Research Article

An (s, S) Production Inventory Controlled Self-Service Queuing System

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We consider a multiserver Markovian queuing system where each server provides service only to one customer. Arrival of customers is according to a Poisson process and whenever a customer leaves the system after getting service, that server is also removed from the system. Here the servers are considered as a standard (s, S) production inventory. Behavior of this system is studied using a three-dimensional QBD process. The condition for checking ergodicity and the steady state solutions are obtained using matrix analytic method. Unlike classical queuing models, the number of servers varies in this model according to an inventory policy.

1. Introduction

In recent years, manufacturing commodities or inventories in anticipation of demand has fallen out of favor considerably. Most of the manufacturers produce items in response to the actual demand. The amount of time taken for the production of an item and the time at which a production order is to be placed are two important parameters in this regard. The standard (s, S) production inventory is an answer to these parameters which is quite different from the usual inventory policies. Most of the studies consider inventory systems in which as many items as needed can be replenished all at once. But in (s, S) production inventory, the replenishment is done on an item-by-item basis. That is, at the time at which the inventory falls to s , the production process is switched on and is switched off at the time instance at which the on-hand inventory reaches S . The production time of an item in the inventory is a random variable with a probability distribution. In production inventory, to know the status of the system both the inventory level and the production status (whether production process is on or off) must be known.

In classical inventory management problems, the service time of an inventory was assumed to be negligible. Inventory models with positive service time have first been investigated by Sigman and Simchi-Levi [1]. Thereafter a lot of works have been carried out in this area. For further details on

inventory models with positive service time we refer to a survey article by Krishnamoorthy et al. [2]. It can be observed that most of the models discussed in the literature have considered queues in which a single server services an inventory. Inventories with/without lead time, perishable inventory, and so forth and attached with a server are also examined with different ordering policies such as (r, Q) , (s, S) and other general randomized policies. The studies of Parthasarathy and Vijayalakshmi [3], Deepak et al. [4], Manuel et al. [5], and Narayanan et al. [6] are some of the examples of the above said models.

In the case of production inventory with positive service time the work of Krishnamoorthy and Narayanan [7] is the first reported work. They have considered $MAP/PH/1$ queue attached with a production inventory. The time for producing an item in the inventory follows a Markovian production scheme.

As mentioned above, most of the works in the literature consider systems where a single server is providing service to arriving customers. Nowadays, self-service inventory facilities are on the rise to get the customer satisfaction. It can be seen that there are many situations where the customer himself services inventory as in retail supermarket or online shopping and so forth. But studies on such self-service inventories are very less. In a self-service facility, there is no server to serve the items but one or more customers get

service simultaneously depending on the availability of the inventory. That is, customers themselves serve the inventory and the time for serving the inventory can be considered as service time. In particular, we can think about a self-serviceable retail outlet which sells a particular type of inventory or an online shopping site exclusively for a single type of inventory. Such a system has the characteristics of a multiserver queuing system.

A self-service queuing system with positive service time was first investigated by Anoop et al. [8]. They have considered a standard (s, S) inventory without lead time as servers and obtained the steady state probabilities, conditional distributions on the system size and inventory level, distribution of the inventory cycle time, and an optimization problem which optimizes the reorder quantity. Nair and Jacob [9] have analyzed a queuing system with retrial of demands by considering the (s, S) inventory as servers.

Production inventory is very relevant when the manufacturer himself is meeting the demands. It could be a street vendor preparing snacks based on the demand for the item. Also it can be the manufacturer of customized items which will be produced depending on the actual demand. In the present work, we try to address a self-service inventory where the inventory is the (s, S) production inventory. We consider the (s, S) production inventory as servers of the queuing system. Unlike in classical queuing models, the number of servers varies in this model according to an inventory policy. The aim of this paper is to study simplest Markovian queuing system with varying number of servers which replenished according to an (s, S) production inventory. It can be observed that there is a strong dependence between the system size and the inventory level. Therefore, a product form solution is not anticipated as described in Krishnamoorthy and Viswanath [10]. We use matrix analytic methods to find the steady state probabilities.

2. Model Description

We consider a queuing system where arrival of customers (demand for an item) follows a Poisson process with rate λ . We have a standard (s, S) production inventory as servers and the service time follows an exponential distribution with parameter μ . Therefore, when there are n customers and i items in the inventory, the effective service rate of the system is $\min(n\mu, i\mu)$. After service completion an item in the inventory as well as a customer leaves from the system which results in the decrement of the system size and the number of servers (inventory). When the inventory level reaches s , the production process is switched on and it is switched off when the inventory level reaches S . The time required to produce an item in the inventory is assumed to be exponentially distributed with parameter α . An arriving customer joins the system with probability 1 if he finds a free server (inventory). But whenever the number of customers exceeds or is equal to the available inventory, an arriving customer joins the system with probability p and he leaves the system without waiting with probability $1 - p$.

We analyze the system by considering a three-dimensional QBD process $\Omega_t = \{N(t), I(t), C(t); t \geq 0\}$,

TABLE 1: Transition rates of $A_2^{(n)}$.

From	To	Rate	Condition
$(n, i, 1)$	$(n - 1, i - 1, 1)$	$\min(n, i)\mu$	For $n > 0, i > 0$
$(n, s + 1, 0)$	$(n - 1, s, 1)$	$\min(n, s + 1)\mu$	For $n > 0$
$(n, i, 0)$	$(n - 1, i - 1, 0)$	$\min(n, i)\mu$	For $n > 0, s + 1 < i \leq S$

where $N(t)$ represents the number of customers in the system at time t , $I(t)$ represents the inventory level at time t , and $C(t)$ represents status of the production process. The state space of the above continuous time Markov chain is given by $\{(n, i, 1); 0 \leq i \leq s\} \cup \{(n, i, c); s + 1 \leq i \leq S - 1; c = 0, 1\} \cup \{(n, S, 0)\}$ for $n \geq 0$. Further $c = 0$ indicates that the production process is off and $c = 1$ indicates that the production process is on.

Proposition 1. *The infinitesimal generator of the QBD process Ω is given by*

$$\Delta = \begin{pmatrix} A_1^{(0)} & A_0^{(0)} & & & & \\ & A_2^{(1)} & A_1^{(1)} & A_0^{(1)} & & \\ & & A_2^{(2)} & A_1^{(2)} & A_0^{(2)} & \\ & & & \ddots & & \\ & & & & A_2^{(S)} & A_1^{(S)} & A_0^{(S)} \\ & & & & & A_2^{(S)} & A_1^{(S)} & A_0^{(S)} \\ & & & & & & \ddots & \\ & & & & & & & \ddots \end{pmatrix}, \quad (1)$$

where the dimension of each block matrix is $\dim = 2S - s$ and the block matrices give the transition rates which are described as follows:

- (a) $A_2^{(n)}, n \geq 1$, represents the transition rate matrices due to service completion at level n (see Table 1).
- (b) $A_1^{(n)}, n \geq 0$, represent the transition rate matrices which leave the first coordinate fixed at level n (see Table 2).
- (c) $A_0^{(n)}, n \geq 0$, represent the transition rate matrices due to arrival of customers at level n (see Table 3).

The proof of Proposition 1 consists of finding the transition rates for the CTMC Ω_t and then arranging in the matrix form. We omit the proof here because it is rather long and trivial.

Proposition 2. *CTMC Ω_t is stable if and only if*

$$\lambda < \frac{\mu}{p} \left[\sum_{j=1}^s j\theta_j + \sum_{j=s+1}^{S-1} j(\theta_{j,0} + \theta_{j,1}) + S\theta_{S,0} \right], \quad (2)$$

where θ_j 's are the solutions of the equation $\Theta A = 0$ with $A = A_2^{(S)} + A_1^{(S)} + A_0^{(S)}$ and $\Theta = (\theta_0, \dots, \theta_s, \theta_{s+1,0}, \theta_{s+1,1}, \dots, \theta_{S-1,0}, \theta_{S-1,1}, \theta_{S,0})$.

Proof. Let $n \geq S$ and $p > 0$. Define $A = A_2^{(S)} + A_1^{(S)} + A_0^{(S)}$. Let

$$\Theta = (\theta_0, \theta_1, \dots, \theta_s, \theta_{s+1,0}, \theta_{s+1,1}, \dots, \theta_{S-1,0}, \theta_{S-1,1}, \theta_{S,0}) \quad (3)$$

TABLE 2: Transition rates of $A_1^{(n)}$.

From	To	Rate	Condition
$(n, i, 1)$	$(n, i + 1, 1)$	α	For $n \geq 0, 0 \leq i < S$
$(n, S - 1, 1)$	$(n, S, 0)$	α	For $n \geq 0$
$(n, i, 1)$	$(n, i, 1)$	$-(\lambda + \alpha + \min(n, i)\mu)$	For $0 \leq n < i$
$(n, i, 1)$	$(n, i, 1)$	$-(p\lambda + \alpha + \min(n, i)\mu)$	For $0 \leq i \leq n$
$(n, i, 0)$	$(n, i, 0)$	$-(p\lambda + \min(n, i)\mu)$	For $s + 1 \leq i \leq S, i \leq n$
$(n, i, 0)$	$(n, i, 0)$	$-(\lambda + \min(n, i)\mu)$	For $s + 1 \leq i \leq S, n \leq i$

TABLE 3: Transition rates of $A_0^{(n)}$.

From	To	Rate	Condition
(n, i, c)	$(n + 1, i, c)$	λ	For $0 \leq n < i, i \geq 0$, and $c = 0, 1$
(n, i, c)	$(n + 1, i, c)$	$p\lambda$	For $0 \leq i \leq n, c = 0, 1$

be the steady state probability vector of the Markov process with generator matrix A . Therefore Θ can be obtained from the following equations:

$$\begin{aligned}\Theta A &= 0, \\ \Theta e &= 1,\end{aligned}\quad (4)$$

where e is a column vector of order $\dim$ with all the elements being 1. The following sets of linear equations are obtained from (4):

$$\begin{aligned}-\alpha\theta_0 + \mu\theta_1 &= 0, \\ \alpha\theta_{j-2} - (\alpha + (j-1)\mu)\theta_{j-1} + j\mu\theta_j &= 0 \\ &\text{for } j = 2, 3, \dots, s,\end{aligned}\quad (5)$$

$$\begin{aligned}-j\mu\theta_{j,0} + (j+1)\mu\theta_{j+1,0} &= 0 \\ &\text{for } j = s+1, s+2, \dots, S-2.\end{aligned}\quad (6)$$

Equations (5) and (6) are simplified as follows:

$$\begin{aligned}\theta_j &= \frac{\alpha^j}{j!\mu^j}\theta_0; \quad \text{for } j = 1, 2, \dots, s, \\ \theta_{j,0} &= \frac{s+1}{j}\theta_{s+1,0}; \quad \text{for } j = s+2, s+3, \dots, S.\end{aligned}\quad (7)$$

Also we have

$$\begin{aligned}\alpha\theta_{s-1} - (\alpha + (s)\mu)\theta_{s,1} + (s+1)\mu[\theta_{s+1,0} + \theta_{s+1,1}] &= 0, \\ \alpha\theta_s - (\alpha + (s+1)\mu)\theta_{s+1,1} + (s+2)\mu\theta_{s+2,1} &= 0, \\ \alpha\theta_{j-2,1} - (\alpha + (j-1)\mu)\theta_{j-1,1} + j\mu\theta_{j,1} &= 0 \\ &\text{for } j = s+3, s+4, \dots, S-1, \\ \alpha\theta_{S-2,1} - (\alpha + (S-1)\mu)\theta_{S-1,1} &= 0, \\ \alpha\theta_{S-1,1} - S\mu\theta_{S,0} &= 0\end{aligned}\quad (8)$$

along with the normalizing condition

$$\sum_{j=1}^s \theta_j + \sum_{j=s+1}^{S-1} (\theta_{j,0} + \theta_{j,1}) + \theta_{S,0} = 1. \quad (9)$$

Equations (7) to (9) can easily be solved and thereby the steady state probability Θ can be obtained.

Neuts [11] has proved that QBD process with generator matrix Δ is stable if and only if

$$\Theta A_0^{(S)} e < \Theta A_2^{(S)} e. \quad (10)$$

Substituting Θ , $A_0^{(S)}$ and $A_2^{(S)}$ in (10), we get

$$\lambda < \frac{\mu}{p} \left[\sum_{j=1}^s j\theta_j + \sum_{j=s+1}^{S-1} j(\theta_{j,0} + \theta_{j,1}) + S\theta_{S,0} \right]. \quad (11)$$

□

3. Steady State Probabilities

When $p = 0$ (loss system), the QBD process reduces to a Markov process with finite state space and the analysis is rather easy. So we omit the discussion of the loss system. Assume that $p > 0$. It is easy to see that the QBD process $\Omega_t = (N(t), I(t), C(t))$ is irreducible. Denote the stationary probabilities of the process by

$$\begin{aligned}\pi(n, i, c) &= \lim_{t \rightarrow \infty} P(N(t) = n, I(t) = i, C(t) = c); \\ n &\geq 0, \quad 0 \leq i \leq S, \quad c = 0, 1.\end{aligned}\quad (12)$$

Define the infinite dimensional steady state probability vector $\Pi = (\pi_0, \pi_1, \pi_2, \dots)$, where each π_j is a $\dim$ dimensional vector described by

$$\begin{aligned}\pi_j &= (\pi(j, 0, 1), \pi(j, 1, 1) \dots \pi(j, s, 1), \pi(j, s+1, 0), \\ &\pi(j, s+1, 1), \dots, \pi(j, S-1, 0), \pi(j, S-1, 1), \\ &\pi(j, S, 0)).\end{aligned}\quad (13)$$

The structure of the infinitesimal generator in this model is similar to the model described by Anoop et al. [8]. The algorithm for calculating the steady state system probabilities is described below.

The process is level independent for $n \geq S$. Therefore the solution is of the form

$$\pi_{s+j} = \pi_s R^j \quad \text{for } j = 1, 2, \dots, \quad (14)$$

where matrix R is the solution of the quadratic equation

$$R^2 A_2^{(S)} + R A_1^{(S)} + A_0^{(S)} = 0. \quad (15)$$

R can be calculated using the following iterative procedure (refer Neuts [11]):

$$R_{n+1} = -\left(A_2^{(S)} + R_n^2 A_0^{(S)}\right) \left(A_1^{(S)}\right)^{-1}; \quad R_0 = \mathbf{0}. \quad (16)$$

Now to find the stationary vectors π_j for $j = 0, 1, \dots, S$ we proceed as follows.

From the equation $\Pi\Delta = 0$ we get

$$\pi_{S-1} A_0^{(S-1)} + \pi_S A_1^{(S)} + \pi_{S+1} A_2^{(S)} = 0. \quad (17)$$

Thus,

$$\pi_S = \pi_{S-1} T_S, \quad \text{where } T_S = -A_0^{(S-1)} \left(A_1^{(S)} + R A_2^{(S)}\right)^{-1}. \quad (18)$$

Proceeding like this, we get

$$\pi_j = \pi_{j-1} T_j,$$

$$\text{where } T_j = -A_0^{(j-1)} \left(A_1^{(j)} + T_{j+1} A_2^{(j+1)}\right)^{-1}, \quad (19)$$

$$\text{for } j = S-1, S-2, \dots, 1.$$

Now we can solve for π_0 from the following system of equations:

$$\begin{aligned} \pi_0 &= \pi_0 U, \\ \pi_0 V e &= 1, \end{aligned} \quad (20)$$

where

$$\begin{aligned} U &= \left(-T_1 A_2^{(1)}\right) \left(A_1^{(0)}\right)^{(-1)}, \\ V &= I + \sum_{i=1}^{S-1} \prod_{k \leq i} T_k + \left(\prod_{k=1}^S T_k\right) (I - R)^{-1}. \end{aligned} \quad (21)$$

Equations (14), (18), (19), and (20) give the steady state solution of the entire system. After calculating the steady state solution, one can find the various characteristics of the system performance.

Remark 3. The expected number of customers in system L_s is

$$\begin{aligned} L_s &= \sum_{n=0}^{\infty} \left(\sum_{i=0}^s n \pi(n, i, 1) \right. \\ &\quad \left. + \sum_{c=0}^1 \sum_{i=s+1}^{S-1} n \pi(n, i, c) + n \pi(n, S, 0) \right). \end{aligned} \quad (22)$$

Remark 4. The expected number of customers waiting in the queue L_q is

$$\begin{aligned} L_q &= \sum_{i=0}^s \sum_{n=i+1}^{\infty} (n-i) \pi(n, i, 1) \\ &\quad + \sum_{i=s+1}^{S-1} \sum_{n=i+1}^{\infty} \sum_{c=0}^1 (n-i) \pi(n, i, c) \\ &\quad + \sum_{n=S+1}^{\infty} (n-S) \pi(n, S, 0). \end{aligned} \quad (23)$$

Remark 5. Expected inventory level L_{inv} is

$$\begin{aligned} L_{\text{inv}} &= \sum_{n=0}^{\infty} \left(\sum_{i=0}^s i \pi(n, i, 1) \right. \\ &\quad \left. + \sum_{c=0}^1 \sum_{i=s+1}^{S-1} i \pi(n, i, c) + S \pi(n, S, 0) \right). \end{aligned} \quad (24)$$

Remark 6. Fraction of the time the production process is on P_{on} is

$$P_{\text{on}} = \sum_{n=0}^{\infty} \sum_{i=0}^{S-1} \pi(n, i, 1). \quad (25)$$

Remark 7. We say that the system is busy when the inventory level is less than or equal to the number of customers in the system. An arriving customer to a busy system may leave without waiting with probability $1 - p$ as described in the model. If the system is not busy, an arriving customer joins the system for service with probability 1. Let P_{busy} be the probability that the system is busy. Then

$$\begin{aligned} P_{\text{busy}} &= 1 - \sum_{i=1}^s \sum_{n=0}^{i-1} \pi(n, i, 1) + \sum_{i=s+1}^{S-1} \sum_{n=0}^{i-1} \sum_{c=0}^1 \pi(n, i, c) \\ &\quad + \sum_{n=0}^{S-1} \pi(n, S, 0). \end{aligned} \quad (26)$$

Remark 8. Effective arrival rate λ_{eff} is

$$\lambda_{\text{eff}} = \lambda (1 - (1 - p) P_{\text{busy}}). \quad (27)$$

Remark 9. Expected waiting time of a customer in the system W_s is

$$W_s = \frac{L_s}{\lambda_{\text{eff}}}. \quad (28)$$

Remark 10. Expected waiting time of a customer in the queue W_q is

$$W_q = \frac{L_q}{\lambda_{\text{eff}}}. \quad (29)$$

4. Optimality of (s, S) Production Inventory with Poisson Demand and Positive Service Time

Supply chain management is an emerging area which deals with production and distribution of commodity. Cost optimization is inevitable in supply chain management. Many researchers have investigated the cost optimization problems. With regard to the model described here, producing more commodities will reduce the waiting time of a customer as well as the shortage cost, but it increases the holding cost. By reducing the production order point s , the holding cost can be minimized but results in the dissatisfaction of customers

and thereby loss of demands. In this situation, a cost function can be defined which optimizes the expected total cost with respect to various inventory parameters.

In this model, customers are being self-served with a production inventory. Assume that the shortage cost incurred by the loss of customers is c_s per customer and the holding cost of the inventory be c_h per inventory. There can be a cost associated with waiting of customers called waiting cost. Denote the waiting cost of a customer by c_w . Since we consider a single stage supply chain model, the other costs like transportation, warehouse, and so forth can be neglected. Thus the total cost of the system consists of the following:

$$\text{Average shortage cost: } T_s = c_s(\lambda - \lambda_{\text{eff}}).$$

$$\text{Average holding cost: } T_h = c_h L_{\text{inv}}.$$

$$\text{Average waiting cost: } T_w = c_w L_q.$$

Thus we have the expected total cost, TC,

$$TC = T_s + T_h + T_w = c_s(\lambda - \lambda_{\text{eff}}) + c_h L_{\text{inv}} + c_w L_q. \quad (30)$$

Here TC is a function of the production order point s when the maximum inventory level S is fixed. Thus an optimum cost TC^* is guaranteed for an optimum production order point s^* . Numerical results are included to illustrate this.

5. Numerical Illustrations

Now we present the numerical results by assigning particular values for the variables to illustrate the behavior of the system.

Table 4 gives the stationary probabilities of the system with $s = 2$ and $S = 5$ for different values of p . Here $\lambda = 2$, $\mu = 3$, and $\alpha = 3$. Various performance measures of the (2, 5) production inventory are listed in Tables 5 and 6. Table 5 gives the effects of service rate on these characteristics while Table 3 gives the effects of production rate. The measures are calculated for the values $\lambda = 2$ and $\alpha = 3$ in Table 5 and $\lambda = 2$ and $\mu = 3$ in Table 6.

It can be observed that as the service rate increases the fraction of the time the production process is on also increases. Probability that the system is busy varies with respect to an increase in service rate or production rate and it depends on the value of p . In other words, the loss of customers ($0 < p < 1$) has an impact on P_{busy} .

Figures 1–6 detail the effect of production order point on various performance measures like L_{inv} , L_q , W_s , P_{on} , P_{busy} , and the loss rate of customers from the system. The loss rate of customers from the system is defined by the difference between the actual arrival rate and the effective arrival rate. That is, loss rate = $\lambda - \lambda_{\text{eff}}$.

The expected total cost is shown in Figure 7 for two different values of p (i.e., $p = 0.8$ and $p = 0.5$) when $c_h = 1$, $c_w = 10$, and $c_s = 10$. It is clear from the figure that for $p = 0.8$, the expected total cost is minimum when the production order point s^* is 5. Similarly the expected total cost is minimum when $s^* = 3$ for $p = 0.5$.

FIGURE 1: Effect of production order point on average number of inventories in the system for $\lambda = 3$, $\mu = 5$, $\alpha = 4$, and $S = 8$.

FIGURE 2: Effect of production order point on average number of customers in the queue for $\lambda = 3$, $\mu = 5$, $\alpha = 4$, and $S = 8$.

FIGURE 3: Effect of production order point on average waiting time in the system for $\lambda = 3$, $\mu = 5$, $\alpha = 4$, and $S = 8$.

TABLE 4: Stationary distributions of $M/M/(2, 5)_p$ production inventory queuing system.

n	Inventory level							
$\downarrow$	0	1	2	(3, 0)	(3, 1)	(4, 0)	(4, 1)	5
(When $p = 1$)								
0	0.023570878	0.047852452	0.077716719	0.057006166	0.061976512	0.056487038	0.037185907	0.055778861
1	0.024822355	0.039284797	0.056183208	0.038259435	0.043415977	0.038004111	0.025577468	0.037658025
2	0.019696824	0.025656673	0.024017735	0.012910347	0.016452625	0.012880807	0.009137528	0.012841079
3	0.014158891	0.016279803	0.011265553	0.002953309	0.005364317	0.002973767	0.002454857	0.003004248
4	0.009767430	0.010466935	0.006074735	0.000675729	0.002329404	0.000555475	0.000699878	0.000579152
5	0.006618010	0.006839790	0.003645597	0.000171047	0.001261194	0.000127200	0.000304902	0.000121941
6	0.004448811	0.004518396	0.002321737	0.000055823	0.000770035	0.000044171	0.000171759	0.000044656
7	0.002980305	0.003002679	0.001520192	0.000025384	0.000496029	0.000022663	0.000107741	0.000024266
8	0.001993551	0.002001301	0.001007287	0.000014562	0.000326616	0.000013964	0.000070313	0.000015263
9	0.001332645	0.001335715	0.000670733	0.000009249	0.000216958	0.000009118	0.000046559	0.000010011
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(When $p = 0.5$)								
0	0.034257750	0.059362526	0.092582634	0.066836851	0.073482486	0.066579902	0.044089492	0.066134238
1	0.018828220	0.045677001	0.064679792	0.044633055	0.050308809	0.044557900	0.029888176	0.044386601
2	0.007684857	0.013685043	0.024088215	0.014903162	0.017637181	0.014915262	0.010244354	0.014938283
3	0.002810841	0.003970403	0.004510621	0.003704683	0.004125032	0.003328799	0.002347414	0.003356255
4	0.000979115	0.001186169	0.000945875	0.000434945	0.000576920	0.000603375	0.000401599	0.000565522
5	0.000333307	0.000368540	0.000232572	0.000049467	0.000104935	0.000053730	0.000044775	0.000079085
6	0.000112224	0.000118038	0.000065614	0.000005605	0.000024864	0.000004977	0.000007460	0.000006341
7	0.000037588	0.000038529	0.000020176	0.000000666	0.000007051	0.000000548	0.000001788	0.000000731
8	0.000012558	0.000012709	0.000006484	0.000000092	0.000002191	0.000000088	0.000000522	0.000000143
9	0.000004191	0.000004215	0.000002127	0.000000016	0.000000709	0.000000021	0.000000165	0.000000040
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(When $p = 0.1$)								
0	0.042391896	0.065221775	0.098458225	0.070151223	0.077810343	0.070120696	0.046686205	0.070029308
1	0.004830091	0.045218022	0.066311062	0.046730926	0.052144341	0.046767482	0.031225679	0.046747131
2	0.000404533	0.002325971	0.022569652	0.015547370	0.017438959	0.015558697	0.010433476	0.015599336
3	0.000030116	0.000109495	0.000693969	0.004233041	0.003327043	0.003435234	0.002203434	0.003437179
4	0.000002122	0.000005155	0.000020555	0.000107589	0.000063755	0.000654103	0.000302508	0.000555848
5	0.000000145	0.000000255	0.000000616	0.000002615	0.000001256	0.000011934	0.000004228	0.000073972
6	0.000000009	0.000000013	0.000000019	0.000000061	0.000000026	0.000000211	0.000000060	0.000000985
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

FIGURE 4: Effect of production order point on the fraction of the time the production is on for $\lambda = 3$, $\mu = 5$, $\alpha = 4$, and $S = 8$. FIGURE 5: Effect of production order point on the loss rate of customers from the system for $\lambda = 3$, $\mu = 5$, $\alpha = 4$, and $S = 8$.

TABLE 5: Effect of the service rate on the performance measures.

μ	L_s	L_q	L_{inv}	P_{on}	P_{busy}
$p = 1$					
3	1.26128283	0.59461616	2.56533672	0.43758628	0.22066624
4	1.03952670	0.53952670	2.52826010	0.47933391	0.20970679
6	0.82683875	0.49350542	2.49046542	0.51814376	0.20343114
10	0.66100046	0.46100046	2.46040819	0.54521573	0.20207694
$p = 0.5$					
3	0.69598270	0.08606313	2.84483098	0.51315689	0.09156230
4	0.53895541	0.07760769	2.80838887	0.54712842	0.09309734
6	0.38022540	0.07030762	2.77155054	0.57740325	0.09722967
10	0.25209999	0.06512298	2.74204104	0.59716321	0.10278435
$p = 0.1$					
3	0.59946945	0.00923442	2.94349927	0.53029764	0.05100771
4	0.45636599	0.00828538	2.90553341	0.55912253	0.05713592
6	0.30945527	0.00747367	2.86721716	0.58444965	0.06478359
10	0.18951233	0.00690174	2.83657169	0.60066953	0.07219290

TABLE 6: Effect of the production rate on the performance measures.

α	L_s	L_q	L_{inv}	P_{on}	P_{busy}
$p = 1$					
3	1.26128283	0.59461616	2.56533672	0.43758628	0.22066624
5	0.72915929	0.06249262	3.38211313	0.32416497	0.04439186
10	0.67803420	0.01136753	3.75348654	0.16908892	0.00963944
15	0.67350435	0.00683769	3.84672816	0.11341264	0.00569199
$p = 0.5$					
3	0.69598270	0.08606313	2.84483098	0.51315689	0.09156230
5	0.66230208	0.01791851	3.42768189	0.33297030	0.02323504
10	0.66181253	0.00450430	3.75999426	0.17039488	0.00516970
15	0.66263988	0.00292540	3.84933937	0.11396409	0.00297136
$p = 0.1$					
3	0.59946945	0.00923442	2.94349927	0.53029764	0.05100771
5	0.63558435	0.00253234	3.45135911	0.33625661	0.01285642
10	0.65208751	0.00077105	3.76426767	0.17122350	0.00214705
15	0.65549660	0.00052887	3.85113541	0.11437079	0.00094205

6. Conclusion

We have considered a queuing model where the servers are regarded as an inventory with (s, S) production inventory policy. The behavior of this system is described by a QBD process where the first S levels are the boundary conditions. Stability condition, steady state distributions, important performance measures, and an optimization problem have been investigated using matrix analytic methods. Detailed analyses of the numerical results are also presented.

Analysis of a self-service inventory using the methods of queuing theory by regarding the inventory as servers is a new framework. The future work includes the generalization of this idea by considering heterogeneous inventories and multistage supply chain.

FIGURE 6: Effect of production order point on the fraction of the time the system is busy for $\lambda = 3$, $\mu = 5$, $\alpha = 4$, and $S = 8$. FIGURE 7: Effect of production order level on the expected total cost when $\lambda = 3$, $\mu = 5$, $\alpha = 4$, $c_h = 1$, $c_w = 10$, $c_s = 10$, and $S = 8$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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