Research Article

Inverse Problem of Air Filtration of Nanoparticles: Optimal Quality Factors of Fibrous Filters

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Application of nanofibers has become an emerging approach to enhance filtration efficiency, but questions arise about the decrease in Quality factor (QF) for certain particles due to the rapidly increasing pressure drop. In this paper, we theoretically investigate the QF of dual-layer filters for filtration of monodisperse and polydisperse nanoparticles. The inverse problem of air filtration, as defined in this work, consists in determining the optimal construction of the two-layer fibrous filter with the maximum QF. In comparison to a single-layer substrate, improved QF values for dual-layer filters are found when a second layer with proper structural parameters is added. The influences of solidity, fiber diameter, filter thickness, face velocity, and particle size on the optimization of QF are studied. The maximum QF values for realistic polydisperse particles with a lognormal size distribution are also found. Furthermore, we propose a modified QF (MQF) accounting for the effects of energy cost and flow velocity, which are significant in certain operations. The optimal MQF of the dual-layer filter is found to be over twice that of the first layer. This work provides a quick tool for designing and optimizing fibrous structures with better performance for the air filtration of specific nanoparticles.

1. Introduction

The capture of nanoparticles from gas streams is critical in many fields, including protective clothing, respirators, air cabin filters, indoor air purifiers, and industrial air purification systems [1]. The filtration performance of fibrous filters is typically measured by the quality factor (QF), also known as the figure of merit, which is the ratio of the penetration rate of particles P to the pressure drop across the filter Δp [2],

$$QF = \frac{-\ln(P)}{\Delta p}. \quad (1)$$

Here, P is equal to $1 - E$, where E is the total filtration efficiency of the filter. A dense and thick filter is often more efficient in particle filtration and has a higher flow resistance or Δp , whereas a dilute and thin filter is more permeable and has a higher P for particles. Hence, improvement of filter

efficiency may compensate for an increase in the pressure drop. In this study, we focus on the evaluation of the filtration performance of fibrous filters, aiming to find the maximum QF with an optimal interplay between P and Δp .

The design of fibrous filters is based on the fact that filtration behavior is highly dependent on filter structure, particle size, and flow pattern. In general, filtration mechanisms that rely on mechanical capture include interception, diffusion, and inertial impaction [2]. A minimum at the most penetrating particle size (MPPS) usually exists between 0.1 and 0.5 μm due to the combined action of these filtration mechanisms, which have different sensitivities to particle size [3]. In comparison to coarse fibers, the use of fine fibers as a filter engenders a smaller MPPS and higher filtration efficiency. However, the pressure drop of filters composed of finer fibers becomes much greater as it decreases with the square of fiber radius, even for extremely fine nanofibers

TABLE 1: Improvement of filtration performance of fibrous filters by morphology modification.

Type	Improved performance	Possible reasons	Reference
Adding beaded nanofibers	Exhibited better filtration performance for aerosol particles ranging from 20 to 300 nm than nanofibers and particle/nanofiber composites	Increased void volume and aspect ratio	[20]
Coating multisize nanoparticles on electrospun nanofibers	Complimentary filtration efficiency (99.989%) and pressure drop (117 Pa) for 300–500 nm NaCl aerosol particles	Rough surface and noncircular cross section of fibers	[21]
Depositing a low amount of carbon nanotube (CNT) films	Filtration efficiency >99% in all cases and meeting the guidelines of HEPA filters; higher pressure drops but better QFs than the substrate	A broad micropore size distribution and a few mesopores between CNTs	[22]
In situ growth of brush-like CNTs on quartz fibers	Pressure drop moderately enhanced, but QF improved fairly well	Slightly reduced pore sizes, much larger specific surface area	[23, 24]

with a slip flow condition [4, 5]. Moreover, a random and inhomogeneous fiber arrangement makes the actual filter more permeable but less efficient in filtration than ordered fiber arrays [6–8]. Therefore, an inhomogeneity factor has been introduced to fit the order-based model to experimental results [9]. The filtration efficiency of filters varies with the face velocity and the MPPS shifts accordingly as well [10].

Utilization of nanofibers has become an emerging approach to enhance air filtration efficiency [11–16]. A composite of microfibers and nanofibers has been found to significantly improve the capture efficiency and QF for MPPS, whereas a single layer of nanofibers only leads to an increase in QF for very large particles [3, 17]. The QF for a dual-layer filter manufactured by the melt-blown technique is much higher than that of the microfiber substrate alone, as the increase in filtration efficiency is not overpaid with an excessive pressure drop [3]. Filtration tests have also indicated that a microfiber substrate layered by a single nanofiber mat of 0.15 μm diameter has a higher QF than conventional fiberglass filters for particles larger than 0.5 μm [18]. The QF for nanoparticles is observed to decline with a decrease in nanofiber diameter whereas the QF for larger particles increases with increasing nanofiber diameter [18]. It is believed that, for nanoparticles, the pressure drop increases more rapidly than the filtration efficiency due to diffusion; for large particles, the filtration efficiency predominated by interception and inertial impaction increases more rapidly than the pressure drop [17]. For a dual-layer filter with mean fiber diameters 14.7 μm and 0.208 μm , respectively, filtration performance has been found to decrease with face velocity [10]. This phenomenon is caused by the much lower likelihood for particles to collide on fibers through diffusion filtration at a smaller face velocity. Enhanced QF has also been observed in a thick layer alone in comparison to multiple thin layers of electrospun nanofibers, as multilayer filters have greater uniformity and smaller fiber sizes [19].

The QF of fibrous filters is also improved by modification of the morphology, including adding beaded nanofibers [20], coating multisize nanoparticles on electrospun nanofibers [21], depositing a low amount of carbon nanotube (CNT) films [22], and in situ growth of brush-like CNTs on quartz fibers [23, 24]. The results of increased QF and the possible explanations are summarized in Table 1.

The filtration quality of a clean filter is not sensitive to the deposition of particles in a short period, but it is strongly affected by particle loading over a long term [1]. With solid particles clogging to form a dust cake in the filter, flow resistance increases exponentially and penetration decreases at a given flow rate [1]. It is interesting to see that the deposited particles are more efficient in filtration than the fibers, as dendrite aggregate is formed with a high specific surface. To take into account continuous and unsteady loading of particles, a more realistic criterion of filtration performance, namely, the filter utility factor (FUF), has been proposed [25]. A cost-effective filter with higher dust-holding capacity has also been designed based on graded fibrous media [25]. The arrangement of a microfiber substrate in front of a nanofiber layer also gives a filter high filtration efficiency in the initial stage and a low increase in drop during continuous loading [26].

From the above discussion we find that there is less concern to design and optimize the fibrous filters to maximize the QF for nanoparticles. Layering of nanofibers onto a microfiber substrate leads to an increased QF over a certain range of particle size, but it may decrease the QF for the other particles. In practical filtration processes, the aim is often to capture particles of a specific size, such as the MPPS, the mean particle size, or the highest proportion particle size. Hence, as an inverse problem, in addition to avoiding any decrease in QF, we attempt in this paper to determine the optimal structural parameters that lead to the maximum QF for these target nanoparticles based on a dual-layer fibrous

FIGURE 1: Illustration of a dual-layer fibrous filter composed of two layers of fibers with the first layer fixed while the second layer varied with different solidity, fiber diameter, and thickness.

filter as illustrated in Figure 1. Here, the first layer is fixed while the second layer varies with different solidity, fiber diameter, and thickness. Furthermore, QF has been found not to be an energy-cost-based index [1]. To this end, we explore the filtration performance of dual-layer fibrous filters on the basis of a modified QF (MQF), by concerning energy cost and flow velocity.

2. Analysis of Theoretical Expressions

This study concerns the filtration of fine particles in clean filters. The assumptions are made that particle clogging is negligible and no particle rebounds from the fiber.

Filter efficiency E is derived on the basis of single fiber efficiency (SFE), making a mass balance for particles in a given filter and integrating from the filter face to the exit [2],

$$E = 1 - \exp \left[\frac{-4cE_t h}{\pi(1-c)d_f} \right], \quad (2)$$

where c is solidity, d_f is fiber diameter, and E_t is total SFE, which is calculated based on the assumption that different filtration mechanisms act independently [2]. E_t is generally expressed as follows:

$$E_t = 1 - (1 - E_D)(1 - E_R)(1 - E_I), \quad (3)$$

where E_D , E_R , and E_I are SFEs caused by diffusion, interception, and inertial impaction, respectively. By taking into

account the aerodynamic slip on the SFE based on diffusion, the following expression is given for E_D [27]:

$$E_D = 2.27Ku^{-1/3}Pe^{-2/3} \left(1 + 0.62KnKu^{-1/3}Pe^{1/3} \right), \quad (4)$$

where Ku is the Kuwabara hydrodynamic factor, which is equal to $Ku = -\ln(c)/2 - 3/4 + c - c^2$ and Kn is the Knudsen number given by $Kn = 2\lambda/d_f$. Here, λ is the mean free path of the gas molecules and the Peclet number Pe is given as $Pe = d_f U/D$, where U is the face velocity and D is the diffusion coefficient of particles. D is widely evaluated by the Stokes-Einstein equation [1]; namely,

$$D = \frac{kTC}{3\pi\mu}, \quad (5)$$

where T is the temperature, k is Boltzmann's constant, C is the slip factor, and μ is the gas viscosity. The expression of C is given by

$$C = 1 + \frac{\lambda}{d_p} \left[2.33 + 0.966 \exp(-0.4985) \frac{d_p}{\lambda} \right]. \quad (6)$$

The SFE due to interception is given as follows [2]:

$$E_R = \frac{(1 + N_R)^{-1} - (1 + N_R) + 2(1 + 1.996Kn)(1 + N_R) \ln(1 + N_R)}{2[-0.75 - 0.5 \ln(c)] + 1.996Kn[-0.5 - \ln(c)]}, \quad (7)$$

where N_R is the ratio of d_p to d_f . The model is accurate when $N_R < 0.2$. The SFE caused by the inertial effect is calculated as the following form [2]:

$$E_I = \frac{JSt}{2Ku^2}, \quad (8)$$

where St is the Stokes number, namely, $St = d_p^2 \rho C U / 18 d_f \mu$, and J is equal to $J = (29.6 - 28c^{0.62})N_R^2 - 27.5N_R^{2.8}$ for $N_R < 0.4$ and $J = 2$ when $N_R > 0.4$.

The pressure drop for nonwoven-like filters composed of randomly layered fibers is calculated as follows [5]:

$$\Delta p = \frac{16\mu c U h (1 + 2Kn)}{d_f^2 \{-0.6 \ln(c) - 0.74 + c - 0.25c^2 + 2Kn[-0.6 \ln(c) - 0.14 + 0.25c^2]\}}. \quad (9)$$

In Figure 2, we examine the effects of filtration mechanisms on the QF of a single-layer fibrous filter. The QFs for diffusion-only, interception-only, inertial impaction-only, and the combined filtration mechanisms are compared at $U = 0.1 \text{ m/s}$, $d_f = 1 \mu\text{m}$, $c = 0.05$, and $h = 1 \text{ mm}$. For nanoparticles (below $0.1 \mu\text{m}$), the QF based on the combined filtration mechanisms decreases with an increase in particle size, approaching the diffusion-only QF. For micro and submicro particles (above $0.1 \mu\text{m}$), the total QF

based on the combined filtration mechanisms increases with an increase in particle size, which is consistent with the interception-only and inertial impaction-only cases. We find that diffusion filtration is dominant for nanoparticles and submicro particles ($< 1 \mu\text{m}$) whereas interception and inertial impaction become more critical to the QF in the range of large particles ($> 3 \mu\text{m}$). It is expected that very small particles are captured through random collision with fibers due to Brownian motion. Larger particles are intercepted by fibers in

FIGURE 2: Effects of filtration mechanisms on the QF of a single-layer filter, including diffusion-only, interception-only, inertial impaction-only, and the combined filtration mechanisms ($U = 0.1$ m/s, $d_f = 1$ μ m, $c = 0.05$, and $h = 1$ mm).

FIGURE 3: Effects of fiber diameter on the QF of a single-layer filter ($U = 0.1$ m/s, $c = 0.05$, and $h = 1$ mm).

the path of the air stream or deviate from the path and then compact fibers due to inertia. The concave curves in Figure 2 indicate that the values of QF vary with particle size under different filtration mechanisms, which is similar to those of filtration efficiency. For improvement of QF, therefore, it is essential to consider all three filtration mechanisms simultaneously.

Capture of particles of a given size is of great interest in specific filtration. In Figure 3, we investigate the effect of fiber diameter on the QF of a single-layer filter, by capturing nanoparticles with diameters 0.01 μ m, 0.05 μ m, and 0.1 μ m. The fiber diameter varies from 0.3 μ m to 10 μ m, the filter thickness is 1 mm, the face velocity is 0.1 m/s, and the solidity is 0.05 . We observe that the QF for particles of 0.01 μ m and 0.05 μ m increases monotonously with the increase in fiber diameter. The reason is that the pressure drop decreases more rapidly than the decrease in filtration efficiency with increasing fiber diameter. For nanoparticles of 0.1 μ m, it is interesting to find the minimum QF from the concave line in Figure 3. This phenomenon reveals the complexity of the

dependence of the QF on the structural parameters, as the decrease in the pressure drop can be outweighed by the decrease in filtration efficiency.

3. QF and MQF of Dual-Layer Filters

In particle filtration by a dual-layer filter, the gas streams are purified layer by layer. Thus, the total penetration of particles is the product of the penetration of the two layers:

$$P_t = P_1 P_2, \quad (10)$$

where P_t , P_1 , and P_2 are the penetrations for the total filter, the first layer, and the second layer, which are given by $1 - E_t$, $1 - E_1$, and $1 - E_2$, respectively. The subscript 1 represents the first layer (e.g., c_1 , h_1 , and d_{f1}) and the subscript 2 indicates in the second layer (e.g., c_2 , h_2 , and d_{f2}). The pressure drops are additive and the total pressure drop is the sum of the two local pressure drops:

$$\Delta p_t = \Delta p_1 + \Delta p_2, \quad (11)$$

where Δp_t , Δp_1 , and Δp_2 are the pressure drops for the total filter, the first layer, and the second layer, respectively.

On the basis of (10) and (11), the QF for the dual-layer filter is given by

$$QF = \frac{-\ln(P_1 P_2)}{\Delta p_1 + \Delta p_2} = \frac{-\ln[(1 - E_1)(1 - E_2)]}{\Delta p_1 + \Delta p_2}. \quad (12)$$

The first layer is set as the control sample, so P_1 and Δp_1 are fixed. And the QF varies with the structural parameters of the second layer, namely, c_2 , h_2 , and d_{f2} .

For polydisperse particles, we obtain the total penetration by considering the distribution of particle size:

$$P = \int_0^\infty f(d_p) \exp\left[\frac{-4cE_t(d_p)h}{\pi(1-c)d_f}\right] d(d_p), \quad (13)$$

where $f(d_p)$ is the probability density function, which follows a lognormal distribution [28, 29]:

$$f(d_p) = \frac{1}{d_p \sigma \sqrt{2\pi}} e^{-[\ln(d_p) - \ln(\langle d_p \rangle)]^2 / 2\sigma^2}, \quad (14)$$

where $\langle d_p \rangle$ is the mean size of particles and σ is the scale parameter.

As mentioned earlier, a good filter should have high filtration efficiency and flow velocity at a low energy consumption. As such, we propose a modified QF (MQF), which is an energy-based indicator of filter performance:

$$MQF = \frac{EU}{G}, \quad (15)$$

where G is the energy cost of the filtration. G is proportional to the pressure drop and the flow flux Q : namely, $G = Q\Delta p$. Here, Q is expressed as $Q = US$, where S is the cross-sectional

area of the filter and is identical for both layers in a given filter system. Then, (15) is rewritten as

$$\text{MQF} = \frac{E}{S\Delta p}. \quad (16)$$

$$U = \frac{d_f}{4} \sqrt{\frac{G}{S\mu h} \frac{\{-0.6 \ln(c) - 0.74 + c - 0.25c^2 + 2\text{Kn}[-0.6 \ln(c) - 0.14 + 0.25c^2]\}}{1 + 2\text{Kn}}}. \quad (17)$$

Thus, U is determined by G for a given filter. Similarly, G can be calculated as a function of U if we fix U . In this study, we consider G as the constraint for optimization of fibrous filters. Substituting (17) into (4), (8), (9), and (16), we have the MQF as follows:

$$\text{MQF} = \frac{1 - P_1 P_2}{S(\Delta p_1 + \Delta p_2)} = \frac{1 - P_1 P_2}{S(\Delta p_1 + \Delta p_2)}, \quad (18)$$

which is a function of G .

4. Results and Discussion

To find the optimal filtration performance, we focus on investigating the QF for a dual-layer filter with the first layer fixed while the second layer varied (Figure 1). The models we employ in this paper, including (4), (7), and (8) for filtration efficiency of single-layer or dual-layer fibrous filters and (9) for pressure drop of fibrous filters, have been well verified by experimental results [5, 18]. The structural parameters of three types of the first, fixed layers are summarized in Table 2. The second layers have different solidity, thickness, and fiber diameter, and the face velocity is set as $U = 0.1$ m/s. We focus mainly on the filtration of particles of a specific size, but we still examine the effect of the polydispersity degree of the particles on the QF. Since the filtration of nanoparticles is less understood [1] and the MPPS is often around 100 nm [3, 17], we focus on the QF of nanoparticles with $d_p = 10$ nm and $d_p = 100$ nm. It is noted that QF is not the only criterion by which the filter quality is measured [17]; the evaluation depends on target requirements and specific operating conditions. Thus, we also explore the MQF by considering the influence of energy cost and flow velocity.

For the filter of Type 1, variations of QF against c_2 and d_{f2} at $U = 0.1$ m/s, $d_p = 10$ nm, $h_1 = 3$ mm, and $h_2 = 1, 3, 9$ mm are shown in Figure 4. The concave curves in Figure 4(a) indicate the optimal filters with the highest QF against c_2 when c_1 is equal to 0.1. The maximum QF results vary with the corresponding values of c_2 , which are around 0.04, 0.05, and 0.06 for $h_2 = 1, 3, 9$ mm, respectively. In addition, the maximum QF increases with the increase in h_2 . It is noted that the three filters have the same QF when $c_2 = c_1 = 0.1$ at the crossover of curves, and the dual-layer filter becomes a single-layer filter with its QF independent of thickness. When $c_2 > 0.1$, the QF values decrease dramatically with the increase in c_2 , revealing the inefficiency of the filters in capturing

MQF is simply equivalent to the ratio of E to Δp for a given filter, while it is calculated by concerning the effects of energy cost and flow velocity.

The two layers have the same U based on conservation of the flow flux and (9); namely,

the particles of $d_p = 10$ nm. This finding is consistent with the experimental results that the highest QF is found within five different values of c_2 for a nanofiber-microfiber dual-layer filter [18]. For the filtration of the particles with $d_p = 100$ nm, Figure 4(b) shows a similar trend of the QF against c_2 . However, the maximum QF values for $d_p = 100$ nm are much smaller than those for $d_p = 10$ nm. Hence, the dual-layer filters with $d_{f1} = d_{f2} = 10 \mu\text{m}$ have a better filtration performance for fine particles, which is similar to that of the single-layer filter. Figures 4(c) and 4(d) show the dependence of QF on d_{f2} for $d_p = 10$ nm and $d_p = 100$ nm, respectively. Here, we have $c_2 = c_1 = 0.1$, $h_1 = 3$ mm, and $h_2 = 1, 3, 9$ mm. In Figure 4(c), the maximum QF results are found against d_{f2} , with the corresponding values of d_{f2} equal to $1.5 \mu\text{m}$, $2 \mu\text{m}$, and $2.5 \mu\text{m}$ for $h_2 = 1, 3, 9$ mm, respectively. When d_{f2} is smaller than $d_{f1} = 10 \mu\text{m}$, the QF decreases considerably with the decrease in d_{f2} . Therefore, the addition of nanofibers onto the substrate or the first layer might not be reasonable in this case, although the filtration efficiency of nanofibers is much higher. In Figure 4(d), the maximum QF values for $d_p = 100$ nm are much lower than those for $d_p = 10$ nm in Figure 4(c). The maximum QFs observed in Figures 4(c) and 4(d) look slightly higher than the QF of a single-layer filter, but selection of the optimized second layer avoids the significant decline in QF caused by using inappropriate fibers. Figure 4 also demonstrates that two layers with close fiber diameters or solidity have higher QF than those with a very broad difference in fiber size or solidity.

Figures 5 and 6 illustrate the dependence of the QF on the structural parameters for the dual-layer filters Type 2 and Type 3, respectively. As for Type 1, their maximum QF results are found at different thickness ratios, with the corresponding values of c_2 slightly smaller than c_1 while the corresponding values of d_{f2} are moderately larger than d_{f1} .

In Figure 7 we study the effects of face velocity on the QF of the dual-layer filter against d_{f2} at $d_p = 100$ nm, $h_1 = h_2 = 3$ mm, $d_{f1} = 10 \mu\text{m}$, and $c_1 = c_2 = 0.1$. The maximum QF values are visible and they increase with the decrease in U . It is feasible to argue that a smaller face velocity ensures more adequate contact between fibers and particles, yielding a higher QF. However, a very small U can slow down the filtration processes even though the QF is satisfactory. Hence, we optimize the filter on the basis of the QF in the desired range of face velocity.

In Figure 8 we compare the QF values between monodisperse and polydisperse particles with the same mean particle

FIGURE 4: Effects of solidity of the second layer on the QF of the dual-layer filter with $d_{f1} = d_{f2} = 10 \mu\text{m}$ and $c_1 = 0.1$, for particles with diameters (a) $d_p = 10 \text{ nm}$ and (b) $d_p = 100 \text{ nm}$, and effects of fiber diameter of the second layer on the QF at $c_1 = c_2 = 0.1$ and $d_{f1} = 10 \mu\text{m}$, for particles with diameters (c) $d_p = 10 \text{ nm}$ and (d) $d_p = 100 \text{ nm}$ ($U = 0.1 \text{ m/s}$, $h_1 = 3 \text{ mm}$, and $h_2 = 1, 3, 9 \text{ mm}$).

TABLE 2: Structural parameters of the first layer in a dual-layer filter for filtration of particles of $d_p = 10 \text{ nm}$ and $d_p = 100 \text{ nm}$ at $U = 0.1 \text{ m/s}$.

Type	c_1	d_{f1}	h_1
1	0.1	$10 \mu\text{m}$	3 mm
2	0.05	$2 \mu\text{m}$	1 mm
3	0.01	$0.4 \mu\text{m}$	0.2 mm

size. The solidity of the second layer c_2 varies in Figure 8(a), with $\langle d_p \rangle = 100 \text{ nm}$, $c_1 = 0.1$, $h_1 = 3 \text{ mm}$, $h_2 = 9 \text{ mm}$, $d_{f1} = d_{f2} = 10 \mu\text{m}$, and $U = 0.1 \text{ m/s}$. Figure 8(b) demonstrates the dependence of QF on the fiber diameter of the second layer d_{f2} , with $\langle d_p \rangle = 10 \text{ nm}$, $c_1 = c_2 = 0.01$, $h_1 = h_2 = 0.2 \text{ mm}$, $d_{f1} = 0.4 \mu\text{m}$, and $U = 0.1 \text{ m/s}$. The particle size follows a lognormal distribution with different scale parameters σ . For the monodisperse particles, the maximum QF results are found for polydisperse particles. With increasing σ and the narrowing distribution of particle size, the maximum

QF values increase as d_{f2} at $c_1 = c_2$ but decrease as c_2 at $d_{f1} = d_{f2}$. It is demonstrated that the value of QF of the dual-layer filter against polydisperse particles is increased by around 10% than that of the fixed single first layer, which is equivalent to the two-layer fibrous filter with the solidity of the second layer extremely small. When σ is very small, the QF curves of for polydisperse particles nearly overlap with those of monodisperse particles, as expected. It is interesting to note that the optimal structural parameters accounting for the maximum QF are close between monodisperse and the polydisperse particles.

The maximum MQF values of the dual-layer filter against the solidity, the fiber diameter, and the thickness of the second layer are shown in Figures 9(a), 9(b), and 9(c), respectively. We set $G = 3.28 \text{ W}$ and $S = 0.005 \text{ m}^2$ for a fibrous filter in a commercial respirator at $U = 0.1 \text{ m/s}$. The values of G and S are determined by the target filters and operating conditions. In Figure 9(a), we have $c_1 = 0.01$, $d_{f1} = 1 \mu\text{m}$, $h_1 = h_2 = 0.2 \text{ mm}$, and $d_p = 100 \text{ nm}$, and the maximum MQF values of the dual-layer filter are found against c_2 at $d_{f2} = 0.5, 1, 2 \mu\text{m}$.

FIGURE 5: Effects of solidity of the second layer on the QF of the dual-layer filter with $d_{f1} = d_{f2} = 2 \mu\text{m}$ and $c_1 = 0.05$, for particles with diameters (a) $d_p = 10 \text{ nm}$ and (b) $d_p = 100 \text{ nm}$, and effects of fiber diameter of the second layer on the QF of the dual-layer filter with $c_1 = c_2 = 0.05$ and $d_{f1} = 2 \mu\text{m}$, for particles with diameters (c) $d_p = 10 \text{ nm}$ and (d) $d_p = 100 \text{ nm}$ ($U = 0.1 \text{ m/s}$, $h_1 = 1 \text{ mm}$, and $h_2 = 0.3, 1, 3 \text{ mm}$).

The maximum MQF results decrease with increases in c_2 and the corresponding values of c_2 vary considerably for different c_1 , which is unlike the QF responses. The optimal MQF of the dual-layer filter is found to be over twice of that of the fixed single first layer. In Figure 9(b), the maximum values of MQF are found against d_{f2} at $c_2 = 0.005, 0.01, 0.02$, and they decline with increasing d_{f2} . The corresponding values of d_{f2} for the maximum MQF are smaller than d_{f1} , which is also unlike that for QF. Finally, in Figure 9(c) the dependence of MQF on h_2 is investigated at $c_2 = 0.005, 0.01, 0.02$ with $d_{f2} = 1 \mu\text{m}$ and $d_{f2} = 0.5, 1, 2 \mu\text{m}$ with $c_1 = 0.01$. The maximum MQF increases with increasing d_{f2} but decreases with the increase in c_2 . Among the different cases, the values of h_2 corresponding to the maximum QF vary considerably, indicating the significance of choosing the appropriate thickness of the dual-layer filter to improve filtration performance. To summarize, the requisite conditions for the optimal QF are based on the fact that the total filtration efficiency has a slower decrease rate before the threshold of structural parameters is

reached but then a faster decrease rate than the total pressure drop at the given energy cost.

5. Conclusion

In this study, the filtration performance of the dual-layer fibrous filter is theoretically studied based on closed-form models. We find the best filter performance in terms of QF for the nanoparticles with a specific size such as the MPPS, the mean particle size, or the highest proportion of a given particle size. The findings are summarized as follows:

- (i) The maximum QF for the capture of monodisperse and polydisperse nanoparticles is found against the variation of solidity, thickness, and fiber diameter of the second layer, when the first layer is fixed.
- (ii) Under a given energy cost, the optimal filters are found with the highest MQF against variations of

FIGURE 6: Effects of solidity of the second layer on the QF of the dual-layer filter with $d_{f1} = d_{f2} = 0.4 \mu\text{m}$ and $c_1 = 0.01$, for particles with diameters (a) $d_p = 10$ nm and (b) $d_p = 100$ nm, and effects of fiber diameter of the second layer on the QF of the dual-layer filter with $c_1 = c_2 = 0.01$ and $d_{f1} = 0.4 \mu\text{m}$, for particles with diameters (c) $d_p = 10$ nm and (d) $d_p = 100$ nm ($U = 0.1$ m/s, $h_1 = 0.2$ mm, and $h_2 = 0.1, 0.2, 0.4$ mm).

FIGURE 7: Effects of face velocity on the QF of a dual-layer filter with varying d_{f2} ($d_p = 100$ nm, $h_1 = h_2 = 3$ mm, $d_{f1} = 10 \mu\text{m}$, and $c_1 = c_2 = 0.1$).

FIGURE 8: Comparison of optimal QF between monodisperse and polydisperse particles against (a) the solidity of the second layer at $\langle d_p \rangle = 100 \text{ nm}$, $c_1 = 0.1$, $h_1 = 3 \text{ mm}$, $h_2 = 9 \text{ mm}$, $d_{f1} = d_{f2} = 10 \mu\text{m}$, and $U = 0.1 \text{ m/s}$; (b) the fiber diameter of the second layer at $\langle d_p \rangle = 10 \text{ nm}$, $c_1 = c_2 = 0.01$, $h_1 = h_2 = 0.2 \text{ mm}$, $d_{f1} = 0.4 \mu\text{m}$, and $U = 0.1 \text{ m/s}$.

FIGURE 9: Maximum values of MQF of the dual-layer filter against (a) solidity ($h_1 = h_2 = 0.2 \text{ mm}$, $c_1 = 0.01$, $d_{f1} = 1 \mu\text{m}$, and $d_{f2} = 0.5, 1, 2 \mu\text{m}$), (b) fiber diameter ($h_1 = h_2 = 0.2 \text{ mm}$, $d_{f1} = 1 \mu\text{m}$, $c_1 = 0.01$, and $c_2 = 0.005, 0.01, 0.02$), and (c) thickness of the second layer, at $c_1 = 0.01$, $d_{f1} = 1 \mu\text{m}$, $d_p = 100 \text{ nm}$, $G = 3.28 \text{ W}$, $S = 0.005 \text{ m}^2$, and $U = 0.1 \text{ m/s}$.

solidity, thickness, and fiber diameter of the second layer when the first layer is fixed.

- (iii) The optimal QF of the dual-layer filter is enhanced by around 10% than that of the fixed first layer, while the maximum MQF can be increased to be over twice in certain cases.
- (iv) The optimal structural parameters accounting for the maximum QF vary with face velocity and particle size distribution.

This work provides a quick tool to design and optimize fibrous filters with better filtration performance of nanoparticles. To avoid expensive computation, current studies are limited to the filter fibers with constant diameter and circular shape and they also assume negligible clogging of particles. The above concerns will be addressed in future work with the assistance of the Computational Fluid Dynamics (CFD) technique. As well, optimization of the fiber surface roughness, the sizes of the nanorods grown on the fiber wall, and the clogging distribution in different filter layers will also be explored to prompt the best filtration performance.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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