Research Article

Generalized Performance Measures of $\bar{X}$ Control Charts Based on Different Sampling Schemes

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Different versions of $\bar{X}$ control chart structure are available under various ranked set strategies. In these control charts, computation of performance measures was carried out through Monte Carlo simulation method (MCSM). In this article, we have defined a generalized structure of $\bar{X}$ control charts under variant sampling strategies followed by derivation of their different performance measures. For the derivation of different performance measures, we have proposed pivotal quantity. For comparative analysis, we have presented results of generalized performance measures by involving numerical method (NM) as computation. We found that values of generalized performance measures based on NM are almost similar to values of performance measures based on MCSM. Also, NM is time efficient and can be considered as an alternative of MCSM.

1. Introduction

Shewhart (1931) proposed control chart under simple random sampling for monitoring the variations (natural or unnatural) in a process's parameters. Afterwards, control charts became popular among researchers and practitioners of various fields. Therefore, different researchers and practitioners constructed improved forms of control charts in order to cope with different situations, for instance, when we have extra information (also termed as auxiliary information) associated with the variable of interest. Then, for such situation, a number of authors proposed and recommended auxiliary information based control charts (see [1–10]).

Salazar & Sinha [1] attempted Shewhart $\bar{X}$ control chart based on ranked set sampling (RSS) for monitoring of location parameter. They utilized the auxiliary information for ranking the units of interest. Muttalak & Al-Sabah [2] extended the idea of Salazar and Sinha [1] by offering control charts under median ranked set sampling (MRSS) and extreme ranked set sampling (ERSS). Abujiya & Muttalak [3] contributed control chart under double ranked set sampling. All of the illustrated studies were designed for Shewhart

$\bar{X}$ control chart based on one point decision rule, that is, if any plotting statistic goes outside the control limits. Later on, Mehmood et al. [4] enhanced the performance of ranked set sampling based control charts for the detection of small-to-moderate shifts by incorporating more runs rules with the existing structures. It is worth mentioning that the common methodology used in the prescribed studies was the computation of performance measures through MCSM. One may think of NM such as quadratic and cubic polynomial (see [11–13]) as an alternate methodology for computing the values of performance measures. The most commonly used performance measures dealing with quality control charts are false alarm rate, power, and average run length (see [14]).

This study is projected to define a generalized design structure of $\bar{X}$ control chart under different ranked set strategies along with derivation of generalized performance measures. For the said purpose, we will propose constant and pivotal quantity. It is important to mention that we are intended to show the significance of NM in terms of producing almost similar results as one can expect through MCSM. This practice may highlight more scope of NM to

researchers especially those who have been working in the direction of quality control charts under variant ranked set schemes. Another purpose of introducing the NM in the place of MCSM is to prove it as time efficient methodology.

The Rest of the article is organized as follows: In Section 2 we will define generalized design structure of $\bar{X}$ control chart under different ranked set strategies. Based on Section 2, we will propose pivotal quantity in Section 3. In Section 4, we will define performance measures under proposed constant and pivotal quantity. Section 5 will contain computation of performance measures under NM as well as MCSM, and results and discussion. Lastly, the study will end up with summary and conclusion.

2. Design Structure

In this section we define a generalized design structure of $\bar{X}$ control chart under different ranked set strategies. The sampling strategies include simple random sampling (SRS), ranked set sampling (RSS), median ranked set sampling (MRSS), extreme ranked set sampling (ERSS), double ranked set sampling (DRSS), double median ranked set sampling (DMRSS), median double ranked set sampling (MDRSS), double extreme ranked set sampling (DERSS), and extreme double ranked set sampling (EDRSS). Now we define the structure as follows.

Let X_{ij} , $i = 1, 2, 3, \dots$, and $j = 1, 2, 3, \dots, n$, denote the j th observation of sample i collected under a sampling strategy T as mentioned above with in-control known mean μ and standard deviation σ . The details of these sampling strategies can be seen in the article of Mehmood et al. [4]. Thus, generalized two sided $\bar{X}$ control chart structure for monitoring the statistic $\bar{X}_T$ is given below:

$$\begin{aligned} UCL &= \mu + Z_{(1-\alpha/2)} k_{2(T,n,\rho_{XY})} \frac{\sigma}{\sqrt{n}}, \\ LCL &= \mu + Z_{(\alpha/2)} k_{2(T,n,\rho_{XY})} \frac{\sigma}{\sqrt{n}}, \end{aligned} \quad (1)$$

where T represents the sampling strategy (as discussed earlier); n indicates the sample size; α is the amount of prefix false alarm rate of a process (see [14]); ρ_{XY} is the amount of correlation between study variable X and concomitant variable Y which can attain different values, i.e., $\rho_{XY} = 1$ when case of perfect sampling strategy is considered and $0 < \rho_{XY} < 1$ when case of imperfect sampling strategy is involved (see [4]); $Z_{(1-\alpha/2)}$, $Z_{(\alpha/2)}$ are $(1 - \alpha/2)$ th and $(\alpha/2)$ th quantiles of standard normal distribution, respectively. Furthermore, $k_{2(T,n,\rho_{XY})}$ is a constant and given below:

$$k_{2(T,n,\rho_{XY})} = \sqrt{\left[(1 - \rho_{XY}^2) + \frac{\rho_{XY}^2}{n} \sum_{i=1}^n \sigma_{(i,T)}^2 \right]}, \quad (2)$$

where $\sigma_{(i,T)}^2$ is the variance of i th order statistic in a sample of size n from standard normal distribution under a ranked set scheme T .

2.1. Construction of Constant. A brief procedure is illustrated for the construction of constant $k_{2(T,n,\rho_{XY})}$ by considering the study of Mehmood et al. [15] as guidance. For readers concern, a brief procedure is illustrated as follows: generate 10^5 random samples of size n from standard normal distribution for the given choice of T , n , and ρ_{XY} followed by calculating the statistic $(\bar{X}_{T,i}, i = 1, 2, 3, \dots, 10^5)$ and standard deviation of statistic $(SD(\bar{X}_{T,i}))$. The resulting values of $k_{2(T,n,\rho_{XY})} = SD(\bar{X}_{T,i}) \sqrt{n}/\sigma$ are given in Table 1 at different choices of T and for some choices of n and ρ_{XY} . Likewise, one may approach other choices of n and ρ_{XY} .

3. Proposed Pivotal Quantity

Before defining the performance measures (see [14]) for structure (1), we have defined a pivotal quantity as follows:

$$\begin{aligned} Q &= \frac{(\bar{X}_{T,i} - \mu)}{\sigma_{\bar{X}_T} = k_{2(T,n,\rho_{XY})} (\sigma/\sqrt{n})} \equiv Z \implies \\ Q &= \frac{(\bar{X}_{T,i} - \mu) \sqrt{n}}{\sigma/\sqrt{n}} \equiv k_{2(T,n,\rho_{XY})} Z, \end{aligned} \quad (3)$$

where Z is standard normal variable. The theoretical justification for the relationship between Q and Z has been shown through Gaussian Kernel Density Plot (see Figures 1–6). The details of Gaussian Kernel Density Plot can be seen in the article of Sheether [16]. In short, Figures 1–6 showed that for a given choice of T , n , and ρ_{XY} the Gaussian Kernel Density Plots of standardized random variable (denoted as Z) and Q random variable almost overlap with each other. This shows that probability distribution of Q random variable can be approximated by the Z random variable.

4. Derivation of Generalized Performance Measures

In this section we define performance measures (see [14]) for structure (1). The performance measures include false alarm rate, power, and average run length.

4.1. False Alarm Rate. False alarm rate (symbolically denoted as α) is a popular measure used for computing the probability that i th sample statistic $\bar{X}_{T,i}$ goes outside the control limits (LCL , UCL) when actually the process is in-control. An expression for representing the definition of α is given below:

$$\alpha = P(\bar{X}_{T,i} > UCL \text{ or } \bar{X}_{T,i} < LCL), \quad (4)$$

$$\alpha = P(\bar{X}_{T,i} > UCL) + P(\bar{X}_{T,i} < LCL),$$

$$\begin{aligned} \alpha &= P \left[\frac{(\bar{X}_{T,i} - \mu) \sqrt{n}}{\sigma} > \frac{(UCL - \mu) \sqrt{n}}{\sigma} \right] \\ &+ P \left[\frac{(\bar{X}_{T,i} - \mu) \sqrt{n}}{\sigma} < \frac{(LCL - \mu) \sqrt{n}}{\sigma} \right], \end{aligned} \quad (5)$$

TABLE 1: values of $k_{2(T,n,\rho_{XY})}$ at a given choice of sampling strategy and some choices of sample size n .

ρ_{XY}	n	SRS	RSS	MRSS	DRSS	DMRSS	MDRSS	ERSS	DERSS	EDRSS
0.75	5	1	0.800	0.778	0.755	0.696	0.718	0.825	0.778	0.775
	6	1	0.778	0.755	0.735	0.688	0.701	0.82	0.775	0.778
	7	1	0.762	0.748	0.725	0.68	0.695	0.802	0.767	0.762
	8	1	0.764	0.738	0.716	0.675	0.687	0.805	0.759	0.769
	9	1	0.753	0.729	0.71	0.671	0.684	0.794	0.753	0.753
	10	1	0.762	0.721	0.707	0.671	0.686	0.794	0.748	0.742
1	5	1.000	0.599	0.537	0.472	0.286	0.368	0.645	0.552	0.540
	6	1.000	0.563	0.494	0.429	0.254	0.328	0.642	0.536	0.554
	7	1.000	0.527	0.460	0.394	0.211	0.290	0.606	0.507	0.507
	8	1.000	0.498	0.431	0.365	0.192	0.267	0.611	0.492	0.522
	9	1.000	0.478	0.407	0.345	0.166	0.242	0.581	0.472	0.488
	10	1.000	0.458	0.389	0.324	0.153	0.226	0.587	0.465	0.502

FIGURE 1: Kernel density plot of Z and Q at $n = 3$, $T = \text{RSS}$, and $\rho_{XY} = 1$. FIGURE 2: Kernel density plot of Z and Q at $n = 6$, $T = \text{RSS}$, and $\rho_{XY} = 1$.

and (5) can be written based on (3):

$$\alpha = P \left[Z k_{2(T,n,\rho_{XY})} > Z_{(1-\alpha/2)} k_{2(T,n,\rho_{XY})} \right] + P \left[Z k_{2(T,n,\rho_{XY})} < Z_{(\alpha/2)} k_{2(T,n,\rho_{XY})} \right], \quad (6)$$

$$\alpha = P \left[Z > \frac{Z_{(1-\alpha/2)} k_{2(T,n,\rho_{XY})}}{k_{2(T,n,\rho_{XY})}} \right] + P \left[Z < \frac{Z_{(\alpha/2)} k_{2(T,n,\rho_{XY})}}{k_{2(T,n,\rho_{XY})}} \right], \quad (7)$$

$$\alpha = P [Z > Z_{(1-\alpha/2)}] + P [Z < Z_{(\alpha/2)}],$$

where $P[Z > Z_{(1-\alpha/2)}]$ is the area between two quantiles points ($z_{(1-\alpha/2)}$ to ∞) of standard normal curve (see [14]). Similarly $P[Z < Z_{(\alpha/2)}]$ is the area between two quantiles points ($-\infty$ to $Z_{(\alpha/2)}$).

$$\alpha = \int_{Z_{(1-\alpha/2)}}^{\infty} f(Z) dz + \int_{-\infty}^{Z_{(\alpha/2)}} f(Z) dz. \quad (8)$$

4.2. Power. Power is defined as the probability that i th sample statistic $\bar{X}_{T,i}$ goes outside the control limits (LCL , UCL) when actually a process is out-of-control. The out-of-control process implies that the in-control mean μ is shifted to new

FIGURE 3: Kernel density plot of Z and Q at $n = 3$, $T = MRSS$, and $\rho_{XY} = 1$.

level $\mu_1 = \mu + \delta\sigma$, where δ denotes the amount of shift. An expression for representing the power $1 - \beta$ is as follows:

$$1 - \beta = P\left[\bar{X}_{T,i} > UCL \text{ or } \bar{X}_{T,i} < LCL\right], \quad (9)$$

$$1 - \beta = P\left[\frac{(\bar{X}_{T,i} - \mu_1) \sqrt{n}}{\sigma} > \frac{(UCL - \mu_1) \sqrt{n}}{\sigma}\right]$$

$$+ P\left[\frac{(\bar{X}_{T,i} - \mu_1) \sqrt{n}}{\sigma} < \frac{(LCL - \mu_1) \sqrt{n}}{\sigma}\right]$$

$$1 - \beta$$

$$= P\left[\frac{(\bar{X}_{T,i} - \mu_1) \sqrt{n}}{\sigma} > Z_{(1-\alpha/2)} k_{2(T,n,\rho_{XY})} - \delta \sqrt{n}\right]$$

$$\cdot P\left[\frac{(\bar{X}_{T,i} - \mu_1) \sqrt{n}}{\sigma} < Z_{(\alpha/2)} k_{2(T,n,\rho_{XY})} - \delta \sqrt{n}\right],$$

$$1 - \beta = P\left[k_{2(T,n,\rho_{XY})} Z > Z_{(1-\alpha/2)} k_{2(T,n,\rho_{XY})} - \delta \sqrt{n}\right]$$

$$+ P\left[k_{2(T,n,\rho_{XY})} Z < Z_{(\alpha/2)} k_{2(T,n,\rho_{XY})} - \delta \sqrt{n}\right],$$

$$1 - \beta = P\left[Z > \frac{Z_{(1-\alpha/2)} k_{2(T,n,\rho_{XY})} - \delta \sqrt{n}}{k_{2(T,n,\rho_{XY})}}\right]$$

$$+ P\left[Z < \frac{Z_{(\alpha/2)} k_{2(T,n,\rho_{XY})} - \delta \sqrt{n}}{k_{2(T,n,\rho_{XY})}}\right],$$

$$1 - \beta = P\left[Z > \frac{Z_{(1-\alpha/2)} k_{2(T,n,\rho_{XY})} - \delta \sqrt{n}}{k_{2(T,n,\rho_{XY})}}\right]$$

$$+ P\left[Z < \frac{Z_{(\alpha/2)} k_{2(T,n,\rho_{XY})} - \delta \sqrt{n}}{k_{2(T,n,\rho_{XY})}}\right],$$

$$1 - \beta = P\left[Z > Z_{(1-\alpha/2)} - \frac{\delta \sqrt{n}}{k_{2(T,n,\rho_{XY})}}\right]$$

$$+ P\left[Z < Z_{(\alpha/2)} - \frac{\delta \sqrt{n}}{k_{2(T,n,\rho_{XY})}}\right],$$

(10)

$$1 - \beta = \int_{Z_{(1-\alpha/2)} - \delta \sqrt{n}/k_{2(T,n,\rho_{XY})}}^{\infty} f(Z) dz$$

(11)

$$+ \int_{-\infty}^{Z_{(\alpha/2)} - \delta \sqrt{n}/k_{2(T,n,\rho_{XY})}} f(Z) dz.$$

4.3. Average Run Length. Average run length is defined as the average number of sample points that must be plotted on a control chart before an out-of-control signal is triggered. Moreover, average run length can be classified as in-control average run length ARL_0 and out-of-control average run length ARL_1 . ARL_0 is defined as the average number of sample points of an in-control process that must be plotted on the control chart before a plotting statistic $\bar{X}_{T,i}$ breaches the control limits (LCL , UCL), whereas ARL_1 is defined as the average number of sample points of an out-of-control process that must be plotted on control chart before a plotting statistic $\bar{X}_{T,i}$ breaches the control limits (LCL , UCL). For computing the ARL_0 and ARL_1 , one can use the results of (11).

$$ARL_0 = \frac{1}{\alpha} = \frac{1}{\int_{Z_{(1-\alpha/2)}}^{\infty} f(Z) dz + \int_{-\infty}^{Z_{(\alpha/2)}} f(Z) dz}. \quad (12)$$

$$ARL_1 = \frac{1}{1 - \beta}$$

$$= \frac{1}{\int_{Z_{(1-\alpha/2)} - \delta \sqrt{n}/k_{2(T,n,\rho_{XY})}}^{\infty} f(Z) dz + \int_{-\infty}^{Z_{(\alpha/2)} - \delta \sqrt{n}/k_{2(T,n,\rho_{XY})}} f(Z) dz}. \quad (13)$$

5. Computation of Performance Measures and Discussion

In this section we have considered (8), (11), (12), and (13) to compute the false alarm rate, power, in-control average run length, and out-of-control average run length. For the said purpose, we have considered a prefix value of false alarm rate = 0.0027, various values of shifts ($\delta = 0, 0.1, 0.2, 0.3, 0.4$, and 0.5), sampling strategies T , sample size n , and ρ_{XY} . Afterwards, for each choice of n , T , and ρ_{XY} we have solved (6), (9), (11), and (12) by using quadratic integration as numerical method (NM), and results are reported in Table 2. The details

TABLE 2: Performance measures through Monte Carlo simulation method (MCSM) and numerical method (NM) for varying choices of T , δ , n , and ρ_{XY} .

(a) $n = 5$ and $\rho_{XY} = 1$

T	Method	δ											
		0.00			0.10			0.20			0.30		
	α	ARL_0	$1 - \beta$	ARL_1	$1 - \beta$	ARL_1	$1 - \beta$	ARL_1	$1 - \beta$	ARL_1	$1 - \beta$	ARL_1	$1 - \beta$
SRS	MCSM	0.0027	370.37	0.0032	312.50	0.0056	178.57	0.0104	96.15	0.0174	57.47	0.0299	33.44
	NM	0.0027	370.37	0.0034	294.12	0.0056	178.57	0.0101	99.01	0.0177	56.50	0.0299	33.44
RSS	MCSM	0.0027	370.37	0.0049	204.08	0.0122	81.97	0.0303	33.00	0.0662	15.11	0.1289	7.76
	NM	0.0027	370.37	0.0047	212.77	0.0122	81.97	0.0300	33.33	0.0659	15.17	0.1285	7.78
MRSS	MCSM	0.0027	370.37	0.0053	188.68	0.0148	67.57	0.0389	25.71	0.0904	11.06	0.1782	5.61
	NM	0.0027	370.37	0.0052	192.31	0.0151	66.23	0.0392	25.51	0.0908	11.01	0.1787	5.60
ERSS	MCSM	0.0027	370.37	0.0046	217.39	0.0105	95.24	0.0252	39.68	0.0538	18.59	0.1031	9.70
	NM	0.0027	370.37	0.0044	227.27	0.0107	93.46	0.0251	39.84	0.0534	18.73	0.1029	9.72
DRSS	MCSM	0.0027	370.37	0.0064	156.25	0.0205	48.78	0.0572	17.48	0.1335	7.49	0.2631	3.80
	NM	0.0027	370.37	0.0063	158.73	0.0201	49.75	0.0571	17.51	0.1340	7.46	0.2634	3.80
DMRSS	MCSM	0.0027	370.37	0.0135	74.07	0.0763	13.11	0.2547	3.93	0.5499	1.82	0.8178	1.22
	NM	0.0027	370.37	0.0133	75.19	0.0759	13.18	0.2552	3.92	0.5502	1.82	0.8181	1.22
MDRSS	MCSM	0.0027	370.37	0.0084	119.05	0.0376	26.60	0.1203	8.31	0.2837	3.52	0.5149	1.94
	NM	0.0027	370.37	0.0085	117.65	0.0372	26.88	0.1199	8.34	0.2841	3.52	0.5154	1.94
DERSS	MCSM	0.0027	370.37	0.0057	175.44	0.0151	66.23	0.0375	26.67	0.0834	11.99	0.1646	6.08
	NM	0.0027	370.37	0.0055	181.82	0.0148	67.57	0.0371	26.95	0.0837	11.95	0.1645	6.08
EDRSS	MCSM	0.0027	370.37	0.0055	181.82	0.0153	65.36	0.0402	24.88	0.0889	11.25	0.1748	5.72
	NM	0.0027	370.37	0.0052	192.31	0.0150	66.67	0.0399	25.06	0.0892	11.21	0.1751	5.71

(b) $n = 8$ and $\rho_{XY} = 1$

T	Method	δ									
		0.00	0.10	0.20	0.30	0.40	0.50				
		α	$1 - \beta$	ARL_0	$1 - \beta$	ARL_1	$1 - \beta$	ARL_1	$1 - \beta$	ARL_1	$1 - \beta$
SRS	MCSM	0.0027	0.0037	270.37	0.0077	129.87	0.0161	62.11	0.0305	32.79	0.0558
	NM	0.0027	0.0038	370.37	0.0076	130.86	0.0159	62.98	0.0309	32.41	0.0562
RSS	MCSM	0.0027	0.0074	370.37	0.0317	31.55	0.0981	10.19	0.2319	4.31	0.4362
	NM	0.0027	0.0077	370.37	0.0314	31.84	0.0976	10.24	0.2323	4.30	0.4358
MRSS	MCSM	0.0027	0.0096	370.37	0.0461	21.69	0.1524	6.56	0.3536	2.83	0.6112
	NM	0.0027	0.0097	370.37	0.0458	21.85	0.1519	6.58	0.3539	2.83	0.6109
ERSS	MCSM	0.0027	0.0058	370.37	0.0196	51.02	0.0532	18.79	0.1248	8.01	0.2456
	NM	0.0027	0.0059	370.37	0.0191	52.44	0.0535	18.68	0.1252	7.99	0.2459
DRSS	MCSM	0.0027	0.0134	370.37	0.0729	13.72	0.2490	4.02	0.5386	1.86	0.8102
	NM	0.0027	0.0131	370.37	0.0732	13.67	0.2489	4.02	0.5382	1.86	0.8098
DMRSS	MCSM	0.0027	0.0628	370.37	0.4792	2.09	0.9232	1.08	0.9980	1.00	1.0000
	NM	0.0027	0.0631	370.37	0.4789	2.09	0.9229	1.08	0.9981	1.00	1.0000
MDRSS	MCSM	0.0027	0.0257	370.37	0.1882	5.31	0.5689	1.76	0.8908	1.12	0.9894
	NM	0.0027	0.0261	370.37	0.1885	5.31	0.5693	1.76	0.8912	1.12	0.9890
DERSS	MCSM	0.0027	0.0081	370.37	0.0322	31.06	0.1017	9.83	0.2397	4.17	0.4497
	NM	0.0027	0.0079	370.37	0.0322	31.09	0.1013	9.87	0.2401	4.16	0.4501
EDRSS	MCSM	0.0027	0.0075	370.37	0.0284	35.21	0.0838	11.93	0.1999	5.00	0.3849
	NM	0.0027	0.0073	370.37	0.0280	35.73	0.0842	11.88	0.2005	4.99	0.3853

(c) $n = 10$ and $\rho_{XY} = 1$

T	Method	δ											
		0.00			0.10			0.20			0.30		
		α	ARL_0	$1 - \beta$	ARL_1	$1 - \beta$	ARL_1	$1 - \beta$	ARL_1	$1 - \beta$	ARL_1	$1 - \beta$	ARL_1
SRS	MCSM	0.0027	370.37	0.0041	243.90	0.0092	108.70	0.0200	50.00	0.0410	24.39	0.0782	12.79
	NM	0.0027	370.37	0.0041	244.14	0.0091	109.97	0.0202	49.61	0.0414	24.17	0.0781	12.81
RSS	MCSM	0.0027	370.37	0.0108	92.59	0.0528	18.94	0.1769	5.65	0.4069	2.46	0.6759	1.48
	NM	0.0027	370.37	0.0106	94.51	0.0528	18.93	0.1769	5.65	0.4066	2.46	0.6753	1.48
MRSS	MCSM	0.0027	370.37	0.0145	68.97	0.0854	11.71	0.2893	3.46	0.5999	1.67	0.8577	1.17
	NM	0.0027	370.37	0.0145	69.09	0.0851	11.75	0.2888	3.46	0.6002	1.67	0.8576	1.17
ERSS	MCSM	0.0027	370.37	0.0072	138.89	0.0272	36.76	0.0828	12.08	0.1991	5.02	0.3800	2.63
	NM	0.0027	370.37	0.0071	140.24	0.0273	36.57	0.0831	12.03	0.1995	5.01	0.3804	2.63
DRSS	MCSM	0.0027	370.37	0.0218	45.87	0.1459	6.85	0.4730	2.11	0.8204	1.22	0.9698	1.03
	NM	0.0027	370.37	0.0215	46.42	0.1462	6.84	0.4728	2.12	0.8198	1.22	0.9701	1.03
DMRSS	MCSM	0.0027	370.37	0.1765	5.67	0.8721	1.15	0.9994	1.00	1.0000	1.00	1.0000	1.00
	NM	0.0027	370.37	0.1765	5.67	0.8725	1.15	0.9993	1.00	1.0000	1.00	1.0000	1.00
MDRSS	MCSM	0.0027	370.37	0.0545	18.35	0.4220	2.37	0.8842	1.13	0.9955	1.00	1.0000	1.00
	NM	0.0027	370.37	0.0548	18.25	0.4218	2.37	0.8846	1.13	0.9953	1.00	1.0000	1.00
DERSS	MCSM	0.0027	370.37	0.0106	94.34	0.0510	19.61	0.1673	5.98	0.3899	2.56	0.6566	1.52
	NM	0.0027	370.37	0.0104	96.18	0.0506	19.77	0.1676	5.97	0.3904	2.56	0.6562	1.52
EDRSS	MCSM	0.0027	370.37	0.0093	107.53	0.0408	24.51	0.1336	7.49	0.3152	3.17	0.5599	1.79
	NM	0.0027	370.37	0.0091	109.36	0.0410	24.39	0.1337	7.48	0.3158	3.17	0.5603	1.78

(d) $n = 5$ and $\rho_{XY} = 0.75$

T	Method	δ											
		0.00			0.10			0.20			0.30		
		α	ARL_0	$1 - \beta$	ARL_1	$1 - \beta$	ARL_1	$1 - \beta$	ARL_1	$1 - \beta$	ARL_1	$1 - \beta$	ARL_1
SRS	MCSM	0.0027	370.37	0.0033	303.03	0.0056	178.57	0.01	100	0.0176	56.82	0.0298	33.56
	NM	0.0027	370.37	0.0034	294.12	0.0056	178.57	0.01	100	0.0177	56.5	0.0299	33.44
RSS	MCSM	0.0027	370.37	0.0038	263.16	0.0075	133.33	0.0154	64.94	0.0299	33.44	0.0545	18.35
	NM	0.0027	370.37	0.0038	263.16	0.0075	133.33	0.0154	64.94	0.0299	33.44	0.0544	18.38
MRSS	MCSM	0.0027	370.37	0.0037	270.27	0.0077	129.87	0.0163	61.35	0.0321	31.15	0.059	16.95
	NM	0.0027	370.37	0.0038	263.16	0.0078	128.21	0.0163	61.35	0.0321	31.15	0.059	16.95
ERSS	MCSM	0.0027	370.37	0.0038	263.16	0.0081	123.46	0.0174	57.47	0.0348	28.74	0.0643	15.55
	NM	0.0027	370.37	0.0039	256.41	0.0082	121.95	0.0174	57.47	0.0348	28.74	0.0644	15.53
DRSS	MCSM	0.0027	370.37	0.004	250	0.0093	107.53	0.0209	47.85	0.0432	23.15	0.0817	12.24
	NM	0.0027	370.37	0.0041	243.9	0.0093	107.53	0.0209	47.85	0.0432	23.15	0.0817	12.24
DMRSS	MCSM	0.0027	370.37	0.0041	243.9	0.0089	112.36	0.0195	51.28	0.0397	25.19	0.0744	13.44
	NM	0.0027	370.37	0.0041	243.9	0.0089	112.36	0.0195	51.28	0.0397	25.19	0.0745	13.42
MDRSS	MCSM	0.0027	370.37	0.0037	270.27	0.0071	140.85	0.0143	69.93	0.0277	36.1	0.05	20
	NM	0.0027	370.37	0.0037	270.27	0.0072	138.89	0.0144	69.44	0.0277	36.1	0.05	20
DERSS	MCSM	0.0027	370.37	0.0037	270.27	0.0078	128.21	0.0162	61.73	0.0321	31.15	0.058	17.24
	NM	0.0027	370.37	0.0038	263.16	0.0078	128.21	0.0163	61.35	0.0321	31.15	0.059	16.95
EDRSS	MCSM	0.0027	370.37	0.0039	256.41	0.0079	126.58	0.0164	60.98	0.0325	30.77	0.0596	16.78
	NM	0.0027	370.37	0.0039	256.41	0.0079	126.58	0.0165	60.61	0.0325	30.77	0.0597	16.75

FIGURE 4: Kernel density plot of Z and Q at $n = 6$, $T = \text{MRSS}$, and $\rho_{XY} = 1$.

FIGURE 5: Kernel density plot of Z and Q at $n = 3$, $T = \text{ERSS}$, and $\rho_{XY} = 0.75$.

of quadratic and cubic polynomial integration can be seen in various articles (see [11–13]). Also, we have computed results of structure (1) through MCSM by taking into account all procedural details from the study of Mehmood et al. [4]. The number of simulations ($N = 10^5$) for implementing the MCSM is motivated from several existing studies such as Mehmood et al. [4] and Mehmood et al. [17]. Besides, we have recorded the time (see Table 3) utilized by both methods for obtaining the power $1 - \beta$. The time required for getting the $1 - \beta$ is dependent on the choice of sampling strategy T , sample size n , and number of simulations N . A method can prove to be time efficient if it consumes minimum time as compared to competitors given that all methods produced almost similar results.

- (i) Table 2 shows that results obtained through both methods (MCSM and NM) are almost close to each other and particularly at large shifts results are similar. The following findings remain true for any choice of T , n , ρ_{XY} , and δ .
- (ii) Behavior of different sampling strategies remains consistent under the both methods as reported and discussed in the study of Mehmood et al. [4].
- (iii) NM has proved to be time efficient which reduced computation burden as compared to MCSM (see Table 3). In more detail, NM has consumed [1, 2] minutes to gain the intended results. In contrast, MCSM has utilized more time, depending on the choice of sampling strategy T , sample size n , and number of simulations N .

- (iv) Structure (1) and equations (8), (11), (12), and (13) represent a generalized form of several studies such as Shewhart (1931) and Montgomery [14] when $T = \text{SRS}$; Salazar & Sinha [1] when $T = \text{RSS}$; Muttlak & Alsabah [2] when $T = \text{RSS}, \text{MRSS}, \text{ERSS}$; Abujyia & Muttlak [3] when $T = \text{DRSS}, \text{DMRSS}, \text{MDRSS}$ and for varying choices of sampling strategy T . Also, structure (1) represents the study of Mehmood et al. [4] under the one point decision rule.

6. Summary and Conclusions

In this study we have defined a generalized design structure of $\bar{X}$ control charts and derived their different performance measures. In this regard, we have proposed constant and pivotal quantity. Afterwards, we have computed values of different generalized performance measures based on numerical method (NM). In conclusion, we have observed that NM has significant scope in terms of producing desired result as one can expect based on MCSM. Also, NM method may be considered as a time efficient methodology for handling performance measures of $\bar{X}$ control chart under ranked set strategies.

Data Availability

No data were used to support this study.

TABLE 3: Time (expressed in minutes) consumed by the MCSM and NM for computing power $1 - \beta$ at different choices of T and n .

T	n	MCSM	NM
SRS	5	[1, 2]	[1, 2]
	6		
	7		
	8		
	9		
	10		
RSS	5	[3, 5]	[1, 2]
MRSS	6		
ERSS	7		
	8		
	9		
	10		
DRSS	5	[14, 60]	[1, 2]
DMRSS	6		
MDRSS	7		
DERSS	8		
EDRSS	9		
	10

FIGURE 6: Kernel density plot of Z and Q at $n = 6$, $T = ERSS$, and $\rho_{XY} = 0.75$.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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