Research Article

Study of a Microbistable Piezoelectric Energy Harvesting

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Received 6 June 2018; Revised 30 August 2018; Accepted 17 September 2018; Published 16 December 2018

Guest Editor: Zhengping Zhou

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A microbistable piezoelectric energy harvester has been developed. The harvester was based on a center-fixed and quadrilateral-free microbistable plate with mass blocks placed at the four corners. Considering the thermoelectromechanical coupling effect, a nonlinear oscillation differential equation was established by Hamilton's principle. Strain gradient theory was applied to consider the size effect, and von Karman theory was used to consider the large deformation effect. The influences of the laying position and area of the piezoelectric layer on the efficiency of energy capture were investigated. The voltage-frequency response of the nonlinear system was investigated, and the snap-through behavior of the bistable plate results in energy harvesting achieving the ideal broaden frequency range before the resonance region.

1. Introduction

Piezoelectric energy harvesters are devices that convert ambient environmental vibration into electrical energy by absorbing ambient vibrations. The bistable plate can trigger snap-through by overcoming the critical load to realize large amplitude oscillations. Therefore, piezoelectric energy harvesting with the characteristics of a bistable plate can improve the energy harvesting efficiency [1]. With the development of microelectromechanical system (MEMS) technology, the size of wireless sensors is simultaneously becoming smaller. To cope with the trend of miniaturization, piezoelectric energy harvesters that provide energy for wireless sensors often tend to be miniaturized [2].

In recent years, bistable piezoelectric energy harvesting has been widely explored. The nonlinear behavior of the bistable plate, such as chaos and the transition between the two steady states, is used to drive the piezoelectric plate to achieve broadband vibration in the nonresonant frequency environment. In 2010, Arrieta et al. [1] proposed a bistable composite laminate with four attached piezoelectric patches and an asymmetric set-up to capture energy in broadband frequency. They found that high energy conversion can be obtained with snap-through between the stable states. In 2012, Betts et al. [3] used an electrical function to study the effects of the aspect ratio, thickness, stacking sequence, and

piezoelectric area on the efficiency of energy capture. In 2013, Betts et al. [4] continued their previous experimental work to understand the dynamic response of the structure, and they used high-speed digital image correlation to identify the dynamic modes and study the energy capture characteristics. Experiments showed that the large deformation caused by snap-through improves the efficiency of energy capture. In 2015, Syta et al. [5] experimentally identified the dynamics of the system response by generated voltage time series, and the frequency spectrum, bifurcation diagrams, and phase portraits were investigated. In 2016, Syta et al. [6] used nonlinear time series analysis methods of the Fourier spectrum and recurrence quantification analysis to experimentally investigate the dynamic response of energy harvesting and relational capturing energy generation.

With the development of MEMS technology, the microbistable structure has been widely investigated. In 2010, Andò et al. [7] considered the nonlinear behavior of a bistable cantilever beam to broaden the spectrum and lower the frequency. They then performed a numerical study based on stochastic differential equations to evaluate the behavior of a MEMS device. In 2016, Medina et al. [8, 9] demonstrated dynamic snap-through of micromechanically initially curved beam structures under electrostatic actuation. Experimental and theoretical results showed that snap-through motion can be achieved by tailored time-dependent electrostatic

FIGURE 1: The energy harvesting structure diagram.

actuation. In the same year, this group constructed a model of a microscale circular curved bistable plate and investigated its reliability, accuracy, and suitability. By applying reasonable low voltages, the actuation feasibility of a microbistable plate with realistic dimensions was demonstrated.

Most reports have discussed triggering of snap-through in macroscale bistable plates. However, few studies have described the snap-through behavior in microscale plates for energy harvesting. In this study, we investigated the effects of the laying position, area, and thickness of the piezoelectric layer on the efficiency of piezoelectric energy harvesting for microbistable piezoelectric energy harvesting. The unique snap-through behavior of the bistable plate was investigated, and several wide frequency ranges were discovered.

2. Device Modeling

With the aim of microbistable piezoelectric energy harvesting, the four corners of the base layer were covered with mass blocks and the pedestal provides harmonic excitation: $w = w_1 \cdot \sin(\omega t)$, where w_1 is the amplitude and ω is the vibration frequency, as shown in Figure 1.

The displacements of any point in the x , y , and z directions are

$$\begin{aligned} u(x, y, z, t) &= u_0(x, y, t) - z \cdot \frac{\partial w}{\partial x}, \\ v(x, y, z, t) &= v_0(x, y, t) - z \cdot \frac{\partial w}{\partial y}, \\ w(x, y, t) &= w_0(x, y, t) + w_1 \sin(\omega t), \end{aligned} \quad (1)$$

where u_0 , v_0 , and w_0 are the displacements of any point on the neutral surface in the x , y , and z directions.

According to von Karman's large deformation theory, the nonlinear strain expressions are given by

$$\begin{aligned} \epsilon_{xx} &= \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2, \\ \epsilon_{yy} &= \frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2, \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \cdot \frac{\partial w}{\partial y}. \end{aligned} \quad (2)$$

The stress expressions for the base layer are obtained from the constitutive relation:

$$\begin{aligned} \sigma_{xx}^S &= \frac{E_S}{1 - \nu_S^2} ((\epsilon_{xx} - \alpha_S \cdot \Delta T) + \nu_S (\epsilon_{yy} - \alpha_S \cdot \Delta T)), \\ \sigma_{yy}^S &= \frac{E_S}{1 - \nu_S^2} ((\epsilon_{yy} - \alpha_S \cdot \Delta T) + \nu_S (\epsilon_{xx} - \alpha_S \cdot \Delta T)), \\ \tau_{xy}^S &= \frac{E_S}{2(1 + \nu_S)} \cdot \gamma_{xy}. \end{aligned} \quad (3)$$

Similarly, the piezoelectric layer stress expressions are defined as

$$\begin{aligned} \sigma_{xx}^P &= \frac{E_P}{1 - \nu_P^2} ((\epsilon_{xx} - \alpha_P \cdot \Delta T) + \nu_P (\epsilon_{yy} - \alpha_P \cdot \Delta T)), \\ \sigma_{yy}^P &= \frac{E_P}{1 - \nu_P^2} ((\epsilon_{yy} - \alpha_P \cdot \Delta T) + \nu_P (\epsilon_{xx} - \alpha_P \cdot \Delta T)), \\ \tau_{xy}^P &= \frac{E_P}{2(1 + \nu_P)} \cdot \gamma_{xy}, \end{aligned} \quad (4)$$

where E is the elastic modulus, ν is Poisson's ratio, α is the thermal expansion coefficient, h_p is the thickness of the piezoelectric layer, e_{31} and e_{32} are the piezoelectric coefficients,

V is the electric potential, superscript S represents the base layer, subscript P represents the piezoelectric layer, and ΔT is the temperature change between the processing and normal operating temperatures. All of the MEMS manufacturing processes, such as sputtering [10], deposition [11], and the sol-gel [12] method, involved high-temperature processing. The difference between the thermal expansion coefficients of the piezoelectric layer and base layer leads to the generation of the original thermal stress.

Suppose that displacements at any point in the middle plane were expressed by

$$\begin{aligned} u_0(x, y) &= d_1x + d_2xy^2 + d_3x^3 + d_4xy^4 + d_5x^3y^2 + d_5x^5, \\ v_0(x, y) &= d_7y + d_8xy^2 + d_9x^3 + d_{10}xy^4 + d_{11}x^3y^2 + d_{12}x^5, \\ w_0 &= \frac{1}{2}(ax^2 + by^2), \end{aligned} \quad (5)$$

where d_i ($i = 1, 2, \dots, 12$) are undetermined coefficients with time t and a and b are the curvatures along the x and y directions, respectively.

The total strain energy within the microplate is composed of two parts. One part is based on the classical continuum mechanics theory and can be expressed as

$$\begin{aligned} U_S^h &= \iiint_{V_S} \frac{1}{2} \left(\sigma_{xx}^S \cdot \varepsilon_{xx} + \sigma_{yy}^S \cdot \varepsilon_{yy} + \tau_{xy}^S \cdot \gamma_{xy} \right) dx dy dz, \\ U_P^h &= \iiint_{V_P} \frac{1}{2} \left(\sigma_{xx}^P \cdot \varepsilon_{xx} + \sigma_{yy}^P \cdot \varepsilon_{yy} + \tau_{xy}^P \cdot \gamma_{xy} \right) dx dy dz. \end{aligned} \quad (6)$$

Based on the strain gradient theory proposed by Mindlin, η_{ijk} ($i, j, k = x, y, z$), and the other part can be expressed as

$$\begin{aligned} U_S^w &= \iiint_{V_S} \left(L_1^S \cdot \eta_{iik} \cdot \eta_{kjj} + L_2^S \cdot \eta_{ijj} \cdot \eta_{ikk} + L_3^S \cdot \eta_{iik} \cdot \eta_{jjk} \right. \\ &\quad \left. + L_4^S \cdot \eta_{ijk} \cdot \eta_{ijk} + L_5^S \cdot \eta_{ijk} \cdot \eta_{kji} \right) dx dy dz, \\ U_P^w &= \iiint_{V_P} \left(L_1^P \cdot \eta_{iik} \cdot \eta_{kjj} + L_2^P \cdot \eta_{ijj} \cdot \eta_{ikk} + L_3^P \cdot \eta_{iik} \cdot \eta_{jjk} \right. \\ &\quad \left. + L_4^P \cdot \eta_{ijk} \cdot \eta_{ijk} + L_5^P \cdot \eta_{ijk} \cdot \eta_{kji} \right) dx dy dz, \end{aligned} \quad (7)$$

where L_i^ξ ($i = 1, 2, \dots, 5$) are the material parameters. The reference value for each material parameter has been obtained by Ramezani [13]:

$$\begin{aligned} L_2^\xi &= \frac{1}{2} l_\xi^2 \lambda_\xi, \\ L_4^\xi &= l_\xi^2 \mu_\xi, \\ L_1^\xi &= L_3^\xi = L_5^\xi = 0, \\ \xi &= S, P, \end{aligned} \quad (8)$$

where l_ξ is the internal length scale parameter depending on the specific material and λ_ξ and μ_ξ are the common Lamé constants:

$$\begin{aligned} \lambda_\xi &= \frac{E_\xi \nu_\xi}{(1 - 2\nu_\xi)(1 + \nu_\xi)}, \\ \mu_\xi &= \frac{E_\xi}{2(1 + \nu_\xi)}, \\ \xi &= S, P. \end{aligned} \quad (9)$$

The potential energy of the system U consists of two parts:

$$U = U_S^h + U_P^h + U_S^w + U_P^w. \quad (10)$$

Because the kinetic energies in the x and y directions are small compared with that in the z direction, they can be ignored [4]. The kinetic energy expression of the structure is

$$T_1 = \iiint_{V_S} \frac{\rho_S}{2} \dot{w}^2 dV_S + \iiint_{V_P} \frac{\rho_P}{2} \dot{w}^2 dV_P, \quad (11)$$

where ρ_S and ρ_P are the densities of each layer of material.

The kinetic energy of the mass block is

$$T_2 = \frac{1}{2} (M_1 \dot{w}_{c1}^2 + M_2 \dot{w}_{c2}^2 + M_3 \dot{w}_{c3}^2 + M_4 \dot{w}_{c4}^2), \quad (12)$$

where $\dot{w}_{ci}$ ($i = 1, \dots, 4$) are the displacement of the four corner points and M_i is the mass of the mass block.

The total kinetic energy of the system is

$$T = T_1 + T_2. \quad (13)$$

The virtual work done by the damping force is

$$\delta W = - \iiint_{V_S} r_S \dot{w} \delta w dV_S - \iiint_{V_P} r_P \dot{w} \delta w dV_S, \quad (14)$$

where r_S and r_P are the damping coefficients.

Because the sizes of the piezoelectric layer in the length and width directions are much larger than those in the thickness direction and the polarization direction is along the thickness direction, the electric displacement D_3 in the thickness direction is much larger than the electric displacement in the other two directions (D_1 and D_2). Thus, D_1 and D_2 can be ignored and D_3 can be expressed as

$$D_3 = e_{31} \varepsilon_{xx} + e_{32} \varepsilon_{yy} - \varepsilon_{33} \frac{V}{h_p}, \quad (15)$$

where V is the electric potential and ε_{33} is the dielectric constant.

Because the electric displacement D_3 does not change with the thickness, the above equation is subjected to homogenization in the thickness direction:

$$\tilde{D}_3 = \frac{\int_{h/2}^{h/(2+h_p)} (e_{31}\epsilon_{xx} + e_{32}\epsilon_{yy} - \epsilon_{33}(V/h_p)) dz}{h_p}. \quad (16)$$

The charge on the electrode can be obtained as follows:

$$Q = -\iint_{\Omega_p} \tilde{D}_3 dx dy = -\frac{1}{h_p} \int_{V_p} \left(e_{31}\epsilon_{xx} + e_{32}\epsilon_{yy} - \epsilon_{33} \frac{V}{h_p} \right) dV_p. \quad (17)$$

According to the generated charge, the current can be obtained by

$$I = -\frac{\partial Q}{\partial t} = \frac{1}{h_p} \int_{V_p} \left(e_{31}\dot{\epsilon}_{xx} + e_{32}\dot{\epsilon}_{yy} - \epsilon_{33} \frac{\dot{V}}{h_p} \right) dV_p. \quad (18)$$

When the external resistance is set to R , the current is

$$I = \frac{V}{R}. \quad (19)$$

From Eqs. (18) and (19), we can obtain

$$-\frac{\partial Q}{\partial t} = \frac{V}{R}. \quad (20)$$

That is,

$$\frac{1}{h_p} \int_{V_p} \left(e_{31}\dot{\epsilon}_{xx} + e_{32}\dot{\epsilon}_{yy} - \epsilon_{33} \frac{\dot{V}}{h_p} \right) dV_p = \int_{V_p} \frac{V}{RV_p} dV_p. \quad (21)$$

The electric equation can be obtained by

$$\dot{V} = \frac{(e_{31}\dot{\epsilon}_{xx} + e_{32}\dot{\epsilon}_{yy} - (h_p V/RV_p))h_p}{\epsilon_{33}}. \quad (22)$$

Substituting the variation of the kinetic energy and potential energy and the virtual force of the damping force into Hamilton's principle gives

$$\int_{T_0}^{T_1} (\delta U - \delta T + \delta W) dt = 0, \quad (23)$$

where t_1 and t_2 are the initial and final moments of the system movement, respectively. The equations about a , b , and d_i can be obtained by Eq. (23). d_i is about the algebraic equation of a and b , which can be expressed in terms of a

and b . By substituting d_i into the first two equations, the system dynamics differential equation can be obtained:

$$\begin{aligned} \ddot{a} = & -m_1 a + m_2 b - m_3 a^2 + m_4 b^2 + m_5 ab + m_6 a^2 b - m_7 ab^2 \\ & + m_8 a \Delta T - m_9 b \Delta T - m_{10} a V + m_{11} b V_1 - m_{12} \Delta T \\ & + m_{13} V - m_{14} r_p \omega \cos(\omega t) - m_{15} r_s \omega \cos(\omega t) \\ & - m_{16} r_p \dot{a} - m_{17} r_s \dot{a} + m_{18} r_p \dot{b} + m_{19} r_s \dot{b} + m_{20} \omega^2 \sin(\omega t), \\ \ddot{b} = & n_1 a - n_2 b + n_3 a^2 - n_4 b^2 + n_5 ab - n_6 a^2 b + n_7 ab^2 \\ & - n_8 a \Delta T + n_9 b \Delta T + n_{10} a V - n_{11} b V + n_{12} \Delta T - n_{13} V \\ & - n_{14} r_p \omega \cos(\omega t) - n_{15} r_s \omega \cos(\omega t) + n_{16} r_p \dot{a} + n_{17} r_s \dot{a} \\ & - n_{18} r_p \dot{b} - n_{18} r_p \dot{b} - n_{19} r_s \dot{b} + n_{20} \omega^2 \sin(\omega t), \end{aligned} \quad (24)$$

where m_i ($i = 1 - 20$) and n_i ($i = 1 - 20$) are the coefficients determined by the material and size parameters.

Equation (24) is connected to electrical Eq. (22) to form a dynamic model for microbistable piezoelectric energy harvesting.

3. Study on the Efficiency of Capturing Energy

To improve the efficiency of energy capture, the effects of the laying position, area, and thickness on the efficiency of the piezoelectric energy were investigated.

3.1. Selection of Layout Location. In this section, we discuss the generated voltage of two types of laying positions. Piezoelectric layers of type I and type II are shown in Figure 2. The size of the base layer is $300 \mu\text{m} \times 300 \mu\text{m} \times 0.7 \mu\text{m}$, and the size of the piezoelectric layer is $200 \mu\text{m} \times 200 \mu\text{m} \times 0.7 \mu\text{m}$, where $S_1 = S_{2-1} + S_{2-2} + S_{2-3} + S_{2-4}$.

The material parameters of the two layers are given in Table 1. The piezoelectric layer material is polyvinylidene fluoride, whose elastic modulus is much smaller than that of the base layer. In scanning the vibration frequency of the pedestal ω , the variation of the voltage with the frequency is obtained (Figure 3).

From Figure 3, for the same excitation and laying area, the natural frequency of type I is lower than that of type II and the peak value of the capturing voltage under the natural frequency is about twice as high as that of type II. This is because under the boundary conditions of center-fixed quadrilateral free, the closer to the center, the greater the strain. To clearly observe the elliptical part of the graph, a local enlarged plot is shown in Figure 3. Therefore, the laying position in the middle has better energy capturing efficiency than at the four sides. Thus, we select type I for the ideal laying position.

3.2. Influence of Thickness and Area of Piezoelectric Layer. In this section, the thickness and area of the piezoelectric layer are discussed for type I when the temperature decreases from the high MEMS processing temperature to room temperature; that is, $\Delta T = -400^\circ\text{C}$. Using I and II laying types, we investigated the influence of the piezoelectric layer area on

FIGURE 2: Type I and type II piezoelectric laying position.

TABLE 1: Material coefficients.

Physical quantity	Value
E_S	190 Gpa
E_P	2.1 Gpa
ν_S	0.2788
ν_P	0.31
α_S	$3 \cdot 10^{-5}/^\circ\text{C}$
α_P	$2.5 \cdot 10^{-6}/^\circ\text{C}$
e_{31}	$2\text{C}\cdot\text{m}^2$
e_{32}	$2\text{C}\cdot\text{m}^2$
ϵ_{33}	$6500\epsilon_0$
ρ_S	$2300\text{kg}/\text{m}^3$
ρ_P	$5670\text{kg}/\text{m}^3$

FIGURE 3: Type I and type II capture voltage versus stimulated frequency.

FIGURE 4: The thickness effect of the layout area on device efficiency.

the energy efficiency of the piezoelectric layer thickness in the range of 0.1–0.9 (Figure 4).

From Figure 4, for the same piezoelectric layer area, the efficiency of energy capture increases with increasing thickness. This is because a thicker piezoelectric layer results in more charge generation and higher capturing voltage. For the same piezoelectric layer thickness, the efficiency of energy capture first increases and then decreases with the increasing area of the piezoelectric layer. In this study, the elastic modulus of the piezoelectric layer is much smaller than that of the base layer. In the initial stage of the increase of the piezoelectric layer area ($0.1 < S_p/S_s < 0.5$), the thermoelectromechanical coupling effect is greater than the effect of

FIGURE 5: Waveform and phase diagrams of a (red circle) and b (blue triangle) in case of no snap-through.

the increased stiffness. As the area increases in the range ($0.5 < S_p/S_s < 0.9$), the increased stiffness effect becomes dominant, system deflection begins to decrease, and the capturing energy efficiency decreases.

4. Nonlinear Frequency Responses

From the above results, type I energy harvesting is best when the area of the piezoelectric patch is 0.5 times the base area and $\Delta T = -400^\circ\text{C}$. Although the efficiency of energy capture increases with the increasing thickness of the piezoelectric layer, the layer cannot be too thick because the stiffness of the structure will increase and deflection will decrease, ultimately leading to lower efficiency of energy capture and snap-through being difficult to achieve. Thus, the thickness is set at $0.7\ \mu\text{m}$.

In this section, we discuss the effect of the snap-through behavior of the bistable plate on the efficiency of energy capture. By fixing the excitation amplitude and scanning the frequency, we obtained the waveforms and phase diagrams of a and b (the curvatures in the x and y directions). These waveforms and phase diagrams reveal that the various snap-through behaviors existing in different frequency

regions allow the system to obtain multiple wide-band energy capture regions before the resonant region.

From Figure 5(a), when the excitation frequency is between 150 and 1500 Hz, the curvatures of the plate in the x and y directions are always the same ($a = b$) and no snap-through behavior occurs. When the frequency is between 1500 and 1600 Hz, the plate is in the state $b > a$ (Figure 5(b)), no snap-through occurs, and the amplitude is small. No snap-through also occurs in the frequency range 1600–2100 Hz, and the plate is in the state $a > b$, as shown in Figure 5(c). However, with increasing frequency, two types of snap-through behavior occur. (1) The states $a > b$ and $a < b$ alternately appear, and there is continuous snap-through behavior, as shown in Figure 6(a). (2) Successive conversion of the two stable states $a = 0$ and $b = 0$ causes large amplitude vibration and enhances the efficiency of energy harvesting, as shown in Figure 6(b). When the excitation frequency is between 2200 and 3100 Hz, no snap-through behavior occurs. The waveform and phase diagram are shown in Figure 5(a).

For microscale energy harvesting, the fundamental frequency is so high that the frequency of vibration in the ambient environment cannot reach the natural frequency

FIGURE 6: Waveform and phase diagrams of a (red circle) and b (blue triangle) in case of snap-through.

and excite resonance. For bistable energy harvesting with the mass block proposed in this paper, although the mass block reduces the natural frequency of the structure, the resonant frequency (6.3 kHz) is also too high compared with the ambient vibration frequency. Therefore, in this study, we make use of the characteristic of large amplitude vibration caused by the snap-through behavior to capture high voltage in several wide frequency ranges far from the resonance region.

To describe the influence of the snap-through behavior of the microbistable plate on the captured voltage, the frequency is plotted against the voltage in Figure 7. The figure shows that 1/3 and 1/2 subharmonic resonances occur when the ambient vibration frequency ω is close to 1/3 and 1/2 times the natural frequency of the microscale bistable energy harvester. There are two types of snap-through phenomena: (1) in the 1/3 subharmonic resonance region (×), continuous snap-through behavior occurs, as shown in Figure 6(a). (2) In the 1/2 subharmonic resonance region (●), the two stable states $a = 0$ and $b = 0$ alternatively occur, as shown in Figure 6(b). The snap-through behavior can produce a higher output voltage and wider bandwidth at a lower frequency, which is crucial for a microscale bistable energy harvester.

5. Conclusion

Considering the size effect, the strain gradient theory is extended to the nonlinear problem of microbistable piezoelectric energy harvesting. Based on the dynamic model established in this paper, a microbistable energy harvester structure was obtained by theoretical analysis. With increasing ambient vibration frequency, the snap-through phenomenon occurs for the microscale bistable energy harvester. Furthermore, the effect of the snap-through behavior on the

FIGURE 7: Voltage frequency response curve.

capturing energy efficiency was investigated, and the aim of capturing more energy in several wide frequency ranges far from the resonance region was realized.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The authors wish to express their gratitude for the supports provided by the National Natural Science Foundation of China under Grant no. 11472019 for this research work.

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