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Research Article

An Investigation of Value Updating Bidders in Simultaneous Online Art Auctions

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Simultaneous online auctions, in which the auction of all items being sold starts at the same time and ends at the same time, are becoming popular especially in selling items such as collectables and art pieces. In this paper, we analyze the characteristics of bidders (*Reactors*) in simultaneous auctions who update their preauction value of an item in the presence of influencing bidders (*Influencers*). We represent an auction as a network of bidders where the nodes represent the bidders participating in the auction and the ties between them represent an *Influencer-Reactor* relationship. We further develop a random effects bilinear model that is capable of handling covariates of both bidder types at the same time and account for higher-order dependence among the bidders during the auction. Using the model and data from a Modern Indian Art auction, we find that *Reactors* tend to update their values on items that have high preauction estimates, bid on items created by high investment risk artists, bid selectively only on certain items, and are more active in the second half of the auction. Implications for the auction house managers are discussed.

1. Introduction

For the last three decades, simultaneous auctions have become one of the most popular auction settings for selling high-priced affiliated private value items, including FCC radio bandwidth spectrum [1], U.S. treasury bills [2], and timber [3]. In these auctions, all the items are sold simultaneously, meaning that their auctions start at the same time and end at the same time. The set of bidders who attend these auctions remain the same throughout the auction event. Such an auction setting is particularly successful in situations where the items are highly complementary and the buyers have a demand for more than one item. For example, broadcasting companies typically need to purchase more than one bandwidth of

the radio spectrum; art collectors buy more than one item of their favorite artist and so on. Recently, this auction format has become popular in online auctions to sell collectables and fine arts [4].

In typical affiliated private value auctions such as art auctions, affiliation theory [5] suggests that some bidders will consult the bid amount of other bidders during the auction and update their preset valuation and their "willingness to pay" for the items they are interested in(Bidders' valuations have some dependence with each other (loosely speaking, one bidder's high value signal makes it more likely that other bidders will exhibit high values), implying that bidders can change valuation if others' bidding behaviors are observable as in open bid format [5].) . Particularly, if the bidders have limited information about the art items, they will consult bids of other bidders as additional item information [6, 7]. Furthermore, if the art items are sold in simultaneous auctions, bidders frequently encounter each other in more than one item. Therefore, it is possible for value-updating bidders to get influenced by the same bidder more than once. How does the characteristics of "known" bidders affect the propensity of value-updating behavior? In this paper, we intend to answer this question. In particular, we analyze the characteristics of such value-updating bidders (termed here as *Reactors*) in the presence of influencing bidders (termed here as *Influencers*). We examine three important questions pertaining to the *Reactors*. In what type of lots (art items) do bidders update their value? Do they bid more in the first half or in the second half of the auction? Are they selective in the lots on which they bid, or do they bid on many lots?

Traditional approaches such as general linear models and logistic regressions are useful in estimating aggregate effects of different characteristics of overbidding bidders and in identifying lots where such behavior can be witnessed but they fall short of fulfilling the goal of this study. Particularly, they are not capable of estimating effects of individual *Reactors* and *Influencers* characteristics that ultimately lead to overbidding of *Reactors*. Further, these models lack the capacity to capture higher-order bidder dependency that exists in simultaneous auctions. Particularly, with bidders competing for multiple items simultaneously, bidders "recognize" each other and engage in anticompetitive practices such as bidder collusion [3, 4]. To some degree, this limitation is due to the data structure available from the auction houses. To overcome this issue, we represent the auction data in the form of a network Y_{ij} of bidders where bidder j (the *Reactor*) has updated his valuation in reaction to the bids of bidder i (the *Influencer*). Using this rich and innovative framework, we examine the characteristics of *Reactors* using a randomeffects dyadic relation model [8]. This is based on a generalized regression framework and is capable of handling covariates of both bidder types. It builds on the social relations model [9, 10] and is capable of specifying random-effects between subsequent bidders. It is proficient in simultaneously considering regressor variables as well as correlation between *Reactors* having the same *Influencers*, between *Reactors* bidding on the same item, and reciprocity between *Reactors* and *Influencers*. In other words, this approach analyzes the bidder characteristics in consideration with the bidder dependence in these simultaneous auctions.

From the auction house manager's perspective, understanding the behavior of bidders is important. Since the rivalry among auction houses has intensified in recent years, much more attention is now given to strengthen relationships with bidders. The underlying quest for all the managers is now to cultivate and promote a strong relationship with bidders and encourage them to participate more in future auctions. *Reactors* play a pivotal role in the success of the auction house. Their value updating behavior typically leads to overbidding and higher price [11] paid for the item, thus playing a critical part in the price formation

process in the auction. Therefore, managers are interested in learning on what and when these bidders bid. (Auction house managers from multiple auction houses were interviewed during our investigation.) To this end, our paper attempts to investigate their characteristics in these simultaneous auctions.

Prior studies on this issue [12, 13], although limited, have mainly focused on understanding the underlying psychological factors that result in value updating behavior. They identified factors including bidder rivalry, social effects, and escalation of commitment that lead to a "win at any cost" mentality of the *Reactors* (when they start bidding irrationally in order to win the item [11]). (This emotional phenomenon is commonly known as "auction fever".) Our paper is essentially an extension of this body of literature. We assume that bidders do update their value during auctions and such behavior is revealed when bidders place a high bid value after using a proxy bidding system earlier in the auction. Proxy bidding is a commonly available feature in most online auction houses where bidders set a maximum amount they are willing to pay, and then let the auction house place proxy bids on their behalf until that price. Along these lines, Ku et al. [11] performed a survey of bidders and found that most of them use proxies to set their maxima.

Another uniqueness of our paper from other online auction research is the context of our study, that is, auctions of high-end fine arts. Selling high-priced art items through online auctions has become a recent trend in the art market. As the demand for fine art has reached its all-time high [14], auction houses and art dealers have found Internet-based auctions as one of the reliable ways to sell art items to a wider group of art lovers. Established online auction houses such as *SaffronArt* (http://www.saffronart.com/), Attinghouse.com, and *AspireArt* (http://www.aspireart.com/), use the simultaneous auction setting in their auctions. Simultaneous online auctions are different from eBay auctions, which are frequently analyzed in academic research. In a simultaneous auction, all the items up for sale are sold *concurrently* to the same group of bidders over a certain period of time. This gives rise to a complex competitive environment, where there is a great level of interdependence between bidders leading to value-updating behavior by some of them. With the art market so hot and with so much at stake for the auction house managers, our paper is well focused in helping these managers develop a better relationship with their customers. From our analysis, we find that *Reactors* typically bid on fewer items, suggesting that they might be the collectors and not the art dealers [15]. They also bid on high-value items and bid more in the second half of the auction than in the first half. We further find that *Reactors* and *Influencers* rarely alter their role during the auction and there is more possibility for *Reactors* to encounter the same *Influencers* over other items than for *Influencers* meeting the same *Reactors* over other items.

In sum, the contribution of our study is threefold. First, we investigate characteristics of value-updating bidders in simultaneous online art auctions. Second, from the technical standpoint, we demonstrate an application of a random-effects dyadic relational model for complex human behavior and emphasize the importance of new and advanced statistical techniques available to the social sciences. And third, we introduce a new approach of considering bid history in the form of a network. This innovative and rich framework will not only allow us to examine the interbidder influence in our study but will also encourage future bidder behavior studies in other auction settings.

The rest of the paper is presented as follows. First, we describe the auction data of our research and discuss our unique approach of representing it as a social network. Second, we discuss the random-effects dyadic relation model and explain how we use it to determine

the characteristics of *Reactors*. Third, we present the results of our investigation. Finally, we discuss the implications of our work and present directions for future research.

2. Auction Data and Bidder Network

Online auctions have become a hot research topic in economics, marketing, management, and statistics. (Laudon and Traver [16] estimate that online auction sales (C2C and B2C) will top $36 billion by 2007. Revenue exceeded $6 billion in 2006 at eBay, the pioneering online auction firm where everything from paperclips to private jets get sold. Even traditional auction houses like Christie's (whose annual revenues are expected to top $4 billion in 2006) are adopting the online model. (http://www.iht.com/articles/2006/07/12/news/auction.php) (http://internet.seekingalpha.com/article/25034)) Particularly, with the availability of detailed bidding data from online auctions, we are now able to investigate bidder behavior and auction characteristics in details that were not possible to explore earlier. For example, in the last decade, a wide range of new studies have looked at interesting auction issues such as price dynamics [17, 18], bidder surplus [17], importance of reference points in auctions [19, 20], herding behavior [21], and forward-looking behavior of bidders [22]. Interestingly, all these studies have focused exclusively on single-item auctions such as eBay, where bidders compete for one item at a time.

Unlike prior studies, this paper investigates simultaneous online auctions where multiple items are sold *concurrently* to a same group of bidders over a certain period of time. We have collected the data from an online auction house called *SaffronArt* (http://www.saffronart.com/). This auction house sells only Modern Indian Art and has become a prominent distribution channel of that genre in recent years. More specifically, the data come from a three-day auction where 199 art lots (a unique piece of art such as a painting, a drawing or a sculpture) were sold. Unlike eBay auctions, these auctions are in simultaneous first-price ascending format. The lots are open at a specific date and time, and they close simultaneously at a specific time. Moreover, to allow bidders to compete for multiple items, the lots are closed sequentially in a group of 20 to 25 lots. For example, lots 1–25 may close at 9:00 AM and lots 26–50 will close at 9:30 AM. Further, to discourage devious online bidder behavior such as sniping the auction has a soft closing time: the closing time extends by three minutes whenever a bid comes during the last three minutes of the auction. (Sniping is a strategic bidding activity where bids are submitted in the last moments of the auction to allow minimal time to other bidders to react to this bid. Such behavior is prominent in eBay auctions as the auction closes promptly at a specific time.), This time extension continues until no one bids during a span of three minutes.

2.1. Modern Indian Art

Modern Indian Art, with over $100 million in auction sales in 2006, is now one of the leading emerging art markets in the world. Although traditional auctions for Modern Indian Art have existed since 1995, it is only since 2000 that the market has exploded, with values realized at auctions growing at a brisk 68.7% annually (coincidentally, this is when *SaffronArt* (http://www.saffronart.com/), the source of our data, started its online auctions of Modern Indian Art). In 2006, online auction sales of Modern Indian Art from *SaffronArt* (http://www.saffronart.com/) ($36.76 million) had more sales (of Modern Indian Art) than the traditional auction houses like Sotheby's ($35.29 million) and Christie's

($33.08 million). Further, *SaffronArt* (http://www.saffronart.com/) sold more art items (537) compared to Sotheby's (484) and Christie's (329) in that year.(In 2005, online auction sales of Modern Indian Art by *SaffronArt* (http://www.saffronart.com/) were $18.06 million, more than that of Sotheby's ($10.49 million) and Christie's ($14.89 million). *SaffronArt* (http://www.saffronart.com/) also sold more art items (390) compared to Sotheby's (276) and Christie's (248) in 2005.) The top ten Indian artists sold 31% of the lots and contributed to 57% of the total value realized at auctions since 1995. Two of these artists are now ranked in the top 100 artists in the world based on their auction sales in 2005. A new set of emerging artists (the new trendsetters, typically born after 1955) have contributed 2% in value and 3% in lots and are becoming increasingly popular, commanding ever higher prices.

2.2. Bidder Influence Network

Identifying *Reactors* in an auction is challenging. First of all, no prior information is available about the private-value distribution of the bidders. These bidders do not reveal the maximum amount they are willing to spend, nor do they explicitly announce their value update at the end of the auction. Luckily, in online auctions, the auction house provides a proxy bidding service to the participants. Proxy bidding is a commonly available feature in most online auctions where bidders set the maximum amount they are willing to pay for the auctioned item, and then let the auction house place proxy bids on their behalf until that price. Bidders using this facility have a predetermined value for the item and use proxy bidding to stay within that value limit [23]. Ku and his colleagues [11] performed a survey of the bidders and found that most of them use proxies to set their maxima. The respondents used terms such as "*maximum personal limit*", "*what we were willing to spend*", "*the most I was willing to bid*", and "*by how I valued it*" to explain their proxy bids. In the same lines, we considered a bidder to be updating his value if he/she reenters the bidding process and places a normal (nonproxy) bid that is higher than his/her earlier proxy bid.

One of the challenges with the available auction (see Figure 1) is how we capture the interbidder dependence during an auction. Particularly, as we frequently observe the same bidders competing for multiple items simultaneously, it is important that we include this auction feature in our analysis. That is, we should consider the influence of the *Influencers* over the *Reactors* in our modeling effort of their characteristics. In order to consider the interbidder dependencies, proper representation of the bidding data is necessary. We transform the bid history into an $N \times N$ bidder influence matrix where N denotes the number of bidders participating in the auction. The value $y_{i,j}$ in each cell of the influence matrix indicates the influence of bidder i over bidder j. We considered the influence measure $y_{i,j}$ as the total number of items in which the *Influencer* (bidder i) has bid between the proxy and nonproxy bid of the *Reactor* (bidder j). Since the auctions are held simultaneously, prior auction studies [3, 4] suggest that bidder-pair with multiple engagements tend to influence each other. Dass and Reddy [4] also showed that such engagements lead to dyadic level bidder effects, which has significant effect on the seller's profit even after controlling for the aggregate competition in the auction. Figure 2 illustrates the network data structure with *Reactors* and *Influencers* as nodes. Here $y_{1,2}$ indicates the number of items in which bidder 1 (an *Influencer*) has placed a bid between the proxy and non-proxy bid of bidder 2 (a *Reactor*).

We also considered two other possible measures of influence for our bidder network $Y_{i,j}$. The first approach considers the order in which the *Influencers* have placed their bids. We assumed that the bidders whose bid is close to that of the non-proxy bid of the *Reactor* will

110
Jagannath panda

High-rise. Bee-rise

Signed and dated in english (verso)
2006
Oil, acrylic and fabric on canvas
78 × 156 in (198.1 × 396.2 cm)

$58140 – 69770
Rs 2500000-3000000

Next valid bid: $ 106600 (Rs 4583800)

Rank	Nick name	Amount ($)	Amount (Rs)	Bid type	Date and time (US EST)
Current highest bio 1	Anonymous 3	99100	4261300		Mar 8 2007 10:47:06 AM
2	Anonymous 25	91600	3938800		Mar 8 2007 10:21:57 AM
3	Racoon	84100	3616300		Mar 8 2007 6:19:29 AM
4	Anonymous 138	76600	3293800		Mar 8 2007 12:54:47 AM
5	Racoon	71600	3078800		Mar 7 2007 8:57:15 AM
6	Anonymous 47	66600	2863800		Mar 7 2007 1:39:24 AM
7	Anonymous 25	61600	2648800		Mar 6 2007 11:51:47 PM
8	Anonymous 3	56600	2433800	Proxy	Mar 6 2007 11:10:21 PM
9	Anonymous 25	51600	2218800		Mar 6 2007 11:10:21 PM
10	Anonymous 3	49100	2111300	Proxy	Mar 6 2007 10:30:00 PM
	Start price	46600	2003800		Mar 6 2007 10:30:00 PM

Figure 1: Bid History from an Online Auction of Modern Indian Art.

provide more value information than the bidders who have placed a bid earlier than that. We captured this notion by computing the difference between the bid ranks (order of the bids) of the *Reactor* and his/her *Influencers*. The second approach considers the reaction time of the *Reactors*. In other words, it considers the difference between the bids of the *Influencers'* and the *Reactors'* normal bid. The shorter the time difference is, the larger is the influence.

Reactors →

Influencers	Bidder 1	Bidder 2	Bidder 3	⋯ Bidder 80
Bidder 1	0	x_{12}	x_{13}	
Bidder 2	x_{21}	0	x_{23}	
Bidder 3 ⋮ Bidder 80	x_{31}	x_{32}	0	

Figure 2: Socio Matrix of Influencers and Reactors.

The longer the time difference is, the weaker is the influence as the *Reactor* may have used other resources to update his value of the item. The second approach yielded the same results as our first approach, and thus their results are excluded from this paper. (The results from these approaches can be obtained from the auctions.)

3. Value Updating Behavior of Bidders

This study assumes that the characteristics of *Influencers* and *Reactors* are homogeneous within the bidder types. Since the bidder influence data are represented in the form of a sociomatrix $Y = [y_{i,j}]$, there are certain modeling constraints that need to be addressed. First, the model needs to accommodate bidder level covariates. The main goal of this paper is to characterize bidders who update their value during a simultaneous auction. Therefore, our modeling framework should be capable of investigating the characteristics that define as *Reactors*. And second, the model should allow second-order dependencies (such as reciprocity and common bidder effects in auctions where two interdependent bidders have common *Influencers* or common *Reactors*). Second-order dependencies are essential in identifying the stability of the bidder characteristics. For example, reciprocity will tell us whether bidders exchange their role as *Reactors* or *Influencers* during the auction. The effects of common *Influencers* and *Reactors* will emphasize how repeated encounters with the same bidders affect bidder decisions. In this paper, we develop a randomeffects dyadic relational model to investigate the characteristics of the bidders.

3.1. Random-Effects Dyadic Relation Model

This modeling approach is based on the works of Ho [24], and Hoff and Ward [8] that specify and analyze random-effects for the originator (*Influencer*) and the recipient (*Reactors*) in a social relations setting. It starts with the description of a simple linear model and builds the complexities around it sequentially. Consider modeling the dyadic influence data $y_{i,j}$ with a linear regression model of the following form:

$$y_{i,j} = \beta' x_{i,j} + \varepsilon_{i,j}, \tag{3.1}$$

where $y_{i,j}$ represents the influence of bidder i over bidder j, and $x_{i,j}$ represents the dyadic level covariates. The regressor $x_{i,j}$ is assumed to have enough individual level bidder information such that the distribution of the errors is invariant under permutations of the unit labels.

This is similar to having an $n \times n$ matrix of errors with a distribution that is invariant under identical row and column permutations; thus $\{\varepsilon_{i,j} : i \neq j\}$ is equal to $\{\varepsilon_{\pi(i),\pi(j)} : i \neq j\}$ in terms of their distribution. This condition is called a weak row-and-column exchangeability [8] of an array. For undirected networks, such as our bidder influence network, it results into a random effects representation of the error term $\varepsilon_{i,j}$ such that it is equal in distribution to $f(\mu, a_i, a_j, \gamma_{i,j})$ where $\mu, a_i, a_j, \gamma_{i,j}$ are independent random variables and f is a function that is specified [25, Theorem 14.11].

Therefore, individual bidders' characteristics and higher-order dependence among the bidders' effects during the auction are captured by an error component of the linear model where the model assumes a covariance structure that is exchangeable under identical permutations of i and j indices of the *Influencers* and *Reactors*, respectively. We model this error as Gaussian, and thus, the joint distribution of $\varepsilon_{i,j}$ is as follows:

$$\varepsilon_{i,j} = a_i + b_j + \gamma_{i,j}, \tag{3.2}$$

where a_i represents the effect of bidder characteristics of *Influencer i*, b_j represents the effect of bidder level characteristics of *Reactor j*, and $(a_i, b_j)' \sim$ multivariate normal [MVN] $(0,\Sigma_{ab})$.

Hence,

$$\begin{bmatrix} a_i \\ b_j \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ba} & \sigma_b^2 \end{bmatrix}\right). \tag{3.3}$$

Additionally, $(\gamma_{i,j} \quad \gamma_{j,i})'$ also follows a multivariate normal distribution with MVN$(0, \Sigma_\gamma)$. Therefore,

$$\begin{bmatrix} \gamma_{i,j} \\ \gamma_{j,i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\gamma^2 & \rho\sigma_\gamma^2 \\ \rho\sigma_\gamma^2 & \sigma_\gamma^2 \end{bmatrix}\right). \tag{3.4}$$

The covariance structure of the error term is therefore

$$\begin{gathered} E\left(\varepsilon_{i,j}^2\right) = \sigma_a^2 + 2\sigma_{ab} + \sigma_b^2 + \sigma_\gamma^2, \qquad E\left(\varepsilon_{i,j}\varepsilon_{j,i}\right) = \rho\sigma_\gamma^2 + 2\sigma_{ab}, \\ E\left(\varepsilon_{i,j}\varepsilon_{k,l}\right) = 0, \qquad E\left(\varepsilon_{i,j}\varepsilon_{i,k}\right) = \sigma_a^2, \\ E\left(\varepsilon_{i,j}\varepsilon_{k,j}\right) = \sigma_b^2, \qquad E\left(\varepsilon_{i,j}\varepsilon_{k,i}\right) = \sigma_{ab}, \end{gathered} \tag{3.5}$$

where σ_a^2 represents the variance in an observation due to the presence of common *Influencer*, σ_b^2 represents the variance in an observation due to the presence of common *Reactors*, and ρ represents the correlation of observations within a *influencer-reactor* pair and serves as a measure of reciprocity or mutuality in the bidder influence data [8]. (This model is also known as the "round-robin" model [9, 10].) To analyze the characteristics differences between *Influencers* and *Reactors* in a particular sample space, the error structure as shown above can be added to a linear predictor in a generalized linear model. Thus,

$$\theta_{i,j} = \beta' x_{i,j} + \varepsilon_{i,j}, \tag{3.6}$$

where $\varepsilon_{i,j} = a_i + b_j + \gamma_{i,j}$.

The above (3.6) is modeled such that the dyadic data are unconditionally dependent, but conditionally independent, given the random effects of *Influencers* and *Reactors*. Moreover, $\beta' x_{i,j} = \beta_0$ as the bidder influence is only modeled in terms of the individual bidder level covariates. Therefore, we consider it as a generalized linear mixed-model with inverse-link function $g(\theta)$. Thus,

$$\begin{gathered} E(y_{i,j} \mid \theta_{i,j}) = g(\theta_{i,j}), \\ p(y_{12}, y_{13}, \ldots, y_{n,n-1} \mid \theta_{12}, \theta_{13}, \ldots, \theta_{n,n-1}) = \prod_{i \neq j} p(y_{i,j} \mid \theta_{i,j}). \end{gathered} \tag{3.7}$$

In our case, a Poisson model with log-link is appropriate, given that we measure the level of influence $y_{i,j}$ as the number of items where bidder i has presumably influenced bidder j to update his/her value. Thus,

$$\begin{gathered} g(\theta_{i,j}) = \varepsilon_{i,j}^{\theta}, \\ p(y_{i,j} \mid \theta_{i,j}) \sim \text{Poisson}\left(e^{\theta_{i,j}}\right). \end{gathered} \tag{3.8}$$

3.2. Incorporating the Effects of Higher-Order Dependence

In auctions, there are possibilities that the effect of intense rivalry initiated between two bidders in the early part of the auction may spillover to the later part of the auction and influence the *Reactors* to update their values. Consider a hypothetical scenario: say bidder A and bidder B have engaged in a bidding-war in the auction. This bidding phenomenon may influence the proxy bidder, say C to update his/her private value and place a higher non-proxy bid (similar to *transitivity)*. Such second-order effects are common in art auctions [4, 15, 26].

In the social networks literature, researchers have used simple functions of latent characteristics vector z_i for *Influencers* and z_j for *Reactors* and added their inner product $z_i' z_j$ to the error model in a fixed effect setting to capture such higher-order effects [8]. They used models of the form $\theta_{i,j} = \beta' x_{i,j} + f(z_i, z_j)$ where the latent characteristics function $f(z_i, z_j)$ is considered either as a distance model $\{f(z_i, z_j) = -|z_i - z_j|\}$ or as a projection model $f(z_i, z_j) = z_i' z_j / |z_j|$. We followed a similar approach and added the inner product $z_i' z_j$ to the error model (3.2). Thus,

$$\varepsilon_{i,j} = a_i + b_j + \gamma_{i,j} + \zeta_{i,j}, \tag{3.9}$$

where $\zeta_{i,j} = z_i' z_j$ is a mean-zero random effect and the random effects a_i, b_j, and $\gamma_{i,j}$ are modeled with the multivariate normal distributions described earlier. The moment properties of the distributions of ζ are $E(\zeta_{i,j}) = 0; E(\zeta_{i,j}^2) = \text{trace} \sum_z^2; E(\zeta_{i,j}\zeta_{j,k}\zeta_{k,i}) = \text{trace} \sum_z^3$ if the z's are modeled as independent k-dimensional MVN random vectors with mean zero and covariance structure matrix as$\sum_z$ and all other second-and third-order moments are equal to zero. Moreover, an orthogonal transformation of the z's leaves $z_i' z_j$invariant, suggesting that $\sum_z$ is a diagonal matrix. For example, lets us consider the case where $\sum_z = \sigma_z^2 I_{k \times k}$ and

the above moments are 0, $k\sigma_z^4$, and $k\sigma_z^6$, respectively. When we add $\zeta_{i,j}$ to the error term, the nonzero second-and third-order moments become

$$\begin{gathered} E\left(\varepsilon_{i,j}^2\right) = \sigma_a^2 + 2\sigma_{ab} + \sigma_b^2 + \sigma_\gamma^2 + K\sigma_z^4, \\ E\left(\varepsilon_{i,j}, \varepsilon_{j,i}\right) = \rho\sigma_\gamma^2 + 2\sigma_{ab} + K\sigma_z^4, \\ E\left(\varepsilon_{i,j}\varepsilon_{j,k}\varepsilon_{k,i}\right) = K\sigma_z^6, \qquad E\left(\varepsilon_{i,j}\varepsilon_{i,k}\right) = \sigma_a^2, \\ E\left(\varepsilon_{i,j}\varepsilon_{k,j}\right) = \sigma_b^2, \qquad E\left(\varepsilon_{i,j}\varepsilon_{k,i}\right) = \sigma_{ab}. \end{gathered} \tag{3.10}$$

The above shows that the effect $\zeta_{i,j} = z_i' z_j$ can be interpreted as a mean-zero random effect that is able to capture the higher-order dependencies found among the bidders in an auction. Marginally, if k increases, the distribution of $\zeta_{i,j}$ will converge to a normal distribution (due to central limit theorem). Jointly, if the ζ's of the *Influencer* and *Reactor* have at least one unit in common, the Markov dependence graph will plot the two bidders as neighbors. If $\zeta's$ are considered as fixed effects, they can be viewed as interaction terms that are highly constrained due to the functional dependence on the z's. For example, if the z_i and z_j are vectors of similar directions and magnitude, then $z_i' z_k$ and $z_j' z_k$ will not be different. This characteristic can be related to *bidder transitivity* as we described earlier. (For more information on how the moments are scaled, please refer to [8, 24, 27].)

Since the inner-product term $z_i' z_j$ is a fixed effect, it can be considered as a reduced-rank interaction term. This is typically known as the *bilinear effect* or *multiplicative interaction*. To include this inner product of the member's latent characteristics in the random effect model, (3.6) is reparameterized as

$$\begin{gathered} \theta_{i,j} = \text{inf}_i + \text{rea}_j + \gamma_{i,j} + z_i' z_j, \\ \text{inf} = \beta_{\text{inf}}' x_{\text{inf},i} + a_i, \\ \text{rea}_i = \beta_{rea}' x_{\text{rea},i} + b_i, \end{gathered} \tag{3.11}$$

where $x_{\text{inf},i}$ are the *Influencer* specific covariates and $x_{rea,i}$ are the *Reactor* specific covariates in the model.

The above model is estimated using a Bayesian estimation process. A Markov Chain Monte Carlo (MCMC) algorithm is used to sample values of the *Influencer* and *Reactor* specific parameters from their posterior distribution. We estimate the parameters for the generalized bilinear regression model by constructing a Markov chain in $\{\beta_{\text{inf}}, \beta_{\text{rea}}, \sum_{ab}, Z, \sigma_z^2, \sum_\gamma\}$ (where Z is a $k \times n$ latent vector matrix) with $p(\beta_{\text{inf}}, \beta_{\text{rea}}, \sum_{ab}, Z, \sigma_z^2, \sum_\gamma \mid Y)$ as the invariant distribution. A Gibbs sampling is used to obtain the chain and to sample θ.

The algorithm followed as per Hoff [8] is as follows.

(1) Sampling of linear effects in the model is as follows

(a) Sample $\text{inf}, \text{rea} \mid \beta_{\text{inf}}, \beta_{\text{rea}}, \sum_{ab}, Z, \theta, \sum_\gamma$ with a linear regression.

(b) Sample $\beta_{\text{inf}}, \beta_{\text{rea}} \mid \text{inf}, \text{rea}, \sum_{ab}$ with a linear regression.

(c) Sample $\sum_{ab}, \sum_\gamma$ from their full conditionals.

(2) Sampling of bilinear effects is as follows

(a) For each bidder $i = 1, 2, \ldots, n$, sample $z_i \mid \{z_j : j \neq i\}, \theta, \beta, s, r, \Sigma_z, \Sigma_\gamma$ with a linear regression.

(b) Sample Σ_z from its full conditional distribution.

(3) Update $\{\theta_{i,j}, \theta_{j,i}\}$ using Metropolis-Hastings step.

(a) Propose $\begin{bmatrix} \theta^*_{i,j} \\ \theta^*_{j,i} \end{bmatrix} \sim N(\begin{bmatrix} \beta' x_{i,j} + a_i + b_j + z_i' z_j \\ \beta' x_{j,i} + a_j + b_i + z_j' z_i \end{bmatrix}, \Sigma_\gamma)$.

(b) Accept $\begin{bmatrix} \theta^*_{i,j} \\ \theta^*_{j,i} \end{bmatrix}$ with probability $p(y_{i,j} \mid \theta^*_{i,j}) p(y_{j,i} \mid \theta^*_{j,i}) / p(y_{i,j} \mid \theta_{i,j}) p(y_{j,i} \mid \theta_{j,i}) \wedge 1$.

The prior distributions of the parameters are taken as follows:

(i) $\beta \sim \mathrm{MVN}(0, 80 \times I_{4\times 4})$,

(ii) Σ_{ab} ~inverse Wishart Distribution $(I_{2\times 2}, 4)$,

(iii) $\sigma_u^2, \sigma_v^2 \sim$ iid inverse gamma(1,1), $\sigma_\gamma^2 = (\sigma_a^2 + \sigma_b^2)/4, \rho = (\sigma_a^2 - \sigma_b^2)/(\sigma_a^2 + \sigma_b^2)$.

We used $K = 2$ dimensions in our analysis. (Five different values of $K = 0$ to $K = 1$ were tested with a fourfold cross-validation procedure as described by Ho in his seminal paper [8]. The predictive performances for all the K values were roughly the same. The biggest improvement in the marginal likelihood criterion was from K = 1 to $K = 2$. Therefore, $K = 2$ was selected.)

4. Bidder Covariates

Reactors are examined in the context of three types of characteristics: lot (item) characteristics, auction characteristics and bidder behavior characteristics.

4.1. Lot Characteristics

Prior work on auctions of fine arts [26, 28] found that both the preauction estimates of the items provided by the auction house and the art type (paper or nonpaper work) are to be significant drivers of auction prices. Before the auction starts, potential bidders are exposed to various types of information about the lots. Lot information and provenance provided by the auction house (in their printed catalogs and in their websites), and comments and suggestions of the art experts (in personal blogs, art magazines, etc.), provide information on the value of the art items such as the estimated price, artist information, and previous auction price history of similar paintings by the artists. The price estimates indicate the value of the items as suggested by the auction house experts (like curators, art specialists, etc.). Mei and Moses [29] found these value estimates to have high correlation with the final realized prices of the art items. Therefore, the higher the preauction estimates, the greater is the tendency for the item to fetch a higher price, and so higher stakes are associated with it. Such high stakes lead to higher bidder propensity to clarify and justify their future bids. Therefore, bidders look for other value signals from different sources and are inclined to change their value belief during the auction of these lots. Thus, we hypothesize that the bidders who act as *Reactors* tend to bid on lots that have high preauction estimates.

The media on which the art item is painted plays an important role in its maintenance and longevity [26]. For example, works on paper are typically of low price as compared to that of works on canvas since paper tends to be more fragile than canvas. Therefore, low financial risk is associated with the purchase of nonpaper items, and thus, we hypothesize that the *Reactors* will tend to update values more on nonpaper items.

4.2. Artist Characteristics

Artist characteristics such as reputation and previous auction history of the artists also play an important role in the valuation of the art items. Established artists are highly reputed and their works are well recognized in the art market. Most of their works have been resold many times in the market and, thus, their value commonly known to all. Therefore, the works of established artists present a low-risk purchase opportunity for the bidders [26]. On the other hand, emerging artists are new to the art market and their works are not well known. Further, not enough works of these artists are sold in the marketplace in order to estimate their values confidently. Therefore, the values of the works of these artists are highly uncertain, making them high-risk purchases. Thus, *Reactors* are hypothesized to react and change their values in lots created by the emerging artists as they tend to seek more information from other bidders.

Like artist reputation, historical market information such as the average price per square inch of the artists' art works or the total number of items sold in the previous year provides a signal to the bidders about the market value of the artist [26]. If the value realized by the works of an artist is low in the previous year, stakes will be high for their art work. Since evaluators are unaware of how the market will react to the works of these artists, they are high-risk purchases for the bidders. Similarly, if fewer items of the artist are sold in the previous year's auctions, less information is available about the artist's present market value. Therefore, it is highly probable for bidders to rely upon other value signals in these lots. Thus, it is hypothesized that *Reactors* will tend to change their values in these lots.

4.3. Bidding Characteristics

During auctions, *Reactors* are assumed to wait and consult bids of others participants (*Influencers*) to reduce any uncertainty they may have about the value of a lot. Thus, we conjecture that *Reactors* will bid more in the second half of the auction than in the first half. Further, these bidders by the virtue of their behavior tend to be selective in the types of lots on which they bid. Their attachment to a particular item is an integral reason for them to update their private value for the lot [13]. Therefore, we hypothesize that they will participate in auctions of fewer lots than other bidders will.

5. Results

The description of the bid data is presented in Table 1. In this particular auction, 199 lots were sold, and 42 bidders were observed to change their value belief for 63 lots during a Modern Indian Art auction held in December 2005. Eighty bidders participated in the auctions of these 63 lots creating 947 bid instances. Thus, an 80×80 bidder matrix is created and only the influence on those forty-two bidders who changed their value belief is considered. Bidder

Table 1: Data description.

No. of lots sold	199
No. of bids	3080
Average no. of bids per lot (range)	15.47 (2, 48)
Average value of the lots (range)	\$56,282 (\$2,850, \$1,351,000)
Average first bid of the lots (range)	\$19,343 (\$650, \$300,000)
Average no. of bidders per lot (range)	6.35 (2, 14)
Average time of bids (scaled 0-1)	0.4998
Average time of entry to the auction	0.5386
Average time of exit to the auction	0.8397

Table 2: Correlation matrix of the bidder covariates.

	($x1$)	($x2$)	($x3$)	($x4$)	($x5$)	($x6$)	($x6$)	($x8$)
Low pre-auction estimates of the lots bid ($x1$)	1							
No. of lots sold by the artists in theprevious year's auction ($x2$)	0.551**	1						
No. of lots by emerging artist ($x3$)	−0.183	−0.182	1					
No. of lots by established artist ($x4$)	0.584**	0.655**	0.210	1				
No. of paperworks ($x5$)	0.127	0.201	0.474**	0.399**	1			
No. of bids placed in the first half of the auction ($x6$)	0.107	0.158	0.359**	0.431**	0.515**	1		
No. of bids placed in the second half of the auctions ($x7$)	0.204	−0.089	0.291**	0.264*	0.116	−0.201	1	
Total no. of unique lots bid ($x8$)	0.239*	0.243*	0.666**	0.673**	0.693**	0.606**	0.304**	1

**Correlation is significant at the 0.01 level (2-tailed).
*Correlation is significant at the 0.05 level (2-tailed).

level covariates are used in the model to determine the characteristics of those who overbid. Table 2 presents the correlation between the bidder level covariates.

A Markov Chain Monte Carlo algorithm was run for the influence data. Each chain was run for 200,000 iterations. However, since a large number of parameters (*Influencer* and *Reactor* specific covariates, and terms capturing higher-order dependencies) are analyzed in the model, only every 50th iteration is stored in order to keep the output file to a reasonable size, as suggested by MacEachernand and Berliner [30]. Outputs from first 20,000 iterations are considered as burn-ins and are not recorded. (A burn-in of 20,000 and capturing every 50th iteration suggests that the model should stabilize by the 400th row of the output. Examining the trace plot, we do find that the model has stabilized well before 400, suggesting that 20,000 burn-in was sufficient. Complete traces are available from the authors.) The posterior means and quantile-based 95% confidence intervals are presented in Tables 3 and 4 where the mean is highlighted with bolded fonts.

Table 3: Posterior Means and Quantile-Based 95% Confidence Interval for *Influencer* and *Reactor* Level Covariates.

Covariates	Influencer	Reactor
Low pre-auction estimates of the lots bid	−0.3861	5.3660
	0.2545	**9.2870**
	0.9042	13.1774
No. of lots sold by the artists in the previous year's auction	−0.5961	−5.8831
	−0.1460	**−3.2870**
	0.3120	−0.6054
No. of lots by emerging artists	−0.1371	−1.6351
	0.2415	**0.8290**
	0.6323	3.0512
No. of lots by established artists	−0.1492	−3.7290
	0.2350	**−1.3695**
	0.6120	0.9690
No. of paper works	−0.4022	−0.6831
	−0.0295	**1.3795**
	0.3471	3.5781
No. of bids placed in the first half of the auction	0.1229	−0.4751
	0.5240	**1.8985**
	0.9260	4.7242
No. of bids placed in the second half of the auction	−0.7030	8.2930
	−0.2420	**11.0610**
	0.2515	7.3632
Total number of unique lots bid	−0.7850	−10.165
	−0.2305	**−6.4200**
	0.3080	−3.2234

Here we will only discuss the results where the 95% confidence interval does not include a zero. From Table 3, we find that the pre-auction estimates have a positive effect for the *Reactors*. This suggests that bidders who typically change their value belief have bid on lots with high estimated values. We also examined the bidder characteristics of the *Influencers* and *Reactors* by comparing their bidding frequency during each half of the auction. Results show that the coefficient for the number of second half bids for the *Reactors* is positive, but there is no effect of the first half bid frequency. This suggests that *Reactors* tend to bid more in the second half of the auction than the first half. We also found that *Influencers* bid more in the first half as compared to the second half of the auction. In addition, we examined the coefficient for the total number of unique lots bid by the two types of bidder. We found that the coefficient for the *Reactors* to be negative (coefficient = − 6.4200) and no effect for the *Influencers*. This suggests that *Reactors* participate in auctions of fewer lots as compared to the *Influencers*. A summary of the results is presented in Table 5.

Estimates of other major parameters (Table 4) suggest that the common influence variance is smaller than the common *Reactor* variance, meaning that the *Influencers* maintain their role throughout the auction. Since these influencing bidders are frequently observed to bid on more lots, they tend to be the art dealers [15]. On the other hand, the common *Reactor* variance is large. This also supports the findings that the *Reactors* bid on fewer items and thus, the same *Reactor* is seldom encountered by the *Influencers*. Error variance was found to

Table 4: Posterior means and quantile-based 95% confidence interval for major parameters of the bilinear-effects model.

Parameters	Posterior mean and C.I.
Common influencer variance	1.4927
	1.9965
	2.7062
Common reactor variance	67.9891
	91.5990
	128.6762
Influencer-Reactor covariance	− 5.1890
	− **2.4560**
	0.1013
Error variance	0.0720
	0.1160
	0.1970
Reciprocity	− 0.7680
	− **0.4960**
	− 0.0399
Variance of latent dimensions	0.0610
	0.1320
	0.3330
Variance of inner dimensions	0.0490
	0.0970
	0.2120

be small, suggesting a good fit for our model. Finally, we find that the reciprocity is negative, which further suggests that the *Reactors* and *Influencers* rarely switch roles during the auction.

6. Implications and Future Directions

This paper investigates the characteristics of bidders (termed here as *Reactors*) who update their private value of the items in the presence of the influencing bidders (termed here as *Influencers*). Traditional approaches such as linear models or logistic regressions, although useful in our context, lack the capacity to capture the inter-bidder effects and analyze *Reactors* and *Influencers* concurrently. To overcome this issue, we represent the auction data as networks of bidders where the nodes represent the bidders participating in the auction and the ties between them represent an *Infuencer-Reactor* relationship. We further develop a random-effects bilinear model capable of handling the covariates of both bidder types at the same time and account for higher-order dependence occurring during an auction.

From the auction house manager's perspective, this study provides a way to identify different types of bidders in terms of how they manage and update their private value towards an item. Such information is vital for auction house managers in strengthening their relationship with their customers and helps them to determine how to attract them to future auctions. For example, our analysis suggests that *Reactors* mostly bid on fewer items, indicating that they might be the art collectors, who, unlike the art dealers, are very selective on the items they desire to purchase [15]. Therefore, there is great possibility and opportunity for the auction house managers to take part in the strategic decisions of the

Table 5: Summary of the findings.

Covariates	Influencers	Reactors
Low pre-auction estimates of the lots bid	No effect	Positive effect suggesting that *Reactors* tend to update values on lots with high pre-auction estimates
Artist's reputation	No effect	Negative effect suggesting that *Reactors* tend to update value on works of artists who have sold fewer works in recent years
No. of paper works	No effect	No effect
Bid frequency at each half of the auction	Positive effect of the first half bid frequency and no effect of the second half bid frequency. This indicates that *Influencers* bid more in the first half as compared to the second half of the auction	Positive effect of second half bid frequency and no effect of the first half bid frequency. This indicates that the *Reactors* tend to bid more in the second half as compared to the first half of the auction
Total no. of unique lots bid	No effect	Negative effect suggesting that *Reactors* tend to bid on small number of lots

Table 6: Managerial Implications.

Findings	Managerial Implications
Reactors bid on fewer items and mostly on high-priced items	– Provide more potable information about the artist and the artwork such as price trend, performance of other items of the artist in the auction – Provide customized support through account managers
Reactors bid more in the second half than in the first half of the auction	– Managers have time (1-2 days in a 3 day auction) to offer solutions to these bidders

bidders and support in their decision making process. Suggestive decisions are shown in Table 6 to support the auction house managers.

As with most research work, this study also has some limitations that need to be acknowledged. First, it is difficult to determine why the *Reactors* have used the proxy bidding system. We used an identification system in the lines of work done by Ku and his colleagues [11], which is tested and proved using a bidder survey. Work by Bapna and his colleagues [23]) found that certain types of bidders in online auctions use proxy bidding as a part of their bidding strategy (referred to as Agent Bidders). They suggested that these bidders place more than 60% of their bids as proxy bids. *Reactors* in our study had less than 20% of their total bids as proxy bids, and thus can be assumed not to be Agent Bidders. We tried two different approaches to identify the level of influence of an *Influencer* over a *Reactor*. Although our measure is grounded on the auction theory, still, further study is needed to develop a better influence measure.

A bidder's value change plays a fundamental role in determining the attractiveness of auctioned items. Therefore, sophisticated models may be developed using value updating

behavior in the future to design more profitable auctions and also help in designing future auctions. Our study contributes to auction house strategy refinement by investigating competition among bidders at the dyadic bidder level. Further studies on this topic are essential to understand the underlying bidding dynamics in auctions.

We hope that this paper will encourage other researchers to start investigating bidder behavior and competition at a more microlevel. We also hope that our paper will promote future applications of bilinear effects models and other advanced techniques in other areas of social sciences.

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