Research Article

Beamforming with Reduced Complexity in MIMO Cooperative Cognitive Radio Networks

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An approach for beamforming with reduced complexity in MIMO cooperative cognitive radio networks (MIMO-CCRN) is presented. Specifically, a suboptimal approach with reduced complexity is proposed to jointly determine the transmit beamforming (TB) and cooperative beamforming (CB) weight vectors along with antenna subset selection in MIMO-CCRN. Two multiantenna secondary users (SU) constitute the desired link, one acting as transmitter (SU TX) and the other as receiver (SU RX) and they coexist with single-antenna primary and secondary users. Some of single antenna secondary users are recruited by desired link as cooperative relay. The maximization of the achievable rates in the desired link is the objective of this work, provided to interference constraints on the primary users are not violated. The objective is achieved by exploiting transmit beamforming at SU TX, cooperation of some secondary users, and cooperative beamforming. Meanwhile, the costs associated with RF chains at the radio front end at SU RX are reduced. Through simulations, it is shown that better performance in the desired link is attained, as a result of cooperation of SUs.

1. Introduction and Related Works

The rigid structure of current spectrum allocation policies creates a bottleneck for rapidly growing wireless users. On the other hand, the Federal Communication Commission (FCC) measurements reveal that most of the licensed frequency bands are either unused or utilized less than 10% of the time [1]. To address the limitations on spectrum usage, the FCC has motivated the use of opportunistic spectrum sharing to make the licensed frequency bands accessible for unlicensed users.

Transmit beamforming with receive combining is one of the simplest approaches to achieve full diversity [2]. Compared with traditional space time codes, beamforming and combining systems provide the same diversity order as well as significantly more array gain [3] at the expense of requiring channel state information at the transmitter.

The issue of transmit beamforming in cognitive radio networks (CRN) and more specifically for secondary users has been investigated from various points of view in [4–7]. In [4], transmit beamforming (TB) is designed for MIMO cognitive radio networks in a single primary user- (PU-) single secondary user (SU) network, to minimize the transmit power of the SU while limiting the interference temperature to PU and achieving the signal-to-interference-plus-noise ratio (SINR) target at SU. The joint problem of TB and power control in CRN have been considered in [5, 6], where the objective is to optimize the SU users sum rate under the interference constraints of PUs. The joint problems of TB at transmitter and antenna subset selection at receiver of a secondary network are considered in [7], where TB is recruited at multiantenna secondary transmitter to maximize the data rates in SU link; meanwhile the interference on PUs is minimized.

The cooperative beamforming (CB) issue in cooperative cognitive radio networks (CCRN) has been discussed in a few papers such as [8–11]. In [8], a number of relays in a dual-hop amplify-and-forward cooperative scheme in cooperative cognitive radio networks (CCRN) are utilized, aiming to maximize the worst signal-to-interference-plus-noise ratio.
of the destinations. The bursty traffic in CRN has been considered in [9], where CB is exploited to access busy time slots or spatial spectrum holes. In [10], the relays comprise a distributed beamformer to beamform the signal towards its destination while maintaining the QoS in the primary user. A cooperative beamforming aided incremental relaying scheme in CRN has been proposed in [11], in which the source and relays can exploit CB to activate packet retransmission in busy time slots or spatial spectrum holes without causing interference to primary users. To the best knowledge of authors, the application of joint TB and CB in CCRN has not ever been investigated. In addition to complexity of the CCRN with joint TB and CB, another reason for not taking advantage of joint TB and CB can be attributed to unavailability of complete channel state information (CSI) at transmitter and relay side. In [12], based on Grassmannian line packing technique (GLP), a method has been presented which does not need CSI at transmitter for TB and works when a limited feedback is available from receiver to transmitter. The beamforming codebook is generated using GLP technique. The transmitter and the receiver preserve the same codebook, which contains, A, beamforming weight vectors for the Grassmannian beamforming (GB). In the GB, the index for the optimal beamforming weight vector, not the vector itself, is fed back from the receiver to the transmitter. Thus, the amount of feedback information can be reduced to \[\log(A)\] bits.

A promising way of capturing a large portion of the channel capacity in MIMO systems at reduced hardware costs and computational complexity is to select optimally a small number of the best antennas from the larger set of antennas available. The performance of systems that use such selection techniques has been shown to be significantly higher than that of the systems using the same number of antennas without any subset selection [13]. However, the number of computations required for such optimal selection grows exponentially with the total number of the antennas available. In [7], an alternative approach to receive antenna selection for capacity maximization is presented that offers near optimal performance at a complexity significantly lower than the schemes in [14]. In this paper a similar method is employed to reduce the complexity of antenna subset selection process.

The motivation of this work is to determine the optimum TB weight vector at the transmitter side, the optimum CB weight vector at relay side, and antenna subset selection in receiver side of a CCRN with reduced complexity. Meanwhile, the proposed system can be considered an effective system to be implemented in one of related standards to CRN (e.g., IEEE 802.22). Moreover, in order to incorporate the CR concept into the commercial wireless communication system, a popular proposal for physical layer is orthogonal frequency-division multiplexing-(OFDM-) based techniques, among which is multicarrier code division multiplexing (MC-CDM). This is because the MC-CDM is more suitable than any other techniques to provide flexible spectrum management, which makes the CRN effectively reuse the frequency resources.

The contributions of this work can be summarized as follows.

(i) Proposing a method with reduced complexity to determine the optimum weight vectors of transmit beamformer, cooperative beamformer, and the best set of antennas in CCRN.

(ii) Proposing an iterative method to obtain the optimum TB weight vector and optimal subset of antennas at receiver.

(iii) Determining the optimal CB weight vector in the virtual array comprised of cooperating relays to remove the interference on PUs.

The rest of the paper is organized as follows. In Section 2, system model is described and the problem is formulated. Section 3 proposes a solution for the problem formulated in Section 2. In Section 4, simulation results are presented and Section 5 concludes the paper.

Notation. Boldface uppercase is used for matrices and boldface lowercase for vectors. det(·), Tr(·), and (·)^H denote the determinant, the trace, and the conjugate transpose operators, respectively. \(I_{M}\) denotes an \(M \times M\) identity matrix. \(\mathcal{C}_{I \times M}^{2\times 2}\) denotes the space of \(I \times u\) matrices with complex entries and \(\mathcal{CN}(0, I)\) represents the distribution of a zero mean circularly symmetric complex Gaussian (ZMCSG) vector with covariance matrix \(I\). \(\text{Diag}()\) is used to represent the \(i\)th row of a matrix, \(H\), respectively. \(\text{Diag}(\cdot)\) gives the diagonal elements of a matrix.

2. System Model

2.1. System Description. The system model is depicted in Figure 1. In the considered system, a secondary network consisting of \(N_{SU} + 2\) secondary users (SUs) coexists with a primary network consisting of \(N_{PU}\) single-antenna primary users (PUs). In the secondary network, there exist two multiantenna and \(N_{SU}\) single-antenna SUs. All users use the same frequency band. However, PUs own the higher priority in using the frequency band. Two multiantenna secondary users (SU TX and SU RX) constitute the desired SU link. The core aim of this work is to maximize the data rates of the desired SU link, using TB at SU TX and CB at cooperating SUs.

The strategy of cooperation of single-antenna SUs with the desired multiantenna SU link is decode-and-forward. Other cooperation strategies can be analyzed in a similar fashion.

A few issues may arise regarding the proposed system model. First of all, the considered scenario is not an artificial one, since there may exist some multiantenna SUs in any practical cognitive radio networks (for e.g., two cognitive base stations) and also it is wise to assume the existence of ubiquitous single-antenna SUs in any cognitive radio network. Therefore, the motivation for providing an applicable solution with reduced complexity for a practical cognitive radio system is assured. Secondly, the cooperation of single-antenna SUs with the desired multiantenna SU link can be rewarded by the mutual cooperation of the multiantenna SU link with them to resolve the issue of the cooperation
incentives of the single-antenna SUs. In order to simplify the analysis in this paper, it is presumed that single-antenna SUs are motivated enough to cooperate with SU TX; that is, a kind of compensation has been provided by the MIMO SU link to keep the single-antenna SUs motivated to cooperate with the MIMO SU link. SU TX is equipped with N antennas that are preceded by a beamformer. It is assumed that channel state information (CSI) is not available at SU TX. Since it is cumbersome to provide the SU TX with the CSI, here we assume that a kind of quantized beamforming, that is, Grassmannian beamforming, is utilized as transmit beamforming scheme at SU TX. As previously stated, the SU TX and the SU RX preserve the same codebook, which contains a number of beamforming weight vectors for the Grassmannian beamforming (GB). The beamforming codebook is generated using Grassmannian line packing technique (GLP) [12]. The GB method works when a limited feedback is available from SU RX to SU TX. Therefore, the optimum TB weight vector, $w_{TB}^{opt}$, is determined at SU RX. Then the index for the optimal TB weight vector, not the vector itself, is fed back, using a few bits of limited feedback, from SU RX to SU TX. SU RX is equipped with M antennas. At SU RX m out of M antennas are selected. Also, the optimum TB weight vector is the one which maximizes the achievable rates, on condition that interference on PUs does not exceed the threshold. Moreover, zero-forcing beamforming is utilized in the virtual MIMO of cooperating SUs, in order not to disturb the existing single-antenna PUs as a result of cooperation. The channels between all nodes are assumed to experience frequency flat Rayleigh fading.

2.2. Structure of SU TX and SU RX. The structure of SU TX and SU RX is depicted in Figure 2. L data symbols on the nth antenna, $[d_{1}^{(m)} \cdots d_{M}^{(m)}]^T$, are code division multiplexed, where L denotes the number of subcarriers of the MC-CDM modulation. Orthogonal Walsh codes are used for spreading and each symbol is multiplied with a length L Walsh code before all spread replicas of the modulated symbols are summed up to form the frequency domain length L vector $x^{m} = [X_{1}^{m} \cdots X_{L}^{m}]^T$; that is, $x^{m} = C_{L}d^{(m)}$, where

$$C_{L} = \begin{pmatrix} C_{L/2} & C_{L/2} \\ C_{L/2} & -C_{L/2} \end{pmatrix}$$

is the normalized Walsh-Hadamard transformation matrix. Note that $L = 2^n$, $n \geq 1$, and $C_{L/2} = 1$. Consequently on each subcarrier we have a MIMO channel with the following input vector: $x_i = [X_{1}^{i} X_{2}^{i} \cdots X_{L}^{i}]^T$ and $1 \leq i \leq L$. The OFDM modulator on each antenna of SU TX consists of an IFFT block, parallel to serial converter followed by a cyclic prefix insertion unit and a digital to analog converter (DAC). The length of the cyclic prefix is assumed to be greater than the largest maximum excess delay over all spatial channels.

2.3. Problem Formulation. The received signal at SU RX can be written as

$$y = H_{SD}w_{TB}x + H_{RD}w_{ZFBF}\hat{x} + i + z,$$  (2)

where $w_{TB}$ and $w_{ZFBF}$ represent the transmit beamforming vector at the SU-TX and zero-forcing beamforming weight vector at the cooperating SU, respectively. $\hat{x}$ stands for the decoded signal at the cooperating SUs. $H_{SD} \in \mathbb{C}^{M \times N}$ and $H_{RD} \in \mathbb{C}^{MN \times N}$ represent the channel coefficients matrix with ZMCSG entries from SU TX to SU RX and from single-antenna SUs (virtual MIMO) to SU RX, respectively; i is interference due to primary users and z represents the white noise (where $z \sim \mathcal{CN}(0, I_{M})$ and $z \in \mathbb{C}^{N \times 1}$). Note that all channel matrices must be considered for each subcarrier, to be more accurate. However, as we aim to find the optimal set of antennas at SU RX, the optimum transmit and cooperative beamformer weight vectors are applied to all of the subcarriers regardless of different channel characteristics of different subcarriers and the dependence of all system parameters on the subcarrier index can be dropped. For simplicity we also assume that the detection process at cooperating SUs is error-free and as a result (2) can be written as

$$y = (H_{SD}w_{TB} + H_{RD}w_{ZFBF})x + i + z.$$  (3)

We assume that the total transmit power is limited to $P_{SU}$; that is,

$$Q_{SU} = E\{w_{TB,\text{opt}}^{H}w_{TB,\text{opt}}x^*x\} = w_{TB,\text{opt}}^{H}w_{TB,\text{opt}}^*P_{x} \leq P_{SU},$$  (4)

where $E\{x^*x\} = P_{x}$ and $Q_{SU}$ stands for the transmit covariance matrix at SU TX. The covariance matrix of the noise and interference at SU RX is given by

$$U = E\{|i|^2 + z^*z\} = I_{M} + H_{PS}H_{PS}^H,$$  (5)

in which $I_{M}$ is $M \times M$ identity matrix and $H_{PS}$ denotes the channel matrix from PUs to SU RX ($H_{PS} \in \mathbb{C}^{M \times N}$). Note that, in (5), it is assumed that i and z are independent random processes. For satisfactory operation of the incumbent PUs in the presence of the SU TX, interference seen at the
PU RX should not exceed a particular threshold, \( P_i \) \((i = 1, 2, \ldots, N_{PU})\):

\[
h_{SP,i}Q_{SU}H_{SP,i}^H \leq P_i, \quad i = 1, 2, \ldots, N_{PU},
\]

(6)

where \( H_{SP} \) is channel matrix from secondary user transmitter (SU TX) to primary users (PUs) and \( h_{SP,i} = H_{SP}(i,:) \) and \( h_{SP,i} \in \mathbb{C}^{1 \times N} \) denotes the \( i \)th row of \( H_{SP} \). The achievable rates of the multiantenna SU link, without cooperation of single-antenna SUs and also all available antennas at SU RX, is given by [15]

\[
R^{(1)} (S, w_{TR, opt}, P_x) = \log_2 \det \left( I_M + H_{SD}Q_{SU}H_{SD}^H U^{-1} \right).
\]

(7)

The achievable rates of the desired SU link at the output of the maximum ratio combiner at SU RX, using the cooperation of single-antenna SUs and also all available antennas at SU RX, can be written as

\[
R^{(2)} (S, w_{TR, opt}, w_{ZFBE, opt}, P_x) = \frac{1}{2} \log_2 \det \left( I_M + H_{SD}Q_{SU}H_{SD}^H U^{-1} + H_{RD}Q_{SU}' H_{RD}^H U^{-1} \right),
\]

(8)

where \( Q_{SU}' = E[w_{ZFBE} w_{ZFBE}^H X^{x^*}] = w_{ZFBE}w_{ZFBE}^H P_x \leq P_{SU} \). The coefficient \( 1/2 \) is due to the fact that cooperative transmission only uses half of the resources (e.g., time slots, frequency bands, etc.). For simplicity, the same transmit power limit, \( P_{SU} \), is assumed at cooperating SUs. Similar to the method presented in [7], we define a diagonal matrix \( S \) (where \( S \in \mathbb{R}^{M \times M} \)):

\[
(S)_{ii} = \begin{cases} 
1 & \text{ith receive antenna selected} \\
0 & \text{otherwise.} 
\end{cases}
\]

(9)

Antennas (in SU RX) that maximize the achievable data rates in the system are selected. The diagonal matrix provides us with the index of selected antennas in the SU RX. Hence, if \( m \) antennas are chosen \((m \leq M)\), new channel matrices, \( \tilde{H}_{SD} \) and \( \tilde{H}_{RD} \), with the same dimension as \( H_{SD} \) and \( H_{RD} \), will have \( M - m \) all-zero rows. Thus, the rate expression at SU RX becomes

\[
R^{(2)} (S, w_{TR, opt}, w_{ZFBE, opt}, P_x) = \frac{1}{2} \log_2 \det \left( I_M + \tilde{H}_{SD}Q_{SU}\tilde{H}_{SD}^H + \tilde{H}_{RD}Q_{SU}' \tilde{H}_{RD}^H \right),
\]

(10)

where \( \tilde{H}_{SD} = \tilde{U}^{-1/2}SH_{SD}, \tilde{H}_{RD} = \tilde{U}^{-1/2}SH_{RD}, \) and \( \tilde{U} \) is defined as follows. With the selected receive antennas we have reduced \( m \times 1 \) interference and noise vectors which give a new interference and noise covariance matrix, \( U_{\text{reduced}} \), of dimension \( m \times m \). This matrix is inflated to form \( \tilde{U} \), an \( M \times M \) matrix, by adding rows and columns of zeros corresponding to the receive antennas not selected.

The PUs are vulnerable to cooperation of single-antenna SUs with SU TX and a relaying scheme must be devised to minimize the alleged interference to other PUs. We propose to use zero-forcing beamforming in virtual antenna array, formed by single-antenna SUs, before their relaying of PU BSs signals. More specifically, this distributed beamforming scheme is aimed not only to remove the interference to other PUs, but also to maximize the data rates from single-antenna SUs (relays) to SU RX. Therefore, it can also be considered as cooperative beamforming. The distributed beamforming can be realized by a virtual antenna array, which can be created by a set of relays in cooperative relaying networks [16].

Figure 2: Structure of SU TX and SU RX.
With this notation, the problem of joint transmit beamforming, cooperative beamforming, and antenna selection can be mathematically explained by

\[
P: \max \frac{1}{2} \log_2 \det \left( \mathbf{I}_M + \mathbf{H}_{SD} \mathbf{Q}_{SU} \mathbf{H}_{SD}^H \mathbf{U}^{-1} + \mathbf{H}_{RD} \mathbf{Q}_{SU} \mathbf{H}_{RD}^H \mathbf{U}^{-1} \right)
\]

subject to

\[
\begin{align*}
\mathbf{S}_{ji} & \in \{0, 1\}, \quad i, j = 1, 2, \ldots, M \\
\mathbf{H}_{SD} \mathbf{Q}_{SU} \mathbf{H}_{SD}^H & \leq \frac{P_{SU}}{P_x} \mathbf{I}_M \\
\mathbf{H}_{RD} \mathbf{Q}_{SU} \mathbf{H}_{RD}^H & \leq \frac{P_{SU}}{P_x} \mathbf{I}_M \\
\mathbf{S} & = m \\
P_x \mathbf{h}_{SP} \mathbf{w}_{TB} \mathbf{H}_{TB}^H \mathbf{h}_{SP} & \leq P_x, \quad i = 1, 2, \ldots, N_{PU} \\
|h_{RP} \mathbf{w}_{ZFBF}| & = 0, \quad i = 1, 2, \ldots, N_{PU}
\end{align*}
\]

where \( h_{RP,i} \), \( i = 1, 2, \ldots, N_{PU} \), denotes the channel coefficients from single-antenna SUs to other single-antenna PUs.

### 3. A Suboptimal Method to Determine the Optimum TB and CB Weight Vectors and Best Antennas

In problem P, four unknown variables must be determined jointly. Due to excessive complexity, in this section we propose a suboptimum solution with reduced complexity to solve P. In this way, problem P is decomposed into two problems, P1 and P2, according to the following:

**P1:** \( \max \log_2 \det \left( \mathbf{I}_M + \mathbf{H}_{SD} \mathbf{Q}_{SU} \mathbf{H}_{SD}^H \mathbf{U}^{-1} \right) \)

subject to

\[
\begin{align*}
\mathbf{S}_{ji} & \in \{0, 1\}, \quad i, j = 1, 2, \ldots, M \\
\mathbf{H}_{SD} \mathbf{Q}_{SU} \mathbf{H}_{SD}^H & \leq \frac{P_{SU}}{P_x} \mathbf{I}_M \\
\mathbf{S} & = m \\
P_x \mathbf{h}_{SP} \mathbf{w}_{TB} \mathbf{H}_{TB}^H \mathbf{h}_{SP} & \leq P_x, \quad i = 1, 2, \ldots, N_{PU} \\
|h_{RP} \mathbf{w}_{ZFBF}| & = 0, \quad i = 1, 2, \ldots, N_{PU}
\end{align*}
\]

**P2:** \( \max \| \mathbf{H}_{RD} \mathbf{w}_{ZFBF} \|^2 \)

subject to

\[
\begin{align*}
\mathbf{H}_{SD} \mathbf{Q}_{SU} \mathbf{H}_{SD}^H & \leq \frac{P_{SU}}{P_x} \mathbf{I}_M \\
|h_{RP} \mathbf{w}_{ZFBF}| & = 0, \quad i = 1, 2, \ldots, N_{PU}
\end{align*}
\]

Moreover, in problem P2, the received signal power at SU RX due to cooperation of single-antenna SUs is aimed to be maximized, instead of achievable data rates at SU RX. This facilitates the finding the optimal zero-forcing beamforming weight vector, as will be discussed soon. In the sequel, possible ways of solving the above problems are discussed.

#### 3.1. Solving P1

**3.1.1. Exhaustive Search.** A straightforward way to solve P1 is to perform an exhaustive search (ES) over all possible combinations of antenna elements and transmit beamforming weight vectors and optimize over \( P_x \). Hence, ES amounts to optimizing \( P_x \) in \( M \times \binom{K}{2} \) times subject to interference and total transmit power constraints, where \( K \) denotes the number codewords in the transmit beamforming codebook matrix. Each optimization of \( P_x \) can be considered a convex problem. However, the need to iterate through all the possible combinations gives a complexity which explodes for higher dimensional systems.

**3.1.2. Convex Approximation.** Problem P1 is highly nonconvex and can be classified as an example of an integer programming problem, since matrix S has only binary elements [17]. The nonconvexity of the problem arises due to the nature of the objective function, interference, and binary constraints. Further, the binary variable renders the problem NP-hard problem. In order to obtain a more computationally efficient approach, we modify the problem in the following way. The binary structure of S can be relaxed so that the antenna selection variable takes on values in the interval 0 to 1. This makes the problem far easier to solve than the original integer program. Finally we note that in this approach the effect of the U matrix cannot be included. This limitation is discussed below. With these changes, P1 can be written as

\[
P3: \max \log_2 \det \left( \mathbf{I}_M + P_x \mathbf{S} \mathbf{H}_{SD} \mathbf{w}_{TB} \mathbf{H}_{TB}^H \mathbf{H}_{SD} \mathbf{S}^H \right)
\]

subject to

\[
\begin{align*}
0 \leq \mathbf{S}_{ji} & \leq 1, \quad i, j = 1, 2, \ldots, M \\
\mathbf{H}_{SD} \mathbf{Q}_{SU} \mathbf{H}_{SD}^H & \leq \frac{P_{SU}}{P_x} \mathbf{I}_M \\
\mathbf{S} & = m \\
P_x \mathbf{h}_{SP} \mathbf{w}_{TB} \mathbf{H}_{TB}^H \mathbf{h}_{SP} & \leq P_x, \quad i = 1, 2, \ldots, N_{PU} \\
|h_{RP} \mathbf{w}_{ZFBF}| & = 0, \quad i = 1, 2, \ldots, N_{PU}
\end{align*}
\]

Note that problem P3 is still nonconvex, due to nonconcavity of the objective function. Thus, we seek a convex approximation (CA) to this problem.

**Proposition 1.** With two of the three variables known, the cost function in problem P3 is concave in the third one and this renders the problem convex in this variable.

**Proof.** Three different cases must be investigated as follows.

(i) When \( S \) and \( P_x \) are known and \( w_{TB} \) is unknown.
(ii) When $w_{TB}$ and $P_x$ are known and $S$ must be specified.
(iii) When $w_{TB}$ and $S$ are known and $P_x$ must be found.

Case 1 (when $S$ and $P_x$ are known and $w_{TB}$ is unknown). In this case, the objective function can be written as

$$g (W_{TB}) = \log_2 \det \left(I_M + P_x S H_{SU} W_{TB} H_{SU}^H S^H \right),$$

where $W_{TB} = w_{TB} w_{TB}^H$ and, considering the constraints, there is no difference between maximizing $W_{TB}$ and $W_{TB}$. According to [17], $g(W_{TB})$ is concave, if and only if $f(t) = g(t W(1)_{TB} + (1-t) W(2)_{TB})$ is concave for every feasible $W(1)_{TB}, W(2)_{TB}$, and $0 \leq t \leq 1$. Therefore $f(t)$ can be written as

$$f(t) = \log_2 \det \left(I_M + P_x S H_{SU} \right) \times (W(2)_{TB} + t (W(1)_{TB} - W(2)_{TB}) ) H_{SU}^H S^H \right),$$

where $A = I_M + P_x S H_{SU} W(2)_{TB} H_{SU}^H S^H$ and $B = P_x S H_{SU} (W(1)_{TB} - W(2)_{TB}) H_{SU}^H S^H$. $f(t)$ can be further manipulated as

$$f(t) = \log_2 \det \left(A^{1/2} (I_M + t A^{-1/2} B A^{-1/2} ) A^{1/2} \right)$$

$$= \sum_{i=1}^{n} \log_2 (1 + t \lambda_i) + \log_2 \det A.$$  

In which $\lambda_1, \ldots, \lambda_n$ denote the eigenvalues of $A^{-1/2} B A^{-1/2}$. Hence,

$$\frac{df(t)}{dt} = \frac{1}{\ln 2} \sum_{i=1}^{n} \frac{\lambda_i}{1 + t \lambda_i},$$

$$\frac{d^2 f(t)}{dt^2} = -\frac{1}{\ln 2} \sum_{i=1}^{n} \frac{\lambda_i^2}{(1 + t \lambda_i)^2}.$$  

Negativity of $d^2 f(t)/dt^2$ results in concavity of $f(t)$ and concavity of $g(W)$ is concluded.

Case 2 ($P_x$ and $w_{TB}$ known and $S$ unknown). First of all, the objective function must be manipulated as

$$\log_2 \det \left(I_M + P_x S H_{SU} w_{TB} w_{TB}^H H_{SU}^H S^H \right) = \log_2 \det \left(1 + P_x w_{TB} w_{TB}^H H_{SU}^H S^H S H_{SU} w_{TB} \right).$$

And, by introducing $S^H S$, the concavity of the objective function is proved in a similar method employed in Case 1.

Case 3 ($S$ and $w_{TB}$ known and $P_x$ unknown). In this case, the objective function can be written as

$$g(P_x) = \log_2 \det \left(I_M + P_x A \right),$$

where $A = S H_{SU} w_{TB} w_{TB}^H H_{SU}^H S^H$. Once again, $f(t)$ is defined as $g(t P(1)_{x} + (1-t) P(2)_{x})$, where $P(1)_{x}$ and $P(2)_{x}$ are any feasible amounts for $P_x$ and $0 \leq t \leq 1$:

$$f(t) = \log_2 \det \left(I_M + (P(2)_{x} + t (P(1)_{x} - P(2)_{x}) ) A \right)$$

$$= \log_2 \det (B + tC),$$

where $B = I_M + P(2)_{x}$ and $C = (P(1)_{x} - P(2)_{x}) A$ and the concavity of the $g(P_x)$ is derived in a similar manner as employed in previous cases.

Thus, to solve P3, we initialize $P_x$ and $w_{TB}$ and optimize over $S$. Then, using the optimum $S$ and the initial value for $P_x$, optimum transmit beamforming weight vector is chosen and ultimately, with $S$ and $w_{TB}$ known, optimum value for transmit power, $P_x$, is obtained. Since elements of $S$ are nonbinary, index of chosen antennas are the $m$-largest diagonal elements of $S$. In the following steps, the proposed procedure for solving the problem is summarized.

A comment on the convergence of the proposed iterative algorithm is in order here. Using a similar approach to [18], we argue that during the $(k+1)^{th}$ iteration we calculate $S^{(k+1)} = \arg \max_x P3(S, w_{TB}^{(k)}, P_x^{(k)})$ and obtain achievable data rate $r_1$.

Then we calculate $w_{TB}^{(k+1)} = \arg \max_{w_{TB}} P3(w_{TB}, w_{TB}^{(k)}, P_x^{(k)})$, $S^{(k+1)}$ giving rate $r_2$. Finally, we evaluate $P_x^{(k+1)} = \arg \max_{P_x} P3(P_x, S^{(k+1)}, w_{TB}^{(k+1)})$ and the corresponding achievable data rate $r_3$. Since $r_1 \leq r_2 \leq r_3$ forms a monotonically increasing sequence which is bounded above (due to transmit power constraint) we conclude that the sequence of achievable data rates converges to a limit. Our simulations indicate that iterating 6 times is almost sufficient to attain an optimum value of P3. Since the problem P3 is not convex in nature, the maximum achievable rates obtained from the proposed iterative rate may not be globally optimum. However, results suggest that the values obtained are robust and are globally optimal most of the time for the parameters and scenario discussed in Section 2.

**Proposed Iterative Method to Solve P1**

**Step 1 (initializations).**

1. Select an initial value for $P_x$ and $P_{SU}$.
2. Select an Initial value for $w_{TB}$ (columns of codebook matrix) which satisfies $\text{Tr}(w_{TB} H_{TB}^H) \leq P_{SU}/P_x$.

**Step 2 (calculation step).**

1. Solve the convex optimization problem and find $S$.
2. Find the optimum $w_{TB}$ which maximizes achievable rates or SNR (depending on the scenario selected) using exhaustive search method.
3. Solve the resultant convex optimization problem, knowing $S$ and $w_{TB}$, to calculate the optimum $P_x$. 

Step 3 (iterative step).

(1) Repeat the Steps 1 and 2 until convergence. The achievable rate at any SNR will be the average of results at that specific SNR.

(2) Using the optimum values of transmit power in each SNR, \( P_x \), find the symbol error rate (using Monte Carlo method).

3.2. Solving P2. After obtaining the optimum value of \( P_x \) and \( S \), it is time to calculate the optimum zero-forcing beamforming weight vector. The cooperating SUs are assumed to be cognitive in the sense that they can obtain the channel state information (CSI) on the channels from themselves to PUs. The objective function of P2 can be written as

\[
\|H_{RD}w_{ZFBF}\|^2 = \sum_{i=1}^{m} |h_{RD,i}w_{ZFBF}|^2, \tag{21}
\]

where \( h_{RD,i} = H_{RD}(i,:) \) denotes the \( i \)th row of \( H_{RD} \), matrix and \( m \) represents the number of selected antennas at SU RX.

Theorem 2. The optimal zero-forcing (ZF) beamforming weight vector which maximizes \( \|H_{RD}w_{ZFBF}\|^2 \) and satisfies the constraints of the problem P2 is one of the orthogonal projections of rows of \( H_{RD} \) onto the orthogonal complementary \( y^\perp \) of the subspace \( y = \text{span}\{h_{CP}^{(2)}, \ldots, h_{CP}^{(N_{PU})}\} \) which maximizes \( \sum_{i=1}^{m} |h_{RD,i}w_{ZFBF}|^2 \). To satisfy the cooperative transmit power constraint, the elements of the optimum ZFBF weight vector have to be multiplied to \( \sqrt{(P_{SU})/P_x |\text{Tr}(w_{ZFBF}w_{ZFBF}^H)|} \).

Proof. The channel vector \( h_{RD,i} \in C^{1 \times N_{SU}} \), \( i = 1, \ldots, m \), can be written as \( h_{RD,i} = a_k^{(i)} e_1 + a_k^{(i)} e_2 + \ldots + a_k^{(i)} e_{N_{SU}} \),

using the \( N_{SU} \) basis vectors \( \{e_1, e_2, \ldots, e_{N_{SU}}\} \) (where \( a_k^{(i)} \sim \text{CN}(0,1), k = 1, 2, \ldots, N_{SU}, i = 1, \ldots, m \) \[19\]). Hence, the matrix \( [e_1, e_2, \ldots, e_{N_{SU}}] = \text{I}_{N_{SU}} \) is \( N_{SU} \times N_{SU} \) identity matrix. Then, we consider the set of basis vectors \( \{\hat{e}_1, \hat{e}_2, \ldots, \hat{e}_{N_{SU}}\} \), which is the orthogonal basis for the subspace \( y = \text{span}\{h_{RP,1}, \ldots, h_{RP,N_{SU}}\} \). Actually, \( y \) is a \( N_{PU} \)-dimensional subspace, for the reason that the probability of the realizations of the independent and continuous random vectors \( h_{RP,1}, \ldots, h_{RP,N_{SU}} \) being interrelated is very much small and thus can be ignored. If \( \{\hat{e}_{N_{SU}+1}, \hat{e}_{N_{SU}+2}, \ldots, \hat{e}_{N_{SU}}\} \) is an orthogonal basis for \( y^\perp \), the orthogonal set \( \{\hat{e}_1, \hat{e}_2, \ldots, \hat{e}_{N_{SU}}\} ) \cup \{\hat{e}_{N_{SU}+1}, \hat{e}_{N_{SU}+2}, \ldots, \hat{e}_{N_{SU}}\} \) is another orthogonal basis for \( C^{1 \times N_{SU}} \). Similarly, \( h_{RD,i} \) can be represented by \( h_{RD,i} = b_k^{(i)} \hat{e}_1 + b_k^{(i)} \hat{e}_2 + \ldots + b_k^{(i)} \hat{e}_{N_{SU}} \). Clearly, \( L = [\hat{e}_1, \hat{e}_2, \ldots, \hat{e}_{N_{SU}}] \) is also a unitary matrix. Moreover, by matrix manipulation we have

\[
[b_k^{(i)}, b_k^{(i)}, \ldots, b_k^{(i)}]^T = LH[d_k^{(i)}, a_k^{(i)}, \ldots, a_k^{(i)}]^T. \tag{22}
\]

Since the random matrix \( L^H \) is unitary and independent with \( h_{RP,i} \), \( d_k^{(i)}, a_k^{(i)}, \ldots, a_k^{(i)} \) has the same distribution as \( [b_k^{(i)}, b_k^{(i)}, \ldots, b_k^{(i)}] \), from (A.22) in \[20\]. As a result,

A comparison between the computational complexity of the optimal and proposed suboptimal methods is presented here. If you choose to solve the problem in (11) using the dual method as an optimal algorithm, \( (2N_{PU} + 3) \) dual variables are updated in every iteration. Using these values, \( M^2 \) function evaluations are performed to find the power allocation. Then, the optimal solution has a complexity of \( O(T(M^2N_{PU})) \), where \( T \) is the number of iterations required to converge, which is usually high \[17\]. In the proposed scheme, every subcarrier in the source side requires not more than \( MN_{PU} \) function evaluations; therefore the complexity of the proposed algorithm is \( O(T(MN_{PU})) \), where \( T \) is the number of

![Figure 3: Achievable data rates versus SNR for different numbers of cooperating SUs, different values of \( \alpha \), and different numbers of selected antennas at SU RX.](image-url)
iterations required to converge for the suboptimal algorithm and is much smaller than $T$, according to simulations.

### 4. Simulation Results

In this section we explain the simulation results based on the proposed solutions for problems $P_1$, $P_2$, and $P_3$. However, before we elaborate the results, we introduce a parameter, $\alpha$, which controls the interference threshold at the PUs. $\alpha$ is chosen so that allowable interference at the PUs is a fraction of PU SNR; that is, $P_i = \alpha \text{SNR}_{\text{PU}}$ at the PUs. To compare the different approaches, we use the measures of achievable rates and symbol error rates. These are the assumptions for the simulations:

(i) achievable rates are determined by averaging over the results obtained from 1000 i.i.d. channels realizations;

(ii) the number of subcarriers of the MC-CDM modulation is assumed to be 16;

**Figure 4:** The effect of cooperative beamforming (zero-forcing beamforming) on the achievable rates of the system.

**Figure 5:** Symbol error rates versus SNR for different values of $\alpha$ and $N_{SU}$.

**Figure 6:** Achievable data rates versus $N_{SU}$ in the multiantenna SU link for different values of $\alpha$ and SNR, when 3 out of 6 antennas are selected at SU RX and CB is employed at cooperating SUs. For dashed lines, $\alpha = 0.2$ and, for solid lines, $\alpha = 0.1$.

**Figure 7:** Comparison between the performance of convex approximation and exhaustive search method.
(iii) CVX package is used along with MATLAB for simulations [21];
(iv) the SU RX is equipped with 6 antennas, while 3 antennas exist at SU TX. The codebook matrix, whose columns are potential transmit beamformer weight vectors, for three antennas at SU TX and three limited feedback bits, that is, eight possible TB weight vectors, is given by [12]:

\[
K = \begin{bmatrix}
1 & 1 & 0 & e^{2\pi j/3} & 1 & e^{4\pi j/3} & 1 & e^{6\pi j/3} & 0 \\
1 & 1 & 0 & 0 & e^{2\pi j/3} & 0 & e^{4\pi j/3} & 0 & 1 \\
0 & 1 & 0 & 0 & e^{2\pi j/3} & 1 & 0 & e^{4\pi j/3} & 0
\end{bmatrix}
\]  \quad (24)

Achievable data rates for different values of \(\alpha\), different numbers of cooperating SUs, and different numbers of antennas at SU RX are presented in Figure 3. CA method is recruited to solve problem P1 and also cooperative beamforming is implemented on virtual MIMO of cooperating SUs. It can be implied from Figure 3 that utilizing the cooperation of single-antenna SUs, larger data rates can be attained. For example, by recruiting 10 SUs as relay and for \(\alpha = 0.1\), achievable data rate is larger than the case with no cooperative SUs and \(\alpha = 0.3\). Note that in both cases, 3 out of 6 available antennas are selected at SU RX. As depicted in Figure 3, even if PUs are able to endure an amount of interference as equal as their receive SNR, that is, \(\alpha = 1\), and also all antennas at SU RX are employed (3 \times 6 MIMO), data rates of the multiple-antenna SU link are far less than the data rates achieved using 20 cooperating SUs. Moreover, the achievable data rates in a 3 \times 6 MIMO system, that is, there are no PUs and thus no power constraint exists, has also been depicted.

In Figure 4, we demonstrate that CB not only removes the interference on PUs, due to cooperation of single-antenna SU with desired SU link, but also leads to an increase in the achievable rates of the desired link. Note that, for all graphs in Figure 4, \(\alpha = 0.1\).

In Figure 5 we consider the effects of TB, CB, and antenna selection on the symbol error rates (SER) of the multi-antenna SU link. CA method is employed to solve problem P1 and 3 out of 6 antennas at SU RX are selected. Clearly, the SER in the multi-antenna SU link diminishes as a result of cooperation of single-antenna SUs. In Figure 6, we plot the achievable data rates versus \(N_{SU}\) in the multi-antenna SU link for different values of and SNR. Also, 3 out of 6 antennas are selected at SU RX and CB is employed at cooperating SUs. It can be implied from Figure 6 that recruiting many cooperative SUs would not result in expectable increase in data rates.

A comparison between the achievable rates of the multi-antenna SU link, using the CA and ES as suboptimal methods and the optimal approach (OA) (using the dual method) has been performed in Figure 7. Evidently, the proposed suboptimal method has near-optimal performance, while the proposed CA method performs very close to ES method, which is very promising.

5. Conclusions

Jointly determining the optimum TB and CB weight vectors and antenna selection in the MIMO-CCRN was considered in this work. The scenario consists of 2 multiantenna SUs and a number of single-antenna SUs and PUs. The problem was formulated and, to achieve a computationally efficient solution with much less complexity, we utilized convex approximation method. It was shown through simulations that using the proposed method, a rather complex problem can be solved with reduced complexity. Furthermore, taking advantage of cooperation of single-antenna secondary users, better data rates are achieved, without imposing intolerable interference on primary users.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References


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