Hindawi
Journal of Probability and Statistics
Volume 2019, Article ID 7519429, 13 pages
https://doi.org/10.1155/2019/7519429

Research Article

Exponentiated Inverse Rayleigh Distribution and an Application to Coating Weights of Iron Sheets Data

Gadde Srinivasa Rao and Sauda Mbwambo

Department of Statistics, the University of Dodoma, Dodoma, PO. Box: 338, Tanzania

Correspondence should be addressed to Gadde Srinivasa Rao; gaddesrao@yahoo.com

Received 6 December 2018; Revised 19 February 2019; Accepted 17 March 2019; Published 1 April 2019

Academic Editor: Alessandro De Gregorio

This article aims to introduce a generalization of the inverse Rayleigh distribution known as exponentiated inverse Rayleigh distribution (EIRD) which extends a more flexible distribution for modeling life data. Some statistical properties of the EIRD are investigated, such as mode, quantiles, moments, reliability, and hazard function. We describe different methods of parametric estimations of EIRD discussed by using maximum likelihood estimators, percentile based estimators, least squares estimators, and weighted least squares estimators and compare those estimates using extensive numerical simulations. The performances of the proposed methods of estimation are compared by Monte Carlo simulations for both small and large samples. To illustrate these methods in a practical application, a data analysis of real-world coating weights of iron sheets is obtained from the ALAF industry, Tanzania, during January-March, 2018. ALAF industry uses aluminum-zinc galvanization technology in the coating process. This application identifies the EIRD as a better model than other well-known distributions in modeling lifetime data.

1. Introduction

In this research life time distribution known as exponentiated inverse Rayleigh distribution (EIRD) was developed and it can be used in reliability estimation and statistical quality control techniques. The Rayleigh distribution is originated from two parameter Weibull distribution and it is appropriate model for life-testing studies. It can be shown by transformation of random variable that if the random variable (r. v) T has Rayleigh distribution, then the r. v. $X = (1/T)$ has an inverse Rayleigh distribution (IRD). Reliability sampling plans of IRD are given in Rosaiah and Kantam [1].

Suppose X is a random variable following inverse Rayleigh distribution with scale parameter σ. Then its pdf, cdf,and reliability function are respectively given by

$$g(x;\sigma) = \frac{2\sigma^2}{x^3} e^{-(\sigma/x)^2}; \quad x \geq 0, \sigma > 0 \tag{1}$$

$$G(x;\sigma) = e^{-(\sigma/x)^2}; \quad x \geq 0, \sigma > 0 \tag{2}$$

$$R(x) = 1 - G(x) = 1 - e^{-(\sigma/x)^2} \tag{3}$$

Generalization of different distribution was discussed in different statistical writings by different authors, mostly applied in reliability estimation, for example, Mudholkar et al. [2], Gupta et al. [3], Nadarajah and Kotz [4], and Mudholkar and Srivastava [5].

Nadarajah and Kotz [4] suggested a method of generating new distribution by using reliability function. Exponentiated Inverse Rayleigh distribution (EIRD) is a generalized form of inverse Rayleigh distribution as suggested by Nadarajah and Kotz [4] as follows:

$$F(x) = 1 - (R(x))^{\alpha} \tag{4}$$

where R(x) in above equation are the reliability function of inverse Rayleigh distribution.

The cumulative density function (CDF) of the EIRD is given by

$$F(x) = 1 - \left(1 - e^{-(\sigma/x)^2}\right)^{\alpha}; \quad x \geq 0,\ \sigma > 0,\ \alpha > 0 \tag{5}$$

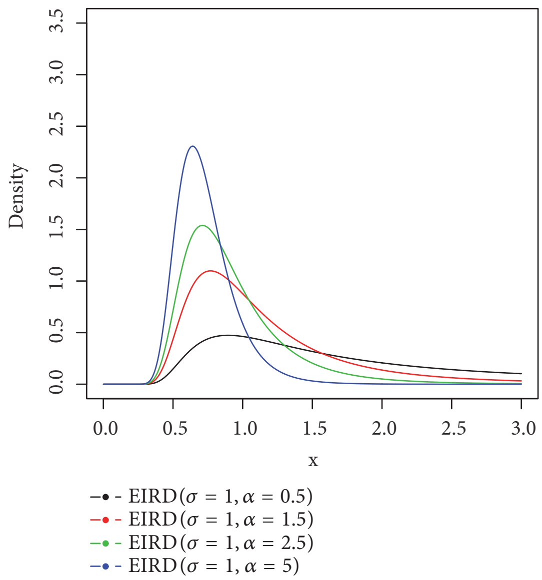


FIGURE 1: Probability density function of EIRD with different shape parameters.

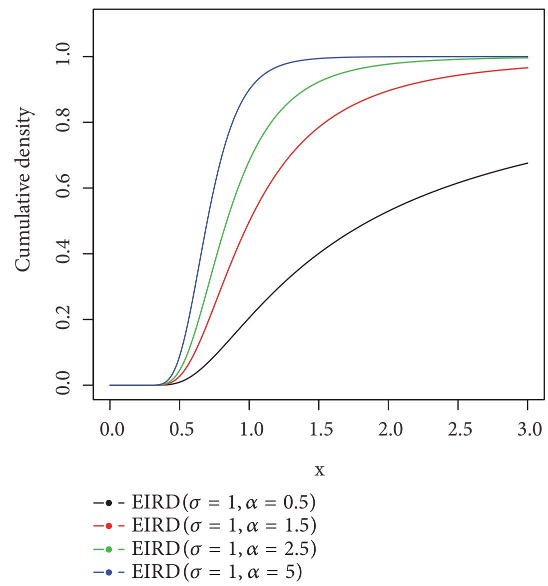


FIGURE 2: Cumulative density function of EIRD with different shape parameters.

where σ is the scale parameter and α is the shape parameter. The probability density function (PDF) of EIRD is

$$f(x) = \frac{2\alpha\sigma^2}{x^3} e^{-(\sigma/x)^2} \left(1 - e^{-(\sigma/x)^2}\right)^{\alpha-1}; \quad x \geq 0,\ \sigma > 0,\ \alpha > 0 \tag{6}$$

The inverse Rayleigh distribution is the particular case of (4) for $\alpha = 1$.

Hence for the exponentiated inverse Rayleigh distribution with the scale parameter σ and shape parameter α will be denoted by EIRD (α, σ).

Reliability and hazard functions of EIRD are as follows:

$$R(x) = 1 - F(x) = \left(1 - e^{-(\sigma/x)^2}\right)^{\alpha} \tag{7}$$

$$h(x) = \frac{f(x)}{R(x)} = 2\alpha\sigma^2 x^{-3} e^{-(\sigma/x)^2} \left(1 - e^{-(\sigma/x)^2}\right)^{-1} \tag{8}$$

Graphs of PDF, CDF, Reliability, and hazard function of EIRD are depicted in Figures 1–4. From the diagram of the PDF it can be shown that the distribution is left skewed and the CDF shows the increasing pattern as we expected. Also by using reliability function, it can be seen that the distribution can be used in lifetime studies since reliability graph tends to decrease as the time increases. The hazard function curve shows that the first increases and then decreases in shape. The lifetime models that present first increase and then decrease shaped failure rates are very useful in survival analysis. Moreover, the hazard rate will in general converge to a constant value. For an example the infant mortality rises to some extent over time and then mortality decreases after the infants got immunity in the body. For greater details, readers may refer to Kotz and Nadarajah [6].

2. Statistical Properties

In this section, we furnish some significant statistical and mathematical properties of the EIRD such as moments, moment generating function, ordered statistics, mode, quantiles, median, skewness, and kurtosis.

2.1. Moments. The r^{th} moment about origin is given by

$$\begin{aligned} \mu_r' &= E(X^r) = \int_0^\infty x^r f(x)\,dx \\ \mu_r' &= \int_0^\infty x^r \frac{2\alpha\sigma^2}{x^3} e^{-(\sigma/x)^2} \left(1 - e^{-(\sigma/x)^2}\right)^{\alpha-1} dx \\ \left(1 - e^{-(\sigma/x)^2}\right)^{\alpha-1} &= \sum_{j=0}^{\infty} \binom{\alpha-1}{j} (-1)^j e^{-(\sigma^2/x^2)j} \end{aligned} \tag{9}$$

Also, $\binom{\alpha-1}{j} = \Gamma(\alpha)/j!\Gamma(\alpha - j)$

$$\begin{aligned} \mu_r' &= \int_0^\infty x^r \frac{2\alpha\sigma^2}{x^3} e^{-(\sigma/x)^2} \sum_{j=0}^{\infty} \binom{\alpha-1}{j} (-1)^j e^{-(\sigma^2/x^2)j} dx \\ &= 2\alpha\sigma^2 \sum_{j=0}^{\infty} \binom{\alpha-1}{j} (-1)^j \int_0^\infty x^{r-3} e^{-\sigma^2/x^2(j+1)} dx \\ \mu_r' &= \alpha \sum_{j=0}^{\infty} \binom{\alpha-1}{j} (-1)^j \frac{\left(\sigma^2 (j+1)\right)^{r/2}}{j+1} \Gamma\left(1 - \frac{r}{2}\right), \quad r < 2 \end{aligned} \tag{10}$$

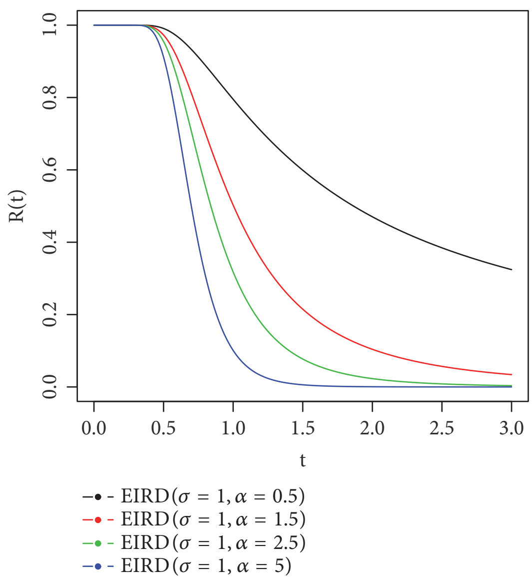


FIGURE 3: Curve of reliability function of EIRD with different shape parameters.

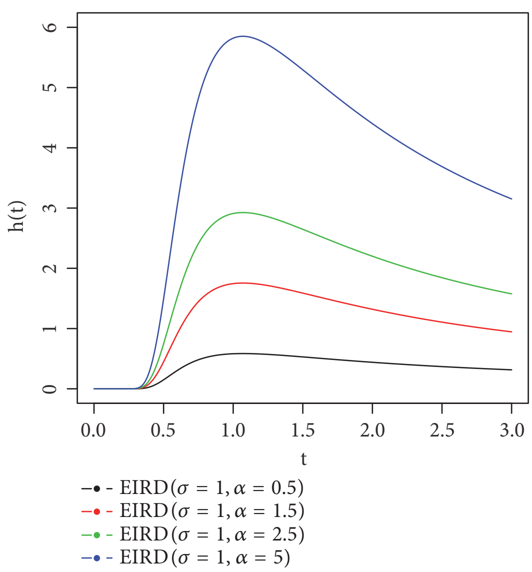


FIGURE 4: Curve of hazard function of EIRD with different values of shape parameter.

Mean of the EIRD can be obtained by putting r=1 in (10).

$$\text{Mean} = \mu_1' = \alpha \sum_{j=0}^{\infty} \binom{\alpha-1}{j} (-1)^j \frac{\left(\sigma^2 (j+1)\right)^{1/2}}{j+1} \Gamma\left(\frac{1}{2}\right). \tag{11}$$

From (10) it can be shown that EIRD has first moment (mean) only and we can only obtain first moment but we cannot obtain variance of this distribution.

2.2. Moment Generating Function. The moment generating function is given by

$$\begin{aligned} M_X(t) &= E\left(e^{tx}\right) = \int_0^{\infty} e^{tx} f(x)\, dx \\ &= \int_0^{\infty} e^{tx} \frac{2\alpha\sigma^2}{x^3} e^{-(\sigma/x)^2} \left(1 - e^{-(\sigma/x)^2}\right)^{\alpha-1} dx \end{aligned} \tag{12}$$

But we know that

$$\left(1 - e^{-(\sigma/x)^2}\right)^{\alpha-1} = \sum_{r=0}^{\infty} \binom{\alpha-1}{r} (-1)^r e^{(\sigma^2/x^2)r} \tag{13}$$

Then (12) becomes

$$\begin{aligned} M_X(t) &= 2\alpha\sigma^2 \sum_{r=0}^{\infty} \frac{(-t)^r}{r!} \int_0^{\infty} x^{r-3} e^{-(\sigma/x)^2 (r+1)} dx \\ &= \alpha \sum_{r=0}^{\infty} \binom{\alpha-1}{r} \frac{(-t)^r}{r!} \frac{\left(\sigma^2 (r+1)\right)^{r/2}}{r+1} \int_0^{\infty} t^{-r/2} e^{-t} dt \end{aligned} \tag{14}$$

Then the moment generating function of EIR distribution is given by

$$\begin{aligned} &M_X(t) \\ &= \alpha \sum_{r=0}^{\infty} \binom{\alpha-1}{r} \frac{(-t)^r}{r!} \frac{\left(\sigma^2 (r+1)\right)^{r/2}}{r+1} \Gamma\left(1 - \frac{r}{2}\right), \\ &\qquad r < 2 \end{aligned} \tag{15}$$

2.3. Order Statistics. Moments of order statistics have great role in quality control testing and reliability to predict time to fail of a certain item by considering few early failures. Suppose $X_{1:m} \leq X_{2:m}, \ldots, \leq x_{m:m}$ are ordered statistics of a random sample $X_1 < X_2 < \ldots < X_m$ drawn from EIRD with CDF $F_X(x)$ and PDF $f_X(x)$, then the PDF of $X_{k:m}$ is given by

$$\begin{aligned} f_{X_{k:m}}(x) &= \frac{m!}{(k-1)!\,(m-k)!} \left[F_X(x)\right]^{k-1} \\ &\quad \cdot \left[1 - F_X(x)\right]^{m-k} f_X(x); \quad k = 1, 2 \ldots, \text{m}. \end{aligned} \tag{16}$$

The PDF of kth order statistics for the EIRD are as follows:

$$\begin{aligned} f_{X_{k:m}}(x) &= \frac{m!}{(k-1)!\,(m-k)!} \frac{2\alpha\sigma^2}{x^3} \\ &\quad \cdot e^{-(\sigma/x)^2} \left(1 - e^{-(\sigma/x)^2}\right)^{\alpha(m-k+1)-1} \\ &\quad \cdot \left(1 - \left(1 - e^{-(\sigma/x)^2}\right)^{\alpha}\right)^{k-1} \end{aligned} \tag{17}$$

2.4. Mode. Mode can be obtained by using the following approach:

$$\begin{aligned} f'(x, \alpha, \sigma) &= 0; \\ f''(x) &< 0 \\ \left[3 + \frac{2\sigma^2}{x^2} + \frac{2(\alpha-1)}{x^2} e^{-(\sigma/x)^2} \left(1 - e^{-(\sigma/x)^2}\right)^{-1}\right] &= 0 \end{aligned} \tag{18}$$

Solution of (18) is called mode of the distribution.

2.5. Quantile and Random Number Generation. Quantiles are very needful for estimation purposes, basically quantile estimators, and also it is used in simulation. The pth quantile of the EIRD is obtained by the following formulae:

$$x_p = Q(p) = \frac{\sigma}{\sqrt{-\log\left(1-(1-p)^{1/\alpha}\right)}} \tag{19}$$

Generally, we have the first three, Q_1=Q(1/4), Q_2=Q(1/2), and Q_3=Q(3/4), that is by substituting value of p=1/4=0.25, p=1/2=0.5, and p=3/4=0.75 in X_p, respectively. Quantile is also used in finding the skewness and kurtosis of the distribution.

Suppose U~Uniform (0,1), then (19) can be used in simulation to generate random number of size n from EIRD as shown below:

$$x_i = \frac{\sigma}{\sqrt{-\log\left(1-(1-u_i)^{1/\alpha}\right)}}; \quad i = 1, 2, \dots n. \tag{20}$$

2.6. Skewness and Kurtosis Using Quantile Approach. There are different methods which are used to find skewness and kurtosis in a certain distribution. The most common method is the one which uses moments of the distribution, but in EIR distribution we have first moment only. Due to this reason the appropriate approach of finding kurtosis and skewness is by using quantiles.

$$\text{Skewness} = \frac{(Q_3-Q_2)-(Q_2-Q_1)}{(Q_3-Q_2)+(Q_2-Q_1)}. \tag{21}$$

Consider, $x_q = \sigma/\sqrt{-\log(1-(1-p)^{1/\alpha})}$.

When $p = 0.25$, we get Q_1, when $p = 0.5$, we get Q_2, and when $p = 0.75$, we get Q_3.

$$\text{Skewness} = \frac{\sigma/\sqrt{-\log\left(1-(0.25)^{1/\alpha}\right)} - 2\sigma/\sqrt{-\log\left(1-(0.5)^{1/\alpha}\right)} + \sigma/\sqrt{-\log\left(1-(0.75)^{1/\alpha}\right)}}{\sigma/\sqrt{-\log\left(1-(0.25)^{1/\alpha}\right)} - \sigma/\sqrt{-\log\left(1-(0.75)^{1/\alpha}\right)}}. \tag{22}$$

Moors [7] suggested a robust alternative measure of kurtosis as follows:

$$\begin{gathered} kurtosis = \frac{(E_7-E_5)-(E_3-E_1)}{(E_6-E_2)}; \\ \text{Where } E_i \text{ is the } i^{th} \text{ octile } E_i = F^{-1}\left(\frac{i}{8}\right). \end{gathered} \tag{23}$$

3. Estimation of Unknown Parameters of EIRD

In this section, different methods which can be used in estimation of parameters α and σ of the EIRD distribution have been discussed. The various methods of estimation discussed are maximum likelihood, ordinary and weighted least square, and percentile based estimation. In literature many authors studied for various distributions to compare different methods of estimation; for more details see Kundu and Raqab [8], Alkasasbeh and Raqab [9], Mazucheli et al. [10], Do Espirito Santo and Mazucheli [11], and Dey et al. [12–17].

3.1. Maximum Likelihood Estimation Method. Maximum likelihood estimation (MLE) method is mostly used in many writings. This is because estimates MLE satisfy many properties of good estimator. Some of the properties are consistency, asymptotic efficiency, and invariance property. Let $X_1, X_2, \dots, X_n$ be random sample of size n drawn from EIR distribution, then the MLE can be obtained as follows:

$$L = 2^n \alpha^n \sigma^{2n} \prod_{i=1}^{n} x_i^{-3} e^{-\sum_{i=1}^{n}(\sigma/x_i)^2} \prod_{i=1}^{n}\left(1-e^{-(\sigma/x_i)^2}\right)^{\alpha-1} \tag{24}$$

$$\begin{aligned} l &= \ln L \\ &= n\ln 2 + n\ln\alpha + 2n\ln\sigma - \sum_{i=1}^{n}\ln x_i^3 - \sum_{i=1}^{n}\left(\frac{\sigma}{x_i}\right)^2 \\ &\quad + (\alpha-1)\sum_{i=1}^{n}\ln\left(1-e^{-(\sigma/x_i)^2}\right) \\ \frac{dl}{d\alpha} &= 0 \Longrightarrow \end{aligned} \tag{25}$$

$$\frac{n}{\alpha} + \sum_{i=1}^{n}\ln\left(1-e^{-(\sigma/x_i)^2}\right) = 0$$

$$\widehat{\alpha} = \frac{-n}{\sum_{i=1}^{n}\ln\left(1-e^{-(\sigma/x_i)^2}\right)}$$

$$\frac{dl}{d\widehat{\sigma}} = 0 \Longrightarrow$$

$$\begin{aligned} &\frac{2n}{\sigma} - 2\sigma\sum_{i=1}^{n}\frac{1}{x_i^2} + 2\sigma(\alpha-1)\sum_{i=1}^{n}\frac{e^{-(\sigma/x_i)^2}}{x_i^2\left(1-e^{-(\sigma/x_i)^2}\right)} = 0 \Longrightarrow \\ &\frac{2n}{\widehat{\sigma}} - 2\widehat{\sigma}\sum_{i=1}^{n}\frac{1}{x_i^2\left(1-e^{-(\widehat{\sigma}/x_i)}\right)} + 2\alpha\widehat{\sigma}\sum_{i=1}^{n}\frac{e^{-(\widehat{\sigma}/x_i)}}{x_i^2\left(1-e^{-(\widehat{\sigma}/x_i)}\right)} \\ &= 0 \end{aligned} \tag{26}$$

Equation (26) above can be solved by iterative procedure like Newton Raphson method to obtain the ML estimator of σ.

Information matrix (I) of EIRD is given by

$$I = \begin{bmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{bmatrix} \quad \text{Where, } I_{11} = E\left[-\frac{\partial^2 \log L}{\partial \alpha^2}\right],\ I_{12} = E\left[-\frac{\partial^2 \log L}{\partial \alpha \partial \sigma}\right],\ I_{21} = E\left[-\frac{\partial^2 \log L}{\partial \sigma \partial \alpha}\right] \text{ and } I_{22} = E\left[-\frac{\partial^2 \log L}{\partial \sigma^2}\right] \tag{27}$$

where

$$\begin{aligned}
\frac{\partial^2 \log L}{\partial \alpha^2} &= -\frac{n}{\alpha^2}; \\
\frac{\partial^2 \log L}{\partial \alpha \partial \sigma} &= 2\sigma \sum_{i=1}^{n} \frac{e^{-(\sigma/x_i)^2}}{x_i^2 \left(1 - e^{-(\sigma/x_i)^2}\right)}; \\
\frac{\partial^2 \log L}{\partial \sigma \partial \alpha} &= 2\sigma \sum_{i=1}^{n} \frac{e^{-(\sigma/x_i)^2}}{\left(1 - e^{-(\sigma/x_i)^2}\right)} \\
\frac{\partial^2 \log L}{\partial \sigma^2} &= -\frac{2n}{\sigma^2} - 2\sum_{i=1}^{n} \left(\frac{1}{x_i}\right)^2 + 2(\alpha - 1) \\
&\quad \left[\sum_{i=1}^{n} \frac{e^{-(\sigma/x_i)^2}}{\left(1 - e^{-(\sigma/x_i)^2}\right)} - 2\sigma \sum_{i=1}^{n} \frac{e^{-(\sigma/x_i)^2}}{x_i^2 \left(1 - e^{-(\sigma/x_i)^2}\right)} \right. \\
&\quad \left. - 2\sigma^2 \sum_{i=1}^{n} \left(\frac{e^{-(\sigma/x_i)^2}}{x_i^2 \left(1 - e^{-(\sigma/x_i)^2}\right)}\right)^2 \right].
\end{aligned} \tag{28}$$

3.2. Ordinary and Weighted Least Square Estimation Methods. This method was suggested by Swain et al. [18], when they used to estimate parameters of Beta distribution. Suppose $X_1, X_2, \ldots, X_n$ are the order statistics of n random samples from EIRD with pdf given above, then the least square estimator (LSE) of the unknown parameters α and σ can be found by differentiating equation (29).

$$S = \sum_{i=1}^{n} \left[1 - \left(1 - e^{-(\sigma/x_i)^2}\right)^{\alpha} - \frac{i}{n+1}\right]^2 \tag{29}$$

With respect to unknown parameters α and σ, solve the following nonlinear equation:

$$\begin{aligned}
&\frac{\partial S}{\partial \sigma} = 0 \Longrightarrow \\
&- \left[\sum_{i=1}^{n} \left(1 - \left(1 - e^{-(\sigma/x_i)^2}\right)^{\alpha} - \frac{i}{n+1}\right)\right] \frac{2\alpha\sigma^2}{x_i^2} \\
&\quad \cdot e^{-(\sigma/x_i)^2} \left(1 - e^{-(\sigma/x_i)^2}\right)^{\alpha-1} = 0 \\
&\frac{\partial S}{\partial \alpha} = 0 \Longrightarrow \\
&- \left[\sum_{i=1}^{n} \left(1 - \left(1 - e^{-(\sigma/x_i)^2}\right)^{\alpha} - \frac{i}{n+1}\right)\right] \\
&\quad \cdot \alpha \left(1 - e^{-(\sigma/x_i)^2}\right)^{\alpha-1} = 0
\end{aligned} \tag{30}$$

The weighted least square estimator (WLSE) can be obtained from the first derivative of the (31) with respect to α and σ and set the result equal to zero.

$$\begin{aligned}
&Q \\
&= \sum_{i=1}^{n} \frac{(n+1)^2 (n+2)}{n-i+1} \left[1 - \left(1 - e^{-(\sigma/x_i)^2}\right)^{\alpha} - \frac{i}{n+1}\right]^2
\end{aligned} \tag{31}$$

Then solve the following obtained nonlinear equation:

$$\begin{aligned}
&\frac{\partial Q}{\partial \sigma} = 0 \Longrightarrow \\
&\sum_{i=1}^{n} \frac{(n+1)^2 (n+2)}{n-i+1} \left[1 - \left(1 - e^{-(\sigma/x_i)^2}\right)^{\alpha} - \frac{i}{n+1}\right] \\
&\quad \cdot \frac{2\alpha\sigma^2}{x_i^2} e^{-(\sigma/x_i)^2} \left(1 - e^{-(\sigma/x_i)^2}\right)^{\alpha-1} = 0 \\
&\frac{\partial Q}{\partial \alpha} = 0 \Longrightarrow \\
&\sum_{i=1}^{n} \frac{(n+1)^2 (n+2)}{n-i+1} \left[1 - \left(1 - e^{-(\sigma/x_i)^2}\right)^{\alpha} - \frac{i}{n+1}\right] \\
&\quad \cdot \alpha \left(1 - e^{-(\sigma/x_i)^2}\right)^{\alpha-1} = 0
\end{aligned} \tag{32}$$

3.3. Percentile Estimation Method. This method was originally proposed by Kao [19, 20]. He supposed that let $p_i = i/(n+1)$ is an estimate of $F(x_i; \alpha, \beta)$, then the percentile estimator (PCE) of α and σ can be found by differentiating

$$Z = \sum_{i=1}^{n} \left(x_i - \frac{\sigma}{\sqrt{-\log\left(1 - (1-p)^{1/\alpha}\right)}}\right)^2 \tag{33}$$

with respect to α and σ and set the result equal to zero

$$\begin{aligned}
&\frac{\partial Z}{\partial \sigma} = 0 \Longrightarrow \\
&\sum_{i=1}^{n} \left(x_{(i)} - \frac{\sigma}{\sqrt{-\log\left(1 - (1-p)^{1/\alpha}\right)}}\right) \\
&\quad \cdot \frac{1}{\sqrt{-\log\left(1 - (1-p)^{1/\alpha}\right)}} = 0
\end{aligned}$$

Table 1: Average bias values of estimates and their MSE (in brackets) when n=20.

Parameters	MLE's	LSE'S	WLSE's	PCE's
$\sigma = 1$	0.0720(0.0474)	-0.0050(0.0637)	0.0065(0.0552)	0.3384(0.5745)
$\alpha = 0.5$	0.0667(0.0359)	0.0226(0.0491)	0.0262(0.0397)	0.0968(0.0578)
$\sigma = 1$	0.0452(0.0225)	-0.0108(0.0295)	-0.0027(0.0258)	-0.0068(0.0347)
$\alpha = 1.5$	0.2957(0.6249)	0.1433(1.7474)	0.1402(1.1410)	0.0733(0.4290)
$\sigma = 1$	0.0419(0.0197)	-0.0109(0.0259)	-0.0032(0.0227)	-0.01400(0.0247)
$\alpha = 2$	0.4445(1.3893)	0.2432(5.4847)	0.2289(3.3600)	0.0736(0.7986)
$\sigma = 1$	0.0400(0.0181)	-0.0109(0.0238)	-0.0034(0.0208)	-0.0172(0.0202)
$\alpha = 2.5$	0.6120(2.6197)	0.3703(14.2642)	0.3383(8.3954)	0.0770(1.3550)
$\sigma = 2$	0.1439(0.1896)	-0.0100(0.2549)	0.0130(0.2208)	0.6767(2.2979)
$\alpha = 0.5$	0.0667(0.0359)	0.0226(0.0491)	0.0262(0.0397)	0.0968(0.0578)
$\sigma = 2$	0.0904(0.0901)	-0.0216(0.1181)	-0.0053(0.1032)	-0.0136(0.1386)
$\alpha = 1.5$	0.2957(0.6250)	0.1433(1.7474)	0.1402(1.1410)	0.0733(0.4290)
$\sigma = 2$	0.0839(0.0790)	-0.0218(0.1037)	-0.0064(0.0906)	-0.0280(0.0989)
$\alpha = 2$	0.4445(1.3893)	0.2432(5.4847)	0.2289(3.3600)	0.0736(0.7986)
$\sigma = 2$	0.0800(0.0723)	-0.0217(0.0952)	-0.0053(0.1032)	-0.0344(0.0807)
$\alpha = 2.5$	0.6120(2.6197)	0.3703(14.2642)	-0.8598(1.8606)	0.0770(1.3550)

Table 2: Average bias values of estimates and their MSE (in brackets) when n=40.

Parameters	MLE's	LSE'S	WLSE's	PCE's
$\sigma = 1$	0.0331(0.0203)	-0.0080(0.0271)	0.0027(0.0231)	0.3920(0.5673)
$\alpha = 0.5$	0.0304(0.0126)	0.0067(0.0154)	0.0122(0.0136)	0.0792(0.0328)
$\sigma = 1$	0.0208(0.0103)	-0.0087(0.0133)	-0.0011(0.0114)	-0.0273(0.0267)
$\alpha = 1.5$	0.1300(0.1877)	0.0375(0.2478)	0.0553(0.2118)	-0.0195(0.2064)
$\sigma = 1$	0.0193(0.0090)	-0.0085(0.0118)	-0.0013(0.0101)	-0.0291(0.0181)
$\alpha = 2$	0.1930(0.3942)	0.0612(0.5325)	0.0856(0.4511)	-0.0505(0.3764)
$\sigma = 1$	0.0184(0.0083)	-0.0083(0.0108)	-0.0014(0.0092)	-0.0292(0.0143)
$\alpha = 2.5$	0.2630(0.7065)	0.0898(0.9731)	0.12067(0.8183)	-0.0819(0.6130)
$\sigma = 2$	0.0661(0.0813)	-0.0159(0.1086)	0.0055(0.0922)	0.7841(2.2692)
$\alpha = 0.5$	0.0304(0.0126)	0.0067(0.0155)	0.0122(0.0136)	0.0792(0.0328)
$\sigma = 2$	0.0415(0.0410)	0.0174(0.0533)	-0.0021(0.0456)	-0.0546(0.1066)
$\alpha = 1.5$	0.1299(0.1877)	0.0375(0.2478)	0.0553(0.2118)	-0.0195(0.2064)
$\sigma = 2$	0.0386(0.0361)	-0.0170(0.0470)	-0.0023(0.0402)	-0.0581(0.0723)
$\alpha = 2$	0.1930(0.3942)	0.0612(0.5325)	0.0856(0.4511)	-0.0505(0.3764)
$\sigma = 2$	0.0368(0.0331)	-0.0167(0.0432)	-0.0021(0.0456)	-0.0585(0.0573)
$\alpha = 2.5$	0.2631(0.7065)	0.0898(0.9731)	-0.9447(1.1011)	-0.0819(0.6130)

$$\frac{\partial Z}{\partial \alpha} = 0 \Longrightarrow$$

$$\sum_{i=1}^{n} \left(x_{(i)} - \frac{\sigma}{\sqrt{-\log\left(1-(1-p)^{1/\alpha}\right)}} \right) \cdot \frac{-(1-p)^{1/\alpha-1}}{2\alpha\left[1-(1-p)^{1/\alpha}\right]\sqrt{-\log\left(1-(1-p)^{1/\alpha}\right)}} = 0 \quad (34)$$

4. Simulation Study for Comparing Estimation Methods

In this part, a Monte Carlo simulation study is conducted to evaluate the performance of different estimation method for estimating unknown parameters of EIRD. The performance of the different estimators is evaluated in terms of mean square error (MSE). The simulation is conducted by using R-software, 10000 random samples of EIRD was generated with values of n = (20, 40, 50, 100) while choosing (α, σ) = (0.5, 1), (1.5, 1), (2, 1), (2.5, 1), (0.5, 2), (1.5, 2), (2, 2), and (2.5, 2). Average bias and MSE values obtained by the method of MLE, LSE, WLSE, and PCE are shown in Tables 1–4. All methods show that they have consistency property since the values of

Table 3: Average bias values of estimates and their MSE (in brackets) when n=50.

Parameters	MLE's	LSE'S	WLSE's	PCE's
$\sigma = 1$	0.0264(0.0154)	-0.0058(0.0215)	0.0036(0.0181)	0.4028(0.5785)
$\alpha = 0.5$	0.0234(0.0091)	0.0046(0.0112)	0.0094(0.0097)	0.0744(0.0281)
$\sigma = 1$	0.0166(0.0078)	-0.0066(0.0105)	0.0001(0.0089)	-0.0327(0.0251)
$\alpha = 1.5$	0.0992(0.1302)	0.0261(0.1717)	0.0422(0.1420)	-0.0405(0.1726)
$\sigma = 1$	0.0155(0.0069)	-0.0065(0.0093)	-0.0002(0.0079)	-0.0330(0.0169)
$\alpha = 2$	0.1470(0.2695)	0.0431(0.3623)	0.0650(0.2962)	-0.0776(0.3123)
$\sigma = 1$	0.0147(0.0063)	-0.0064(0.0085)	-0.0003(0.0072)	-0.0311(0.0130)
$\alpha = 2.5$	0.1999 (0.4770)	0.0636(0.6509)	0.0915(0.5277)	-0.1094(0.5007)
$\sigma = 2$	0.0527(0.0615)	-0.0116(0.0860)	0.0072(0.0722)	0.8057(2.3139)
$\alpha = 0.5$	0.0234(0.0091)	0.0046(0.0112)	0.0094(0.0097)	0.0744(0.0281)
$\sigma = 2$	0.0333(0.0312)	-0.0133(0.0421)	0.0002(0.0358)	-0.0654(0.1003)
$\alpha = 1.5$	0.0992(0.1302)	0.0261(0.1718)	0.04226(0.0142)	-0.0405(0.1726)
$\sigma = 2$	0.0309(0.0275)	-0.0130(0.0371)	-0.0003(0.03150)	-0.0660(0.0674)
$\alpha = 2$	0.1469(0.2695)	0.0431(0.3623)	0.0650(0.2962)	-0.0776(0.3123)
$\sigma = 2$	0.0295(0.0252)	-0.0128(0.0341)	0.0002(0.0358)	-0.0621(0.0518)
$\alpha = 2.5$	0.1999(0.4770)	0.0636(0.6509)	-0.9579(1.0577)	-0.1094(0.5007)

Table 4: Average bias values of estimates and their MSE (in brackets) when n=100.

Parameters	MLE's	LSE'S	WLSE's	PCE's
$\sigma = 1$	0.0123(0.0071)	-0.0037(0.0102)	0.0026(0.0083)	0.4887(0.6267)
$\alpha = 0.5$	0.0110(0.0040)	0.0016(0.0051)	0.0051(0.0044)	0.0732(0.0201)
$\sigma = 1$	0.0078(0.0037)	-0.0038(0.0051)	0.0007(0.0042)	-0.0465(0.0217)
$\alpha = 1.5$	0.0464(0.0546)	0.0096(0.0732)	0.0218(0.0608)	-0.0759(0.1151)
$\sigma = 1$	0.0073(0.0033)	-0.0037(0.0045)	0.0005(0.0037)	-0.0398(0.0133)
$\alpha = 2$	0.0686(0.1113)	0.0162(0.1506)	0.0332(0.1244)	-0.1157(0.2009)
$\sigma = 1$	0.0070(0.0030)	-0.0036(0.0041)	0.0004(0.0034)	-0.0343(0.0095)
$\alpha = 2.5$	0.0931(0.1943)	0.0242(0.2650)	0.0462(0.2181)	-0.1487(0.3103)
$\sigma = 2$	0.2463(0.02854)	-0.0073(0.0407)	0.0052(0.0333)	0.9773(2.5066)
$\alpha = 0.5$	0.0110(0.0040)	0.0016(0.0051)	0.0051(0.0044)	0.0732(0.0201)
$\sigma = 2$	0.0156(0.0148)	-0.0076(0.0202)	0.0013(0.0168)	-0.0930(0.0868)
$\alpha = 1.5$	0.0464(0.0546)	0.0096(0.0732)	0.0218(0.0608)	-0.0759(0.1151)
$\sigma = 2$	0.0146(0.0131)	-0.0074(0.0178)	0.0010(0.0148)	-0.0799(0.0533)
$\alpha = 2$	0.0686(0.1113)	0.0162(0.1506)	0.0332(0.1244)	-0.1157(0.2009)
$\sigma = 2$	0.0139(0.0120)	-0.0072(0.0164)	0.0013(0.0168)	-0.0687(0.0379)
$\alpha = 2.5$	0.0931(0.1943)	0.0242(0.2650)	-0.9783(1.0173)	-0.1487(0.3103)

average bias and MSE decrease as the sample size increases. Based on the value of MSE, the method of MLE shows good performance since it has small value of MSE compared to other methods.

We have also obtained bias and MSE estimates of reliability function and hazard rate functions using MLE. In Table 5 we present the corresponding estimates of reliability and hazard rate functions. From Table 5, it can be seen that the value bias and MSE of hazard and reliability are very small, which indicates that the method is good. Also the values of bias and MSE tend to decrease as sample size increases, which implies that the method is consistent. The entire simulation is done using R software and R code is given in the Appendix.

5. Industrial Application

In this section we investigate two data sets of coating weights (gm/m^2) from ALAF industry, Tanzania. ALAF industry is the part of Safal group which is the major producer of the most accepted and trusted steel roofing brand. Safal group is operated in the 11 countries which are found in the Eastern and Southern part of the African continent. Safal group brought a major advanced coating technology in Africa with four coating mills located at Kenya, Uganda, Tanzania, and South Africa. ALAF industry is the one of the coating mills of Safal group, and then it deals with improving the quality steel roofing. There are several processes which are done in order to improve the quality of steel roofing; one of those

Table 5: Bias and MSE of reliability and hazard functions obtained from different values of n, σ, and α.

n	Parameter		RF		HF	
	σ	α	Bias	MSE	Bias	MSE
20	1.0	0.5	-0.0084	0.0062	0.0301	0.0077
	1.0	1.5	0.0017	0.0003	0.1378	0.1375
	1.0	2.5	0.0009	0.0000	0.2873	0.5794
	2.0	0.5	-0.0028	0.0079	0.0212	0.0050
	2.0	1.5	-0.0044	0.0028	0.1103	0.0928
	2.0	2.5	0.0012	0.0005	0.2356	0.3965
40	1.0	0.5	-0.0045	0.0031	0.0138	0.0028
	1.0	1.5	0.0010	0.0001	0.0608	0.0419
	1.0	2.5	0.0005	0.0000	0.1238	0.1585
	2.0	0.5	-0.0019	0.0038	0.0099	0.0019
	2.0	1.5	-0.0024	0.0015	0.0491	0.0295
	2.0	2.5	0.0007	0.0003	0.1025	0.1134
50	1.0	0.5	-0.0033	0.0025	0.0106	0.0020
	1.0	1.5	0.0008	0.0001	0.0464	0.0292
	1.0	2.5	0.0004	0.0000	0.0941	0.1074
	2.0	0.5	-0.0013	0.0030	0.0077	0.0014
	2.0	1.5	-0.0018	0.0012	0.0376	0.0208
	2.0	2.5	0.0006	0.0002	0.0780	0.0777
100	1.0	0.5	-0.0016	0.0012	0.0050	0.0009
	1.0	1.5	0.0004	0.0001	0.0217	0.0123
	1.0	2.5	0.0002	0.0000	0.0439	0.0439
	2.0	0.5	-0.0007	0.0015	0.0036	0.0006
	2.0	1.5	-0.0009	0.0006	0.0176	0.0089
	2.0	2.5	0.0003	0.0001	0.0365	0.0322

Table 6: Coating weight by chemical method on Tcs and Bcs.

Tcs
36.8 47.2 35.6 36.7 55.8 58.7 42.3 37.8 55.4 45.2 31.8 48.3 45.3 48.5 52.8 45.4 49.8 48.2 54.5 50.1 48.4 44.2 41.2 47.2 39.1 40.7 40.3 41.2 30.4 42.8 38.9 34.0 33.2 56.8 52.6 40.5 40.6 45.8 58.9 28.7 37.3 36.8 40.2 58.2 59.2 42.8 46.3 61.2 58.4 38.5 34.2 41.3 42.6 43.1 42.3 54.2 44.9 42.8 47.1 38.9 42.8 29.4 32.7 40.1 33.2 31.6 36.2 33.6 32.9 34.5 33.7 39.9
Bcs
45.5 37.5 44.3 43.6 47.1 52.9 53.6 42.9 40.6 34.1 42.6 38.9 35.2 40.8 41.8 49.3 38.2 48.2 44.0 30.4 62.3 39.5 39.6 32.8 48.1 56.0 47.9 39.6 44.0 30.9 36.6 40.2 50.3 34.3 54.6 52.7 44.2 38.9 31.5 39.6 43.9 41.8 42.8 33.8 40.2 41.8 39.6 24.8 28.9 54.1 44.1 52.7 51.5 54.2 53.1 43.9 40.8 55.9 57.2 58.9 40.8 44.7 52.4 43.8 44.2 40.7 44.0 46.3 41.9 43.6 44.9 53.6

processes is coating process. ALAF industry uses aluminum-zinc galvanization technology in the coating process. We analyze two data sets to illustrate the model validity on EIRD. For the first data set of 72 observations on coating weight by chemical method on top center side (TCS) and for the second data set of 72 observations on coating weight by chemical method on bottom center side (BCS), the data sets are presented in Table 6.

5.1. Model Validity and Selection Criteria. To verify that this EIRD model is suitable for the data sets which have been used, we have used negative log-likelihood value, Akaike information criteria (AIC), Bayesian information criteria (BIC), K-S distance, and p value,

where

$$AIC = -2\ell\left(\widehat{\theta}\right) + 2q,$$
$$BIC = -2\ell\left(\widehat{\theta}\right) + q\log(n) \tag{35}$$

where $\ell(\widehat{\theta})$ denotes the log-likelihood at MLEs, q is the number of parameters, and n is a sample size.

The values of goodness-of-fit measures, the values of negative likelihood obtained by MLE method, and their standard errors (in brackets) of the models fitted are given in the Tables 7 and 8.

The strength of fit of EIRD is compared with the other four distributions, namely: IRD, RD, IWD, and GIED, and the results in Tables 7 and 8 show that EIRD has the highest p value compared with the four distributions. Also histograms

Table 7: Statistics of the goodness-of-fit, MLE, and standard errors (SE) for Bcs.

Model	-2ℓ	AIC	BIC	K-S	P-value	Parameter	Estimates (SE)
EIRD	497.14	501.14	505.69	0.089	0.6	α	18.2307 (5.4750)
						σ	78.5609 (4.0494)
IRD	596.16	598.16	600.43	0.39	1.00E-09	σ	41.8013 (2.4632)
RD	595.14	597.12	599.41	0.36	3.00E-08	σ	31.4872 (1.8553)
IWD	520.72	524.72	529.27	0.16	0.04	α	4.8144 (3.3126e-02)
						σ	4.8266e+07 (8.3886e+03)
GIERD	495.17	499.17	503.73	0.12	0.3	α	505.6008 (279.3040)
						σ	289.2005 (26.8417)

Table 8: Statistics of the goodness of fit, MLE, and standard errors (SE) for Tcs.

Model	-2ℓ	AIC	BIC	K-S	P-value	Parameter	Estimates (SE)
EIRD	503.66	507.66	512.22	0.061	1	α	13.1767 (3.5726)
						σ	73.0053 (3.7615)
IRD	593.60	595.60	597.88	0.37	9.00E-09	σ	40.8745 (2.4086)
RD	593.79	595.79	598.06	0.36	1.00E-08	σ	31.0157 (1.8276)
IWD	512.73	516.73	521.28	0.18	0.02	α	4.0927 (3.3893e-02)
						σ	2.3069e+06 (8.3925e+03)
GIERD	505.78	509.78	514.33	0.089	0.6	α	245.9686 (118.6173)
						σ	252.0199 (23.1886)

Table 9: The goodness of fit statistics for parameter estimated under various methods for Tcs.

Method	$\hat{\alpha}$	$\hat{\sigma}$	$-\ell$	K-S	p-value
MLE	13.1805	73.0100	236.9226	0.0612	0.9502
LSE	10.3468	69.7534	238.6676	0.0544	0.9835
WLSE	11.0723	70.6468	238.0821	0.0552	0.9805
PCE	11.8341	71.6815	237.7409	0.0611	0.9510

Table 10: The goodness of fit statistics for parameter estimated under various methods for Bcs.

Method	$\hat{\alpha}$	$\hat{\sigma}$	$-\ell$	K-S	p-value
MLE	18.2130	78.5517	237.9956	0.0887	0.6230
LSE	18.5736	79.1091	238.6264	0.0965	0.5139
WLSE	17.7225	78.6110	238.8775	0.0986	0.4853
PCE	17.1623	77.9676	238.4305	0.0913	0.5852

and theoretical densities and Q-Q plot verify how well used data set fits to EIRD compared with other four distributions in Figures 5 and 6. The data set of coating weights for March, 2018 from ALAF industry is used to obtain the results.

The parameters of EIRD are estimated by using four different estimation methods, known as, maximum likelihood, least square, weighted least square, and percentile estimation method. The efficiency of the estimation methods differs from one data set to another as shown in Tables 9 and 10. For example least square estimation method is the best among the others in TCs while maximum likelihood is the best estimation method for BCs.

6. Conclusions

EIRD which is derived from this study performs well; from the diagram of the PDF it can be shown that the distribution is positively skewed and the CDF shows the increasing pattern as other distributions. Also by using reliability function it can be seen that the distribution can be used in lifetime studies since reliability graph tends to decrease as the time increases. The hazard function shows the upside down bath-tub curve shape. The unique characteristic of the distribution has only one moment and kurtosis and skewness are found in terms of quantile.

Four methods of estimation was used in parameter estimation; the methods are maximum likelihood, least square, weighted least square, and percentile estimation. From the simulation study it is observed that the method of maximum likelihood is the best compared to other methods since it has minimum value of MSE. Also findings revile that all methods are consistent since the values of bias and MSE decrease as sample size increases. The data set of coating weights for March, 2018 from ALAF industry is used to study the performance of the proposed distribution. It is shown that EIRD is better performed more than the existing distributions namely: IRD, RD, IWD, and GIED.

Appendix

R codes for estimating parameters of exponentiated inverse Rayleigh distribution

```
# Probability density function

deird<-function(x,sigma,alpha){
```

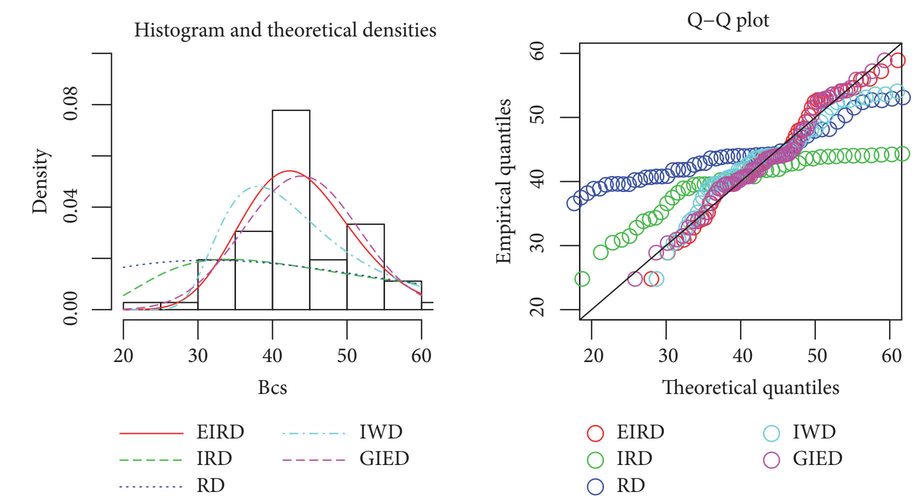


FIGURE 5: Histograms and theoretical densities and Q-Q plot for BCS.

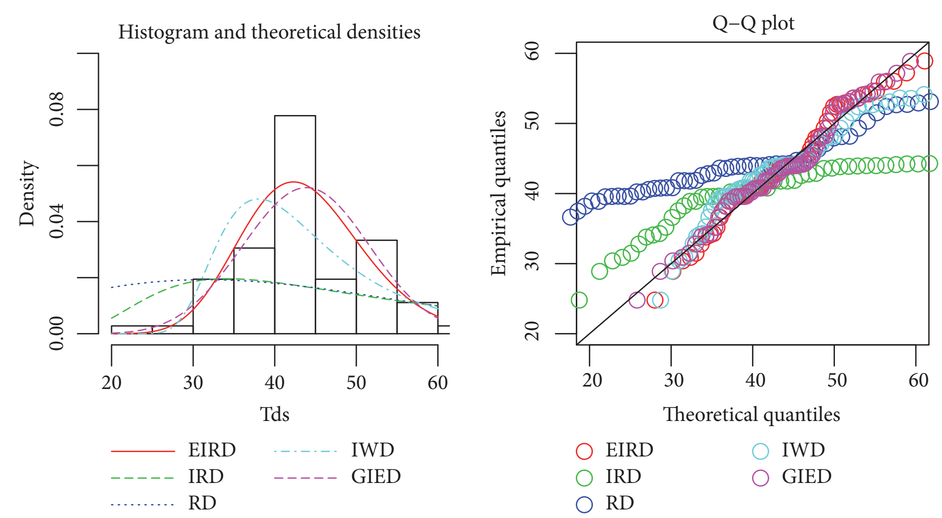


FIGURE 6: Histograms and theoretical densities and Q-Q plot for TCS.

```
  p<-(2*alpha*sigma^2/x^3)*exp(-
(sigma/x)^2)*(1-    exp(-(sigma/x)^2))^(alpha-1)
return(p)
}
# cumulative distribution function
peird<-function(x,sigma,alpha){
  d<-1-(1-exp(-(sigma/x)^2))^alpha
  return(d)
}
# quantile function
qeird<-function(p,sigma,alpha){
  q<-sigma/(-log(1-(1-p)^(1/alpha)))^(1/2)
  return(q)
}
# random number generation
reird<-function(n,sigma,alpha){
  x<-qeird(p=runif(n),sigma,alpha)
  return(x)
}
# Pdf Curves
x <- seq(0, 3, by=.001)
plot(x,   deird(x,   1,0.5),   type="l",   ylim=c(0,3.5),
ylab="Density",

     main=" ",lwd=3)

lines(x, deird(x, 1,1.5), col=2,lwd=3)
lines(x, deird(x, 1.0, 2.5), col=3,lwd=3)
```

```
lines(x, deird(x, 1.0, 5.0), col=4,lwd=3)
#legend(3,1.7, legend = c("exp(1)", "exp(2)", "exp(3)"), pch=c(20,20,20),col=c(1,2,3))
    legend(par('usr')[2], par('usr')[4], xjust=1,
c(c(as.expression(substitute(EIRD(sigma==1, alpha==0.5))),
        as.expression(substitute(EIRD(sigma==1,alpha==1.5))),
        as.expression(substitute(EIRD(sigma==1,alpha==2.5))),
        as.expression(substitute(EIRD(sigma==1,alpha==5.0))))),
   lty =c(2,2,2,2), lwd =c(3,3,3,3), pch=c(20,20,20,20), col=c(1,2,3,4))


# CDF curves
x <- seq(0, 3, by=.001)
plot(x, peird(x, 1,0.5), type="l", ylim=c(0,1.1), ylab="Cumulaive density",
   main=" ",lwd=2)
lines(x, peird(x, 1,1.5), col=2,lwd=3)
lines(x, peird(x, 1.0, 2.5), col=3,lwd=3)
lines(x, peird(x, 1.0, 5.0), col=4,lwd=3)
#legend(3,1.7, legend = c("exp(1)", "exp(2)", "exp(3)"), pch=c(20,20,20),col=c(1,2,3))
legend(1.5,0.4, c(c(as.expression(substitute(EIRD(sigma==1,alpha==0.5))),
        as.expression(substitute(EIRD(sigma==1,alpha==1.5))),
        as.expression(substitute(EIRD(sigma==1,alpha==2.5))),
        as.expression(substitute(EIRD(sigma==1, alpha==5.0))))),
   lty =c(2,2,2,2), lwd =c(3,3,3,3), pch=c(20,20,20,20), col=c(1,2,3,4))


# Reliability curves
x <- seq(0, 3, by=.001)
plot(x, rteird(x, 1,0.5), type="l", ylim=c(0,1.0), ylab="R(t)",xlab="t",
   main=" ",lwd=3)
lines(x, rteird(x, 1,1.5), col=2,lwd=3)
lines(x, rteird(x, 1.0, 2.5), col=3,lwd=3)
lines(x, rteird(x, 1.0, 5.0), col=4,lwd=3)
#legend(3,1.7, legend = c("exp(1)", "exp(2)", "exp(3)"), pch=c(20,20,20),col=c(1,2,3))
legend(par('usr')[2], par('usr')[4], xjust=1,
   c(c(as.expression(substitute(EIRD(sigma==1, alpha==0.5))),
        as.expression(substitute(EIRD(sigma==1, alpha==1.5))),
        as.expression(substitute(EIRD(sigma==1, alpha==2.5))),
        as.expression(substitute(EIRD(sigma==1, alpha==5.0))))),
   lty =c(2,2,2,2), lwd =c(3,3,3,3), pch=c(20,20,20,20), col=c(1,2,3,4))

# Hazard function curves
x <- seq(0, 3, by=.001)
plot(x, heird(x, 1,0.5), type="l", ylim=c(0,6), ylab="h(t)",xlab="t",
   main=" ",lwd=3)
lines(x, heird(x, 1,1.5), col=2,lwd=3)
lines(x, heird(x, 1.0, 2.5), col=3,lwd=3)
lines(x, heird(x, 1.0, 5.0), col=4,lwd=3)
#legend(3,1.7, legend = c("exp(1)", "exp(2)", "exp(3)"), pch=c(20,20,20),col=c(1,2,3))
legend(par('usr')[2], par('usr')[4], xjust=1,
   c(c(as.expression(substitute(EIRD(sigma==1, alpha==0.5))),
        as.expression(substitute(EIRD(sigma==1, alpha==1.5))),
        as.expression(substitute(EIRD(sigma==1, alpha==2.5))),
        as.expression(substitute(EIRD(sigma==1, alpha==5.0))))),
   lty =c(2,2,2,2), lwd =c(3,3,3,3), pch=c(20,20,20,20), col=c(1,2,3,4))


### To find estimation
EIRD_MLE<-function(B,n,psigma,palpha){
  require(maxLik)
 old = options(digits=20)
set.seed(2018) #Setting seed
 o<-0
 ite<-0
 mps <- matrix(nrow=B,ncol=2)


rIR<-function(n,sigma,alpha) {
```

```
 U<-runif(n,0,1)
 t<-sigma/sqrt(-log(1-((1-U)^(1/alpha))))
 return(t)
 }


LL_eird1<-function(theta,x){
   n<-length(x)
   sigma<-theta[1]
   alpha<-theta[2]
   -n*log(2)-n*log(theta[2])-2*n*log(theta[1])
+3*sum(log(x))+sum((theta[1]/x)^2)-
   (theta[2]-1)*sum(log(1-exp(-(theta[1]/x)^2)))
 }
mles<-optim(c(55,8.52), LL_eird1,x=Tds,
hessian=TRUE,control= list(maxit=10000))
 ################## Monte Carlo Simulation
#######################
while(o<B)
 {
 amps<-NULL
 x<-rIR(n, psigma, palpha)
 mles<-try(optim(c(psigma, palpha), LL_eird1,x=x,
hessian=TRUE,control=
  list(maxit=10000))$par)
 if(is.double(mles[1]) & mles[2]>0){
     o<-o+1
     mps[o,] <- mles
#       cat(o,"  ",ite+1, "\n")
   }
   ite<-ite+1
 }
old = options(digits=4)
 mediaemv<-c(mean(mps[,1]), sd(mps[,1]),
mean(mps[,2]), sd(mps[,2]))
 round(mediaemv,4)
 sbias<-mean(mps[,1])-psigma
 abias<-mean(mps[,2])-palpha
 smse<-sbias^2+(sd(mps[,1]))^(2)
 amse<-abias^2+(sd(mps[,2]))^(2)
 print(c(n,psigma, palpha,sbias,abias,smse,amse))
 }
```

Abbreviations

EIRD: Exponentiated Inverse Rayleigh Distribution
r.v.: Random variable
PDF: Probability Density Function
CDF: Cumulative Density Function
MLE: Maximum Likelihood Estimator
LSE: Least Square Estimator
PCE: Percentile Estimator
WLSE: Weighted Least Square Estimator
MSE: Mean Square Error.

Data Availability

The used data sets are given in the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Methodology and computations are done by GS Rao and writing and data collection done by M. Sauda. Both authors read and approved the final manuscript.

References

[1] K. Rosaiah and R. R. L. Kantam, "Acceptance sampling based on the inverse rayleigh distribution," *Economic Quality Control*, vol. 20, no. 2, pp. 277–286, 2005.

[2] G. S. Mudholkar, D. K. Srivastava, and M. Friemer, "The exponentiated weibull family: a reanalysis of the bus-motor-failure data," *Technometrics*, vol. 37, no. 4, pp. 436–445, 1995.

[3] R. C. Gupta, P. L. Gupta, and R. D. Gupta, "Modeling failure time data by Lehman alternatives," *Communications in Statistics -Theory and Methods*, vol. 27, no. 4, pp. 887–904, 1998.

[4] S. Nadarajah and S. Kotz, "The exponentiated type distributions," *Acta Applicandae Mathematicae*, vol. 92, no. 2, pp. 97–111, 2006.

[5] G. S. Mudholkar and D. K. Srivastava, "Exponentiated Weibull family for analyzing bathtub failure-rate data," *IEEE Transactions on Reliability*, vol. 42, no. 2, pp. 299–302, 1993.

[6] S. Kotz and S. Nadarajah, *Extreme Value Distributions: Theory and Applications*, London, UK, World Scientific, 2000.

[7] J. J. A. Moors, "A quantile alternative for kurtosis," *Journal of the Royal Statistical Society, Series D (The Statistician)*, vol. 37, no. 1, pp. 25–32, 1988.

[8] D. Kundu and M. Z. Raqab, "Generalized Rayleigh distribution: different methods of estimations," *Computational Statistics & Data Analysis*, vol. 49, no. 1, pp. 187–200, 2005.

[9] M. R. Alkasasbeh and M. Z. Raqab, "Estimation of the generalized logistic distribution parameters: comparative study," *Statistical Methodology*, vol. 6, no. 3, pp. 262–279, 2009.

[10] J. Mazucheli, F. Louzada, and M. E. Ghitany, "Comparison of estimation methods for the parameters of the weighted Lindley distribution," *Applied Mathematics and Computation*, vol. 220, no. 1, pp. 463–471, 2013.

[11] A. do Espirito Santo and J. Mazucheli, "Comparison of estimation methods for the Marshall–Olkin extended Lindley

distribution," *Journal of Statistical Computation and Simulation*, vol. 85, no. 17, pp. 3437–3450, 2014.

[12] S. Dey, T. Dey, and D. Kundu, "Two-parameter Rayleigh distribution: different methods of estimation," *American Journal of Mathematical and Management Sciences*, vol. 33, no. 1, pp. 55–74, 2014.

[13] S. Dey, S. Ali, and C. Park, "Weighted exponential distribution: properties and different methods of estimation," *Journal of Statistical Computation and Simulation*, vol. 85, no. 18, pp. 3641–3661, 2015.

[14] S. Dey, T. Dey, S. Ali, and M. S. Mulekar, "Two-parameter Maxwell distribution: Properties and different methods of estimation," *Journal of Statistical Theory and Practice*, vol. 10, no. 2, pp. 291–310, 2016.

[15] S. Dey, D. Kumar, P. L. Ramos, and F. Louzada, "Exponentiated Chen distribution: Properties and estimation," *Communications in Statistics - Simulation and Computation*, vol. 46, no. 10, pp. 8118–8139, 2017.

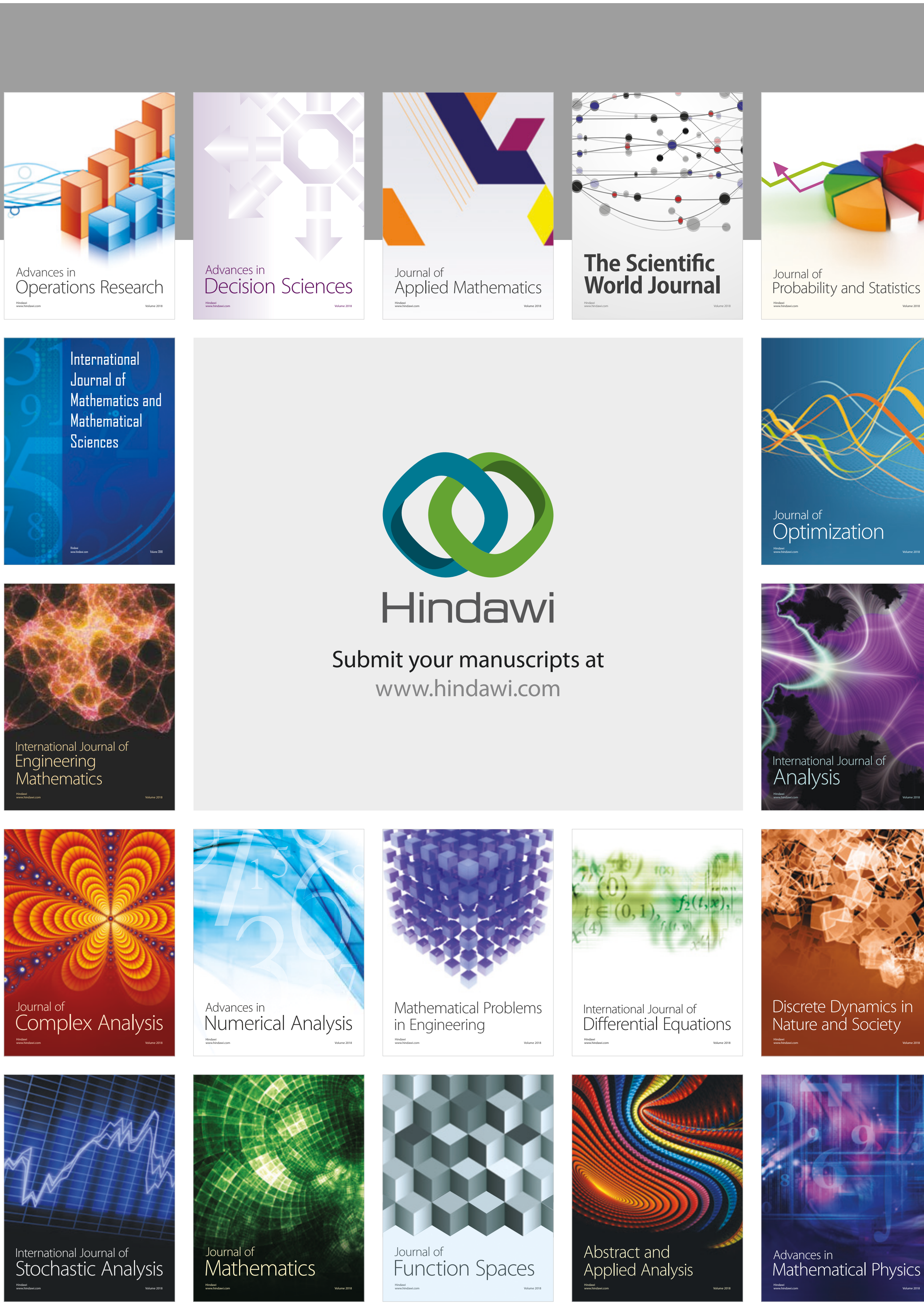
[16] S. Dey, B. Al-Zahrani, and S. Basloom, "Dagum distribution: properties and different methods of estimation," *International Journal of Statistics and Probability*, vol. 6, no. 2, pp. 74–92, 2017.

[17] S. Dey, E. Raheem, S. Mukherjee, and H. K. Ng, "Two parameter exponentiated Gumbel distribution: properties and estimation with flood data example," *Journal of Statistics and Management Systems*, vol. 20, no. 2, pp. 197–233, 2017.

[18] J. Swain, S. Venkatraman, and J. Wilson, "Least squares estimation of distribution function in Johnson's translation system," *Journal of Statistical Computation and Simulation*, vol. 29, pp. 271–297, 1988.

[19] J. H. K. Kao, "Computer methods for estimating weibull parameters in reliability studies," *IRE Transactions on Reliability and Quality Control*, vol. 13, pp. 15–22, 1958.

[20] J. H. K. Kao, "A graphical estimation of mixed Weibull parameters in life-testing of electron tubes," *Technometrics*, vol. 1, no. 4, pp. 389–407, 1959.


Advances in
Operations Research
Advances in
Decision Sciences
Journal of
Applied Mathematics
The Scientific
World Journal
Journal of
Probability and Statistics
International
Journal of
Mathematics and
Mathematical
Sciences
Hindawi
Submit your manuscripts at
www.hindawi.com
Journal of
Optimization
International Journal of
Engineering
Mathematics
International Journal of
Analysis
Journal of
Complex Analysis
Advances in
Numerical Analysis
Mathematical Problems
in Engineering
International Journal of
Differential Equations
Discrete Dynamics in
Nature and Society
International Journal of
Stochastic Analysis
Journal of
Mathematics
Journal of
Function Spaces
Abstract and
Applied Analysis
Advances in
Mathematical Physics