Research Article

On a Power Transformation of Half-Logistic Distribution

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A new continuous distribution on the positive real line is constructed from half-logistic distribution, using a transformation and its analytical characteristics are studied. Some characterization results are derived. Classical procedures for the estimation of parameters of the new distribution are discussed and a comparative study is done through numerical examples. Further, different families of continuous distributions on the positive real line are generated using this distribution. Application is discussed with the help of real-life data sets.

1. Introduction

Distributions defined on the positive real line are widely used in modeling of survival data. The Weibull, Pareto, and Exponential distributions have major roles in fitting data sets in the fields of computer science, engineering, biology, and so forth. Half-logistic distribution (HLD) is another life distribution used in reliability analysis by many researchers. The half-logistic random variable studied by Balakrishnan [1, 2] has survival function

$$\bar{F}(x) = \frac{2}{1 + e^x}, \quad x > 0. \quad (1)$$

By imparting the location and scale parameters, its probability density function (pdf) is

$$f(x; \mu, \sigma) = \frac{2e^{((x-\mu)/\sigma)}}{\sigma \cdot (1 + e^{((x-\mu)/\sigma)})^2}, \quad x \geq \mu, \sigma > 0. \quad (2)$$

In the past few years many researchers have paid much attention to this distribution and several generalizations have been introduced. Srinivasa Rao et al. [3], Olapade [4–6], Cordeiro et al. [7], Kantam et al. [8], and so forth are some of the recent works in this area.

It is well known that through power transformation, the Weibull is an extension of exponential while the power function distribution is that of uniform. Hence, it is of interest to know what would be the distribution of similar power

transformation of half-logistic distributions. Motivated by this, in the present paper, we introduce a new continuous distribution on the positive real line using a transformation of half-logistic random variable. We study the properties and applications of this so-called generalization of half-logistic distribution.

The remaining part of the paper is organized as follows. In Section 2, Power Half-Logistic Distribution is introduced and its properties are studied. In Section 3, some characterization results are derived. Estimation of the parameters is done in Section 4 and numerical illustrations are given therein. Section 5 deals with extensions of this new transformed distribution. Application to real data sets is considered in Section 6 followed by a concluding section at the end.

2. Power Half-Logistic Distribution

Let Y be a random variable following half-logistic distribution with survival function

$$\bar{F}_Y(y) = \frac{2}{1 + e^{\beta y}}, \quad y > 0, \beta > 0. \quad (3)$$

Consider the transformation, $X = Y^{1/\alpha}$. Then the survival function of X is

$$\bar{F}_X(x) = \frac{2}{1 + e^{\beta x^\alpha}}, \quad x > 0, \alpha, \beta > 0 \quad (4)$$

FIGURE 1: pdfs of PHLD for various values of the parameters.

and its pdf is

$$f(x) = \frac{2\alpha\beta e^{\beta x^\alpha} x^{\alpha-1}}{(1 + e^{\beta x^\alpha})^2}, \quad x > 0, \alpha > 0, \beta > 0. \quad (5)$$

Hereinafter we call the random variable X with pdf (5) as the Power Half-Logistic Distribution (PHLD).

The graphical form of the pdf for various values of α and β is given in Figure 1 ($\beta = 0.8$ and $\beta = 1$). When β is fixed and α is increasing in $(0, 1)$, the pdf becomes more convex. But when α is > 1 it becomes concave. Another characteristic is that for $0 < \alpha < 1$, the distribution is heavy tailed but for $\alpha > 1$, the distribution is light tailed as compared to HLD (see Figure 2). So it can be used to model data sets having tail probability less or greater than HLD. The plots of survival function and hazard rates for different values of α when $\beta = 0.5$ and 1.5 are shown in Figures 3 and 4, respectively, for an alternate view of the behavior of the distribution.

Next we explore the analytical properties of the PHLD, deriving its moments, median, quantiles, hazard function, and log odds function, and summarize them below.

Properties. (1) The s th moment $E(X^s) = (2s/(\beta^s \cdot \alpha))\Gamma(s/\alpha) \sum_{j=0}^{\infty} ((-1)^j / (j+1)^{s/\alpha})$.

(2) Median $= ((\log 3)/\beta)^{1/\alpha}$.

(3) The p th quantile is $((1/\beta) \log((1+p)/(1-p)))^{1/\alpha}$.

(4) Hazard rate $r(x) = \alpha\beta e^{\beta x^\alpha} x^{\alpha-1} / (1 + e^{\beta x^\alpha})$. It can be seen in Figure 4 that for $\alpha > 1$, the distribution has increasing failure rate (IFR), but for $0 < \alpha < 1$ the distribution has decreasing failure rate (DFR).

(5) The log odds function is $\log[(e^{\beta x^\alpha} - 1)/2]$.

It may be noted that in a recent study on the exponentiated half-logistic family by Cordeiro et al. [7] a special case of it called half-logistic Weibull distribution has been just

mentioned without any elaborate study. They proposed a new exponentiated half-logistic (EHL) family as a competitive alternative for lifetime data analysis. For any parent continuous distribution G they defined the corresponding EHL- G distribution with distribution function $[(1 - \bar{G}(x)^\lambda)/(1 + \bar{G}(x)^\lambda)]^\alpha$. This new family extends several common distributions such as Frechet, normal, log-normal, Gumbel, and log-logistic distributions. It is interesting to observe that PHLD is the same as the distribution pointed out there when $G(x) = 1 - \exp(-x^\beta)$.

Next we derive some characterization results of PHLD.

3. Characterizations

In the first characterization we establish a relationship between the PHLD and Weibull distribution.

Result 1. Suppose $\bar{F}(x)$ and $\bar{G}(x)$ are survival functions with respective pdfs f and g . Then in the equation

$$\frac{d}{dx} \left(\frac{1}{\bar{G}(x)} \right) = \frac{1}{2} \frac{d}{dx} \left(\frac{1}{\bar{F}(x)} \right) \quad (6)$$

or

$$\frac{g(x)}{(\bar{G}(x))^2} = \frac{1}{2} \frac{f(x)}{(\bar{F}(x))^2}, \quad (7)$$

$g(x)$ has PHLD if, and only if, $f(x)$ is Weibull.

Proof. Suppose the pdf $g(x)$ is PHLD with the form as in (5) and then substituting in (7) and further on integrating, we get

$$\bar{F}(x) = e^{-\beta x^\alpha}, \quad (8)$$

which is the survival function of Weibull random variable.

FIGURE 2: Comparison of PHLD, HLD, and Weibull densities.

Conversely assuming $f(x)$ as Weibull with pdf

$$f(x) = e^{-\beta x^\alpha} \alpha \beta x^{\alpha-1} \quad (9)$$

and substituting in (7), we get

$$\bar{G}(x) = \frac{2}{1 + e^{\beta x^\alpha}}, \quad (10)$$

which is the survival function of PHLD.

Result 2. The function $g(x)$ in

$$x^{1/\alpha} \frac{g(x^{1/\alpha})}{x} = \alpha f(x) \quad (11)$$

is the pdf of PHLD if, and only if, $f(x)$ is the pdf of HLD.

Proof. The proof easily follows. $\square$

Result 3. For a survival function $\bar{G}(x)$, the functional equation

$$f((x^\alpha + y^\alpha)^{1/\alpha}) = f(x) f(y) \quad (12)$$

(a variant of Cauchy's equation) is satisfied by $f(x) = 2/(\bar{G}(x) - 1)$ if, and only if, $\bar{G}(x) = 2/(1 + e^{cx^\alpha})$.

Proof. Suppose $f(x) = 2/(\bar{G}(x) - 1)$ satisfies the given functional equation.

Then there exists a constant c such that $f(x) = e^{cx^\alpha}$ (see [9]). Hence, $\bar{G}(x) = 2/(1 + e^{cx^\alpha})$.

FIGURE 3: Survival function of PHLD ($\beta = 0.5$ and 1.5). FIGURE 4: Hazard rate of PHLD ($\beta = 0.5$ and 1.5).

Converse part easily follows by assuming $\bar{G}(x) = 2/(1 + e^{cx^\alpha})$. $\square$

One may derive further characterizations of PHLD by taking the $(1/\alpha)$ th power of half-logistic variables in the results of Olapade [4], which described some characterizations of half-logistic distribution.

4. Estimation of the Parameters

We use the following three common methods for estimation purpose. Numerical illustrations are also done subsequently.

4.1. Maximum Likelihood Estimation. Suppose a sample of size n is taken from PHLD with density function (5). By taking logarithms and finding the derivative with respect to α

and β we have two nonlinear equations which can be solved simultaneously and numerically.

We have

$$\begin{aligned}\frac{\partial \log L}{\partial \alpha} &= \frac{n}{\alpha} + \sum_{i=1}^n \beta x_i^\alpha \log x_i + \sum_{i=1}^n \log x_i \\ &\quad - 2 \sum_{i=1}^n \frac{1}{1 + e^{\beta x_i^\alpha}} \log x_i x_i^\alpha \beta e^{\beta x_i^\alpha}, \quad (13) \\ \frac{\partial \log L}{\partial \beta} &= \frac{n}{\beta} + \sum_{i=1}^n x_i^\alpha - \frac{2}{1 + e^{\beta x_i^\alpha} x_i^\alpha}.\end{aligned}$$

4.2. Method of Moments. Method of moment estimation is another common method used for estimation of parameters. Equating the first and second raw moments to corresponding central moments, the following are the equations obtained:

$$\begin{aligned}\frac{\sum_{i=1}^n x_i}{n} &= \frac{2}{\alpha \beta} \Gamma\left(\frac{1}{\alpha}\right) \sum_{j=0}^{\infty} \frac{(-1)^j}{(j+1)^{1/\alpha}}, \\ \frac{\sum_{i=1}^n x_i^2}{n} &= \frac{4}{\alpha \beta^2} \Gamma\left(\frac{2}{\alpha}\right) \sum_{j=0}^{\infty} \frac{(-1)^j}{(j+1)^{2/\alpha}}.\end{aligned} \quad (14)$$

4.3. Least Square Method. Least square estimation method involves the least squares regression to estimate the two parameters based on the linearized PHLD distribution function. For details of this procedure see Krishnaiah [10]. The basis of this method is the transformation of PHLD survival function

$$\bar{F}(x) = \frac{2}{1 + e^{\beta x^\alpha}}, \quad (15)$$

in the form

$$\log x = \frac{-\log \beta}{\alpha} + \frac{1}{\alpha} \log \left[\log \left(\frac{2}{1 - F(x)} - 1 \right) \right]. \quad (16)$$

On putting $Y = \log x$ and $X = \log[\log((2/(1 - F(x))) - 1)]$, it becomes a linear function of X and Y in the form

$$Y = \frac{1}{\alpha} (-\log \beta) + \frac{1}{\alpha} X. \quad (17)$$

Note that $1/\alpha$ is the slope of this equation and $(1/\alpha) \cdot (-\log \beta)$ is the intercept.

Let $x_1 < x_2 < \dots < x_n$ be the times of failure arranged in ascending order and n is the sample size. Then $F(x_i)$ is estimated as in Zaka and Akhter [11], using Bernards' median rank method given by

$$\hat{F}(x_i) = \frac{i - 0.3}{n + 0.4}. \quad (18)$$

Now the least square estimates of α and β are

$$\begin{aligned}\hat{\alpha} &= \frac{n \sum X_i^2 - (\sum X_i)^2}{n \sum (X_i Y_i) - \sum X_i \sum Y_i}, \\ \hat{\beta} &= e^{(-1/n)(\hat{\alpha} \sum Y_i - \sum X_i)},\end{aligned} \quad (19)$$

where X_i and Y_i are the values corresponding to the ordered failure times x_i .

TABLE 1: Parameter estimates.

Sample size	Parameters α, β	Method	Estimates $\hat{\alpha}, \hat{\beta}$	K-S distance	p value
$n = 100$	0.5, 0.8	MLE	0.5052, 0.8049	0.03	1.0
		LSE	0.4973, 0.8069	0.04	1.0
$n = 50$	0.5, 0.8	MLE	0.5140, 0.8039	0.06	1.0
		LSE	0.5008, 0.8122	0.06	1.0
$n = 20$	0.5, 0.8	MLE	0.5441, 0.7908	0.15	0.9831
		LSE	0.5107, 0.8200	0.15	0.9831
$n = 100$	1.2, 1.0	MLE	1.1948, 1.0082	0.03	1.0
		LSE	1.2218, 1.0012	0.03	1.0
$n = 50$	1.2, 1.0	MLE	1.2363, 0.9993	0.04	1.0
		LSE	1.1968, 1.0147	0.04	1.0
$n = 20$	1.2, 1.0	MLE	1.2968, 1.0093	0.1	1.0
		LSE	1.2270, 1.0323	0.1	1.0

4.4. Numerical Examples. Samples of sizes 100, 50, and 20 are generated from PHLD for different values of parameters. The standard method of generation in R-programming is used for the generation of samples. We repeat this process 1000 times and compute simulated average, standard errors, confidence intervals, and coverage probabilities in each case. Comparison of the maximum likelihood estimation (MLE) and least square estimation (LSE) methods mentioned above is done. The computations are performed using R-programme and results are shown in Table 1. Kolmogorov-Smirnov (K-S) statistic and corresponding p values are used for comparing the estimation methods. The 95% confidence intervals for the parameters using maximum likelihood estimates are also constructed in Table 2. Value of the K-S statistic is the same for both methods in most of the cases. Also p value is the same in both cases. As there is no clear supremacy of a method over the other, we suggest MLE method since it is more prevalent and the estimates have better appealing properties. The distribution functions are considered for the generated sequence for given parameter values and also using estimated parameters and those functions are plotted in Figure 5. Similarly histograms and superimposed density curves for estimated values of the parameters are shown in Figure 6. From this we conclude that the above two methods of estimation are in agreement. Coverage probabilities for the parameters for different sample sizes are given in Table 3 and it is clear that they are higher in the case of LSE method.

TABLE 2: Confidence interval for the parameters.

Sample size	Parameters α, β	Estimates $\hat{\alpha}, \hat{\beta}$	Confidence intervals	MSE
$n = 100$	0.5	0.5052	(0.4219, 0.5885)	0.0018
	0.8	0.8049	(0.60095, 1.0089)	0.0108
$n = 50$	0.5	0.5140	(0.3879, 0.6400)	0.0041
	0.8	0.8039	(0.5302, 1.0776)	0.0194
$n = 20$	0.5	0.5441	(0.3176, 0.7707)	0.0133
	0.8	0.7908	(0.3631, 1.2186)	0.0476
$n = 100$	1.2	1.1948	(1.0142, 1.4293)	0.0112
	1.0	1.0082	(0.7976, 1.2049)	0.0107
$n = 50$	1.2	1.2363	(0.9437, 1.5288)	0.0228
	1.0	0.9993	(0.7065, 1.2920)	0.0223
$n = 20$	1.2	1.2968	(0.7858, 1.8088)	0.0679
	1.0	1.0093	(0.5248, 1.4938)	0.0611

TABLE 3: Coverage probabilities for the parameters based on estimation methods.

Method	Sample size	Parameters $\alpha = 0.5, \beta = 0.8$	Parameters $\alpha = 1.2, \beta = 1$
MLE	$n = 100$	0.875, 0.864	0.834, 0.832
LSE		0.942, 0.955	0.944, 0.963
MLE	$n = 50$	0.844, 0.812	0.734, 0.724
LSE		0.946, 0.95	0.939, 0.951
MLE	$n = 20$	0.836, 0.529	0.575, 0.593
LSE		0.94, 0.956	0.942, 0.955

5. Extensions of PHLD

Several types of extensions are possible using this distribution. Some of them are very much related with already existing distributions in the literature. These are discussed in this section.

5.1. Log Power Half-Logistic Distributions. Consider the distribution of some log transformations and we call the distributions obtained as log Power Half-Logistic Distributions. When $Y = e^X$ and X has PHLD, then

$$f(y) = \frac{2\alpha\beta e^{\beta(\log y)^\alpha} (\log y)^{\alpha-1}}{y(1 + e^{\beta(\log y)^\alpha})^2}, \quad y > 1. \quad (20)$$

This distribution is called as log positive Power Half-Logistic Distribution.

In a similar way we define the distribution of $Y = e^{-X}$ as log negative Power Half-Logistic Distribution with support $0 < y < 1$.

If $Y = e^{X^\alpha}$, where $X \sim \text{PHLD}$, the pdf of Y is given by

$$g(y) = \frac{2}{(1+y)^2}, \quad y > 1. \quad (21)$$

Similarly the pdf of $Z = e^{-X^\alpha}$ is

$$g(z) = \frac{2}{(1+z)^2}, \quad 0 < z < 1. \quad (22)$$

So we get two distributions with the same structure but defined at two disjoint intervals $[1, \infty)$ and $[0, 1]$ which is the characteristic of a nonnegative random variable with respect to log transformations. These are having the same form as we transform half-logistic distribution by the transformations $Y = e^X$ and $Z = e^{-X}$.

Immediately, we have the following result, which may be exploited for generating random variables from PHLD.

Result 4. If $X \sim U(0, 1)$, then the random variable $Z = [(1/\beta) \log(a - X)/X]^{1/\alpha}$ has truncated PHLD (TPHLD).

Proof. If $X \sim U(0, 1)$, then

$$\begin{aligned} P(Z \geq z) &= P\left(\left[\left(\frac{1}{\beta}\right) \log\left(\frac{a - X}{X}\right)\right]^{1/\alpha} \geq z\right) \\ &= \frac{a}{1 + e^{\beta z^\alpha}}, \end{aligned} \quad (23)$$

which is a new distribution with density function

$$\begin{aligned} f(x) &= \frac{\alpha a \beta e^{\beta x^\alpha} x^{\alpha-1}}{(1 + e^{\beta x^\alpha})^2}; \\ x &> \left[\frac{1}{\beta} \log(a - 1)\right]^{1/\alpha}, \quad \alpha > 0, \beta > 0, a \geq 2. \end{aligned} \quad (24)$$

□

We call this as Truncated Power Half-Logistic Distribution (TPHLD). Note that when $\alpha = 1$, we have the Truncated Half-Logistic Distribution; when $a = 2, \alpha = 1$, it gives HLD; and when $a = 2, \alpha > 0$, it gives PHLD.

5.2. Families of Distributions Generated from PHLD. We have, in the literature, quite a few families of distributions generated from Beta and Gamma distributions (see [7, 12, 13]). This type of distributions is generalizations of many existing families. Here we generate families of distributions from the PHLD. A detailed study of this type of distributions, its properties, applications, and so forth is not attempted in this paper for brevity, but would be carried out in future.

FIGURE 5: Distribution functions when $n = 20, 50, 100$.

Define a new transformation

$$G_1(x) = \int_0^{\phi(x)} d(H(x)), \quad (25)$$

where $H(x)$ is the distribution function of PHLD or its generalizations and $\phi(x)$ takes different forms of $\bar{G}(x)$, the survival function of a random variable. This transformation gives us very interesting results as summarized in Table 4. Note that when $\beta = 1$, $\bar{G}(x)$ in Result number 1 in Table 4 has Marshall and Olkin [14] form with parameter 2. So a general structure is needed for constructing Marshall-Olkin form with parameter α . This is explained in the following remark.

Remark 1. Consider a new distribution, called General Power Half-Logistic Distribution (GPHLD), by adding a skewness parameter γ ,

$$h(x) = (1 + \gamma) \frac{\gamma \beta \alpha e^{\beta x^\alpha} x^{\alpha-1}}{(1 + \gamma e^{\beta x^\alpha})^2}; \quad x > 0, \alpha > 0. \quad (26)$$

The survival function is

$$\bar{H}(x) = \frac{1 + \gamma}{1 + \gamma e^{\beta x^\alpha}}. \quad (27)$$

FIGURE 6: Fitted density curves based on estimation.

Using this in (25) we get a new family of life distributions, with survival function

$$\bar{H}(x) = \frac{\bar{G}(x)^\beta (1 + \gamma)}{\gamma + \bar{G}(x)^\beta} \quad (28)$$

which can be considered as a generalization of Marshall and Olkin [14] (M-O) form with survival function $\alpha \bar{F}(x)/(1 - \alpha \bar{F}(x))$, $\alpha > 0$. Let $\beta = 1$ in (28); then corresponding density function is given by $h(x) = (1 + \gamma)\gamma g(x)/(\gamma + \bar{G}(x))^2$ and

hazard rate is $(\gamma/(\gamma + \bar{G}(x)))r(x)$ where $r(x)$ is the hazard rate function of $g(x)$. We can see that the parameters γ in (28) and α in M-O are related as $\gamma = 1/(\alpha - 1)$.

Interestingly we have noted that (27) is the Weibull-geometric distribution introduced by Barreto-Souza et al. [15] with parameter θ in $[-1, \infty)$.

Remark 2. Result number 3 of Table 4 is obtained by taking the γ th power of distribution function of PHLD (called as Type I PHLD) which is the same as in Cordeiro et al. [7].

TABLE 4: Families of distributions generated.

Result number	$H(x), x > 0, \alpha > 0, \beta > 0$	$\phi(x)$	$\overline{G}_1(x)$
1	$1 - \frac{2}{1 + e^{\beta x^\alpha}}$	$(-\log \overline{G}(x))^{1/\alpha}$	$\frac{2\overline{G}^\beta}{1 + \overline{G}^\beta}$
2	$1 - \frac{2}{1 + e^{\beta x^\alpha}}$	$\frac{G}{\overline{G}}$	$\frac{2}{1 + e^{\beta(G/\overline{G})^\alpha}}$
3	$\left[1 - \frac{2}{1 + e^{\beta x^\alpha}}\right]^y$	$(-\log \overline{G}(x))^{1/\alpha}$	$1 - \left[\frac{1 - \overline{G}(x)^\beta}{1 + \overline{G}(x)^\beta}\right]^y$
4	$1 - \left(\frac{2}{1 + e^{\beta x^\alpha}}\right)^y$	$(-\log \overline{G}(x))^{1/\alpha}$	$\left[\frac{2\overline{G}(x)^\beta}{1 + \overline{G}(x)^\beta}\right]^y$

TABLE 5: Fitting based on data set 1.

Distribution	Parameter estimates	Log likelihood	K-S distance	p value
Weibull	$\hat{\alpha} = 2.08, \hat{\beta} = 0.0001$	-113.6902	0.1304	0.9897
PHLD	$\hat{\alpha} = 1.77, \hat{\beta} = 0.0006$	-114.0720	0.2609	0.4143
Type I HLD	$\hat{\alpha} = 3.41, \hat{\beta} = 0.0350$	-113.0407	0.2174	0.6487
GPHLD	$\hat{\alpha} = 2.01, \hat{\beta} = 0.0002, \hat{\gamma} = 4.96$	-113.7000	0.1304	0.9897

TABLE 6: Fitting based on data set 2.

Distribution	Parameter estimates	Log likelihood	K-S distance	p value
Weibull	$\hat{\alpha} = 5.78, \hat{\beta} = 0.06$	-15.2070	0.2063	0.1367
PHLD	$\hat{\alpha} = 5.05, \hat{\beta} = 0.13$	-13.8273	0.1587	0.4055
Type I HLD	$\hat{\alpha} = 5.04, \hat{\beta} = 0.13$	-13.8300	0.1587	0.4055
GPHLD	$\hat{\alpha} = 3.202, \hat{\beta} = 0.695, \hat{\gamma} = 0.064$	-12.0336	0.0952	0.9375

Remark 3. $\overline{G}_1(x)$ in Result number 4 of Table 4 is a member of the Lehmann family of distributions and this is Type II PHLD.

6. Applications

In this section we use three sets of real-life data to fit the distributions. The analysis is done using R-programming software. The first set is discussed by Gupta and Kundu [16] in the fitting of exponentiated exponential distribution.

Data Set 1. The first data set is taken from Lawless [17, page 98]. The data are the number of million revolutions before failure for each of the 23 ball bearings in the life test and they are 17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.64, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, and 173.40. We consider the Weibull, Type I HLD (Kantam et al. [8]), PHLD, and GPHLD for this particular data set.

The likelihood value as noted in Table 5 is greatest for Type I HLD and also based on p values of K-S statistic, we conclude that Type I HLD is a good fit for the data.

Data Set 2. This data set is from Smith and Naylor [18] representing strengths of 1.5 cm glass fibres. The data set is 0.55, 0.93, 1.25, 1.36, 1.49, 1.52, 1.58, 1.61, 1.64, 1.68, 1.73, 1.81, 2,

0.74, 1.04, 1.27, 1.39, 1.49, 1.53, 1.59, 1.61, 1.66, 1.68, 1.76, 1.82, 2.01, 0.77, 1.11, 1.28, 1.42, 1.5, 1.54, 1.6, 1.62, 1.66, 1.69, 1.76, 1.84, 2.24, 0.81, 1.13, 1.29, 1.48, 1.5, 1.55, 1.61, 1.62, 1.66, 1.7, 1.77, 1.84, 0.84, 1.24, 1.3, 1.48, 1.51, 1.55, 1.61, 1.63, 1.67, 1.7, 1.78, and 1.89 (see [19]). They have fitted different distributions to this data set. When we use this data set for the four distributions, Weibull, Type I HLD, PHLD, and GPHLD, the results are as follows.

Morais and Barreto-Souza [19] have shown that the Weibull-geometric distribution is better fit to this data set. The log-likelihood values, K-S distance, and p values in Table 6 reveal that GPHLD is better than the other three models (see Remark 1 in this context).

Data Set 3. This data set is of camber of 497 lead wires taken from Leone et al. [20]. Cooray et al. [21] considered this data and fitted folded logistic distribution. They got log-likelihood = -1698.24, K-S distance = 0.06, and p value = 0.32. Results are illustrated in Table 7 and it is clear that PHLD is the most suitable for this data set.

7. Conclusions

A new distribution on the positive real line is constructed using power transformation on half-logistic distribution.

TABLE 7: Fitting based on data set 3.

Distribution	Parameter estimates	Log likelihood	K-S distance	p value
Weibull	$\hat{\alpha} = 1.860, \hat{\beta} = 0.0059$	-1692.670	0.0795	0.10
PHLD	$\hat{\alpha} = 1.595, \hat{\beta} = 0.0180$	-1692.180	0.0785	0.10
Type I HLD	$\hat{\alpha} = 2.095, \hat{\beta} = 0.1450$	-1698.953	0.0845	0.06
GPHLD	$\hat{\alpha} = 1.824, \hat{\beta} = 0.0069, \hat{\gamma} = 9.934$	-1692.484	0.0986	0.02

Analytical properties, some characterizations, and estimation of the parameters are done. New families of distributions are generated from this new distribution which generalizes many existing families of distributions. Applications are discussed with the help of three data sets. The properties, characteristics, and applications of the newly generated families of distributions are further topics for future research work. Some of these families are generated using the odds function.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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