

## Research Article

# Reliability and Sensitivity Analysis of a Repairable System with Warranty and Administrative Delay in Repair

Mohamed S. El-Sherbeny <sup>1,2</sup> and Zienab M. Hussien<sup>1</sup>

<sup>1</sup>Department of Mathematics, Faculty of Sciences and Arts-Rabigh Campus, King Abdulaziz University, Jeddah, Saudi Arabia

<sup>2</sup>Department of Mathematics, Faculty of Science, Helwan University, Cairo, Egypt

Correspondence should be addressed to Mohamed S. El-Sherbeny; [m\\_el\\_sherbeny@yahoo.com](mailto:m_el_sherbeny@yahoo.com)

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The purpose of this study is to analyze the behavior of some industrial systems in light of the cost-free warranty policy. According to this policy, we assume that the repairman is not always present in the system. When the active unit fails, the repairman will be called to visit the system; however, administrative procedures may delay the visit for some time. Once on the system, the repairman first inspects whether the fault is caused by the user or not and whether it is repairable or not. According to product warranty laws, the repairman carries out the repair or replacement of the faulty unit. The failure time, administrative delay time, inspection time, and repair time are assumed taken as a negative exponential distribution. The system model is analyzed by the supplementary variable technique and Laplace transform, as various performance metrics of system efficiency have been obtained. The sensitivity and relative sensitivity analyses for the system parameters have also been performed. Finally, an illustrative example is taken to illustrate the efficiency of the system.

## 1. Introduction

In the present time, warranty is a necessary condition for selling many industrial products. Warranty is defined as the mandatory contract afforded by a manufacturer with regard to the sale of a product. In other words, the warranty is a legal responsibility in the following cases: early failure of the product or the disability of the product to perform its function. When selling a product, the warranty covers either free repair or free replacement. Rules of the warranty are classified as follows: (1) the manufacturer is not legally responsible for the product in case of failure due to incorrect and careless use or deterioration of the product by the user; this makes the customer responsible for the cost of the product repair; (2) the customer is not legally responsible for the product in the following cases: (a) manufacturing defects and (b) errors in software; this makes the manufacturer responsible for free repair or free replacement of the product.

More recently, users have distinguished single-unit systems due to their ability to afford the cost and ingrained

reliability. Therefore, reliability models of single-unit systems have been examined by researchers, such as Kumar et al. [1], Du et al. [2], Goel and Mumtaz [3], Ram and Kadyan [4], Attia et al. [5], and Malhotra and Taneja [6], under different conditions from failure and repair policies without interest paying much attention to any warranty policies and service agreement. The only way to ensure the efficiency of the product sold is after-sales services related to warranty policy. Therefore, the longer warranty leads to best reliability and provides convenience for the users. Kadyan and Ramniwas [7] calculated the cost benefit of the single-unit system and derived some reliability measures when applying warranty policy on the system. The profit function of industrial system with the presence of a cost-free warranty policy is obtained by Ram and Garg [8]. Effect warranty policies on remanufactured products are discussed by Alqahtani and Gupta [9]. Ram et al. [10] study system consists of a single unit with inspection for the feasibility of repair beyond the warranty. Ram et al. [11] presented the concept of preventive maintenance (PM) beyond warranty

for a single-unit system. Providing warranty to the system for a certain period of operation is one of the effective ways to ensure the reliability of a system. Therefore, Mo et al. [12] presented a new warranty policy where the buyer invests in PM costs within the product life cycle to reduce losses due to production failures. Huang et al. [13] presented an approach using preventive maintenance with extended policy. Chopra and Ram [14] derived the reliability measures of the system subjected to two types of failure, and under this effect, two different types of repair are modeled using Gumbel–Hougaard family copula.

In this article, we discussed the administrative delay, that is, the time associated with processes or tasks not directly related to the recovery or repair of the failed equipment, such as processing requests, short-term unavailability of repair facilities, or delays in calling the repairman due to higher priorities. Therefore, the idea of administrative delay is very important because the administrative delay of failed equipment leads to more delays in expected production and more complaints from customers, which makes it more difficult for organizations to serve their customers. However, the concept of a single-unit system with administrative delay in the repair under warranty has not previously appeared in the literature.

In light of the previously mentioned observations, this article analyzes a single-unit repairable system with administrative repair delay under warranty using the supplementary variables technique. The cost of repairs during the warranty period will be borne by the manufacturer, but the warranty does not cover product failure due to damage caused by the user, such as misuse, accident, and physical damage, during the warranty period. If the active unit is down, the technician has to be called to the system, which takes some time to reach the system due to some administrative procedures. In such systems, there is a single server that decides whether the error is caused by the user or not, and whether it can be repaired or replaced after the inspection process. All times of the system follow a negative exponential distribution. The expressions for some economic measures, such as reliability, mean time to system failure, and availability, were derived to illustrate system behavior and evolution.

## 2. Model and Assumptions

The system under study has a single active unit with a single service repair facility. We, specifically, have the following assumptions.

- (1) The system contains a single active unit for working
- (2) There is a single repairer that is not always available with the system to perform repair or replacement and inspection of a failing unit
- (3) When the active unit failed, the repairman must be called to visit the system that takes some time to come to the system due to some administrative actions
- (4) As soon as the repairman arrives for the failed unit, he first inspects that appliance and decides whether

or not the fault was caused by the user and can be repaired or replaced

- (5) The cost of repair or replacement for the failed unit during warranty is borne by the manufacturer, provided malfunctions are not due to the users
- (6) The distributions of all times in the system are negative exponential
- (7) The system works as new after repair or replacement

## 3. Model Development

**3.1. State Specification.** To discuss the behavior of the system at any time  $t$ , we can describe this system according to the following states:

$S_0/S_1$ : the unit is operative during the working period within/out of the warranty

$S_2/S_6$ : the unit is failure mode and waiting for repairman due to administrative delay within/out of the warranty

$S_3/S_7$ : the failed unit under inspection within/out of the warranty

$S_4/S_8$ : the failed unit under replacement within/out of the warranty

$S_5/S_9$ : the failed unit under repair within/out of the warranty

### 3.2. Notations

$\lambda_1/\lambda_2$ : the constant failure rate of the unit within the warranty period/out of the warranty period

$\alpha$ : constant rate of accomplishment of the warranty period

$\mu_1/\mu_4$ : repair rate of the unit in warranty period/out of the warranty period

$\mu_2/\mu_3$ : replacement rate of the unit in warranty period/out of the warranty period

$\beta_1/\beta_2$ : rate of administrative delay time for getting the repairman available within warranty period/out of the warranty period

$\gamma/\eta$ : inspection rate of the failed unit within warranty period/out of the warranty period

$\theta_1$ : the probability that the failed unit will be replaced within the warranty period, according to the inspector's decision

$\theta_2$ : the probability that the failed unit will be repaired within the warranty period, according to the inspector's decision

$\theta_3$ : the probability that the failed unit will be replaced outside the warranty period, due to customer misuse

$\theta_4$ : the probability that the failed unit will be repaired outside the warranty period, due to customer misuse

$\varphi_1$ : the probability that the failed unit will be repaired outside the warranty period, according to the decision

of the inspector at the completion of the warranty period

$\varphi_2$ : the probability that the failed unit will be replaced outside the warranty period, according to the decision of the inspector at the completion of the warranty period

$\pi_0(t)/\pi_1(t)$ : the probability that at time  $t$  the system is in good state in warranty period/beyond warranty period

$\pi_2(t)/\pi_6(t)$ : the probability that at time  $t$  the system is in a failed state and getting the repairman available in warranty period/beyond the warranty period

$\pi_3(t)/\pi_7(t)$ : the probability that the system will undergo an inspection process at time  $t$  to determine whether a failed unit should be replaced or repaired within/beyond the warranty period

$\pi_4(t)/\pi_8(t)$ : the probability of replacement of the failed unit by the decision of the inspector within/outside the warranty period

$\pi_5(t)/\pi_9(t)$ : the probability of repair of the failed unit by the decision of the inspector within/outside the warranty period

### 4. Analysis of the System

**4.1. Reliability.** The system reliability  $R(t)$  is the probability that the product or system functions adequately for a specified period  $(0, t]$ . To deduce an expression for the reliability of the system, we using the Markov process to the up states, namely,  $S_0$  and  $S_1$ , as shown in Figure 1. We derive the following differential equations for reliability:

$$\left[ \frac{d}{dt} + \lambda_1 + \alpha \right] \pi_0(t) = 0, \tag{1}$$

$$\left[ \frac{d}{dt} + \lambda_2 \right] \pi_1(t) = \alpha \pi_0(t). \tag{2}$$

The initial conditions are as follows:

$$\pi_i(0) = \begin{cases} 1, & i = 0, \\ 0, & i \neq 0. \end{cases} \tag{3}$$

Using the Laplace transform of (1) and (2) and using also the initial condition, we get

$$[s + \lambda_1 + \alpha] \pi_0(s) = 1, \tag{4}$$

$$[s + \lambda_2] \pi_1(s) = \alpha \pi_0(s). \tag{5}$$

From (4), we get

$$\pi_0(s) = \frac{1}{(s + \lambda_1 + \alpha)}. \tag{6}$$

By substituting (6) into (5), we get

$$\pi_1(s) = \frac{\alpha}{(s + \lambda_1 + \alpha)(s + \lambda_2)}. \tag{7}$$

Then, the Laplace transformation formula of the reliability  $R(s)$  of the system is

$$R(s) = \pi_0(s) + \pi_1(s) = \frac{(s + \lambda_2) + \alpha}{(s + \lambda_1 + \alpha)(s + \lambda_2)}. \tag{8}$$

Taking the inverse Laplace transform for (8), we get

$$R(t) = L^{-1}\{R(s)\} = \frac{\lambda_1 - \lambda_2}{\lambda_1 - \lambda_2 + \alpha} \exp\{-(\lambda_1 + \alpha)t\} + \frac{\alpha}{\lambda_1 - \lambda_2 + \alpha} \exp\{-\lambda_2 t\}. \tag{9}$$

**4.2. Mean Time to System Failure (MTTF).** MTTF is a measurement of reliability for nonrepairable systems. It is the mean time predictable until the first fail of the system. By using (9), we get

$$MTTF = \int_0^\infty R(t) dt = \frac{(\lambda_2 \lambda_1 - \lambda_2^2 + \alpha \lambda_1 + \alpha^2)}{\lambda_2 (\lambda_1 - \lambda_2 + \alpha) (\lambda_1 + \alpha)}. \tag{10}$$

**4.3. System Availability.** Availability  $(Av(t))$  is the probability that the system (or unit) will be inoperative in the long term. Starting from the states of the system as in Figure 1, we can describe the differential equations in each state of this system and are summarized as follows:

$$\begin{aligned} \left[ \frac{d}{dt} + \lambda_1 + \alpha \right] \pi_0(t) &= \mu_2 \pi_4(t) + \mu_1 \pi_5(t), \\ \left[ \frac{d}{dt} + \lambda_2 \right] \pi_1(t) &= \alpha \pi_0(t) + \mu_3 \pi_8(t) + \mu_4 \pi_9(t), \\ \left[ \frac{d}{dt} + \beta_1 \right] \pi_2(t) &= \lambda_1 \pi_0(t), \\ \left[ \frac{d}{dt} + \gamma \right] \pi_3(t) &= \beta_1 \pi_2(t), \\ \left[ \frac{d}{dt} + \mu_2 \right] \pi_4(t) &= \theta_1 \gamma \pi_3(t), \\ \left[ \frac{d}{dt} + \mu_1 \right] \pi_5(t) &= \theta_2 \gamma \pi_3(t), \\ \left[ \frac{d}{dt} + \beta_2 \right] \pi_6(t) &= \lambda_2 \pi_1(t), \\ \left[ \frac{d}{dt} + \eta \right] \pi_7(t) &= \beta_2 \pi_6(t), \\ \left[ \frac{d}{dt} + \mu_3 \right] \pi_8(t) &= \theta_3 \gamma \pi_3(t) + \varphi_2 \eta \pi_7(t), \\ \left[ \frac{d}{dt} + \mu_4 \right] \pi_9(t) &= \theta_4 \gamma \pi_3(t) + \varphi_1 \eta \pi_7(t). \end{aligned} \tag{11}$$

The initial conditions are as follows:

$$\pi_i(0) = \begin{cases} 1, & i = 0, \\ 0, & i \neq 0. \end{cases} \quad (12)$$

Taking Laplace transform of all the previous equations, we obtain the following:

$$[s + \lambda_1 + \alpha]\pi_0(s) = 1 + \mu_2\pi_4(s) + \mu_1\pi_5(s), \quad (13)$$

$$[s + \lambda_2]\pi_1(s) = \alpha\pi_0(s) + \mu_3\pi_8(s) + \mu_4\pi_9(s) \quad (14)$$

$$[s + \beta_1]\pi_2(s) = \lambda_1\pi_0(s), \quad (15)$$

$$[s + \gamma]\pi_3(s) = \beta_1\pi_2(s), \quad (16)$$

$$[s + \mu_2]\pi_4(s) = \theta_1\gamma\pi_3(s), \quad (17)$$

$$[s + \mu_1]\pi_5(s) = \theta_2\gamma\pi_3(s), \quad (18)$$

$$[s + \beta_2]\pi_6(s) = \lambda_2\pi_1(s), \quad (19)$$

$$[s + \eta]\pi_7(s) = \beta_2\pi_6(s), \quad (20)$$

$$[s + \mu_3]\pi_8(s) = \theta_3\gamma\pi_3(s) + \varphi_2\eta\pi_7(s), \quad (21)$$

$$[s + \mu_4]\pi_9(s) = \theta_4\gamma\pi_3(s) + \varphi_1\eta\pi_7(s). \quad (22)$$

From (13)–(22), we obtain

where,

$$\pi_0(s) = C(s), \quad (23)$$

$$\pi_1(s) = C_1(s), \quad (24)$$

$$\pi_2(s) = \frac{\lambda_1}{s + \beta_1}C(s), \quad (25)$$

$$\pi_3(s) = \frac{\beta_1\lambda_1}{(s + \gamma)(s + \beta_1)}C(s), \quad (26)$$

$$\pi_4(s) = \frac{\theta_1\gamma\beta_1\lambda_1}{(s + \gamma)(s + \beta_1)(s + \mu_2)}C(s), \quad (27)$$

$$\pi_5(s) = \frac{\theta_2\gamma\beta_1\lambda_1}{(s + \gamma)(s + \beta_1)(s + \mu_1)}C(s), \quad (28)$$

$$\pi_6(s) = \frac{\lambda_2}{s + \beta_2}C_1(s), \quad (29)$$

$$\pi_7(s) = \frac{\beta_2\lambda_2}{(s + \eta)(s + \beta_2)}C_1(s), \quad (30)$$

$$\pi_8(s) = A(s) + A_1(s)C_1(s), \quad (31)$$

$$\pi_9(s) = A_2(s) + A_3(s)C_1(s), \quad (32)$$

$$\begin{aligned} C(s) &= \frac{(s + \mu_1)(s + \mu_2)(s + \gamma)(s + \beta_1)}{(s + \lambda_1 + \alpha)(s + \mu_1)(s + \mu_2)(s + \gamma)(s + \beta_1) - (\mu_2\theta_1(s + \mu_1) + \mu_1\theta_2(s + \mu_2))(\gamma\beta_1\lambda_1)}, \\ A(s) &= \frac{\theta_3\gamma\beta_1\lambda_1 C(s)}{(s + \mu_3)(s + \gamma)(s + \beta_1)}, \\ A_1(s) &= \frac{\varphi_2\eta\beta_2\lambda_2}{(s + \mu_3)(s + \eta)(s + \beta_2)}, \\ A_2(s) &= \frac{\theta_4\gamma\beta_1\lambda_1 C(s)}{(s + \mu_4)(s + \gamma)(s + \beta_1)}, \\ A_3(s) &= \frac{\varphi_1\eta\beta_2\lambda_2}{(s + \mu_4)(s + \eta)(s + \beta_2)}, \\ C_1(s) &= \frac{\alpha C(s) + \mu_3 A(s) + \mu_1 A_2(s)}{s + \lambda_2 - \mu_3 A_1(s) - \mu_1 A_3(s)}. \end{aligned} \quad (33)$$

It is worth mentioning that

$$\sum_{i=0}^9 \pi_i(s) = \frac{1}{s}. \quad (34)$$

Using (23)–(34), the Laplace transformation formula of the availability  $Av(s)$  is given by

$$Av(s) = \pi_0(s) + \pi_1(s) = C(s) + C_1(s). \quad (35)$$

Hence, the steady-state availability  $Av(\infty)$  of the system is

$$Av(\infty) = \lim_{s \rightarrow 0} \{s(\pi_0(s) + \pi_1(s))\}. \quad (36)$$



system reliability  $R(t)$ , mean time to system failure (MTTF), and steady-state availability are impacted by changing the values of the several system parameters. Moreover, in Subsection 5.2, some numerical results for sensitivity and relative sensitivity analysis for the proposed system are discussed.

**6.1. System Reliability Analysis.** First, we analyze graphically the effects of various system parameters on  $R(t)$ , shown in Figures 2–5. In Figure 2, we can observe that the system reliability increases as  $\lambda_1$  decreases. Remember that the reliability is the probability that the system is working correctly over a period of time. Figure 4 shows that the system reliability increases when the failure rate of the active unit  $\lambda_2$  decreases. We observe that the system reliability depends on the change of the system parameters  $\lambda_1$ ,  $\lambda_2$ , and  $\alpha$ . Therefore, we conclude from Figure 3 that the system reliability increases when  $\alpha > \lambda_1$ . By following the same reason previously mentioned, the higher the system warranty completion rate  $\alpha$  and the lower the failure rate  $\lambda_2$ , the higher the system reliability, as shown in Figure 5.

Next, we present some numerical results of the mean time to system failure MTTF to discuss the influence of various system parameters on the behavior of the mean time to system failure, which are shown in Tables 1 and 2. In Table 1, the influence of parameter  $\alpha$  on MTTF is shown as follows.

- (1) In the first column of this table, the value of MTTF decreases when  $\lambda_1 < \lambda_2$ , and  $\alpha$  varies from 0.2 to 1.4.
- (2) In the second column of the same table, the value of MTTF is constant when  $\lambda_1 = \lambda_2$ , and  $\alpha$  varies from 0.2 to 1.4.
- (3) Finally, in the third column, we show that the value of MTTF increases when  $\lambda_1 > \lambda_2$ , and  $\alpha$  varies from 0.2 to 1.4.

Table 2 shows the effect of system parameters  $\lambda_1$  and  $\lambda_2$  on MTTF. It is observed that the MTTF decreases with increasing  $\lambda_1$  and  $\lambda_2$ , and this is quite obvious at  $\alpha = 0.6$ .

Finally, we display the numerical results for the steady-state availability. We fixed  $\beta_1 = 0.4; \beta_2 = 0.6; \eta = 0.4; \theta_3 = 0.25; \theta_4 = 0.25; \mu_4 = 0.6; \mu_5 = 0.4; \varphi_2 = 0.4$  as the base in this study. First, Table 3 shows how the steady-state availability of the system is affected by the change of  $\alpha$  values as follows.

- (1) In the first and second columns of this table, the value of the steady-state availability decreases when  $\lambda_1 \leq \lambda_2$ , and  $\alpha \in \{0.1, \dots, 0.9\}$ .
- (2) In the third column, we show that the value of the steady-state availability increases when  $\lambda_1 \leq \lambda_2$ , and  $\alpha \in \{0.1, \dots, 0.9\}$ .

Next, the changes in the failure rate of an operating unit under warranty (out of warranty),  $\lambda_1$  ( $\lambda_2$ ), are calculated and the results are presented in Table 4. From Table 4, it can be seen that the steady-state availability of the system decreases when both  $\lambda_1$  and  $\lambda_2$  increase and  $\alpha = 0.6$ .

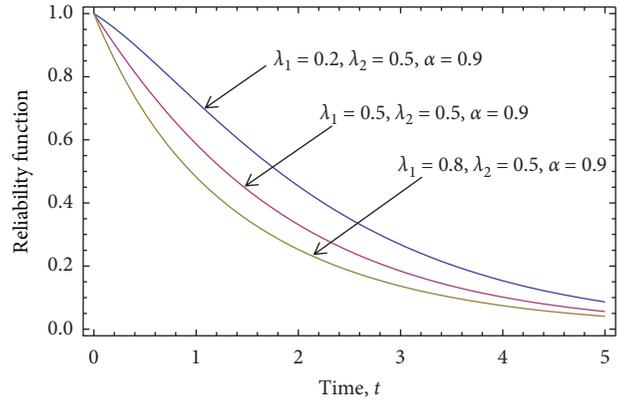


FIGURE 2: System reliability versus failure rate  $\lambda_1$ .

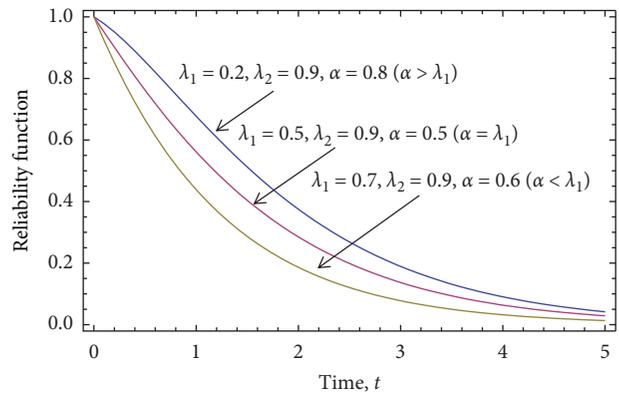


FIGURE 3: System reliability versus failure rate  $\lambda_1$  and rate of completion of warranty  $\alpha$ .

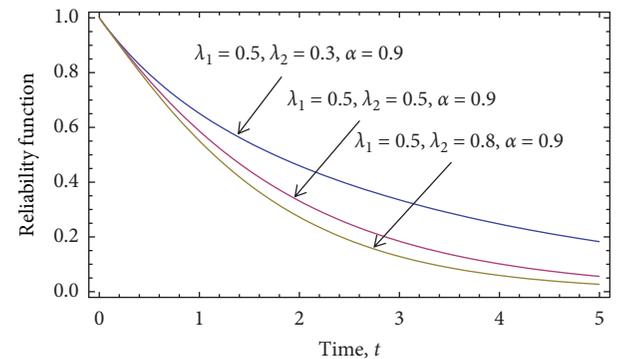


FIGURE 4: System reliability versus failure rate  $\lambda_2$ .

**6.2. Numerical Results for Sensitivity Analysis and Relative Sensitivity Analysis.** In this part, the numerical results are given for both the sensitivity and relative sensitivity analysis. First, the sensitivity analysis of  $R(t)$  caused by various parameters is presented, and the results are displayed in Figure 6. From Figure 6, it can be seen that  $\lambda_1$  has a significant effect on  $R(t)$ , the parameter  $\lambda_2$  has a moderate effect on  $R(t)$ , and the parameter  $\alpha$  has a small influence on  $R(t)$ , because this curve is close to the horizontal axis. Moreover, the signs of the sensitivity of  $R(t)$  for  $\lambda_1$ ,  $\lambda_2$ , and  $\alpha$

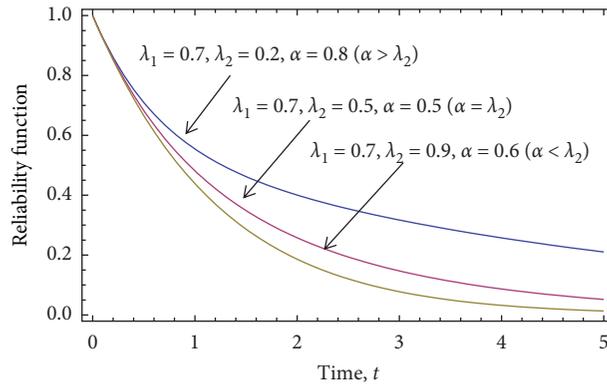


FIGURE 5: System reliability versus failure rate  $\lambda_2$  and rate of completion of warranty  $\alpha$ .

TABLE 1: The mean time to system failure (MTTF) for different values of  $\alpha$ .

$\alpha$	$\lambda_1 < \lambda_2$	$\lambda_1 = \lambda_2$	$\lambda_1 > \lambda_2$
	$\lambda_1 = 0.3, \lambda_2 = 0.6$	$\lambda_1 = 0.3, \lambda_2 = 0.3$	$\lambda_1 = 0.6, \lambda_2 = 0.3$
0.2	2.66667	3.33333	2.08333
0.4	2.38095	3.33333	2.33333
0.6	2.22222	3.33333	2.5
0.8	2.12121	3.33333	2.61905
1.0	2.05128	3.33333	2.70833
1.2	2.00	3.33333	2.77778
1.4	1.96078	3.33333	2.83333

TABLE 2: The mean time to system failure (MTTF) for different values of parameters  $\lambda_1$  and  $\lambda_1$  when  $(\alpha = 0.6)$ .

$\lambda_1$	$\lambda_2 = 0.3$ MTTF	$\lambda_2$	$\lambda_1 = 0.3$ MTTF
0.1	4.28571	0.1	7.77778
0.2	3.75	0.2	4.44444
0.3	3.33333	0.3	3.33333
0.4	3.0	0.4	2.77778
0.5	2.72727	0.5	2.44444
0.6	2.5	0.6	2.22222
0.7	2.30769	0.7	2.06349
0.8	2.14286	0.8	1.94444

TABLE 3: The steady-state availability for different values of  $\alpha$  when  $(\beta_1 = 0.4, \beta_2 = 0.6, \eta = 0.4, \theta_3 = 0.25, \theta_4 = 0.25, \mu_3 = 0.4, \mu_4 = 0.6, \varphi_2 = 0.4)$ .

$\alpha$	$\lambda_1 < \lambda_2$	$\lambda_1 = \lambda_2$	$\lambda_1 > \lambda_2$
	$\lambda_1 = 0.3, \lambda_2 = 0.6$	$\lambda_1 = 0.3, \lambda_2 = 0.3$	$\lambda_1 = 0.6, \lambda_2 = 0.3$
0.1	0.319149	0.526316	0.210526
0.2	0.283688	0.467836	0.233918
0.3	0.265957	0.438596	0.250627
0.4	0.255319	0.421053	0.263158
0.5	0.248227	0.409357	0.272904
0.6	0.243161	0.401003	0.280702
0.7	0.239362	0.394737	0.287081
0.8	0.236407	0.389864	0.292398
0.9	0.234043	0.385965	0.296896

are negative, which implies that the incremental changes of these parameters worsen the system reliability  $R(t)$ . The results of relative sensitivity on reliability are almost the

same as those of sensitivity analysis, but the high levels of each curve are quite different from each other as shown in Figure 7.

TABLE 4: The steady-state availability for different values of  $\lambda_1$  and  $\lambda_1$  when  $(\alpha = 0.6, \beta_1 = 0.4, \beta_2 = 0.6, \eta = 0.4, \theta_3 = 0.25, \theta_4 = 0.25, \mu_3 = 0.4, \mu_4 = 0.6, \varphi_2 = 0.4)$ .

$\lambda_1$	$\lambda_2 = 0.3$ $Av(\infty)$	$\lambda_2$	$\lambda_1 = 0.3$ $Av(\infty)$
0.1	0.299065	0.1	0.466472
0.2	0.249221	0.2	0.29304
0.3	0.213618	0.3	0.213618
0.4	0.186916	0.4	0.168067
0.5	0.166147	0.5	0.138528
0.6	0.149533	0.6	0.11782
0.7	0.135939	0.7	0.102498
0.8	0.124611	0.8	0.0907029

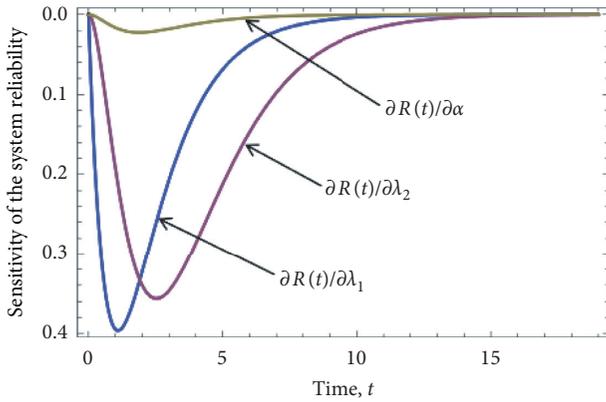


FIGURE 6: Sensitivity analysis on reliability function  $t \in [0, 20]$ .

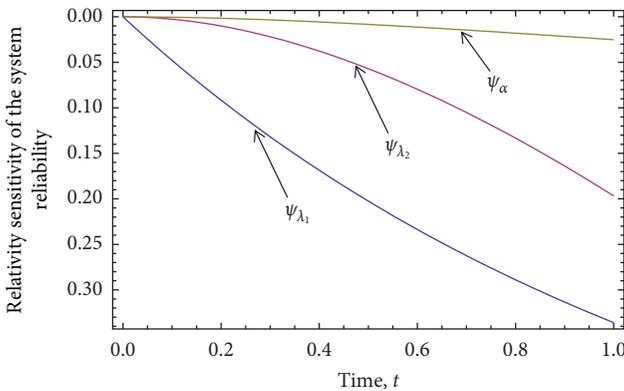


FIGURE 7: Relative sensitivity analysis on reliability function  $t \in [0, 1]$ .

Next, the sensitivity and relative sensitivity analyses of MTTF are performed with various parameters. The sensitivity and relative sensitivity analyses of MTTF with three parameters are tabulated in three tables. Tables 5–7 show that the signs of the sensitivity of MTTF with parameters  $\lambda_1$ ,  $\lambda_2$ , and  $\alpha$  are negative, indicating that the incremental changes in these parameters degrade MTTF. Furthermore, by looking at the absolute values in these tables, the order of impact on the MTTF with each parameter can be determined. It can be observed that the order of sensitivity on MTTF is  $\lambda_2 > \lambda_1 > \alpha$  and the order of relative sensitivity on MTTF is  $\lambda_2 > \lambda_1 > \alpha$ .

TABLE 5: Sensitivity and relative sensitivity analysis of MTTF with  $\alpha$ .

$\alpha$	$\lambda_1 = 0.4, \lambda_2 = 0.6$	
	$\eta_\alpha$	$\omega_\alpha$
0.1	-1.33333	-0.0571429
0.3	-0.680272	-0.0952381
0.5	-0.411523	-0.10101
0.7	-0.275482	-0.0979021
0.9	-0.197239	-0.0923077
1.1	-0.148148	-0.0862745

TABLE 6: Sensitivity and relative sensitivity analysis of MTTF with  $\lambda_1$ .

$\lambda_1$	$\alpha = 0.4, \lambda_2 = 0.6$	
	$\eta_{\lambda_1}$	$\omega_{\lambda_1}$
0.1	-6.66667	-0.2
0.3	-3.40136	-0.428571
0.5	-2.05761	-0.555556
0.7	-1.37741	-0.636364
0.9	-0.986193	-0.692308
1.1	-0.740741	-0.733333

TABLE 7: Sensitivity and relative sensitivity analysis of MTTF with  $\lambda_2$ .

$\lambda_2$	$\alpha = 0.4, \lambda_1 = 0.3$	
	$\eta_{\lambda_2}$	$\omega_{\lambda_2}$
0.2	-14.2857	-0.666667
0.4	-3.57143	-0.5
0.6	-1.5873	-0.4
0.8	-0.892857	-0.333333
1.0	-0.571429	-0.285714
1.2	-0.396825	-0.25

### 7. Conclusions

In this paper, we studied the behavior of the system, which consists of a single-unit operating under a free repair or replacement warranty. To solve the differential equations of the system under study, we used the Laplace transform technique. We provided the explicit expressions for system reliability  $R(t)$ , mean time to system failure (MTTF), and steady-state availability  $Av(\infty)$ . The numerical results show

that the influence of the failure rates  $\lambda_1$ ,  $\lambda_2$  cannot be neglected for a repairable system with changing completion of warranty rate  $\alpha$ . Both the sensitivity and relative sensitivity analyses of the system reliability  $R(t)$  and mean time to system failure (MTTF) are also derived. The results show that the sensitivities of  $R(t)$  are significant only with  $\lambda_1$ ,  $\lambda_2$ , and  $\alpha$ . Finally, it can be shown that the order of sensitivity on (MTTF) is  $\lambda_2 > \lambda_1 > \alpha$  and the order of relative sensitivity on (MTTF) is  $\lambda_2 > \lambda_1 > \alpha$ . From the previous results, it can be seen that the parameter  $\lambda_2$  is the most important parameter affecting both performance measures, so its value should be well observed.

## Data Availability

The data used to support the findings of this study are included within the article in the references.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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