Research Article

A New Resistance Formulation for Carbon Nanotubes

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A new resistance formulation for carbon nanotubes is suggested using fractal approach. The new formulation is also valid for other nonmetal conductors including nerve fibers, conductive polymers, and molecular wires. Our theoretical prediction agrees well with experimental observation.

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1. INTRODUCTION

We know from Ohm's law that the current flows down a voltage gradient in proportion to the resistance in the circuit. Current is therefore expressed in the following form:

$$I = \frac{E}{R}, \quad (1)$$

where I is the current, E is the voltage, R is the resistance. The resistance, R , in (1) is expressed in the form

$$R = \frac{kL}{A} = \frac{kL}{\pi r^2}, \quad (2)$$

where A is the area of the conductor, L is its length, r is the radius of the conductor, and k is the resistance parameter.

Equation (2) is actually valid only for metal conductors where there are plenty of electrons in the conductor. The exponent, 2, in (2) can be interpreted as the fractal dimension of the section.

For nonconductors (e.g., nerve fibers [1, 2], conductive polymers [3], charged electrospun jets [4–6]), we suggested a modified resistance formulation discussed in the next section.

2. ALLOMETRIC MODEL

The resistance for Ohm conductor (see Figure 1) scales as

$$R_C \propto \frac{L}{A} \propto L^{+1} r^{-2}. \quad (3)$$

So for the Ohmic bulk conduction current, we have

$$I_c \propto R_c^{-1} \propto L r^2, \quad (4)$$

which corresponds to

$$I_c = k\pi r^2 L V, \quad (5)$$

where V is the applied electric field.

The resistance for surface convection (see Figure 2), which occurs in electrospinning and charged flow [7, 8], scales as

$$R_s \propto r^{-1}. \quad (6)$$

For the surface convection current, we have

$$I_s \propto r, \quad (7)$$

which corresponds to [4]

$$I_s = 2\pi r \sigma u, \quad (8)$$

where σ is surface density of the charge.

For SWNTs and other nonmetal materials, we suggest the following scaling relation [9]:

$$R \propto L^d r^{-(1+D)}, \quad (9)$$

where D is the fractal dimension of its perimeter of the section of the carbon nanotubes, d is the fractal dimension of longitudinal length. When $D = 1$ (infinite smoothness

FIGURE 1: Resistance for Ohmic conductor: $R_c = kL/A \propto L^{+1}r^{-2}$, where r is the radius of the conductor.

FIGURE 2: Resistance for surface convection: $R \propto r^{-1}$.

of the section perimeter) and $d = 1$ (infinite continuity of the wall), (9) turns out to be (2). When $D = 0$ and $d = 1$, (9) is valid for the surface convection current (Figure 2). Sundqvist et al. [10] found the resistance of SWNTs does not follow what metal conductors do, and suggested the following formulation:

$$R(L) \propto \exp\left(\frac{L}{L_0}\right) \quad (10)$$

which is different from our scaling model, (9).

3. FRACTAL DIMENSION

The fractal dimension is defined as [11, 12]

$$D_f = \frac{\ln M}{\ln N}, \quad (11)$$

where M is the number of new units within the original unit with a new dimension, N is the ratio of the original dimension to the new dimension.

Consider the well-known Koch curve as illustrated in Figure 3, we have $M = 4$ and $N = 3$, so the fractal dimension reads $D_f = \ln 4 / \ln 3$.

For single-walled carbon nanotubes, we consider a special case of (6,6) CNTs as illustrated in Figure 4. To calculate the fractal dimension of its perimeter of the section of the carbon nanotube, we have $M = 2$, and $N = \sqrt{3}$ as illustrated in Figure 4(b), resulting in

$$D = \frac{\ln 2}{\ln(\sqrt{3})} = 1.26. \quad (12)$$

Similarly, to calculate the fractal dimension of longitudinal length of the carbon nanotube, we have $M = 4$, and $N = \sqrt{3}$ as illustrated in Figure 4(c), yielding the following fractal dimension:

$$d = \frac{\ln 4}{\ln(\sqrt{3})} = 2.52. \quad (13)$$

Our prediction, therefore, reads

$$R = \frac{aL^{2.52}}{r^{2.26}}, \quad (14)$$

where a is a material constant, just like k in (2).

FIGURE 3: Koch curve.

(a)

(b) Cross-section boundary curve

(c) Wall boundary

FIGURE 4: Fractal boundary of carbon nanotube.

FIGURE 5: Sundqvist et al.'s experiment [10]. Resistance ($\text{k}\Omega$) versus length (μm) for SWNTs.

In order to verify our theoretical prediction, we have to reanalyze Sundqvist et al.'s experiment data [10]. It is obvious that $R = 0$ when $L = 0$. But in Sundqvist et al.'s experiment, we found that $R(0) \approx 50 \text{ k}\Omega$; this is the error due to the contact resistance at the tip, so the initial error (the contact resistance) is taken away from every obtained data, the modified experimental data is illustrated in Figure 5.

4. CONCLUSION

In conclusion, the paper represents a novel attempt to characterize the relationship between the resistance and length of carbon nanotubes using fractal approach. We find our prediction agrees well with the experimental data, and the results might find some potential applications in future.

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