1. Introduction

Self-assembled semiconductor QDs have been a subject of numerous investigations owing to their promising potential application in optoelectronic devices [1–3]. The exploitation of this kind of nanostructures towards the improvement of the devices performance mainly relies on the ability to control their size and uniformity. Accordingly a lot of works have been deployed for best optimization of the QDs properties by varying growth temperature [4], growth rate [5–7], and capping conditions [8–10]. Nevertheless, the self-organization nature of the InAs/GaAs QDs makes their properties strongly dependent on the fabrication conditions [11–14]. Various techniques have been used to evaluate the morphology of nanostructures after capping, but all have their limitations [15]. For example, while it is well recognized that decreasing the growth rate enhances the dots size and decreases their surface density [6, 7], no clear picture has been established yet concerning the impact of this parameter on the evolution of the buried dots aspect ratio. Furthermore, intense theoretical and computational efforts have so far been expended in order to understand the evolution of the self-assembled QD's electronic structures dependence on several parameters such as shape [16], composition [17], and wetting layer thickness [18]. Most of these works were mainly focused on fundamental investigation of the QDs electronic structure. We propose in this work to extend the numerical modeling to a more practical use by coupling it to PL measurements to gain useful estimation on the buried dots average size in a facile and nondestructive way.

In this context, we report on a coupled numerical and high excitation power photoluminescence measurements method allowing an accurate assessment of the buried QDs size and study its evolution as a function of the growth rate.

2. Theoretical Considerations

The theoretical calculations have been performed by COMSOL Multiphysics software [19] where the single band effective mass Schrödinger equation has been solved for a lens shaped InAs QDs using the finite element methods (FEM). A schematic representation of the modeled QD structure is...
shown by Figure 1 where the three-dimensional QD can be constituted by a rotation of $2\pi$ around $z$-axis.

The steady state Schrödinger equation within the single band effective mass approximation can be given by

$$\frac{-\hbar^2}{2m^*} \Delta \psi(\vec{r}) + V(\vec{r}) \psi(\vec{r}) = E \psi(\vec{r}),$$

(1)

where $\hbar$, $m^*$, $E$, $V$, and $\psi$ are, respectively, the reduced Plank’s constant, the electron (hole) effective mass, confined energy state, confining potential, and wave function. $\vec{r}$ is the coordinate vector in spherical symmetry depending on the variables $(r, \phi, z)$, where $r$ is the radial coordinate perpendicular to the growth direction, $z$ is the axial coordinate, and $\phi$ is the azimuthal angle ranging from 0 to $2\pi$.

Equation (1) can be written in 3D cylindrical coordinates as follows:

$$\frac{-\hbar^2}{2m^*} \left( \frac{\partial^2 \chi}{\partial r^2} + \frac{1}{r} \frac{\partial \chi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \chi}{\partial \phi^2} + \frac{\partial^2 \chi}{\partial z^2} \right) \chi(r, \phi, z)$$

$$+ V \chi(r, \phi, z) = E \chi(r, \phi, z),$$

(2)

by considering that the wave function is separable on

$$\chi(r, \phi, z) = \chi(r, z) \Phi(\phi), \text{ where } \Phi(\phi) \propto e^{i n \phi}.$$  (3)

To satisfy the condition $\Phi(\phi) = \Phi(\phi + 2\pi)$, $n$ must be integer.

These conditions permit, after straightforward simplification process, rewriting (2) in the following form:

$$\frac{-\hbar^2}{2m^*} \left( \frac{\partial^2 \chi_n}{\partial r^2} + \frac{1}{r} \frac{\partial \chi_n}{\partial r} + \frac{\partial^2 \chi_n}{\partial z^2} \right) \chi_n(r, z)$$

$$+ \left( V + \frac{1}{r^2} \frac{\hbar^2}{2m^*} n^2 \right) \chi_n(r, z)$$

(4)

$$- \frac{\hbar^2}{2m^*} \frac{1}{r} \frac{\partial \chi_n}{\partial r} = E_n \chi_n(r, z).$$

The impact of strain has been taken into account [20] in this model. The potential energy ($V$) for electrons (holes) is taken to be zero inside the QD and equal to the strained conduction (valence) band discontinuity outside the QD. The evaluated electron and holes confined energies allow the estimation of the $n$th noncorrelated electron-hole optical transition energy by the following relation:

$$E_n = e_n + E_{\text{InAs}}^g + h_n,$$

(5)

where $E_{\text{InAs}}^g$ is the strained InAs band gap energy. $e_n$ and $h_n$ are, respectively, the confined electrons and holes energies.

The material parameters used in this work are taken from [21].

To ensure reliable calculations, an adaptive mesh refinement has been adopted providing more grids in the active material in and around the QD and the WL. Dirichlet boundary conditions have been applied for the external boundaries where the wave function should vanish and Newman condition to the internal boundary ensuring the continuity of the wave function. More details on the calculation procedure can be found elsewhere [22].

3. Experiments

The studied samples consist of InAs QDs layer with 2.4 ML nominal thickness grown at 500°C by molecular beam epitaxy on semi-insulating (001) GaAs substrate. The samples differ on the InAs material deposition rates being varied from 0.013 ML/s to 0.069 ML/s. The QDs are covered by 50nm of GaAs at 500°C with a relatively high growth rate (0.98 ML/s) for all samples to minimize the In/Ga intermixing. The optical properties are evaluated by photoluminescence (PL) spectroscopy using a 514.5 nm line of an Ar+ laser. The PL emission was dispersed by a spectrometer and detected by an InGaAs photodetector using a conventional lock-in technique. More details on the growth procedure, photoluminescence, and atomic force microscopy investigations of these structures can be found elsewhere [7].

4. Results and Discussion

Figure 2(a) shows typical excitation power dependent PL spectra measured at 10 K from the single layer QDs deposited with a growth rate of 0.069 ML/s. This spectra unarguably show that the excited states get progressively populated upon increasing the excitation power density leading to the appearance of two excited states emission bands. The manifestation of multiple peaks arising from the state filling due to the three-dimensional carrier confinement allows an accurate estimation of the buried QDs size by adjusting the calculated transition energies to the experimental values. Typical envelope function for the ground states (GS) and first (1st ES) and second (2nd ES) excited states are shown in Figure 2(b).

The numerically driven emission energies deduced from (5) can be tuned by changing the QDs AR. Good matching between the theoretical and experimental emission bands within some meV yielded an estimation of the QDs average height of 3.3 nm and base diameter of 29 nm.

The PL investigation of the QDs fabricated at different growth rate has also revealed a state filling effect with varying the excitation power density. Typical PL spectra showing the emission bands relative to the ground state (GS) and...
Figure 2: (a) Typical 10K PL spectra recorded at various excitation density at 80 W cm$^{-2}$ (squares), 20 W cm$^{-2}$ (circles), and 4 W cm$^{-2}$ (triangles). These spectra show the state filling effects with increasing the excitation power. The highest excitation density spectra show three emission bands deconvoluted by three Gaussians, namely, the optical transition energies arising from GS, 1st ES, and 2nd ES. (b) Electron envelop function calculated using the simulated buried dot size for the GS, 1st ES, and 2nd ES.

two excited states are given by Figure 3. The PL emission bands are found to be red shifted with decreasing the InAs material’s deposition rate testifying an increased dots size. Additionally, for the same laser excitation power density, the 1st ES and GS emission peaks’ intensity ratio increases from 0.5 for a growth rate of 0.069 ML/s to up to 1.3 for QDs deposited at 0.013 ML/s. The relative enhancement of the 1st ES emission peaks’ intensity indicates an increased number of the excess carriers available to populate the higher energy levels prior to their radiative recombination. This signifies also that the number of required photo carriers to saturate the GS energy levels decreases with decreasing the QDs deposition rate. The observed behavior, consequently, gives evidence of a pronounced decrease of the QDs’ density. Accordingly, for high growth rate, the QDs are constrained to be prematurely formed because of the limited atom surface diffusion length [5–7]. However, for lower growth rate, the number of incoming surface atoms get reduced which increases their diffusion length. It becomes possible for them to reach existing islands where they get incorporated instead of being constrained to form additional islands. This induces a reduction of the QDs density and an increase of their size in conformity with AFM observations. Indeed, morphological studies have shown a monotonic decrease of the QDs density with a pronounced height enhancement with decreasing the growth rate [6, 7].

Knowing that the same InAs nominal material thickness has been deposited for all samples, the observed PL behavior is well explained by the decrease of the QDs density in favor of the formation of bigger dots’ size. However, quantitative information on the realistic size evolution of the buried dots’ size is missing considering the changes that may occur to the covered dots upon the deposition of the capping layer material [23, 24].

By considering a lens shaped QD with 1 ML thick pure InAs WL, as stated in the theoretical consideration part, we
Table 1: Experimental and calculated optical transition energies for different growth rates. Also presented are the simulated QDs height, base diameter, and corresponding aspect ratio.

<table>
<thead>
<tr>
<th>Growth rate (ML/s)</th>
<th>GS (eV)</th>
<th>1st ES (eV)</th>
<th>2nd ES (eV)</th>
<th>h (nm)</th>
<th>D (nm)</th>
<th>AR</th>
</tr>
</thead>
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<tr>
<td>0.013</td>
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<td>1.14</td>
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<td>1.246</td>
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<tr>
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<td>1.126</td>
<td>1.197</td>
<td>1.193</td>
<td>1.26</td>
<td>1.264</td>
</tr>
</tbody>
</table>

Figure 3: 10K PL spectra obtained from InAs/GaAs QD samples obtained at 10K with an excitation density of 80 W/cm² for InAs deposition rate 0.069 ML/s (red line), 0.043 ML/s (green line), 0.026 ML/s (blue line), and 0.013 ML/s (black line).

The good agreement within some meV allows gaining realistic size estimation for buried QDs. The results show also a monotonic decrease of the dots AR with increasing the growth rate. The proposed method succeeds in giving the same asymptotic evolution of the QDs height with much smaller values. The observed large height difference between buried and uncapped QDs can have several cumulative reasons. Indeed, the AFM tip artifact can lead to an overestimation of the uncovered QDs size [25]. Additionally, the capping process has been shown to induce a shrinkage of the QDs size [26]. It is also worth mentioning that neglecting the intermixing effects in the present model might provide smaller QDs size [22].

Considering the relative variation of the QDs height and base diameter, the QDs height seems to be the most affected parameter by the growth rate variation within the adopted range in this work.

5. Conclusion

The correlation between high excitation density PL measurements and numerical solving of single particle one band Schrödinger equation in the frame of the effective mass approximation allowed estimating a realistic buried QDs
size with application to truck the evolution of the covered InAs/GaAs QDs size and aspect ratio dependence on the growth rate. We found that the enhancement of the QDs size induced by the lowering of the growth rate is mainly due to the increase of the average dots height.

The proposed method is however general and can be applied to any QDs system if provided the ability to measure the optical transition energies issued from the ground and excited states.

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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