Research Article

Ranking All DEA-Efficient DMUs Based on Cross Efficiency and Analytic Hierarchy Process Methods

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The aim of this paper is to present an original approach for ranking of DEA-efficient DMUs based on the cross efficiency and analytic hierarchy process (AHP) methods. The approach includes two basic stages. In the first stage using DEA models the cross efficiency value of each DEA-efficiency DMU is specified. In the second stage, the pairwise comparison matrix generated in the first stage is utilized to rank scale of the units via the one-step process of AHP. The advantage of this proposed method is its capability of ranking extreme and nonextreme DEA-efficient DMUs. The numerical examples are presented in this paper and we compare our approach with some other approaches.

1. Introduction

Data envelopment analysis (DEA) is a tool for evaluation and measuring of the efficiency of a set of decision making units (DMUs) that consume multiple inputs and produce multiple outputs, first introduced by Charnes, Cooper, and Rhodes (CCR) [1] and extended by Banker, Charnes, and Cooper (BCC) [2]. DEA is used to establish a best practice group from among a set of observed units and to identify the units that are inefficient when compared to the best practice group [3–6]. One important issue in DEA which has been studied by many DEA researchers is to discriminate between efficient DMUs. Several authors have proposed methods for ranking the best performers ([7–14] among others). Perhaps the most widely known and applied ranking method is the super-efficiency DEA model. Developed by Andersen and Petersen [13], the main idea of this approach is to evaluate DMU after this performer itself is excluded from the reference set. The problem with super-efficiency DEA is that under certain conditions infeasibility occurs which limits the applicability of the technique (for details see [15]). Wu [4] utilized the concept of fuzzy for ranking of DMUs in the traditional DEA models. For this purpose, firstly the DMUs are evaluated by the CCR and the cross efficiency models. Secondly, a fuzzy preference relation is established. Finally, a priority vector

of the preference is constructed and is used for ranking DMUs. Jahanshahloo et al. [8] proposed a ranking system based on changing reference set. In the proposed ranking system, the evaluation for efficient DMUs is dependent on the efficiency changes of all inefficient units due to its absence in the reference set while the estimate for inefficient DMUs depends on the influence of the exclusion of each efficient unit from the reference set. One popular method to rank multiple alternatives with respect to multiple criteria is analytical hierarchy process (AHP) method developed by Saaty [16]. The analytic hierarchy process is a multicriteria decision making technique and was used for complete ranking of DMUs by Sinuany-Stern et al. [17] and Jablonsky [18, 19] and references cited therein. Sinuany-Stern et al. [17] presented an AHP/DEA methodology for ranking DMUs, which integrates DEA and the AHP. In this AHP/DEA methodology, DEA was first run for each pair of DMUs separately to create a pairwise comparison matrix, which was then solved using the eigenvector method to produce a full ranking for all DMUs, efficient and inefficient. Their method has some problems, such as illogic comparing of two DMUs in a DEA model (see [20] for details). Alirezaee and Sani [20] extended Sinuany-Stern's approach and proposed a new two-stage AHP/DEA methodology for ranking DMUs that removes these problems. They used together with the efficiency of

units the influence rate of each unit to the other units in order to determine the elements of pairwise comparison matrix. Jablonsky [18] used an AHP model with interval pairwise comparisons for the evaluation and classification of efficient units and compares the results with super-efficiency DEA scores. In [19], Jablonsky presented an original procedure for ranking of DMUs in DEA models based on combination of AHP and DEA principles. He/she considered the DMUs as alternatives and the ratios of output/input as criteria. An extensive review of ranking models in DEA is given in Adler et al. [12]. The aim of this paper is to propose a method for ranking extreme and nonextreme efficient DMUs using cross efficiency and AHP methods. Our approach includes two stages. In the first step using DEA models the cross efficiency value of each DEA-efficiency DMU is specified. In the second stage, the pairwise comparison matrix generated in the first step is utilized to rank scale of the units via the AHP. This paper is organized as follows. Section 2 presents some basic DEA models and reviews some ranking method. In Section 3 we briefly review the AHP. Section 4 gives a brief introduction to the cross efficiency evaluation. The new ranking method for DEA-efficient DMUs is developed in Section 5. Numerical examples are examined in Section 6. The paper is concluded in Section 7.

2. Data Envelopment Analysis Models

Consider a set of n DMUs which is associated with m inputs and s outputs. Particularly, DMU _{j} ($j \in J = 1, \dots, n$) consumes amount x_{ij} of input i and produces amount y_{rj} of output r . Let $X_j = (x_{1j}, \dots, x_{mj})$ in which $X_j \geq 0$ and $X_j \neq 0$ and $Y_j = (y_{1j}, \dots, y_{sj})$ in which $Y_j \geq 0$ and $Y_j \neq 0$. The production possibility set (PPS) of CCR model is defined as follows:

$$T_{\text{CCR}} = \left\{ (X, Y) \mid X \geq \sum_{j=1}^n \lambda_j X_j, Y \leq \sum_{j=1}^n \lambda_j Y_j, \lambda_j \geq 0, j \in J \right\} \quad (1)$$

and similarly the production possibility set of BCC model is defined as follows:

$$T_{\text{BCC}} = \left\{ (X, Y) \mid X \geq \sum_{j=1}^n \lambda_j X_j, Y \leq \sum_{j=1}^n \lambda_j Y_j, \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0, j \in J \right\}. \quad (2)$$

The input-oriented BCC and input-oriented CCR models, corresponding to DMU _{p} , $p \in J$, are given by (3) and (4), respectively:

$$\begin{aligned} \min \quad & \theta - \epsilon \left(\sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \right) \\ \text{s.t.} \quad & \sum_{j \in J} \lambda_j y_{rj} - s_r^+ = y_{rp}, \quad r = 1, \dots, s \\ & \sum_{j \in J} \lambda_j x_{ij} + s_i^- = \theta x_{ip}, \quad i = 1, \dots, m \end{aligned}$$

$$\begin{aligned} \sum_{j \in J} \lambda_j &= 1 \\ \lambda_j &\geq 0, \quad j \in J \\ s_i^- &\geq 0, \quad i = 1, \dots, m \\ s_r^+ &\geq 0, \quad r = 1, \dots, s \\ \theta &\text{ free} \end{aligned} \quad (3)$$

$$\begin{aligned} \min \quad & \theta - \epsilon \left(\sum_{i=1}^m t_i^- + \sum_{r=1}^s t_r^+ \right) \\ \text{s.t.} \quad & \sum_{j \in J} \lambda_j x_{ij} + t_i^- = \theta x_{ip}, \quad i = 1, \dots, m \\ & \sum_{j \in J} \lambda_j y_{rj} - t_r^+ = y_{rp}, \quad r = 1, \dots, s \\ & \lambda_j \geq 0, \quad j \in J \\ & t_i^- \geq 0, \quad i = 1, \dots, m \\ & t_r^+ \geq 0, \quad r = 1, \dots, s \\ & \theta \text{ free}, \end{aligned} \quad (4)$$

where ϵ is non-Archimedean small and positive number and s_i^- , s_r^+ , t_i^- , and t_r^+ , $i = 1, \dots, m$, $r = 1, \dots, s$, are called slack variables belonging to $\mathbb{R}^{\geq 0}$. Note that s_i^- and t_i^- represent input excesses; also s_r^+ and t_r^+ represent output shortfalls. Also θ and λ_j , $j \in J$, are real numbers and $\lambda_j \in \mathbb{R}^{\geq 0}$. Models (3) and (4) are called envelopment forms (with non-Archimedean number).

DMU _{p} is said to be *strong efficient* (CCR-efficient) if and only if $\theta^* = 1$ and $\mathbf{t}^{+*} = \mathbf{0}$ and $\mathbf{t}^{-*} = \mathbf{0}$. Where the superscript (*) indicates optimality. In similar manner the BCC-efficient DMUs can be defined.

The dual of model (4) (without ϵ , i.e., $\epsilon = 0$), which is called multiplier form (corresponding to DMU _{k}), is as follows:

$$\begin{aligned} \max \quad & \theta_{kk} = \sum_{r=1}^s u_{rk} y_{rk} \\ \text{s.t.} \quad & \sum_{r=1}^s u_{rk} y_{rj} - \sum_{i=1}^m v_{ik} x_{ij} \leq 0, \quad j = 1, \dots, n \\ & \sum_{i=1}^m v_{ik} x_{ik} = 1, \\ & u_{rk} \geq 0, \quad r = 1, \dots, s \\ & v_{ik} \geq 0, \quad i = 1, \dots, m. \end{aligned} \quad (5)$$

The AP model is as follows [13]:

$$\begin{aligned} \min \quad & \theta \\ \text{s.t.} \quad & \sum_{j \in J - \{p\}} \lambda_j y_{rj} \geq y_{rp}, \quad r = 1, \dots, s \\ & \sum_{j \in J - \{p\}} \lambda_j x_{ij} \leq \theta x_{ip}, \quad i = 1, \dots, m \\ & \lambda_j \geq 0, \quad j \in J - \{p\} \\ & \theta \text{ free.} \end{aligned} \quad (6)$$

Jahanshahloo's method corresponding inefficient DMU_a is as follows [8]:

$$\begin{aligned}
\min \quad & \theta - \epsilon \left(\sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \right) \\
\text{s.t.} \quad & - \sum_{j \in J - \{b\}} \lambda_j x_{ij} + \theta x_{ia} - s_i^- = 0, \quad i = 1, \dots, m \\
& \sum_{j \in J - \{b\}} \lambda_j y_{rj} - s_r^+ = y_{ra}, \quad r = 1, \dots, s \\
& \lambda_j \geq 0, \quad j \in J - \{b\} \\
& s_i^- \geq 0, \quad i = 1, \dots, m \\
& s_r^+ \geq 0, \quad r = 1, \dots, s \\
& \theta, \text{ free.}
\end{aligned} \tag{7}$$

The efficiency of strong efficiency DMU_b will be denoted by Ω and will be given by

$$\Omega_b = \frac{\sum_{a \in J_n} \partial_{a,b}}{\tilde{n}}, \tag{8}$$

in which J_n is the set of nonstrong efficiency DMUs and $\tilde{n}$ is the number of nonstrong efficiency DMUs.

Jahanshahloo et al. [21] used l_1 -norm in order to rank the extremely efficient DMUs in DEA models with constant and variable returns to scale, and the proposed method can remove the difficulties arising from AP and MAJ [11] models. Their proposed model is as follows:

$$\begin{aligned}
\min \quad & \Gamma_p^c(X, Y) = \sum_{i=1}^m |x_i - x_{ip}| + \sum_{r=1}^s |y_r - y_{rp}| \\
\text{s.t.} \quad & \sum_{j \in J - \{p\}} \lambda_j x_{ij} \leq x_i, \quad i = 1, \dots, m \\
& \sum_{j \in J - \{p\}} \lambda_j y_{rj} \geq y_r, \quad r = 1, \dots, s \\
& x_i \geq 0, \quad i = 1, \dots, m \\
& y_r \geq 0, \quad r = 1, \dots, s \\
& \lambda_j^p \geq 0, \quad j \in J - \{p\}.
\end{aligned} \tag{9}$$

As mentioned earlier, Sinuany-Stern's approach [17] is a two-stage model for fully ranking model. In the first stage Sinuany-Stern run the following CCR model for each pair of DMUs A and B:

$$\begin{aligned}
\max \quad & E_{AA} = \sum_{r=1}^s u_r y_{rA} \\
\text{s.t.} \quad & \sum_{r=1}^s u_r y_{rB} - \sum_{i=1}^m v_i x_{iB} \leq 0, \quad j = 1, \dots, n \\
& \sum_{i=1}^m v_i x_{iA} = 1,
\end{aligned}$$

$$\begin{aligned}
& \sum_{r=1}^s u_r y_{rA} \leq 1, \\
& u_r \geq 0, \quad r = 1, \dots, s \\
& v_i \geq 0, \quad i = 1, \dots, m.
\end{aligned} \tag{10}$$

E_{AA} is the efficiency score of unit A. The cross efficiency of DMU A is as follows:

$$E_{BA} = \frac{\sum_{r=1}^s u_r^* y_{rB}}{\sum_{i=1}^m v_i^* x_{iB}} \tag{11}$$

in which u_r^* and v_i^* are the optimal weights of model (10). In the similar manner E_{AB} and E_{AA} can be computed. The pairwise comparison matrix $\mathbf{A} = [a_{ij}]$ ($i, j = 1, \dots, n$) is constructed as follows:

$$a_{ij} = \frac{E_{ii} + E_{ij}}{E_{jj} + E_{ji}}, \quad a_{ii} = 1. \tag{12}$$

In the second stage using a one-step process (Step 3 in Section 3), the vector weights of the pairwise comparison matrix $\mathbf{A}$ are computed. Based on these weights, the efficient and inefficient DMUs are ranked.

Alirezaee and Sani [20] proposed a two-stage AHP/DEA methodology for ranking DMUs. In the first stage they create the pairwise comparison matrix as follows.

First, using CCR model (4) the relative efficiency of each DMU is determined. Then, each DMU is evaluated by the following model:

$$\begin{aligned}
\min \quad & \theta \\
\text{s.t.} \quad & \sum_{j=1, j \neq k}^n \lambda_j y_{rj} \geq y_{rk}, \quad r = 1, \dots, s \\
& \sum_{j=1, j \neq k}^n \lambda_j x_{ij} \leq \theta x_{ik}, \quad i = 1, \dots, m \\
& \lambda_j \geq 0, \quad j = 1, \dots, n, \quad j \neq k \\
& \theta \text{ free.}
\end{aligned} \tag{13}$$

This model measures efficiency DMU_p after excluding DMU_k from PPS. The pairwise comparison matrix $\mathbf{A}$ is constructed as follows:

$$\mathbf{A} = [a_{ij}], \quad a_{ij} = \frac{TE^i + \{TE_i^j - TE^j\}}{TE^j + \{TE_j^i - TE^i\}}, \quad i, j = 1, \dots, n, \tag{14}$$

in which TE^i and TE_j^i are the objective function of (4) and (13) at optimality, respectively.

In the second stage, similar to AHP/DEA method, using a one-step process of AHP, the DMUs are ranked. In this paper we rank DMUs in the CCR model; in a similar way one can also rank DMUs in the BCC model.

TABLE 1: Random index.

Matrix order	1	2	3	4	5	6	7	8	9
RI	0.00	0.00	0.58	0.9	1.12	1.24	1.32	1.41	1.45

FIGURE 1: Three-level hierarchy.

3. Analytical Hierarchy Process (AHP)

AHP is a powerful tool for analysis of complex decision problems. AHP is employed for ranking a set of alternatives or for the selection of the best in a set of alternatives with respect to multiple criteria. In this section a brief discussion of AHP is provided. For more details see Saaty [16].

Application of AHP to a decision problem involves four steps [23, 24].

Step 1 (structuring of the decision problem into a hierarchical model). The first step is to decompose a decision problem into elements. In its simplest form (Figure 1), this structure comprises a goal at the topmost level, criteria (and subcriteria) at the intermediate levels, while the lowest level contains a set of the alternatives. A simple AHP model (Figure 1) has three levels (goal, criteria, and alternatives).

Step 2 (making pairwise comparisons and obtaining the judgment matrix). In this step, the elements of a particular level are compared with respect to a specific element in the immediate upper level. The more important (or more attractive) element receives higher rating (between 1 and 9) than another one that has the less important element [16]. The comparisons are used to form a matrix of pairwise comparisons called the judgment (pairwise comparison) matrix $\mathbf{A}$. Each entry a_{ij} of the judgment matrix satisfied the three rules: $a_{ij} > 0$; $a_{ij} = 1/a_{ji}$; and $a_{ii} = 1$ for all i, j . In fact, the number a_{ij} shows the relative importance of alternative A_i in comparison with alternative A_j with respect to criterion C_j or the relative importance of criteria C_i in comparison with criterion C_j with respect to goal. If $a_{ij} = a_{ik} \cdot a_{kj}$, for all the entries of the matrix, then the matrix is said to be consistent. If the property does not hold for all the entries, the level of inconsistency can be captured by a measure called consistency ratio [16] (see next step).

Step 3 (local weights and consistency of comparisons). In this step, local weights of the elements are calculated from

the judgment matrices using the eigenvector method (EVM). The normalized eigenvector corresponding to the principal eigenvalue of the judgment matrix provides the weights of the corresponding elements. Though EVM is followed widely in traditional AHP computations, other methods are also suggested for calculating weights, including the logarithmic least-square technique (LLST) [25, 26], goal programming [27], and recently DEAHP method [15, 16, 20, 22, 24–33]. When EVM is used, consistency ratio (CR) can be computed. The consistency index (CI) is calculated as follows:

$$CI = \frac{\lambda_{\max} - 1}{n - 1} \quad (15)$$

in which n is the number of criteria and $\lambda_{\max}$ is the maximum eigenvalue of the judgment matrix. Then, the consistency ratio (CR) is calculated as the ratio of consistency index and random index (RI); that is,

$$CR = \frac{CI}{RI}. \quad (16)$$

Random index is the consistency index of a randomly generated reciprocal matrix from the scales 1 to 9. Table 1 shows the value of RI sorted by the order of matrix. For a consistent matrix $CR = 0$ and $\lambda_{\max} = n$. A value of CR less than 0.1 is considered acceptable because human judgments need not be always consistent, and there may be inconsistencies introduced because of the nature of scale used. In the inconsistency case, $\lambda_{\max} \geq n$. If CR for a matrix is more than 0.1, judgments should be elicited once again from the DM till he/she gives more consistent judgments.

Remark 1. If the pairwise comparisons matrix is as follows:

$$\mathbf{A} = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \dots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \dots & \frac{w_2}{w_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \dots & \frac{w_n}{w_n} \end{bmatrix}, \quad (17)$$

then any weight derivation method, when applied to the above matrix $\mathbf{A}$, should generate the true weights $w_1, w_2, \dots, w_n$.

Step 4 (aggregation of weights across various levels to obtain the final weights of alternatives). In order to obtain final weights of the decision alternatives (elements at the lowest level), the local weights of elements of different levels are aggregated. For example, the final weight of alternative A_p

is computed using the following hierarchical (arithmetic) aggregation rule in traditional AHP:

$$\begin{aligned} &\text{Final weight of } A_p \\ &= \sum_j \left[(\text{Local weight of } A_p \text{ with respect to } C_j) \right. \\ &\quad \left. \times (\text{Local weight of criterion } C_j) \right]. \end{aligned} \quad (18)$$

By definition, the weights of alternatives and importance of criteria are normalized. The final weights represent the rating of the alternatives in achieving the goal of the problem.

4. Cross Efficiency Evaluation

The cross efficiency evaluation was first proposed by Sexton et al. [30] and was later investigated by Doyle and Green [34, 35]. It is often calculated as a two-stage process. In the first stage, the optimal weights of inputs and outputs are calculated for each DMU using the classical DEA formulation, for example, CCR model (5). Let u_{rk}^* ($r = 1, 2, \dots, s$) and v_{ik}^* ($i = 1, \dots, m$) be the optimal solution to the LP (5). Then, $\theta_{kk}^* = \sum_{r=1}^s u_{rk}^* y_{rk}$ is referred to as the optimistic efficiency or CCR efficiency of DMU_k, which reflects the self-evaluation of DMU_k, whereas θ_{jk} is referred to as a cross efficiency value of DMU_j, which reflects the peer-evaluation of DMU_k to DMU_j ($j = 1, \dots, n$; $j \neq k$). Consider

$$\theta_{jk} = \frac{\sum_{r=1}^s u_{rk}^* y_{rj}}{\sum_{i=1}^m v_{ik}^* x_{ij}}, \quad k = 1, \dots, n, \quad j = 1, \dots, n. \quad (19)$$

Therefore, each DMU will have multiple different efficiency scores, whose average reflects the overall performance of the DMU. Based on average cross efficiencies, DMUs can be compared and ranked.

In general, the optimal weights obtained using classical DEA in the first stage are multiple solutions. Therefore, the values θ_{kj}^* will change depending on these values in the second stage. In the second stage, these drawbacks are reduced and a suitable set of weights preserving the efficiency values obtained by DEA is selected for each DMU. To reduce this undesirable case, there are some model suggestions for preserving the self-efficiency scores, θ_{kk}^* , obtained for each DMU. Sexton et al. [30] suggested introducing a secondary goal to optimize the input and output weights while keeping the CCR efficiency unchanged. According to Sexton et al. [30], the secondary goals could be, for example, either aggressive or benevolent. In the aggressive context, DMU_k chooses the optimal solutions such that it maximizes self-efficiency and at a secondary level minimizes the other DMUs cross efficiency levels. The benevolent secondary objective would be to maximize all DMUs cross efficiency rankings. The most widely used secondary goals were suggested by Doyle and Green [34]. Wang and Chin [31] propose several new data envelopment analysis (DEA) models for cross efficiency evaluation by introducing a virtual ideal DMU (IDMU) and a virtual anti-ideal DMU (ADMU). These new DEA models determine the input and output weights

by minimizing the distance of a DMU from IDMU or maximizing the distance between the DMU and ADMU or both of them simultaneously, without the use of the input and output information of the other DMUs as a secondary goal. Örkücü and Bal [33] presented goal programming models that could be used in the second stage of the cross evaluation. See also [31] for new alternative models for DEA cross efficiency evaluation. In this paper we present another secondary goal to the choice of weights among the alternative optimal solutions. For this aim, we first find the alternative optimal solutions corresponding to all extreme DEA-efficient DMUs using the algorithm proposed by Jahanshahloo et al. [36]. Then, the any nonnegative convex linear combination of these alternative optimal solutions can be considered as the chosen weights corresponding to DMU_k (in this paper we consider the arithmetic mean of the alternative optimal solutions). The advantage of our approach is that in second stage it yields the positive weights (see Remark 2). We emphasize that one may choose another secondary goal to determine the input and output weights.

Remark 2. Any convex linear combination of optimal solutions of the following LP is optimal too:

$$\begin{aligned} &\max \quad cx \\ &\text{s.t.} \quad Ax \leq b \\ &\quad \quad x \geq 0 \end{aligned} \quad (20)$$

in which A is the $m \times n$ matrix and x , c , and b are the $n \times 1$ and $m \times 1$ vectors, respectively.

5. Proposed Method

In this section we combine the cross efficiency method with AHP technique and provide a two-stage method for ranking all DEA-efficient DMUs using cross efficient approach and AHP technique. In the first stage, we obtain the CCR-efficient DMUs and the optimal solutions of the CCR model (5) corresponding to them. Note that model (5) may have the multiple solutions. In this case, we apply the proposed algorithm in [36] as secondary goal. Suppose $(u_{1k}^*, u_{2k}^*, \dots, u_{sk}^*, v_{1k}^*, v_{2k}^*, \dots, v_{mk}^*)$ are the optimal solutions (weights) of LP (5) corresponding to DMU_k. The efficiency score of CCR-efficient DMU_j using the weights of DMU_k is as follows:

$$\theta_{jk} = \frac{\sum_{r=1}^s u_{rk}^* y_{rj}}{\sum_{i=1}^m v_{ik}^* x_{ij}}, \quad k = 1, \dots, n, \quad j = 1, \dots, n. \quad (21)$$

Also, θ_{jk} is referred to as a cross efficiency value of DMU_j, which reflects the peer-evaluation of DMU_k to DMU_j ($j = 1, \dots, n$; $j \neq k$). Furthermore, if the input-output weights of the LP (5) are not unique, the arithmetic mean of the alternative optimal solutions corresponding to DMU_k is considered as the optimal weights of DMU_k in order to obtain positive weights (see Remark 2). In the second stage, using a one-step process of AHP (Step 3 in Section 3), the vector weights of the pairwise comparison matrix, generated

TABLE 2: Example 1: DMUs' data (extracted from [[22], p. 689]).

Branch	Input			Output		
	Staff	Computer terminals	Space m ²	Deposits	Loans	Charge
1	0.9503	0.70	0.1550	0.1900	0.5214	0.2926
2	0.7962	0.60	1.0000	0.2266	0.6274	0.4624
3	0.7982	0.75	0.5125	0.2283	0.9703	0.2606
4	0.8651	0.55	0.2100	0.1927	0.6324	1.0000
5	0.8151	0.85	0.2675	0.2333	0.7221	0.2463
6	0.8416	0.65	0.5000	0.2069	0.6025	0.5689
7	0.7189	0.60	0.3500	0.1824	0.9000	0.7158
8	0.7853	0.75	0.1200	0.1250	0.2340	0.2977
9	0.4756	0.60	0.1350	0.0801	0.3643	0.2439
10	0.6782	0.55	0.5100	0.0818	0.1835	0.0486
11	0.7112	1.00	0.3050	0.2117	0.3179	0.4031
12	0.8113	0.65	0.2550	0.1227	0.9225	0.6279
13	0.6586	0.85	0.3400	0.1755	0.6452	0.2605
14	0.9763	0.80	0.5400	0.1443	0.5143	0.2433
15	0.6845	0.95	0.4500	1.0000	0.2617	0.0982
16	0.6127	0.90	0.5250	0.1151	0.4021	0.4641
17	1.0000	0.60	0.2050	0.0900	1.0000	0.1614
18	0.6337	0.65	0.2350	0.0591	0.3492	0.0678
19	0.3715	0.70	0.2375	0.0385	0.1898	0.1112
20	0.5827	0.55	0.5000	0.1101	0.6145	0.7643

in the first stage, are calculated. The DEA-efficient DMUs are ranked based on these weights. The pairwise comparison matrix $\mathbf{A}$ is generated by θ_{ij} as follows:

$$\mathbf{A} = [\theta_{ij}] \quad \text{in which,} \quad (22)$$

$$a_{ij} = \frac{\theta_{ii} + \theta_{ij}}{\theta_{jj} + \theta_{ji}}, \quad i, j = 1, \dots, n, \quad (\theta_{ii} = \theta_{ii}^*).$$

Moreover, since the pairwise comparison matrices are constructed from objective data in our case, we are not concerned about the consistency of $\mathbf{A}$.

Remark 3. It is evident that $a_{ii} = 1$, $a_{ij} = 1/a_{ji}$, $i, j = 1, \dots, n$.

It should be noted that since we intend to rank CCR-efficient DMUs, it is enough to compute the values of a_{ij} for each pair of CCR-efficient DMUs DMU_i and DMU_j . After determining the pairwise comparison matrix $\mathbf{A}$, AHP technique is applied for ranking CCR-efficient DMUs. In this paper we use the eigenvector method (EVM) for determining eigenvalue and eigenvector of the pairwise comparison matrix $\mathbf{A}$.

6. Numerical Examples

In this section, two numerical examples are examined to illustrate the proposed method. Comparisons with other existing procedures will also be made. In Example 2 we compare the proposed method with two other ranking methods which are based on AHP.

Example 1. We evaluated with our method the data of 20 branch banks of Iran. This data was previously analyzed by Amirteimoori and Kordrostami [22] and is listed in Table 2. Running the DEA model (4) will result in seven efficient units as 1, 4, 7, 12, 15, 17, and 20. Using the above-mentioned method the optimal weights corresponding to CCR-efficient DMU_j , $j = 1, 4, 7, 12, 15, 17, 20$, are chosen. The results are summarized in Table 3. The cross efficiency and pairwise comparison matrices of seven CCR-efficient DMUs are shown in Tables 4 and 5, respectively. For this pairwise comparison matrix, $\lambda_{\max} = 7.0111825$, $\text{CI} = 0.00186375$, and $\text{CR} = 0.00141 < 1$. So, pairwise comparison matrix is consistent enough. The corresponding eigenvector (weights) is shown in the last column of Table 5. Table 6 shows a comparison of our proposal and some other ranking approaches. All these approaches are implemented in input-oriented version under the condition of CRS. As reported in Table 6, DMU_4 is the most efficient one in our method. According to the results, the rankings of DMUs by the four methods are almost similar; in particular, the results of our method are more similar to the method [21].

Example 2. Consider a performance assessment problem investigated by Sinuany-Stern et al. [17], which includes eight DMUs to be evaluated in light of two inputs and two outputs. The data set is provided in Table 7, together with the CCR-efficiencies of the eight DMUs, where the CCR efficiencies evaluate DMUs A , B , and D to be all DEA efficient and they need to be further differentiated.

Table 8 shows the results obtained using the ranking methods of Alirezaee and Sani [20] and Sinuany-Stern et al.

TABLE 3: Example 1: the optimal weights obtained by proposed method.

DMU	u_1	u_2	u_3	v_1	v_2	v_3
1	$\frac{2743104820616200}{7}$	$\frac{1145504368295450}{7}$	$\frac{203344811696175}{7}$	$\frac{86942634280800}{7}$	$\frac{86942634280800}{7}$	$\frac{6674020361903369}{7}$
4	$\frac{10557780612150395395}{2}$	$\frac{7137450867780576185}{6}$	$\frac{8614624057487049745}{3}$	$\frac{15106565712832042015}{6}$	$\frac{15589621335236010925}{6}$	$\frac{99578464810982125259}{20}$
7	$\frac{192476232067007701700}{53}$	$\frac{3007266838159896400}{4510281972189568425}$	$\frac{31962169004909158050}{53}$	$\frac{129835340277577355800}{53}$	$\frac{159334283058868224250}{53}$	$\frac{35696050063119836479}{53}$
12	$\frac{4090713978428108425}{17}$	$\frac{4510281972189568425}{17}$	$\frac{228787181284072450}{17}$	$\frac{3200631113384367575}{17}$	$\frac{1036164623791167125}{17}$	$\frac{12048173865672775077}{34}$
15	$\frac{422111584566960050}{47}$	$\frac{1647910063938525750}{47}$	$\frac{1461021310131305000}{47}$	$\frac{2289837544488904650}{47}$	$\frac{2192290568305410050}{47}$	$\frac{2546169695477270243}{47}$
17	$\frac{43696890644910227500}{27}$	$\frac{32066814438604442500}{27}$	$\frac{3549650609396840000}{27}$	$\frac{217430715355767500}{27}$	$\frac{2798202153611424125}{3}$	$\frac{76056230540954147200}{27}$
20	$\frac{65310366646698125}{10656566179635625}$	$\frac{10656566179635625}{70175639377223750}$	$\frac{70175639377223750}{104635477353783125}$	$\frac{104635477353783125}{54469492763181.25}$	$\frac{54469492763181.25}{6814920193906181.25}$	$\frac{6814920193906181.25}{6814920193906181.25}$

TABLE 4: Example 1: cross efficiency matrix for CCR-efficient DMUs.

DMU	Target DMU						
	1	4	7	12	15	17	20
1	1	0.49504029	0.53660363	0.67987789	0.50878889	0.74467050	0.36909990
4	0.95525399	1	0.81873718	0.84047040	0.89121676	0.92400072	0.94263179
7	0.68426795	0.80156597	1	0.98769702	0.85624403	0.85682199	0.88697995
12	0.83162917	0.71092883	0.87483547	1	0.75165252	0.91722504	0.68657179
15	0.97375065	0.91292479	0.92595846	0.89933688	1	0.84347305	0.93897325
17	0.94554521	0.41786480	0.78137007	0.97194231	0.54846678	1	0.25490089
20	0.33804338	0.65282037	0.79256331	0.62361928	0.68029207	0.49238820	1

TABLE 5: Example 1: pairwise comparison matrices of seven CCR-efficient DMUs.

	DMU ₁	DMU ₄	DMU ₇	DMU ₁₂	DMU ₁₅	DMU ₁₇	DMU ₂₀	Weight
DMU ₁	1	0.7646271	0.9123273	0.9171495	0.7644273	0.8967514	1.0232103	0.33493
DMU ₄	1.3078269	1	1.0095312	1.0757141	0.9886519	1.3569705	1.1753435	0.42144
DMU ₇	1.0960978	0.9905587	1	1.0601981	0.9638027	1.0423561	1.0526712	0.38601
DMU ₁₂	1.0903347	0.9296149	0.9432199	1	0.9222442	0.9722520	1.0387729	0.36912
DMU ₁₅	1.3081688	1.0114783	1.0375567	1.0843114	1	1.1905150	1.1539501	0.41546
DMU ₁₇	1.115136	0.7369356	0.9593650	1.0285398	0.8399725	1	0.8408676	0.34737
DMU ₂₀	0.9773161	0.8508150	0.9499641	0.9626742	0.8665885	1.1892478	1	0.36284

TABLE 6: Example 1: results for comparing.

DMU _k	Proposed method		Model (6)		Model (6) (AP)		Model (9) l_1 -norm	
	Weight	Rank	Ω_b	Rank	θ^*	Rank	$\Gamma(X, Y)$	Rank
1	0.33493	7	0.675	6	1.1009	7	0.0156	7
4	0.42144	1	0.708	2	1.9336	2	0.4802	2
7	0.38601	3	0.698	3	1.1737	5	0.1915	3
12	0.36912	4	0.677	5	1.1095	6	0.0589	6
15	0.41546	2	0.790	1	4.9139	1	0.7965	1
17	0.34737	6	0.677	5	1.3484	3	0.1760	4
20	0.36284	5	0.680	4	1.1849	4	0.1077	5

TABLE 7: Example 2: DMUs' data (extracted from [17]).

DMU	Input		Output		CCR efficiency
A	20	12	60	36	1
B	10	15	30	45	1
C	15	12	30	36	0.999
D	5	70	15	80	1
E	3	9	3	9	0.510
F	9	18	1	18	0.402
G	63	19	8	19	0.330
H	22	73	1	3	0.022

[17] and the proposed method. By our method DMU A has the best performance. In contrast, DMU A has the worst performance by Alirezaee's method. In view of inputs and

outputs of the efficient DMUs, it seems that our method gives more reasonable result than Alirezaee's method.

7. Conclusion

In this paper we propose a two-stage method for ranking all extreme and nonextreme efficient DMUs based on cross efficiency and AHP technique. In the first stage, cross efficiency score of each DEA-efficient DMU is calculated. In general, the optimal input-output weights obtained by classical DEA are not unique; therefore, the cross efficiency scores depending on these weights are also not unique. To remove this problem we first obtain all alternative optimal solutions of DEA model (5) and then the arithmetic mean of them is considered as a secondary goal. In the second stage, using a one-step process of AHP, the DEA-efficient DMUs are ranked. It seems that the proposed method gives more reasonable results, because

TABLE 8: Example 2: the results of ranking methods.

Sinuany's method		Alirezaee's method		Proposed method	
B	0.20994	B	0.1993	A	0.591675
A	0.153001	D	0.1929	B	0.582900
D	0.14140	A	0.1883	D	0.556908

we consider all alternative weights corresponding to DEA-efficient DMUs. Also, our method can be extended to interval cross efficiency and interval AHP. We suggest as future works a deeper analysis in this subject. Finally, in this paper we rank CCR-efficient DMUs; in a similar way one can also rank BCC-efficient DMUs.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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