_n to $\{1, 2, 3, \ldots, 3n-2\}$ by

$$\chi(u_\alpha) = 3\alpha, \quad 1 \le \alpha \le n-1 \text{ and},$$
$$\chi(v_\alpha) = -2 + 3\alpha, \quad 1 \le \alpha \le n. \tag{26}$$

Therefore,

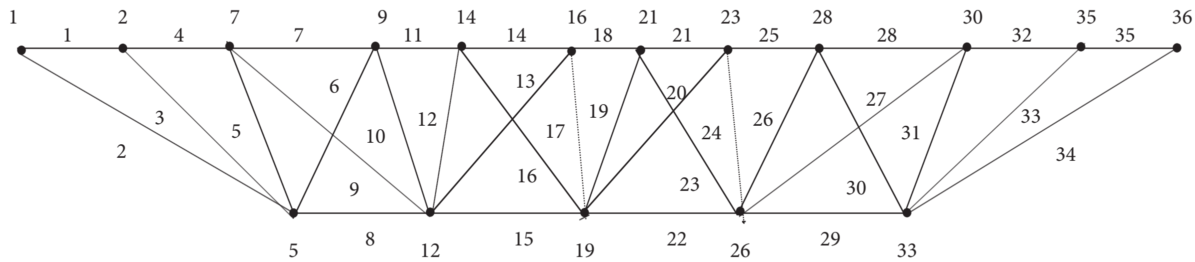

FIGURE 7: A classical mean labeling of $L(M(P_7))$.

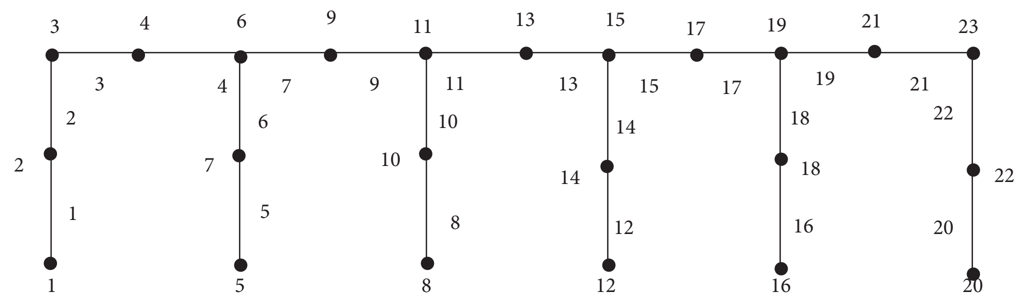

FIGURE 8: A classical mean labeling of $S(P_6 \circ K_1)$. Therefore, a classical mean labeling of $S(P_6 \circ K_1)$ is given by Figure 8.

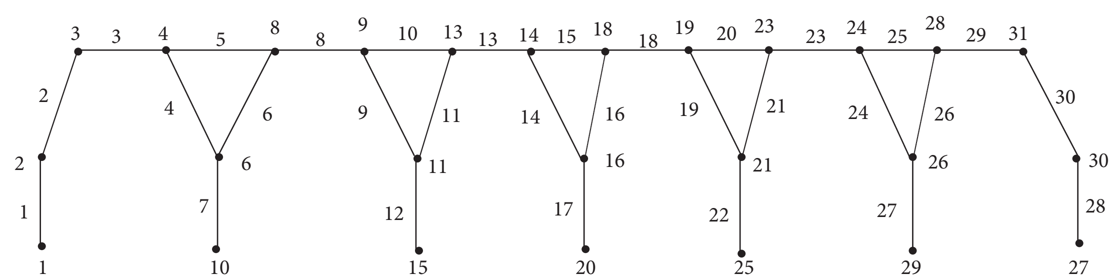

FIGURE 9: A classical mean labeling of $L(S(P_7 \circ K_1))$.

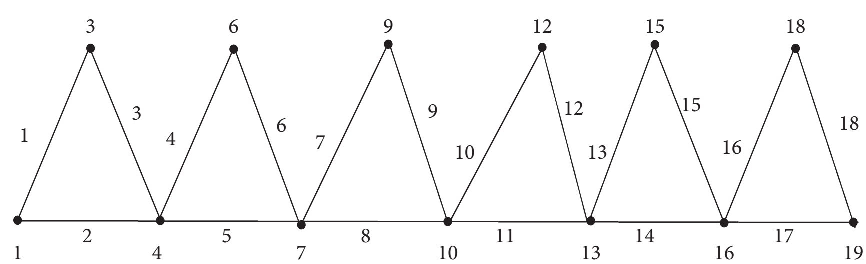

FIGURE 10: A classical mean labeling labeling of T_7.

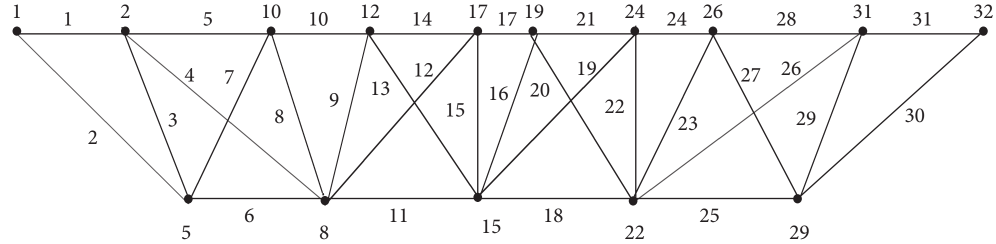

FIGURE 11: A classical mean labeling labeling of $L(T_6)$.

$$\begin{aligned}\chi^*(u_\alpha v_\alpha) &= 3\alpha - 2, \quad 1 \le \alpha \le n-1,\\ \chi^*(u_\alpha v_{\alpha+1}) &= 3\alpha, \quad 1 \le \alpha \le n-1 \text{ and},\\ \chi^*(v_\alpha v_{\alpha+1}) &= -1 + 3\alpha, \quad 1 \le \alpha \le n-1.\end{aligned} \tag{27}$$

Hence, a classical mean labeling of T_7 is given by Figure 10.

Let $V(L(T_n)) = \{x_\alpha\colon 1 \le \alpha \le n\} \cup \{x'_\alpha\colon 1 \le \alpha \le n-2\} \cup \{y'_\alpha\colon 1 \le \alpha \le n-1\}$ and $E(L(T_n)) = \{y_\alpha y_{\alpha+1}, x_\alpha x'_{\alpha-1}, x_{\alpha'} x_{\alpha+2}\colon 1 \le \alpha \le n-2\} \cup \{y_\alpha x_{\alpha+1}, y_\alpha x'_{\alpha-1}, x_\alpha y_\alpha\colon 1 \le \alpha \le n-1\} \cup \{x_1 x_2, x_2 x'_1\}$.

Case i. $n \ge 3$ in the graph $L(T_n)$. Develop a mapping χ from the line graph of T_n to $\{1, 2, 3, \ldots, 7n-10\}$ by

$$\begin{aligned}\chi(x_\alpha) &= \begin{cases}\alpha, & 1 \le \alpha \le 2,\\ 11, & \alpha = 3 \text{ and } \alpha = n,\\ 12, & \alpha \ne 3 \text{ and } \alpha = n,\\ 7\alpha - 9, & 4 \le \alpha \le n-1,\\ 7\alpha - 10, & \alpha = n,\end{cases}\\ \chi(x'_\alpha) &= 7\alpha + 3, \text{ for } 1 \le \alpha \le n-2 \text{ and},\\ \chi(y_\alpha) &= \begin{cases}5, & \alpha = 1.\\ 7\alpha - 6, & 2 \le \alpha \le n-1.\end{cases}\end{aligned} \tag{28}$$

Therefore,

$$\begin{aligned}\chi^*(y_\alpha y_{\alpha+1}) &= \begin{cases}6, & \alpha = 1,\\ 7\alpha - 3, & 2 \le \alpha \le n-2,\end{cases}\\ \chi^*(y_\alpha x_{\alpha+1}) &= \begin{cases}3, & \alpha = 1,\\ 7\alpha - 5, & 2 \le \alpha \le n-1,\end{cases}\\ \chi^*(x_\alpha x'_{\alpha-1}) &= \begin{cases}5, & i = 2,\\ 7\alpha - 7, & 3 \le \alpha \le n-2,\end{cases}\\ \chi^*(y_\alpha x'_{\alpha-1}) &= -6 + 7\alpha, \quad \text{for } 2 \le \alpha \le n-1,\\ \chi^*(x_\alpha y_\alpha) &= \begin{cases}2\alpha, & 1 \le \alpha \le 2,\\ -8 + 7\alpha, & 3 \le \alpha \le n-1,\end{cases}\\ \chi^*(x'_\alpha x_{\alpha+2}) &= \begin{cases}10, & \alpha = 1,\\ 7\alpha + 3, & 2 \le \alpha \le n-2,\end{cases}\\ \chi^*(x_1 x_2) &= 1 \text{ and } \chi^*(x_2 x'_1) = 5.\end{aligned} \tag{29}$$

Case ii. $n = 2$ in the graph $L(T_n)$. For $n = 2$, the graph $L(T_n)$ is a cycle C_3 and by Theorem 2, the result follows. Hence, from Figure 11, the triangular snake T_n and its line graph $L(T_n)$ are classical mean graphs, for $n \ge 2$. □

7. Conclusion

In this paper, it is found that the line graph operation preserves the classical meanness property for some standard graphs. Further investigation can be done to analyze the preservation of the classical meanness property by the line graph operation for other graphs.

Data Availability

No data were used to support the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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