Mathematical Simulation of Temperature Profiles within Microwave Heated Wood Made for Wood-Based Nanocomposites

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High intensive microwave pretreatment is a new method to modify wood for the fabrication of wood-based nanocomposites. Based on the physical law on heat transfer, a mathematical model to describe the temperature profiles within wood heated by high intensive microwave was established and simulated in this research. The results showed that the temperature profiles within wood were related to microwave heating methods; the temperature inside wood firstly increased and then gradually decreased along the direction of microwave transmission when the unilateral microwave heating was applied, and the temperature difference along the thickness direction of wood was very significant; the temperature with wood firstly increased and then gradually decreased from the woods surface to interior when the bilateral microwave heating was applied. Compared with the unilateral microwave heating, bilateral microwave heating is a better microwave heating method for the more uniform wood microwave pretreatment.

1. Introduction

Microwave pretreatment is a technique in the forest product industry to improve process efficiency and product performance. Intensive microwave energy usually generates high internal steam pressure that could rupture weaker elements of wood structure and create the microvoids in the radial-longitudinal planes of wood [1, 2]. As a result, wood permeability is increased and the fluid movement during drying is considerably facilitated. This type of modification is a new technology that has been evaluated in recent years for different purposes. Some studies reported that the several properties of wood were improved after pretreatment. For example, the preservative absorption of wood could be elevated by factors of 10–14, and the drying rate was 5–10 times higher, even that the microwave pretreated wood could be used for fabrication of the new functional materials or wood-based nano-composites [3–13]. During microwave pretreatment, the temperature distribution within wood is a very important factor to impact the effect of microwave pretreatment of wood. The more even temperature profiles, the more even micro-voids within wood, and the better modification effect for wood. Compared with conventional heating method, microwave heating is a different heating method [14–17]. As conventional heating method was applied, the temperature within wood was usually lower than that of the wood surface. There are some different results of the temperature profile of wood during microwave heating. Some scholars believe that the internal temperature of wood was higher than the outside temperature in microwave heating process [18, 19], while other scholars reported that the highest temperature appeared inside wood and a few millimeters away from the surface rather than in the wood surface or the internal wood [20, 21]. Which one is the fact? Based on Lambert’s law, a mathematical model to describe the temperature profiles of wood microwave heated with the unilateral and bilateral microwave source was established and simulated in order to provide references for reasonable design of microwave heating device and effective control of microwave pretreatment process.
2. Model Development

The heat within wood during microwave heating can result from two aspects. One part is the absorption of microwave by wood, which can be described by absorption power of microwave, and the other is the heat transfer through wood. Thus, the temperature profile of wood during microwave heating can be simulated if the heat transfer model of microwave heating and its corresponding initial and boundary conditions were established within wood. Wood is regarded as a homogeneous material in this study in order to make analysis and simulate more effectively.

2.1. Heat Transfer Model. When wood was vertically and evenly radiated by the unilateral and bilateral microwave source (Figure 1), the absorbed power of wood in microwave electromagnetic field can be written as follows [22]:

\[ p = 5.56 f E^2 \varepsilon \tan \delta \cdot 10^{-11}, \]  

(1)

where \( p \) is the power absorbed by per unit of volume wood, w/m\(^3\); \( f \) is microwave frequency, Hz; \( E \) is electric field intensity in microwave electromagnetic field, V/m; \( \varepsilon \) is the dielectric constants of wood; \( \tan \delta \) is loss angle tangent. Based on the above equation, the stronger the electric field intensity and the higher the microwave frequency, the greater microwave power the absorbed by wood.

When microwave got into the wood, the energy on wood surface was the most intensive. With the microwave penetrating inside the wood, energy attenuated in an exponential form, while microwave field released energy to the wood. As the energy of microwave field weakened with increased depth of microwave getting into the wood, the wood thickness was limited when microwave heating was applied for wood drying. During microwave heating, the depth of microwave penetrating the wood can be described by Lambert’s law. For one-dimensional unilateral microwave source radiation, if the direction of the microwave transmission is \( x \)-direction and microwave field distributes in the interval \([0, d]\), the following relation can be obtained [22]:

\[ I(x) = I_0 \exp(-bx). \]  

(2)

Here, \( I_0 \), \( I(x) \) are microwave intensity on wood surface and within wood with a depth of \( x \) from wood surface, respectively, and \( b \) is absorption coefficient of microwave by wood, which can be regarded as a constant. In microwave field, the square of microwave electric field intensity is proportional to microwave intensity \( (E^2 \propto I) \), the equation is as follows:

\[ E^2 = E_0^2 \exp(-bx), \]  

(3)

where \( E_0 \) is electric field intensity of microwave on wood surface.

If the temperature of wood distribution is \( T[x, t] \), \( t \) is time. One-dimensional heat conduction equation is available based on Fourier’s law [23]:

\[ q(x) = -\lambda \frac{dT}{dx}, \]  

(4)

where \( q \) is the heat flux density, namely, the heat flowing through per unit of area in per unit of time along \( x \)-direction; \( \lambda \) is thermal conductivity of wood, which usually is a constant. The absorbed heat through heat transfer in a slice with an area of \( A \) and a thickness of \( dx \) during \( dt \) can be written as follows:

\[ Q_2 = qA \, dt \, (q + dq) \, A \, dt = -A \, dq \, dt. \]  

(5)

While the heat obtained by absorbing the microwave directly is

\[ Q_1 = pA \, dx \, dt = 5.56 \, fE^2 \varepsilon \tan \delta \cdot 10^{-11} \, A \cdot dx \, dt. \]  

(6)

Based on the heat balance principle:

\[ CA \, dx \, dT = Q_1 + Q_2, \]  

(7)

where \( C \) is the heat capacity of per unit of volume wood, J/m\(^3\)°C.

Based on (1), (3), (4), (5), (6), and (7), the following equation can be simplified:

\[ C \rho \frac{dT}{dt} = 5.56 \, fE^2 \varepsilon \tan \delta \cdot 10^{-11} \cdot \exp(-bx) + \lambda \frac{\partial^2 T}{\partial x^2}. \]  

Equation (8) is the heat conduction equation for wood microwave heating. Because \( q_0 = 5.56 \, fE^2 \varepsilon \tan \delta \cdot 10^{-11} \), which
can be regulated by the changed parameters of microwave generating device (such as power and frequency), then (8) can be simplified as follows:

\[
\frac{C\rho}{\lambda} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{q_0}{\lambda} \exp(-b x). \tag{9}
\]

When using bilateral microwave sources for wood modification pretreatment (Figure 1(b)), the heat conduction equation can be obtained by using the same method as mentioned above:

\[
\frac{C\rho}{\lambda} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{q_0}{\lambda} \times \left[ \exp\left(-b\left(\frac{d}{2} + x\right)\right) + \exp\left(-b\left(\frac{d}{2} - x\right)\right) \right]. \tag{10}
\]

Partial differential equations (9), (10) described the temperature profiles within wood when the unilateral microwave source and bilateral microwave sources were applied, respectively, and the temperature profiles can be simulated based on the equations and the initial and boundary conditions.

2.2. Initial Condition. The initial condition of heat transfer model is relatively simple, because a heat balance existed between the wood and the surrounding environment before heating, so the wood temperature is the same as the ambient temperature, when the room temperature is \(T_0\); therefore, the initial condition for unilateral microwave heating is as follows:

\[
T(x, 0) = T_0, \quad x \in [0, d]. \tag{11}
\]

The initial condition for bilateral microwave source heating is

\[
T(x, 0) = T_0, \quad x \in \left[-\frac{d}{2}, \frac{d}{2}\right]. \tag{12}
\]

2.3. Boundary Conditions. In order to solve the mathematical model, the boundary equations of heat transfer within wood during microwave heating should be developed.

When the unilateral microwave source was applied for heating, the boundary equations for heat transfer on \(x = 0\) and \(x = d\) (wood surface) are as follows:

\[
\alpha \left(T(0, t) - T_0\right) = \lambda \frac{\partial T}{\partial x} \bigg|_{x=0},
\]

\[
\alpha \left(T(d, t) - T_0\right) = -\lambda \frac{\partial T}{\partial x} \bigg|_{x=d}. \tag{13}
\]

When the bilateral microwave source was applied for heating, the boundary equations for heat transfer on \(x = -d/2\) and \(x = d/2\) (wood surface) are as follows:

\[
\alpha \left[T\left(-\frac{d}{2}, t\right) - T_0\right] = \lambda \frac{\partial T}{\partial x} \bigg|_{x=-d/2},
\]

\[
\alpha \left[T\left(\frac{d}{2}, t\right) - T_0\right] = -\lambda \frac{\partial T}{\partial x} \bigg|_{x=d/2}. \tag{14}
\]

where \(\alpha\) is the convective heat transfer coefficient of air, \(\text{w/(m}^2\cdot\text{C})\).

2.4. Heat Conduction Model. In summary, a heat conduction model to describe the temperature distribution within wood was established as follows:

Unilateral microwave heating:

\[
\frac{C\rho}{\lambda} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{q_0}{\lambda} \exp(-b x),
\]

\[
T(x, 0) = T_0, \quad x \in [0, d],
\]

\[
\lambda \frac{\partial T}{\partial x} \bigg|_{x=0} = \alpha \left[T(0, t) - T_0\right],
\]

\[
\lambda \frac{\partial T}{\partial x} \bigg|_{x=d} = -\alpha \left[T(d, t) - T_0\right]. \tag{15}
\]

Bilateral microwave heating:

\[
\frac{C\rho}{\lambda} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{q_0}{\lambda} \left[\exp\left(-b\left(\frac{d}{2} + x\right)\right) + \exp\left(-b\left(\frac{d}{2} - x\right)\right)\right],
\]

\[
T(x, 0) = T_0, \quad x \in \left[-\frac{d}{2}, \frac{d}{2}\right],
\]

\[
\lambda \frac{\partial T}{\partial x} \bigg|_{x=-d/2} = \alpha \left[T\left(-\frac{d}{2}, t\right) - T_0\right],
\]

\[
\lambda \frac{\partial T}{\partial x} \bigg|_{x=d/2} = -\alpha \left[T\left(\frac{d}{2}, t\right) - T_0\right].
\]

3. Simulation and Discussion

Based on the mathematical model of heat conduction and the given parameters mentioned above, using the MATLAB Software, the temperature profile of wood during microwave heating can be simulated. In the mathematical simulation, the thickness of sawn timber was \(d = 0.02\) m, ambient temperature was \(T = 25^\circ\text{C}\), and microwave radiation power per unit of area was \(q_0 = 10^7\) W/m\(^2\) (unilateral microwave source). When using bilateral microwave sources, the radiation power of each microwave source was \(q_0 = 5 \times 10^6\) W/m\(^2\), which means the total microwave radiation intensity of bilateral microwave sources is the same as that of the unilateral microwave source. According to the references [24, 25], the values of thermal conductivity of wood, heat capacity per unit of volume, and the heat transfer coefficient between the surface of wood and environment are selected as follows, respectively:

\[
\lambda = 0.16\ W/\left(\text{m} \cdot \text{C}\right),
\]

\[
C = 1.354 \times 10^6\ J/\left(\text{m}^3 \cdot \text{C}\right), \tag{16}
\]

\[
\alpha = 20\ W/\left(\text{m}^2 \cdot \text{C}\right).
\]
Figures 2 and 3 show temperature profiles within wood when the unilateral microwave source was applied for heating. It appears that the temperature distribution through wood is more uniform at the first stage of microwave heating. However, the temperature difference within wood increased dramatically with the increased microwave heating time; The temperature of wood surface facing the microwave source was higher than that of wood core layers; the lowest temperature appeared on the surface which was away from the side of microwave source; Due to the heat dissipated from wood surface to the surrounding environment, the highest temperature did not appear on the wood surface facing the microwave source, but occurred at a distance of several millimeters from the surface exposed to microwave source (in the region of approximately 1 to 2 mm away from the wood surface). This simulation result was consistent with the experimental results of Piotr et al. [20, 21] and the others [25]. Therefore, without considering the differences in microwave absorption within wood, a temperature field with a higher temperature in wood interior and a lower one on wood surface cannot be established if the unilateral microwave source was applied for modification pretreatment. The temperature distribution within wood increased and then gradually decreased along the direction of microwave transmission, and there is a temperature gradient within wood, which is not good for the even modification pretreatment of wood.

The temperature profiles within wood heated by the bilateral microwave source were shown in Figures 4 and 5. From the figure, it can be found that the temperature curves on both sides of wood showed a symmetrical shape along the central symmetry layer of wood and the temperature of wood interior was lower than that of the wood surface. Moreover, the temperature within wood firstly increased and then decreased from the wood surface to interior. Similar to the unilateral microwave heating, the peak temperature neither appeared on wood surface facing the microwave source nor on the wood interior but occurred at a distance of several millimeters from the wood surface. Compared with the unilateral microwave source heating, the temperature distribution is more uniform for the bilateral microwave source heating.

Based on the simulation results above, without considering the differences in microwave absorption within wood, the temperature profiles within wood were related to the methods of microwave heating directly. Compared with the unilateral microwave source heating, the temperature distribution is more uniform for the bilateral microwave source heating. Here, it must be stressed that wood is the nonhomogeneous material, and the difference of dielectric properties of wood are significant within the different areas of wood, which results in a difference on the microwave absorption ability. Therefore, the temperature distribution within wood heated by microwave may be in fact more complex.

4. Conclusions

Based on the physical law on heat transfer, a mathematical model to describe the temperature profiles within wood microwave heated with deferent methods was established and simulated in this research. The results showed that the temperature profiles inside wood were related to the methods of microwave heating during modification pretreatment. The temperature within wood firstly increased and then gradually
Figure 5: Temperature profile of wood microwave heating (bilateral microwave heating, gray).

decreased along the direction of microwave transmission when the unilateral microwave source was applied for wood microwave pretreatment. The temperature inside wood firstly increased and then gradually decreased from wood surface to interior when the bilateral microwave source were applied for wood microwave pretreatment. Compared with the unilateral microwave source heating, the temperature distribution is more uniform for the bilateral microwave sources heating. Therefore, the effects of the number of microwave source and microwave transmission direction on uniformity of microwave heating should be taken into account during the design of wood microwave heating device.

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