Estimation of Population Mean in Chain Ratio-Type Estimator under Systematic Sampling

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Received 7 September 2015; Accepted 11 October 2015

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A chain ratio-type estimator is proposed for the estimation of finite population mean under systematic sampling scheme using two auxiliary variables. The mean square error of the proposed estimator is derived up to the first order of approximation and is compared with other relevant existing estimators. To illustrate the performances of the different estimators in comparison with the usual simple estimator, we have taken a real data set from the literature of survey sampling.

1. Introduction and Literature Review

Incorporating the knowledge of the auxiliary variables is very important for the construction of efficient estimators for the estimation of population parameters and increasing the efficiency of the estimators in different sampling design. Using the knowledge of the auxiliary variables, several authors have proposed different estimation technique for the finite population mean of the study variable; Cochran [1], Tripathy [2], Kadiar and Cingi [3, 4], Singh et al. [5], Khan and Arunachalam [6], Lone and Tailor [7], Khan [8], Khan and Hussain [9], and Khan et al. [10] have worked on the estimation of population parameters using auxiliary information.

In the present paper, we will work on the estimation of the population mean of the study variable; Cochran [1], Tripathy [2], Kadiar and Cingi [3, 4], Singh et al. [5], Khan and Arunachalam [6], Lone and Tailor [7], Khan [8], Khan and Hussain [9], and Khan et al. [10] have worked on the estimation of population parameters using auxiliary information.

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\( e_0 = (\bar{y}_{ys} - \bar{y})/\bar{Y}, e_1 = (\bar{x}_{ys} - \bar{X})/\bar{X}, \) and \( e_2 = (\bar{z}_{ys} - \bar{Z})/\bar{Z}, \) such that \( E(e_i) = 0, \) for \( i = 0, 1, \) and 2.

The first order of approximation of the above errors terms is given by

\[
\begin{align*}
E(e_0^2) &= \theta \rho_y^2 C_y^2, \\
E(e_1^2) &= \theta \rho_x^2 C_x^2, \\
E(e_2^2) &= \theta \rho_z^2 C_z^2,
\end{align*}
\]

\( E(e_0 e_1) = \theta k C_x^2 \sqrt{\rho_y^2 \rho_x^2}, \)

\( E(e_0 e_2) = \theta k \rho_y^2 C_x^2 \sqrt{\rho_x^2 \rho_z^2}, \)

\( E(e_1 e_2) = \theta k \rho_x^2 C_z^2 \sqrt{\rho_z^2 \rho_x^2}, \)

where

\[
\begin{align*}
\lambda &= \left( \frac{N - 1}{nN} \right), \\
\rho_{yx} &= \frac{S_{yx}}{S_y S_x}, \\
\rho_{yz} &= \frac{S_{yz}}{S_y S_z}, \\
\rho_{xz} &= \frac{S_{xz}}{S_x S_z}, \\
k &= \frac{\rho_{yx} C_y}{C_x}, \\
k^* &= \frac{\rho_{yx} C_y}{C_z}, \\
\rho_y^* &= \left\{ 1 + (n - 1) \rho_y \right\}, \\
\rho_x^* &= \left\{ 1 + (n - 1) \rho_x \right\}, \\
\rho_z^* &= \left\{ 1 + (n - 1) \rho_z \right\}, \\
\rho^{**} &= \frac{\rho_y}{\rho_x}, \\
\rho^{**} &= \frac{\rho_x}{\rho_z}, \\
\rho^{**} &= \frac{\rho_x}{\rho_z}, \\
\rho^{**} &= \frac{\rho_x}{\rho_z}, \\
\rho^{**} &= \frac{\rho_x}{\rho_z},
\end{align*}
\]

where \( \rho_y, \rho_x, \) and \( \rho_z \) are the intraclass correlation among the pair of units for the variables \( y, x, \) and \( z, \) respectively.

The variance of the usual unbiased estimator for population mean is

\[
\text{Var} (\hat{\theta}_0) = \lambda \bar{Y} \rho^{**2} C_y^2 C_z.
\]
2. Proposed Estimator

In this section, we have proposed the following regression in ratio-cum-product type estimator for the unknown population mean under systematic sampling:

\[ t_m = \bar{y}_{sy} \left( \frac{X}{X + b_{xy} (\bar{x}_y - X)} \right) \delta_1 \]

\[ \cdot \left( \frac{Z + b_{yz} (\bar{z}_y - Z)}{Z} \right) \delta_2, \]  

(14)

where \( \delta_1 \) and \( \delta_2 \) are the unknown constants, whose values are to be found for the minimum mean square error.

The mean square error (MSE) of the estimator up to first order of approximation is

\[
\text{MSE}(t_m) = \lambda \overline{Y}^2 \left[ \rho_y^2 C_y^2 + \delta_1^2 \rho_{yx}^2 \rho_x^2 C_x^2 + \delta_2^2 \rho_{yz}^2 \rho_z^2 C_z^2 - 2 \delta_1 \beta_{yx} k C_x \sqrt{\rho_y^2 \rho_x^2} - 2 \delta_2 \beta_{yz} k C_z \sqrt{\rho_y^2 \rho_z^2} \right. \\
\left. - 2 \delta_1 \delta_2 \beta_{yx} \beta_{yz} k^2 C_x^2 \sqrt{\rho_y^2 \rho_x^2 \rho_z^2} \right].
\]  

(15)

On differentiating (15), with respect to \( \delta_1 \) and \( \delta_2 \), we obtain the minimum mean squared error of the estimator \( t_m \), which is given by

\[
\text{MSE}(t_m) = \lambda \overline{Y}^2 \rho_y^2 \left[ C_y^2 - k^2 C_z^2 - \frac{(k^* k^{**} C_y^2 - k C_z^2)^2}{(C_x^2 - k^2 C_z^2)} \right],
\]  

(16)

where the optimum values are

\[ \delta_1 = \sqrt{\rho_y^2} (k C_y^2 - k^* k^{**} C_z^2) / \beta_{yx} \sqrt{\rho_x^2 (C_x^2 - k^2 C_z^2)} \]

and

\[ \delta_2 = (\sqrt{\rho_y^2} / \beta_{yz}) \sqrt{\rho_z^2} |k^* (k C_x^2 - k^* k^{**} C_z^2) / (C_x^2 - k^2 C_z^2)| - k^*). \]

3. Comparison

In this section, we have compared the MSE of the proposed estimator with the MSEs of simple estimator, Swain [14] estimator, Shukla [16] estimator, Singh et al. [20] estimators, and Tailor et al. [25] estimator and found some theoretical conditions under which the proposed estimator will always perform better:

(i) By (16) and (3), \( \text{MSE}(t_m) \leq \text{MSE}(t_0) \) if

\[
\left[ k_x^2 C_z^2 + \frac{(k^* k^{**} C_y^2 - k C_z^2)^2}{(C_x^2 - k^2 C_z^2)} \right] \geq 0.
\]  

(17)

(ii) By (16) and (5), \( \text{MSE}(t_m) \leq \text{MSE}(t_1) \) if

\[
\left[ \rho_y^2 \left\{ k_x^2 C_z^2 + \frac{(k^* k^{**} C_y^2 - k C_z^2)^2}{(C_x^2 - k^2 C_z^2)} \right\} + \rho_x^2 C_z^2 \left( 1 - 2k^* \sqrt{\rho_z^2} \right) \right] \geq 0.
\]  

(18)

(iii) By (16) and (6), \( \text{MSE}(t_m) \leq \text{MSE}(t_2) \) if

\[
\left[ \rho_y^2 \left\{ k_x^2 C_z^2 + \frac{(k^* k^{**} C_y^2 - k C_z^2)^2}{(C_x^2 - k^2 C_z^2)} \right\} + \rho_z^2 C_z^2 \left( 1 - 2k^* \sqrt{\rho_y^2} \right) \right] \geq 0.
\]  

(19)

(iv) By (16) and (8), \( \text{MSE}(t_m) \leq \text{MSE}(t_3) \) if

\[
\left[ k_x^2 C_z^2 C_y^2 + \frac{(k^* k^{**} C_y^2 - k C_z^2)^2}{(C_x^2 - k^2 C_z^2)} \right] \geq 0.
\]  

(20)

(v) By (16) and (10), \( \text{MSE}(t_m) \leq \text{MSE}(t_4) \) if

\[
\left[ \rho_y^2 \left\{ k_x^2 C_z^2 + \frac{(k^* k^{**} C_y^2 - k C_z^2)^2}{(C_x^2 - k^2 C_z^2)} \right\} + \frac{\rho_x^2 C_z^2}{4} \left( 1 - 4k^* \sqrt{\rho_z^2} \right) \right] \geq 0.
\]  

(21)

(vi) By (16) and (11), \( \text{MSE}(t_m) \leq \text{MSE}(t_5) \) if

\[
\left[ \frac{\rho_z^2 C_x^2}{4} \left( 1 + 4k^* \sqrt{\rho_z^2} \right) + \rho_y^2 \left\{ k_x^2 C_z^2 + \frac{(k^* k^{**} C_y^2 - k C_z^2)^2}{(C_x^2 - k^2 C_z^2)} \right\} \right] \geq 0.
\]  

(22)

(vii) By (16) and (13), \( \text{MSE}(t_m) \leq \text{MSE}(t_6) \) if

\[
\left[ \rho_y^2 C_x^2 \left( 1 - 2k^* \sqrt{\rho_z^2} \right) + \rho_z^2 C_x^2 \left( 1 - 2k^* \sqrt{\rho_z^2} \right) \right] \geq 0.
\]  

(23)
Table 1: The percent relative efficiency of different estimators with respect to \( t_0 \).

<table>
<thead>
<tr>
<th>Estimator</th>
<th>( t_0 )</th>
<th>( t_1 )</th>
<th>( t_2 )</th>
<th>( t_3 )</th>
<th>( t_4 )</th>
<th>( t_5 )</th>
<th>( t_6 )</th>
<th>( t_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSE ( (t_0) )</td>
<td>1455.08</td>
<td>373.32</td>
<td>768.06</td>
<td>45.52</td>
<td>820.09</td>
<td>1044.42</td>
<td>187.08</td>
<td>22.74</td>
</tr>
<tr>
<td>PRE ( (t_0, t_0) )</td>
<td>100.00</td>
<td>389.62</td>
<td>189.45</td>
<td>3196.57</td>
<td>177.43</td>
<td>139.32</td>
<td>777.79</td>
<td>6400.00</td>
</tr>
</tbody>
</table>

4. Numerical Comparison

For comparing the theoretical efficiency conditions of the different estimators numerically, we have used the following real data set.

Population 1 (source: Tailor et al. [25]). Consider

\[
\begin{align*}
N &= 15, \\
n &= 3, \\
\bar{X} &= 44.47, \\
\bar{Y} &= 80, \\
\bar{Z} &= 48.40, \\
C_y &= 0.56, \\
C_x &= 0.28, \\
C_z &= 0.43, \\
S^2_y &= 2000, \\
S^2_x &= 149.55, \\
S^2_z &= 427.83, \\
S_{yx} &= 538.57, \\
S_{yz} &= -902.86, \\
S_{xz} &= -241.06, \\
\rho_{yx} &= 0.9848, \\
\rho_{yz} &= -0.9760, \\
\rho_{xz} &= -0.9530, \\
\rho_y &= 0.6652, \\
\rho_x &= 0.707, \\
\rho_z &= 0.5487.
\end{align*}
\]

(24)

For the percent relative efficiencies (PREs) of the estimator, we use the following formula and the results are shown in Table 1:

\[
\text{PRE}(t_\alpha, t_0) = \frac{\text{MSE}(t_0)}{\text{MSE}(t_\alpha)} \times 100, \text{ for } \alpha = 0, 1, 2, 3, 4, 5, 6, \text{ and } m.
\]

5. Conclusion

A chain ratio-type estimator is proposed under double sampling scheme using two auxiliary variables, and the properties of the proposed estimator are derived up to first order of approximations. Both theoretically and empirically, it has been shown that the recommended estimator performed better than the other competing estimators in terms of higher percent relative efficiency. Hence, looking on the dominance nature of the proposed estimator may be suggested for its practical applications.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

The authors are thankful to the anonymous learned referees for their valuable suggestions regarding the improvement of the paper.

References


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