Power Optimization of Tilted Tomlinson-Harashima Precoder in MIMO Channels with Imperfect Channel State Information

H. Khaleghi Bizaki, M. Khaleghi H., and S. M. Razavizadeh

***Abstract*— This paper concentrates on the designing of a robust Tomlinson-Harashima Precoder (THP) over Multiple-Input Multiple-Output (MIMO) channels in wireless communication systems with assumption of imperfect Channel State Information (CSI) at the transmitter side. With the assumption that the covariance matrix of channel estimation error is available at the transmitter side, we design a THP that presents robustness against channel uncertainties. In the proposed robust THP, the transmit power is further minimized by using the Tilted constellation concept. This power minimization reduces the Inter-Channel Interference (ICI) between sub-channels and furthermore, recovers some part of the THP's power loss. The Bit Error Rate (BER) of the proposed system is further improved by using a power loading technique. Finally, the simulation results compare the performance of our proposed robust THP with a conventional MIMO-THP.**

***Index Terms*—MIMO Channel, Tomlinson-Harashima Precoder, Minimum Mean Squared Error (MMSE) Precoder, Tilted Constellation, Imperfect Channel State Information (CSI), Power Loading.**

I. Introduction

Recently, due to impressive capacity of Multiple-Input Multiple-Output (MIMO) channel, many companies and researches have been attracted toward designing of transceivers for MIMO systems and specially robust transceivers in the case of channel uncertainties. It is well known that Inter Channel Interference (ICI) is one of the important problems of the MIMO systems. It is possible to pre-eliminate the ICI at the transmitter side by using precoding techniques, and consequently reduce transmission power. Due to channel estimation errors, channel variations during time, and channel feedback errors, the assumption of perfect CSI at the transmitter side is not always true and hence the CSI at the transmitter side has uncertainties [1]. Due to these uncertainties, the ICI will be increased, and consequently led to BER growth. Linear precoder in [2] designed for imperfect CSI, but it could only slightly reduce uncertainties effects by using statistical information of the channel. Nonlinear Tomlinson-Harashima Precoder (THP) in [3] has considered for imperfect CSI assumption, and we combine it with the tilted constellation method in our work.

In this paper, we propose design of a robust MIMO-THP that can reduce uncertainties effects more than the proposed methods in [2] and [3], and subsequently can further reduce BER. Similar to [3] we utilize second-order statistics of the uncertainties of the MIMO channels at the transmitter side. Moreover, by using tilted constellation concept in [4], we can significantly reduce transmit power. It should be noted that, transmit power reduction can decrease ICI between sub-channels and in addition recover some parts of the THP's power loss. This ICI reduction between sub-channels is one of the main challenges in the designing of our precoder.

Tilted constellation method is introduced in [4] for SISO channel. Based on this idea, we have extended it for the MIMO channel. Here, the power reduction is done by tilting or rotating the ordinary constellation through a proper angle. The desired angle is one that minimizes the transmit power and is selected from a set of possible angles. The results show that, the power reduction will be greater at high SNRs values.

To do the Tilted constellation method, we use different angles for tilting the constellation in each antenna. Then, we find a set of proper angles, which are selected in a manner to minimize the transmit power in all antennas. The transmit power reduction can reduce the transmission block power, lower than the power of modulated symbols in each block. Finally, we can send proper angles to the receiver within the main data frame. We will show that, the proposed method outperforms the conventional MIMO-THP in most SNRs and also in all symbol block lengths.

We can adjust transmit power of each antenna when the CSI (perfect/imperfect) is available at the transmitter side which is known as power loading [5]. It should be noted that, in the MIMO systems some of the corresponding parallel sub-channels (corresponding to each antenna) might have very low channel condition or might be useless for sometimes [5]. In this situation, the transmitter can adjust transmit power of each antenna by redistributing the available transmit power to get a better average error rate [5]. Moreover, by utilize power loading, we improve the BER performance of the Tilted

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MIMO-THP. We also study the complexity of our proposed method with respect to some previous methods.

This paper is organized as follows. In Section II, the system model of the Tilted MIMO-THP in imperfect CSI scenario is introduced. In Section III, the proposed robust Tilted MIMO-THP is described for imperfect CSI. Section IV presents the power loading technique to improve the performance of the robust precoder. In section V the simulation results are given to show the performance of the proposed method and its comparison with the conventional MIMO-THP. Finally, conclusions are drawn in Section VI.

II. System Model

In this paper, a MIMO communication system with n_T transmit antenna at the base station and n_R users at the receivers side, each equipped with one receive antenna is considered. The block diagram of considered system is shown in Fig.1.

The n_T input data symbols and the transmitted signals are denoted by vector $\boldsymbol{a}$ and vector $\boldsymbol{x}$, respectively. Both these vectors are of size $n_T \times 1$. The channel matrix is modeled as $\boldsymbol{H} = \boldsymbol{H_0} + \Delta\boldsymbol{H}$, where $\boldsymbol{H_0}$ is the $n_R \times n_T$ estimated channel matrix whose elements are independent identically distributed (i.i.d.) zero mean complex Gaussian random variables with variance $1-\rho^2$ (i. e, $\boldsymbol{H_0} \sim CN(0, 1-\rho^2)$) and $\Delta\boldsymbol{H}$ is an $n_R \times n_T$ channel uncertainties matrix whose elements are i.i.d. zero mean complex Gaussian random variables with variance ρ^2 $(i, e, \Delta\boldsymbol{H} \sim CN(0, \rho^2))$. Thus, $\boldsymbol{H}$ can be modeled as $\boldsymbol{H} \sim CN(0,1)$. The matrix $\boldsymbol{T}$ is the proper rotation angles matrix which is named as Tilting matrix, to minimize the transmission power. The received signal $\boldsymbol{y}$ can be denoted as:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{n}$ is the noise vector whose elements are assumed to be i.i.d. complex Gaussian random variables with zero mean and variance σ^2 (i.e. $\boldsymbol{n} \sim CN(0, \sigma^2 \boldsymbol{I_R})$).

As we see in Fig. 1, the n_T dimensional vector $\boldsymbol{a}$ correspond to input data and n_T dimensional vector $\boldsymbol{x}$ represent transmitted signals. The feedback matrix $\boldsymbol{B}$ is added to pre-eliminate the interference from previous users and the tilting matrix $\boldsymbol{T}$ is added to reduce the transmit power. Finally, in order to keep the power of transmitted symbols within the original constellation boundary we employ modulo operator at the forward loop. The boundary region of the constellation is related to the order of modulation constellation, which for rectangular *M*-ary QAM modulation is $t = 2\sqrt{M}$. Using Tilted constellation, the signal constellation is further rotated by a set of appropriate angles; represented by the diagonal elements of the Tilting matrix $\boldsymbol{T}$; to reduce the ICI and power loss of THP.

The resultant signal $\hat{\boldsymbol{x}}$ is then passed through a unitary feed-forward filter $\boldsymbol{F}$, to eliminate the residual interference. Finally, the precoded signal is sent through the MIMO channel. All processes to eliminate the interference are performed at the transmitter side, hence receivers at the user side are left with some simple operations including power

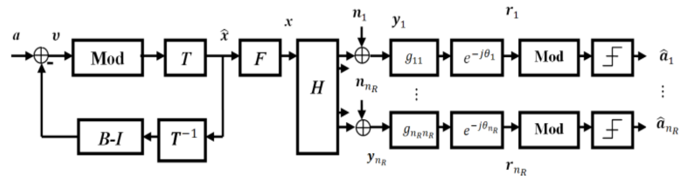

Fig. 1. The proposed Tilted MIMO-THP in decentralized scenario.

scaling (i.e. elements of the diagonal matrix $\boldsymbol{G}$), de-modulo operation, constellation tilting with reverse angle and a single user detection.

By using the second-order statistical of the uncertainties of the MIMO channels at the transmitter side, we design a THP that presents robustness against channel uncertainties. We can describe difference between impressive vector $\boldsymbol{v}$ and input vector to the decision block as [3]:

$$\boldsymbol{e} = \boldsymbol{r} - \boldsymbol{v} = [\boldsymbol{T^{-1}}\boldsymbol{G}(\boldsymbol{H_0} + \boldsymbol{\Delta H})\boldsymbol{F} - \boldsymbol{B}]\hat{\boldsymbol{x}} + \widetilde{\boldsymbol{n}} \tag{2}$$

MMSE solution should minimize the signal error as:

$$\begin{cases} \arg\min_{\boldsymbol{B},\boldsymbol{F},\boldsymbol{G}} \mathrm{E}\|[[\boldsymbol{T^{-1}}\boldsymbol{G}(\boldsymbol{H_0} + \boldsymbol{\Delta H})\boldsymbol{F} - \boldsymbol{B}]\hat{\boldsymbol{x}} + \widetilde{\boldsymbol{n}}]\|^2 \\ \quad \text{so that} \qquad \mathrm{E}\|\hat{\boldsymbol{x}}\|^2 \le \mathrm{P_T} \end{cases} \tag{3}$$

where $\mathrm{P_T}$ is the total transmitted power. Since, we assume the matrix $\boldsymbol{F}$ is unitary, the power constraint guaranteed also.

Instead of analysis of (3) we can use orthogonality property as follow [3]:

$$\mathrm{E}[\boldsymbol{e}\boldsymbol{r}^H] = 0 \tag{4}$$

Hence from (4) we have:

$$\boldsymbol{G}\Phi_{\boldsymbol{rr}} = \boldsymbol{B}\Phi_{\boldsymbol{xr}} \tag{5}$$

We can calculate $\Phi_{\mathbf{rr}}$ and $\Phi_{\mathbf{xr}}$ using (1) as:

$$\Phi_{\boldsymbol{rr}} = E[\boldsymbol{r}\boldsymbol{r}^H] = \boldsymbol{\sigma_x^2}(\boldsymbol{H_0}\boldsymbol{H_0}^H + \boldsymbol{\zeta I} + \boldsymbol{C_{\Delta H}}) \tag{6}$$

$$\Phi_{\boldsymbol{xr}} = E[\boldsymbol{x}\boldsymbol{r}^H] = \boldsymbol{\sigma_x^2}(\boldsymbol{F}^H\boldsymbol{H_0}^H) \tag{7}$$

By substituting (6) and (7) in (5), and after some manipulations, we have:

$$\boldsymbol{F}^H = \boldsymbol{B}^{-1}\boldsymbol{G}(\boldsymbol{H_0}\boldsymbol{H_0}^H + \boldsymbol{\zeta I} + \boldsymbol{C_{\Delta H}})\boldsymbol{H_0}^{-H} \tag{8}$$

Since $\mathbf{F}$ is unitary, we have

$$\boldsymbol{R}\boldsymbol{R}^H = (\boldsymbol{H_0}\boldsymbol{H_0}^H + \boldsymbol{\zeta I} + \boldsymbol{C_{\Delta H}})\boldsymbol{H_0}^{-H}\boldsymbol{H_0}^{-1}(\boldsymbol{H_0}\boldsymbol{H_0}^H + \boldsymbol{\zeta I} + \boldsymbol{C_{\Delta H}}) \tag{9}$$

where $\boldsymbol{R} = \boldsymbol{G}^{-1}\boldsymbol{B}$. Matrix $\boldsymbol{R}$ can be obtained by doing the

Cholesky factorization of (9). We can calculate matrixes $\boldsymbol{G}$, $\boldsymbol{B}$ and $\boldsymbol{F}$ as:

$$\boldsymbol{G} = diag\left(\boldsymbol{1}/\boldsymbol{r}_{11}, \dots, \boldsymbol{1}/\boldsymbol{r}_{n_T n_T}\right)$$
$$\boldsymbol{B} = \boldsymbol{G}\left(\boldsymbol{H_0}\boldsymbol{H_0}^H + \boldsymbol{\zeta I} + \boldsymbol{C_{\Delta H}}\right)\boldsymbol{H_0}^{-H} = \boldsymbol{GR} \tag{10}$$
$$\boldsymbol{F} = \boldsymbol{H_0}^{-1}(\boldsymbol{H_0}\boldsymbol{H_0}^H + \boldsymbol{\zeta I} + \boldsymbol{C_{\Delta H}})\, \boldsymbol{R}^{-H}$$

Substituting $\boldsymbol{G}$, $\boldsymbol{B}$ and $\boldsymbol{F}$ in (10) the error covariance matrix is equal to:

$$\Phi_{\boldsymbol{ee}} = E[\boldsymbol{ee}^H] = \boldsymbol{\sigma_x^2 G}(\boldsymbol{\zeta^2 H_0}^{-H}\boldsymbol{H_0}^{-1} + \boldsymbol{\zeta I} + \boldsymbol{C_{\Delta H}})\boldsymbol{G}^H \tag{11}$$

where the scalar $\boldsymbol{\zeta}$ represents the $\frac{1}{SNR} = \frac{\sigma_n^2}{\sigma_x^2}$.

It can be seen in (11), while the channel has uncertainty, the error covariance matrix has an additional term of $\boldsymbol{\sigma_x^2 G C_{\Delta H} G}^H$ with respect to its counterpart in perfect CSI case.

In this paper, we assume the above relations and matrixes as the conventional robust MIMO-THP, i.e. without using Tilted constellations.

III. Robust Tilted MIMO-THP

As mentioned before, in the general model of Fig. 1, each diagonal element of the Tilting matrix $\boldsymbol{T}$ determines the rotation angle of its corresponding antenna signal.

In conventional THP, the main constellation is only extended by the modulo operator on its boundary regions. However, in the Tilted-THP, the signal constellation is further rotated by a set of appropriate angles. To do this, first a block of symbols with length $n_T \times N$ is divided to n_T groups each of length N. Then, for every group of symbols which is transmitted from each antenna, the transmitter chooses an appropriate angle from a set of possible angles in such a way that the transmitted power is minimized based on some predefined criteria. From [4], the ordinary constellation is tilted by Q possible angles as $\theta_q; q = 1, \dots, Q$, then the optimal angle θ_q^* is selected by the following equation:

$$q^* = \arg\min_{q=1,\dots,Q} \left\|\mathrm{Mod}[\boldsymbol{a} - \boldsymbol{i}e^{-j\theta_q}]\right\| \tag{12}$$

where $\boldsymbol{a}$ is the group of symbol in each transmit antenna and $\boldsymbol{i}$ illustrates the interference sequence at each antenna due to the previous antennas and is calculated similar to the conventional THP. The difference between the conventional (i.e. un-Tilted) and Tilted THP is in rotating of the transmitted signal of the previous antennas by their optimal angles. Transmitted power and resulting optimal angle of each antenna can be calculated as [4]

$$P_{Tilted}(Q) = \mathrm{E}\left\{\min_{q=1,\dots,Q} \boldsymbol{P}_q\right\} \tag{13}$$
$$\boldsymbol{P}_q = \frac{1}{N}\sum_{i=1}^{N} \left\|Mod[\boldsymbol{a} - \boldsymbol{i}e^{-j\theta_q}]\right\|^{\boldsymbol{2}}$$

The vector $\boldsymbol{a}_1$, which is transmitted from the first transmit antenna, is not affected by any interference signal, that's why we transmit $\boldsymbol{a}_1$ without any interference. The optimal tilted angle θ_1 is resulted by minimizing the transmitted power of first antenna, i.e. ;

$$\boldsymbol{x_1} = \boldsymbol{a}_1 e^{j\theta_1} \tag{14}$$

For $\boldsymbol{a}_2$, which is transmitted from the second antenna, the optimal transmit angle achieve by computing the minimum transmitted power similar to the $\boldsymbol{a}_1$ but here, there is an interference due to the $\boldsymbol{a}_1$, i.e. $\boldsymbol{b}_{21}$, and hence

$$\boldsymbol{x}_2 = (Mod(\boldsymbol{a}_2 - \boldsymbol{b}_{21}\boldsymbol{x}_1 e^{-j\theta_2}))\boldsymbol{e}^{j\theta_2} \tag{15}$$

Similarly, for the k^{th} antenna, we have

$$\boldsymbol{x}_k = (Mod(\boldsymbol{a}_k - \textstyle\sum_{j=1}^{k-1} \boldsymbol{b}_{kj}\boldsymbol{x}_j e^{-j\theta_k}))e^{j\theta_k} \tag{16}$$

where θ_k is the optimal tilted angle for the k^{th} antenna. At the receiver side, the received signal for the k^{th} antenna can be shown as,

$$\boldsymbol{r}_k = (\boldsymbol{b}_k\boldsymbol{x} + \boldsymbol{n}_k)e^{-j\theta_k} \tag{17}$$

Where $\boldsymbol{b}_k$ denoting the k^{th} row of the matrix $\boldsymbol{B} = [\boldsymbol{b}_1, \dots, \boldsymbol{b}_{k,\dots}]^{\boldsymbol{T}}$. By using (16), (17) can be written as,

$$\boldsymbol{r}_k = (\ \boldsymbol{x}_k + \textstyle\sum_{j=1}^{k-1} \boldsymbol{b}_{kj}\boldsymbol{x}_j + \boldsymbol{n}_k)\ e^{-j\theta_k} \tag{18}$$

Since $b_{kk} = 1$, hence after modulo operator we obtain

$$\widehat{\boldsymbol{a}}_k = Mod(\boldsymbol{r}_k)$$
$$= Mod\left((\boldsymbol{a}_k e^{j\theta_k} - \sum_{j=1}^{k-1} \boldsymbol{b}_{kj}\boldsymbol{x}_j + \sum_{j=1}^{k-1} \boldsymbol{b}_{kj}\boldsymbol{x}_j + \boldsymbol{n}_k)e^{-j\theta_k}\right)$$
$$= Mod\left((\boldsymbol{a}_k e^{j\theta_k} + \boldsymbol{n}_k)e^{-j\theta_k}\right) = Mod\left(\boldsymbol{a}_k + \boldsymbol{n}_k e^{-j\theta_k}\right) \tag{19}$$

As shown [3], in the Tilted constellation case, the matrix $\boldsymbol{B}$ is similar to the untilted or regular MIMO-THP case. Its difference is only in rotation of the phase of the noise. Since the noise has circularly symmetric complex Gaussian distribution, therefore, for the conventional case, all equations and results for the decomposition of matrices in the Tilted constellation case are the same.

If we use the Tilted constellation for detecting the original symbols directly, the performance will be degraded due to rotation and reduction of minimum distance between symbols. Hence we should send the optimal angles to the receiver to detecting the desired symbols. In the receiver, first, the symbols are reversely rotated at each antenna and then, are detected in a way similar to the conventional THP.

IV. Power Loading

The SER of MIMO-THP in imperfect CSI can be approximated as follows [6]:

$$SER \approx \sum_{k=1}^{K}\left(1-\frac{1}{\sqrt{M_k}}\right)Q\left(\sqrt{\frac{3}{M_k-1}\frac{(P_{Tilted})_k}{\sigma_e^2}}\right) \qquad (20)$$

where $(P_{Tilted})_k$ is the transmitted power of k^{th} transmit antenna and σ_e^2 is noise power[3]

$$\sigma_e^2 = [\Phi_{ee}]_{kk} = \frac{1}{|r_{kk}|^2}\left(\sigma_n^2 + \sum_{j=1}^{K}(P_{Tilted})_j\left|\delta_{kj}\right|^2 + \beta_k\right) \quad (21)$$

Here, $\delta_{ij} = [\boldsymbol{\Delta H}]_{ij}$, $\beta_k = \sigma_n^4\sum_{j=1}^{K}\left|h_{kj}\right|^2/(P_{Tilted})_j$ and $h_{ij} = [\boldsymbol{H}^{-H}\boldsymbol{H}^{-1}]_{ij}$. With the assumption of small error, i.e. $\alpha(P_{Tilted})_j \le \sigma_n^2; \forall j$ [3], we can approximate the σ_e^2 value as follows:

$$\sigma_e^2 = \frac{1}{|r_{kk}|^2}\left(\sigma_n^2 + \alpha(P_{Tilted})_T + \beta_k\right) \qquad (22)$$

where $(P_{Tilted})_T$ is the total tilted power. At the above equation, we assumed the worst-case (i.e. $\alpha = \max_{\boldsymbol{B},\boldsymbol{F},\boldsymbol{G}}\left|\delta_{ij}\right|^2; \forall i,j$). We distribute the power to minimize SER in imperfect CSI by using the Lagrange method, so that can be written as follows:

$$L = \sum_{k=1}^{K}\left(1-\frac{1}{\sqrt{M_k}}\right)Q\left(\sqrt{(P_{Tilted})_k\beta_k}\right) - \lambda\left(K - \sum_{k=1}^{K}(P_{Tilted})_k\right) \quad (23)$$

Hence as in [3]

$$\beta_k = \frac{3}{M_k-1}\frac{1}{\sigma_n^2+\alpha(P_{Tilted})_T+\beta_k} \qquad (24)$$

Unfortunately, there is not a closed for the solution of the above optimization problem, so we should use some suboptimal solution as in [3].

By Solving $\partial L/\partial(P_{Tilted})_k = 0$ for $(P_{Tilted})_k$, we have:

$$(P_{Tilted})_k = \frac{1}{\beta_k}W\left(\frac{\beta_k}{A_k}\Lambda\right) \qquad (25)$$

Where $A_k = \frac{M_k}{(\sqrt{M_k}-1)^2}\frac{1}{\beta_k}$, $\beta_k = \frac{3}{M_k-1}\frac{|r_{kk}|^2}{\sigma_n^2+\alpha(P_{Tilted})_T+\beta}$, $\Lambda = cte$ and W(x) is real Lambert function [3]

Thus, the power vector $\boldsymbol{P}$= $[(P_{tilted})_1, \ldots, (P_{tilted})_{n_T}]$ Can be obtained by using the following unique repeated solution[3]:

I. A small positive value Λ is chosen so that:

$$\sum_{k=1}^{K}\frac{1}{A_k} \le P_T$$

II. The value $\hat{P}_T$ is calculated:

$$\hat{P}_T = \sum_{k=1}^{K}\frac{1}{\beta_k}W\left(\frac{\beta_k}{A_k}\Lambda\right)$$

III. If $\hat{P}_T$ is not close enough to P_T, Λ multiply $P_T/\hat{P}_T$ and be returned to the stage II.

IV. Based on the (25) vector $\boldsymbol{P}$ is calculated.

This algorithm is present as pseudo code in Table 1 .

Table 1: The proposed power loading algorithm

```
Λ=0.01;  % initial value for Λ
P_Total = nT;  % Total transmit power
Calculate A_k, β_k, Λ
Loop: Calculate P̂
If P_transmit - P̂ →0 then P_transmit = P̂ end
Elseif P̂ > P_transmit , Break;
Else   Λ =(P_T/P̂_T)× Λ ,  goto loop
end
```

Now, we discuss about the complexity of the proposed method. We realize that the proposed method add some parts to the conventional THP method. To compare the complexities of the proposed method and the conventional THP, we only consider those parts that are changed in our method. For example, for the case of $n_R = n_T = 4$, the complexity of the conventional THP is $15n$ at the transmitter side and is $8n$ at the receiver side while for the proposed Tilted-THP method this complexity is $25n$ at the transmitter side and is $12n$ at the receiver side ,where $n = N/n_T$ is the length of the input symbol to each antenna. As it can be seen, the increased complexity is suitable. On the other hand, as we will show in the next section in simulation results, the performance gain that we attain by the proposed method is more considerable with respect to the above complexity increasing.

V. Simulation Results

To evaluate the performance of our proposed method, we use some simulations. To do this we assume a MIMO-THP system with 4 transmit and 4 receive antennas with 4-QAM modulation. We assume that the channel estimation error covariance ρ =0.1 and 0.05. In our figures, the labels ‘conv’, ‘Rob’ and ‘P.L.’ are corresponding to conventional, Robust and Power Loading methods, respectively.

Fig. 2 shows the transmitted power of the MMSE-THP system versus the number of tilted angles. In this figure, the SNR is assumed to be 10dB. It can be seen that by increasing the number of available tilted angles, the transmitted power reduces until it converges to a constant floor .Also, it can be seen that by decreasing the block length, the power reduction will be more and the power converges faster to its floor. The reason of this floor is that we assumed the matrix $\boldsymbol{F}$ is unitary.

As can be seen, for example, the transmitted power for the conventional MIMO-MMSE-THP for Q=25 is 4.91 but for the Tilted constellation and with the same parameters, the transmitted power decrease to 4 and 4.24 when the block length increase to 5 and 10, respectively.

Fig.3. shows BER of the MMSE and Robust-Tilted-MMSE precoders versus the Eb/No for different block lengths and for ρ = 0.05 with and without the power loading. As it is seen in this figure, with the Robust-Tilted THP, the BER decreases compared to the conventional one. This BER reduction is more when we use the power loading. Moreover, because of more power reduction in smaller block lengths, the BER reduction will be more in smaller block length. For example, for SNR=30dB and N=4 in the conventional MIMO-MMSE-THP, we have BER=0.0028 while in the robust Tilted constellation MIMO-MMSE-THP for with and without power loading we have BER=0.0015 and BER=0.0004, respectively.

Fig.4. show BER of the MMSE and Robust-Tilted-MMSE precoders versus the Eb/No for different block lengths and for ρ = 0.1 with and without the power loading. As it is seen in this figure, with the Robust-Tilted constellation, BER decreased compared to the conventional MIMO-THP which is similar to the previous figure. Again, BER reduction is more when we use the power loading. Due to more power reduction of Tilted constellation for shorter block length, the resultant BER has reduced more for shorter block lengths. More channel uncertainties with respect to the previous figure lead to BER incensement. For example, at SNR=30dB in the conventional MIMO-MMSE-THP, BER=0.011 while in the robust Tilted constellation MIMO-MMSE-THP for N=4 with and without power loading BER=0.005 and BER=0.0012, respectively.

VI. Conclusion

In this paper, we proposed a Robust-Tilted-MIMO-THP scheme for MIMO channels based on MMSE criterion with the assumption that the imperfect CSI is available at the transmitter side. We used second-order terms of the uncertainties of the MIMO channels at the transmitter to design a robust precoder. In the proposed robust method, we minimized the transmit power by using the Tilted constellation. The power minimization will result in ICI reduction between sub-channels and in addition recovering some of the THP's power loss. Consequently, this ICI reduction and power recovering result led to BER improvement, especially at high SNRs. The transmission power is more reduced by utilizing more Tilted angles and smaller symbol block length. Also, we demonstrated that the achieved performance gain by using the proposed method is more considerable with respect to its higher complexity. Finally, we showed that we can improve our precoder performance further, by utilizing power loading.

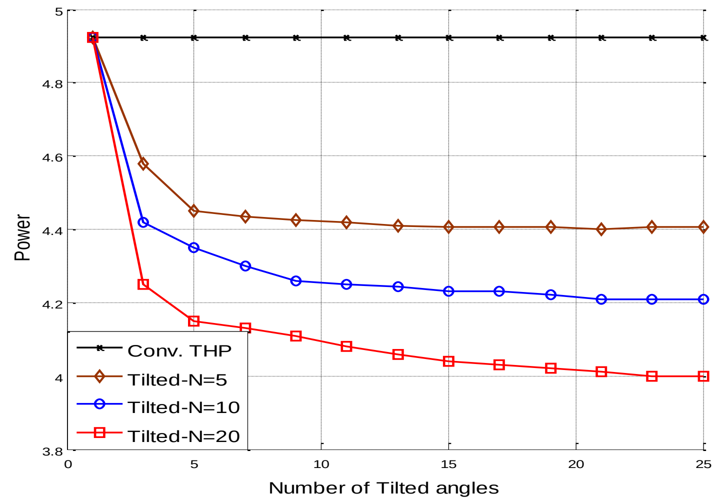

Fig. 2. Transmitted power vs. the number of Tilted angles in different block length N=5,10,20. (SNR=10dB, $n_T = n_R$ =4, 4QAM)

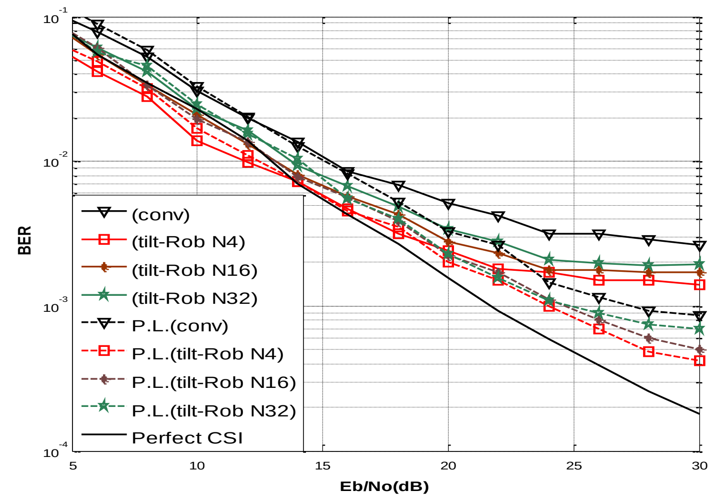

Fig. 3. BER vs. SNR and several block length in MIMO-MMSE-THP with and without Power Loading (ρ=0.05).

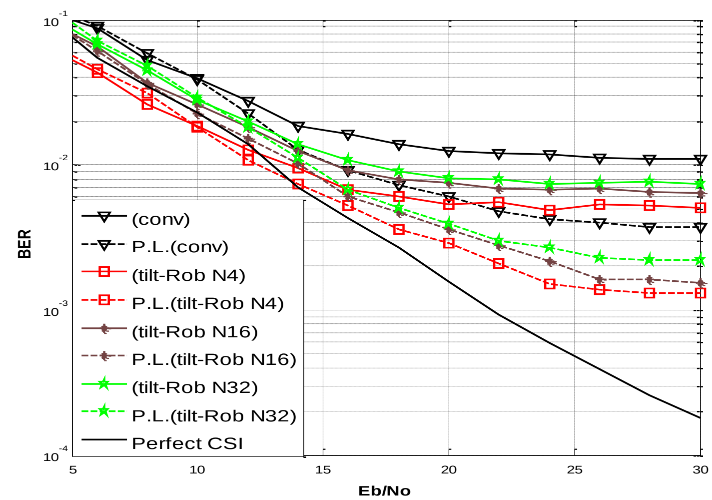

Fig. 4. BER vs. SNR and several block length in MIMO-MMSE-THP with and without Power Loading (ρ=0.1).

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