Research Article

An Improved Class of Chain Ratio-Product Type Estimators in Two-Phase Sampling Using Two Auxiliary Variables

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This paper presents a technique for estimating finite population mean of the study variable in the presence of two auxiliary variables using two-phase sampling scheme when the regression line does not pass through the neighborhood of the origin. The properties of the proposed class of estimators are studied under large sample approximation. In addition, bias and efficiency comparisons are carried out to study the performances of the proposed class of estimators over the existing estimators. It has also been shown that the proposed technique has greater applicability in survey research. An empirical study is carried out to demonstrate the performance of the proposed estimators.

1. Introduction

The use of auxiliary information for estimating population mean of the study variable has greater applicability in survey research. It is utilized at the estimation stage and design stage to obtain an improved estimator compared to those not utilizing auxiliary information. The use of ratio and product strategies in survey sampling solely depends upon the knowledge of population mean $\bar{X}$ of the auxiliary variable X .

The ratio estimator was developed by Cochran [1] to estimate the population mean $\bar{Y}$ of the study variable Y by using information on auxiliary variable X , positively correlated with Y . The ratio estimator is most effective when the relationship between Y and X is linear through the origin and the variance of Y is proportional to X . Robson [2] defined a product estimator that was revisited by Murthy [3]. The product estimator is used when the auxiliary variable X is negatively correlated with the study variable Y .

When the population mean $\bar{X}$ of the auxiliary variable X is not known before the start of a survey, then a first-phase sample of size n' is selected from the population of size N on which only the auxiliary variable X is measured in order to furnish a good estimate of $\bar{X}$. And then a

second-phase sample of size n is selected from the first-phase sample of size n' on which both the study variable Y and the auxiliary variable X are measured. This procedure of selecting the samples from the given population is known as two-phase sampling (or double sampling). The concept of double sampling was first introduced by Neyman [4]. Some contribution to two-phase sampling has been made by Sukhatme [5], Hidirolou and Sarndal [6], Fuller [7], Hidirolou [8], Singh and Vishwakarma [9], and Sahoo et al. [10].

We can use either one or two (or more than two) auxiliary variables while estimating population mean of the study variable; keeping this fact, Chand [11] introduced chain ratio estimators. This led various authors including Kiregyera [12], Singh and Upadhyaya [13], Prasad et al. [14], Singh et al. [15], Singh and Choudhury [16], and Vishwakarma and Gangele [17] to modify the chain type estimators and discuss their properties.

When the population mean $\bar{Z}$ of another auxiliary variable Z which has a positive correlation with X (i.e., $\rho_{XZ} > 0$) is known and if $\rho_{YX} > \rho_{YZ} > 0$, then it is advisable to estimate $\bar{X}$ by $\bar{X} = \bar{x}'(\bar{Z}/\bar{z}')$, which would provide a better estimate of $\bar{X}$ as compared to $\bar{x}'$.

The usual chain type ratio and product estimators of $\bar{Y}$ under double sampling scheme using two auxiliary variables X and Z are given, respectively, by

$$\begin{aligned}\bar{y}_R^{\text{dc}} &= \bar{y} \frac{\bar{x}' \bar{Z}}{\bar{x} \bar{Z}'} \\ \bar{y}_P^{\text{dc}} &= \bar{y} \frac{\bar{x} \bar{Z}'}{\bar{x}' \bar{Z}}.\end{aligned}\quad (1)$$

Singh and Choudhury [16] suggested the following exponential chain type ratio and product estimators of $\bar{Y}$ under double sampling scheme using two auxiliary variables X and Z :

$$\begin{aligned}\bar{y}_{\text{Re}}^{\text{dc}} &= \bar{y} \exp \left\{ \frac{(\bar{x}'/\bar{Z}') \bar{Z} - \bar{x}}{(\bar{x}'/\bar{Z}') \bar{Z} + \bar{x}} \right\}, \\ \bar{y}_{\text{Pe}}^{\text{dc}} &= \bar{y} \exp \left\{ \frac{\bar{x} - (\bar{x}'/\bar{Z}') \bar{Z}}{\bar{x} + (\bar{x}'/\bar{Z}') \bar{Z}} \right\},\end{aligned}\quad (2)$$

where $\bar{x}'$ and $\bar{Z}'$ are the sample means of X and Z , respectively, based on the first-phase sample of size n' drawn from the population of size N with the help of Simple Random Sampling Without Replacement (SRSWOR) scheme. Also, $\bar{y}$ and $\bar{x}$ are the sample means of Y and X , respectively, based on the second-phase sample of size n drawn from the first-phase sample of size n' with the help of SRSWOR scheme.

2. Proposed Estimator

It has been theoretically established that, in general, the linear regression estimator is more efficient than the ratio (product) estimator except when the regression line of Y on X passes through the neighborhood of the origin, in which the efficiencies of these estimators are almost equal. However, owing to stronger intuitive appeal, survey statisticians favour the use of ratio and product estimators. Further, we note that, in many practical situations, the regression line does not pass through the neighborhood of the origin. In these situations, the ratio estimator does not perform well as the linear regression estimator. Considering this fact, Singh and Ruiz Espejo [18] made an attempt to improve the performance of these estimators and suggested the following ratio-product type estimator for population mean $\bar{Y}$ under double sampling scheme using single auxiliary variable X :

$$\bar{y}_{\text{RP}}^d = \bar{y} \left[\alpha \frac{\bar{x}'}{\bar{x}} + (1 - \alpha) \frac{\bar{x}}{\bar{x}'} \right], \quad (3)$$

where α is a real constant.

We propose the following exponential chain ratio-product type estimator for population mean $\bar{Y}$ under double sampling scheme using two auxiliary variables X and Z :

$$\begin{aligned}\bar{y}_{\text{RPe}}^{\text{dc}} &= \bar{y} \left[\alpha \exp \left\{ \frac{(\bar{x}'/\bar{Z}') \bar{Z} - \bar{x}}{(\bar{x}'/\bar{Z}') \bar{Z} + \bar{x}} \right\} \right. \\ &\quad \left. + (1 - \alpha) \exp \left\{ \frac{\bar{x} - (\bar{x}'/\bar{Z}') \bar{Z}}{\bar{x} + (\bar{x}'/\bar{Z}') \bar{Z}} \right\} \right],\end{aligned}\quad (4)$$

where α is a real constant to be determined such that the Mean Square Error (MSE) of the proposed estimator $\bar{y}_{\text{RPe}}^{\text{dc}}$ is minimum. For $\alpha = 1$, $\bar{y}_{\text{RPe}}^{\text{dc}} \rightarrow \bar{y}_{\text{Re}}^{\text{dc}}$, whereas, for $\alpha = 0$, $\bar{y}_{\text{RPe}}^{\text{dc}} \rightarrow \bar{y}_{\text{Pe}}^{\text{dc}}$.

Remark. It is noted that the proposed estimator in (4) is a special case of the class of estimators $\bar{y}_{\text{class}} = \bar{y}H(\bar{x}, \bar{Z}')$ proposed by Srivastava [19], where $H(\cdot)$ is a parametric function such that $H(\bar{x}' \mid s_1, \bar{Z}) = 1$ and satisfies certain regularity conditions defined in Srivastava [19].

3. Bias and MSE of the Proposed Estimator

To obtain the Bias and Mean Square Error (MSE) of the proposed estimator $\bar{y}_{\text{RPe}}^{\text{dc}}$, we consider

$$\begin{aligned}\bar{y} &= \bar{Y}(1 + e_0), & \bar{x} &= \bar{X}(1 + e_1), \\ \bar{x}' &= \bar{X}(1 + e_1'), & \bar{Z}' &= \bar{Z}(1 + e_2'),\end{aligned}\quad (5)$$

such that

$$E(e_0) = E(e_1) = E(e_1') = E(e_2') = 0, \quad (6)$$

where $|e_0| < 1$, $|e_1| < 1$, $|e_1'| < 1$, $|e_2'| < 1$.

Let C_Y , C_X , and C_Z be the coefficients of variation of Y , X , and Z , respectively. Also, let ρ_{YX} , ρ_{YZ} , and ρ_{XZ} be the correlation coefficients between Y and X , Y and Z , and X and Z , respectively. Then, we have

$$\begin{aligned}E(e_0^2) &= f_1 C_Y^2, & E(e_1^2) &= f_1 C_X^2, \\ E(e_1'^2) &= f_2 C_X^2, & E(e_2'^2) &= f_2 C_Z^2, \\ E(e_0 e_1) &= f_1 \rho_{YX} C_Y C_X, & E(e_0 e_1') &= f_2 \rho_{YX} C_Y C_X, \\ E(e_0 e_2') &= f_2 \rho_{YZ} C_Y C_Z, \\ E(e_1 e_1') &= f_2 C_X^2, & E(e_1 e_2') &= f_2 \rho_{XZ} C_X C_Z, \\ E(e_1' e_2') &= f_2 \rho_{XZ} C_X C_Z,\end{aligned}\quad (7)$$

where

$$\begin{aligned}f_1 &= \left(\frac{1}{n} - \frac{1}{N} \right), & f_2 &= \left(\frac{1}{n'} - \frac{1}{N} \right), \\ f_3 &= f_1 - f_2 = \left(\frac{1}{n} - \frac{1}{n'} \right),\end{aligned}$$

$$C_Y^2 = \frac{S_Y^2}{\bar{Y}^2}, \quad C_X^2 = \frac{S_X^2}{\bar{X}^2}, \quad C_Z^2 = \frac{S_Z^2}{\bar{Z}^2},$$

$$\rho_{YX} = \frac{S_{YX}}{S_Y S_X}, \quad \rho_{YZ} = \frac{S_{YZ}}{S_Y S_Z}, \quad \rho_{XZ} = \frac{S_{XZ}}{S_X S_Z},$$

$$S_Y^2 = \frac{1}{(N-1)} \sum_{i=1}^N (Y_i - \bar{Y})^2, \quad S_X^2 = \frac{1}{(N-1)} \sum_{i=1}^N (X_i - \bar{X})^2,$$

$$S_Z^2 = \frac{1}{(N-1)} \sum_{i=1}^N (Z_i - \bar{Z})^2,$$

$$\begin{aligned}
S_{YX} &= \frac{1}{(N-1)} \sum_{i=1}^N (Y_i - \bar{Y})(X_i - \bar{X}), \\
S_{YZ} &= \frac{1}{(N-1)} \sum_{i=1}^N (Y_i - \bar{Y})(Z_i - \bar{Z}), \\
S_{XZ} &= \frac{1}{(N-1)} \sum_{i=1}^N (X_i - \bar{X})(Z_i - \bar{Z}).
\end{aligned} \tag{8}$$

Now, expressing the estimator $\bar{y}_{\text{RPe}}^{\text{dc}}$ in terms of e_0, e_1, e'_1 , and e'_2 and neglecting the terms of e_0, e_1, e'_1 , and e'_2 involving degree greater than two, we get

$$\begin{aligned}
\bar{y}_{\text{RPe}}^{\text{dc}} &= \bar{Y} \left[1 + \alpha (e'_1 - e'_2 - e_1 + e_0 e'_1 - e_0 e'_2 - e_0 e_1) \right. \\
&\quad - \frac{\alpha}{2} (e_1'^2 - e_2'^2 - e_1^2) + e_0 \\
&\quad - \frac{1}{2} (e'_1 - e'_2 - e_1 + e_0 e'_1 - e_0 e'_2 - e_0 e_1) \\
&\quad + \frac{1}{4} (e_1'^2 - e_2'^2 - e_1^2 - e'_1 e'_2 + e_1 e'_2 - e_1 e'_1) \\
&\quad \left. + \frac{1}{8} (e_1'^2 + e_2'^2 + e_1^2) \right]
\end{aligned} \tag{9}$$

To the first degree of approximation, the Bias and Mean Square Error (MSE) of the proposed estimator $\bar{y}_{\text{RPe}}^{\text{dc}}$ are given by

$$\begin{aligned}
B(\bar{y}_{\text{RPe}}^{\text{dc}}) &= \bar{Y} \left[\frac{(4\alpha - 1)}{8} \right. \\
&\quad \times \{f_3 C_X^2 + f_2 C_Z^2\} - \frac{(2\alpha - 1)}{2} \\
&\quad \left. \times \{f_3 \rho_{YX} C_Y C_X + f_2 \rho_{YZ} C_Y C_Z\} \right],
\end{aligned} \tag{10}$$

$$\begin{aligned}
\text{MSE}(\bar{y}_{\text{RPe}}^{\text{dc}}) &= \bar{Y}^2 \left[f_1 C_Y^2 + \frac{(2\alpha - 1)^2}{4} \{f_3 C_X^2 + f_2 C_Z^2\} \right. \\
&\quad \left. - (2\alpha - 1) \{f_3 \rho_{YX} C_Y C_X + f_2 \rho_{YZ} C_Y C_Z\} \right].
\end{aligned} \tag{11}$$

To the first degree of approximation, the expressions for Bias and Mean Square Error (MSE) of the estimators $\bar{y}_R^{\text{dc}}, \bar{y}_P^{\text{dc}}, \bar{y}_{\text{Re}}^{\text{dc}}, \bar{y}_{\text{Pe}}^{\text{dc}}$, and $\bar{y}_{\text{RP}}^d$ are, respectively, given by

$$\begin{aligned}
B(\bar{y}_R^{\text{dc}}) &= \bar{Y} [f_3 C_X^2 + f_2 C_Z^2 - f_3 \rho_{YX} C_Y C_X - f_2 \rho_{YZ} C_Y C_Z], \\
B(\bar{y}_P^{\text{dc}}) &= \bar{Y} [f_3 \rho_{YX} C_Y C_X + f_2 \rho_{YZ} C_Y C_Z],
\end{aligned}$$

$$\begin{aligned}
B(\bar{y}_{\text{Re}}^{\text{dc}}) &= \bar{Y} \left[\frac{3}{8} \{f_3 C_X^2 + f_2 C_Z^2\} \right. \\
&\quad \left. - \frac{1}{2} \{f_3 \rho_{YX} C_Y C_X + f_2 \rho_{YZ} C_Y C_Z\} \right], \\
B(\bar{y}_{\text{Pe}}^{\text{dc}}) &= \bar{Y} \left[\frac{-1}{8} \{f_3 C_X^2 + f_2 C_Z^2\} \right. \\
&\quad \left. + \frac{1}{2} \{f_3 \rho_{YX} C_Y C_X + f_2 \rho_{YZ} C_Y C_Z\} \right], \\
B(\bar{y}_{\text{RP}}^d) &= \bar{Y} [\alpha f_3 C_X^2 - (2\alpha - 1) f_3 \rho_{YX} C_Y C_X],
\end{aligned} \tag{12}$$

$$\begin{aligned}
\text{MSE}(\bar{y}_R^{\text{dc}}) &= \bar{Y}^2 [f_1 C_Y^2 + f_3 C_X^2 + f_2 C_Z^2 \\
&\quad - 2f_3 \rho_{YX} C_Y C_X - 2f_2 \rho_{YZ} C_Y C_Z],
\end{aligned}$$

$$\begin{aligned}
\text{MSE}(\bar{y}_P^{\text{dc}}) &= \bar{Y}^2 [f_1 C_Y^2 + f_3 C_X^2 + f_2 C_Z^2 \\
&\quad + 2f_3 \rho_{YX} C_Y C_X + 2f_2 \rho_{YZ} C_Y C_Z],
\end{aligned}$$

$$\begin{aligned}
\text{MSE}(\bar{y}_{\text{Re}}^{\text{dc}}) &= \bar{Y}^2 \left[f_1 C_Y^2 + \frac{1}{4} \{f_3 C_X^2 + f_2 C_Z^2\} \right. \\
&\quad \left. - \{f_3 \rho_{YX} C_Y C_X + f_2 \rho_{YZ} C_Y C_Z\} \right],
\end{aligned}$$

$$\begin{aligned}
\text{MSE}(\bar{y}_{\text{Pe}}^{\text{dc}}) &= \bar{Y}^2 \left[f_1 C_Y^2 + \frac{1}{4} \{f_3 C_X^2 + f_2 C_Z^2\} \right. \\
&\quad \left. + \{f_3 \rho_{YX} C_Y C_X + f_2 \rho_{YZ} C_Y C_Z\} \right],
\end{aligned}$$

$$\begin{aligned}
\text{MSE}(\bar{y}_{\text{RP}}^d) &= \bar{Y}^2 [f_1 C_Y^2 + 4\alpha^2 f_3 C_X^2 \\
&\quad - 4\alpha f_3 \{C_X^2 + \rho_{YX} C_Y C_X\} \\
&\quad + f_3 \{C_X^2 + 2\rho_{YX} C_Y C_X\}].
\end{aligned} \tag{13}$$

3.1. Optimum Value of α . As we know, α is determined so as to minimize the Mean Square Error (MSE) of the estimators $\bar{y}_{\text{RP}}^d$ and $\bar{y}_{\text{RPe}}^{\text{dc}}$. So, the optimum values of α , for which $\text{MSE}(\bar{y}_{\text{RP}}^d)$ and $\text{MSE}(\bar{y}_{\text{RPe}}^{\text{dc}})$ are minimum, are obtained by using the following conditions:

$$\begin{aligned}
\frac{\partial}{\partial \alpha} \text{MSE}(\bar{y}_{\text{RP}}^d) &= 0, \\
\frac{\partial}{\partial \alpha} \text{MSE}(\bar{y}_{\text{RPe}}^{\text{dc}}) &= 0.
\end{aligned} \tag{14}$$

The optimum value of α , which minimizes the Mean Square Error (MSE) of the estimator $\bar{y}_{\text{RP}}^d$, is given by

$$\alpha_{\text{opt}} = \frac{1}{2} \left[1 + \rho_{YX} \frac{C_Y}{C_X} \right]. \tag{15}$$

The optimum value of α , which minimizes the Mean Square Error (MSE) of the estimator $\bar{y}_{RPe}^{dc}$, is given by

$$\alpha_{opt} = \frac{f_3 (2\rho_{YX}C_Y C_X + C_X^2) + f_2 (2\rho_{YZ}C_Y C_Z + C_Z^2)}{2(f_3 C_X^2 + f_2 C_Z^2)}. \quad (16)$$

Substituting the value of α from (15) in (13), we get the minimum MSE of $\bar{y}_{RP}^d$ as

$$MSE(\bar{y}_{RP}^d)_{min} = \bar{Y}^2 [f_1 C_Y^2 - f_3 \rho_{YX}^2 C_Y^2]. \quad (17)$$

Substituting the value of α from (16) in (11), we get the minimum MSE of $\bar{y}_{RPe}^{dc}$ as

$$\begin{aligned} MSE(\bar{y}_{RPe}^{dc})_{min} \\ = \bar{Y}^2 \left[f_1 C_Y^2 - \frac{(f_3 \rho_{YX} C_Y C_X + f_2 \rho_{YZ} C_Y C_Z)^2}{f_3 C_X^2 + f_2 C_Z^2} \right]. \end{aligned} \quad (18)$$

4. Efficiency Comparisons

It is well known that the Bias and variance of the usual unbiased estimator $\bar{y}$ for population mean in SRSWOR are

$$B(\bar{y}) = 0, \quad (19)$$

$$V(\bar{y}) = f_1 S_Y^2 = f_1 \bar{Y}^2 C_Y^2. \quad (20)$$

From (11), (13), and (20), we have

(i) $MSE(\bar{y}_{RPe}^{dc}) < V(\bar{y})$, if

$$\alpha < \frac{4(f_3 \rho_{YX} C_Y C_X + f_2 \rho_{YZ} C_Y C_Z) + f_3 C_X^2 + f_2 C_Z^2}{2(f_3 C_X^2 + f_2 C_Z^2)}, \quad (21)$$

(ii) $MSE(\bar{y}_{RPe}^{dc}) < MSE(\bar{y}_R^{dc})$, if

$$\alpha < \frac{4(f_3 \rho_{YX} C_Y C_X + f_2 \rho_{YZ} C_Y C_Z) - (f_3 C_X^2 + f_2 C_Z^2)}{2(f_3 C_X^2 + f_2 C_Z^2)}, \quad (22)$$

(iii) $MSE(\bar{y}_{RPe}^{dc}) < MSE(\bar{y}_P^{dc})$, if

$$\alpha < \frac{4(f_3 \rho_{YX} C_Y C_X + f_2 \rho_{YZ} C_Y C_Z) + 3(f_3 C_X^2 + f_2 C_Z^2)}{2(f_3 C_X^2 + f_2 C_Z^2)}, \quad (23)$$

(iv) $MSE(\bar{y}_{RPe}^{dc}) < MSE(\bar{y}_{Re}^{dc})$, if

$$\alpha < \frac{2(f_3 \rho_{YX} C_Y C_X + f_2 \rho_{YZ} C_Y C_Z)}{f_3 C_X^2 + f_2 C_Z^2}, \quad (24)$$

(v) $MSE(\bar{y}_{RPe}^{dc}) < MSE(\bar{y}_{Pe}^{dc})$, if

$$\alpha < \frac{2(f_3 \rho_{YX} C_Y C_X + f_2 \rho_{YZ} C_Y C_Z) + f_3 C_X^2 + f_2 C_Z^2}{f_3 C_X^2 + f_2 C_Z^2}, \quad (25)$$

(vi) $MSE(\bar{y}_{RPe}^{dc}) < MSE(\bar{y}_{RP}^d)$, if

$$\alpha < \frac{4(f_3 \rho_{YX} C_Y C_X - f_2 \rho_{YZ} C_Y C_Z) + (3f_3 C_X^2 - f_2 C_Z^2)}{2(3f_3 C_X^2 - f_2 C_Z^2)}. \quad (26)$$

The range of α provides enough scope for choosing many estimators that are more efficient than the above considered estimators.

5. Empirical Study

To examine the merits of the proposed estimator of $\bar{Y}$, we have considered the following natural population datasets.

Population I (source: Cochran [20]) is shown as follows:

Y: number of “placebo” children,

X: number of paralytic polio cases in the “placebo” group,

Z: number of paralytic polio cases in the “not inoculated” group

$N = 34$, $n' = 15$, $n = 10$, $\bar{Y} = 4.92$, $\bar{X} = 2.59$, and $\bar{Z} = 2.91$,

$\rho_{YX} = 0.7326$, $\rho_{YZ} = 0.6430$, $\rho_{XZ} = 0.6837$, $C_Y^2 = 1.0248$, $C_X^2 = 1.5175$, and $C_Z^2 = 1.1492$.

Population II (source: Murthy [21]) is shown as follows:

Y: area under wheat in 1964,

X: area under wheat in 1963,

Z: cultivated area in 1961,

$N = 34$, $n' = 10$, $n = 7$, $\bar{Y} = 199.44$, $\bar{X} = 208.89$, and $\bar{Z} = 747.59$,

$\rho_{YX} = 0.9801$, $\rho_{YZ} = 0.9043$, $\rho_{XZ} = 0.9097$, $C_Y^2 = 0.5673$, $C_X^2 = 0.5191$, and $C_Z^2 = 0.3527$.

Here, we have computed

(i) the Absolute Relative Bias (ARB) of different suggested estimators of $\bar{Y}$ using the formula

$$ARB(\cdot) = \left| \frac{\text{Bias}(\cdot)}{\bar{Y}} \right|, \quad (27)$$

(ii) the Percentage Relative Efficiencies (PREs) of different suggested estimators of $\bar{Y}$ with respect to $\bar{y}$ using the formula

$$PRE(\cdot, \bar{y}) = \frac{V(\bar{y})}{MSE(\cdot)} \times 100. \quad (28)$$

TABLE 1: Absolute Relative Bias (ARB) of different estimators of $\bar{Y}$.

Estimators	Population I	Population II
$\bar{y}$	0.0000	0.0000
$\bar{y}_R^{dc}$	0.0369	0.0042
$\bar{y}_P^{dc}$	0.0564	0.0513
$\bar{y}_{Re}^{dc}$	0.0068	0.0079
$\bar{y}_{Pe}^{dc}$	0.0165	0.0198
$\bar{y}_{RP}^d$	0.0222	0.0008
$\bar{y}_{RPe}^{dc}$	0.0058	0.0243

TABLE 2: Percentage Relative Efficiencies (PREs) of different estimators of $\bar{Y}$ with respect to $\bar{y}$.

Estimators	Population I	Population II
$\bar{y}$	100	100
$\bar{y}_R^{dc}$	136.91	730.81
$\bar{y}_P^{dc}$	*	*
$\bar{y}_{Re}^{dc}$	184.36	259.55
$\bar{y}_{Pe}^{dc}$	*	*
$\bar{y}_{RP}^d$	133.95	156.96
$\bar{y}_{RPe}^{dc}$	189.27	763.30

*Data is not applicable.

6. Conclusion

It is observed from Table 1 that,

(i) for population I,

$$\begin{aligned} \text{ARB}(\bar{y}) &< \text{ARB}(\bar{y}_{RPe}^{dc}) < \text{ARB}(\bar{y}_{Re}^{dc}) < \text{ARB}(\bar{y}_P^{dc}) \\ &< \text{ARB}(\bar{y}_{RP}^d) < \text{ARB}(\bar{y}_R^{dc}) < \text{ARB}(\bar{y}_P^{dc}), \end{aligned} \quad (29)$$

(ii) for population II,

$$\begin{aligned} \text{ARB}(\bar{y}) &< \text{ARB}(\bar{y}_{RP}^d) < \text{ARB}(\bar{y}_R^{dc}) < \text{ARB}(\bar{y}_{Re}^{dc}) \\ &< \text{ARB}(\bar{y}_{Pe}^{dc}) < \text{ARB}(\bar{y}_{RPe}^{dc}) < \text{ARB}(\bar{y}_P^{dc}). \end{aligned} \quad (30)$$

From Table 2, we see that the Percentage Relative Efficiency (PRE) of the proposed estimator $\bar{y}_{RPe}^{dc}$, for populations I and II, is more as compared to all other existing estimators, that is, usual unbiased estimator $\bar{y}$, chain type ratio estimator $\bar{y}_R^{dc}$, chain type product estimator $\bar{y}_P^{dc}$, exponential chain type ratio estimator $\bar{y}_{Re}^{dc}$, exponential chain type product estimator $\bar{y}_{Pe}^{dc}$, and ratio-product type estimator $\bar{y}_{RP}^d$.

Finally, from Tables 1 and 2, we conclude that the proposed estimator $\bar{y}_{RPe}^{dc}$ (based on two auxiliary variables X and Z) is a more appropriate estimator in comparison to other existing estimators as it has appreciable efficiency as well as lower relative bias.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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