Research Article

A Robust DS Combination Method Based on Evidence Correction and Conflict Redistribution

Fang Ye, Jie Chen, and Yuan Tian

College of Information and Communication Engineering, Harbin Engineering University, Harbin 150001, China

Correspondence should be addressed to Yuan Tian; tianyuan347@126.com

Received 6 November 2017; Accepted 8 May 2018; Published 5 June 2018

Academic Editor: Giuseppe Maruccio

Copyright © 2018 Fang Ye et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

To eliminate potential evidence conflicts, an effective and accurate DS combination method is addressed in this paper. DS evidence theory is an outstanding information fusion approach with valid uncertainty treatment. Nevertheless, there are some limitations of the usage of the DS evidence theory. On the one hand, due to the complexity of a combat measurement environment and the inconsistency of sensor capabilities, sensor sources have enormous uncertainty, which would inevitably cause conflicts for evidence combination. On the other hand, DS combination rule realizes the unity property of fusing results with a compulsive normalization, which unavoidably leads to conflicting situations. To solve the possible evidence conflicts in a multisensor fusion system, we raise a robust DS combination method based on evidence correction and conflict redistribution. Firstly, two corrected indexes—the reliability index and consistency index—are separately addressed with the introduction of the Matusita distance function and closeness degree function. After the evidence modification based on two correction indexes, the conflicts caused by unreliable sensor sources are solved. Then, based on the corrected evidences, we put forward a weighted assignment of conflicting mass where the weight index lies on the evidence credibility. As the normalization step is abolished, the conflict redistribution strategy avoids the conflicts caused by straightforward normalization. Through comprehensive conflict management, the proposed DS combination method can not only guarantee the rationality and availability of fusing results, but also enhance the reliability and robustness of a multisensor system. Finally, three combination experiments with different conflicting degrees illustrate the advantage and superiority of the novel combination method for conflict management. Consequently, the innovation of the novel algorithm is verified.

1. Introduction

Multisensor information fusion is a hot issue, which has been employed in various military and civil fields. Single-source data are often incomplete and unstable, which violates the highly precise requirement in practical applications [1, 2]. Contrarily, information fusion can combine the rich and diverse information from a multisensor system, and obtain accurate and creditable outputs. Clearly, a multisensor system can collect multiclass information from homogeneous or heterogeneous sensors, and further integrate these information to form more accurate, reliable, and complete decision-making results compared to a single-sensor system [3]. Therefore, a multisensor system overcomes the limitations and the inert zone of a single-sensor system, which improves the quantity of system outputs.

A multisensor information fusion system has two overt advantages:

1. The information of a single sensor is unilateral, incomplete, possibly imprecise, or erroneous. Different sensors have diverse accuracy and measurement categories. Through a comprehensive fusion of this rich information, a multisensor system will produce more reliable and accurate decision-making results.

2. Different sensors can provide different information characteristics. Multisensor information can receive
multielement information characteristics. Thus, a multisensor system has a better anti-interference ability.

Uncertainty inevitably exists in a multisensor system. On the one hand, under the complex monitoring environment and unbalanced sensor sensitivities, environmental noise and man-made interference seriously affect the precision and reliability of sensor information [4]. On the other hand, due to the limited accuracy of sensors, several sensors cannot measure the real target information. Hence, incomplete, uncertain, imprecise, and inconsistent information generally occurs in a multisensor system [5].

The DS evidence theory (DS theory) is an outstanding uncertainty reasoning method for a multisensor system [6, 7]. The DS theory can obtain reasonable fusion results even with an error sensor, which strengthens system robustness [8, 9]. In view of its excellent uncertainty management nature, the DS theory has been diffusely applied in many areas, for example, decision making [10, 11], target recognition [12, 13], multisource classification [14, 15], fault diagnosis [16, 17], and supplier selection problem [18]. Besides, a new self-interference suppression idea based on the DS theory in the two-way full-duplex MIMO relay is presented in [19]; a joint DS detection for superposition modulation in a nonorthogonal multiple access channel with a multiple antenna configuration is proposed in [20]; and the DS theory is applied to outcome analysis in radiation oncology in [21]. Accordingly, with the improvement and development of the DS theory, its application prospect will be more extensive.

The application of the DS theory in a multisensor system also has its shortcomings [22]. Firstly, the different levels of performance of sensors, cluster, and interference of a complex combat environment directly lead to conflicts among evidences. When evidences are highly conflicting, the fusing results obtained by the DS combination method are normally contrary to common sense. Secondly, although the DS combination rule ensures the unity of mass function, the peremptory normalization limits the usage of the DS theory. When the conflicting factor is close to 1, evidences are highly conflicting. The DS theory cannot obtain reasonable fusing results at highly conflicting situations as the denominator of the DS combination rule is approximate to 0. These counterintuitive phenomena of the DS theory are called paradoxes.

To address the paradoxes, scholars have carried out relevant research. Schubert [23] developed a discounted proportion method for conflict management by exploring the relationship between complementary information and conflicting evidence. Yang et al. [24] quantified the evidence classification based on a core vector method. Lin et al. [25] proposed a weighted evidence combination based on the Mahalanobis distance function, which can efficiently solve high evidence conflicts. Odgerel and Lee [26] introduced a modified combination method based on partial conflict measurement by using the absolute difference between two evidences, which obtains logically acceptable results compared to the DS combination rule. Inspired by the evolutionary game theory, Deng et al. [27] considers a biological and evolutionary perspective to study the combination of evidences. Bi et al. [28] measured the evidence conflicts correctly after analyzing various improved methods.

According to the above analyses, we proposed a comprehensive conflict management algorithm. Firstly, via adopting the Matusita distance function [29] and closeness degree function, the evidence reliability index and evidence consistency index are, respectively, addressed. Through the correction of potentially conflicting evidences based on the reliability index and consistency index, the conflicts caused by unreliable sensor sources are solved. Then, based on the weighted mass function, we designed a rational mass assignment of conflicting probability instead of directly employing the DS combination rule. Through the effective conflict redistribution, the conflicts caused by straightforward normalization are avoided.

The paper is arranged as follows. The summary of the DS theory is presented in Section 2. Section 3 introduces the sources of conflicts in the DS theory to demonstrate its application limitations. Section 4 emphasizes the improved DS combination method based on evidence correction and conflict redistribution, where an implementation example is given. Experiments and discussions are exhibited in Section 5, and Section 6 concludes this paper.

2. DS Evidence Theory

As the generalization of the probability theory and Bayesian reasoning, the DS evidence theory can obtain fusing results without the requirement of prior knowledge and conditional probability. Based on the accumulation of evidences, a multisensor system can get valid and accurate fusing results. With the following strong theoretical derivation, the DS theory can effectively handle the system uncertainty.

2.1. Frame of Discernment. The frame of discernment (frame) contains $M$ mutually exclusive and exhaustive hypotheses.

\[
\Theta = \{H_1, H_2, \ldots, H_M\},
\]

where $H_i (i = 1, 2, \ldots, M)$ is the $i$th hypothesis that reflects the $i$th potential result of the multisensor system.

Accordingly, we can derive the power set $2^\Theta$ of DS theory.

\[
2^\Theta = \{\emptyset, \{H_1\}, \{H_2\}, \ldots, \{H_M\}, \{H_1, H_2\}, \ldots, \{H_1, H_2, \ldots, H_M\}, \Theta\},
\]

where $\emptyset$ is the empty set.

It is clearly seen in (2) that the power set $2^\Theta$ has $2^M$ propositions, and any proposition $H \subseteq \Theta$ satisfies $H \in 2^\Theta$.

2.2. Mass Function. Evidences in DS theory are acquired by multisensor information. Mass function (mass) is a function $m : 2^\Theta \rightarrow [0, 1]$ that satisfies (3) and (4).

\[
m(\emptyset) = 0,
\]

\[
\sum_{H \subseteq \Theta} m(H) = 1,
\]

for $H \in 2^\Theta$. 

\[
\sum_{H \subseteq \Theta} m(H) = 1,
\]
Obviously, mass \( m(H) \) represents the initial support degree of evidence \( m \) to proposition \( H \). Equation (3) reflects that mass satisfies the nonnegativity property, and (4) ensures the unity property of mass.

2.3. Uncertainty Representation. Then, the belief function and plausibility function on \( 2^\Theta \) are deduced.

\[
Bel(H) = \sum_{A \subseteq H} m(A), \\
Pl(H) = \sum_{A \supseteq \emptyset} m(A).
\]

(3)

Equations (6) and (7) separately reflect the lower and upper bounds of mass. The uncertainty interval \([Bel(H), Pl(H)]\) is expressed in Figure 1.

2.4. DS Combination Rule. DS combination rule can synthesize multisensor information to obtain effective and accurate decision-making results. Assuming that the frame of a multisensor system is \( \Theta = \{H_1, H_2, \ldots, H_M\} \), the DS combination rule of two evidences \( m_1 \) and \( m_2 \) is defined.

\[
m(H) = \frac{1}{1-K} \sum_{H_i; H_j \neq \emptyset} m_1(H_i) \cdot m_2(H_j), \quad H \neq \emptyset,
\]

(6)

\[
m(\emptyset) = 0,
\]

where \( K \) is the conflicting factor that reflexes the conflicting degree of evidences \( m_1 \) and \( m_2 \). \( 1 - K \) is the normalized factor that ensures the unity property of fused mass.

\[
K = \sum_{H_i; H_j \neq \emptyset} m_1(H_i) \cdot m_2(H_j).
\]

(7)

Equation (6) reveals that the essence of DS combination rule is the direct sum operation of evidences \( m_1 \) and \( m_2 \). Thus, the DS combination rule is also expressed as \( m = m_1 \oplus m_2 \). The diagrammatic sketch of \( m = m_1 \oplus m_2 \) is described in Figure 2.

In Figure 2, the combined pane represents the fusing mass of proposition \( H \). Apparently, the DS combination rule in (6) conforms to both commutative law and associate law.

\[
m_1 \oplus m_2 = m_2 \oplus m_1,
\]

\[
(m_1 \oplus m_2) \oplus m_3 = m_1 \oplus (m_2 \oplus m_3).
\]

(8)

Accordingly, the DS combination rule of multisensor evidences can be simply extended (details are shown in [7]).

3. Sources of Conflicts

Although DS theory has rigorous theoretical foundation, its practical usages have certain limitations. Since Zadeh presented the fusing paradoxes [30], research about conflict management has become one of the mainstream trends in decades [31]. Through the summarizing of existing references, recent studies can be classified into two mainstreams: methods based on the correction of original evidence sources and methods based on the modification of the DS combination rule [22]. Notably, two kinds of methods have different understandings and views on the conflicting sources.

3.1. Typical Conflicting Paradoxes

3.1.1. Completely Conflicting Paradox. In the multisensor system, assume that the frame is \( \Theta = \{A, B, C\} \), and there are two evidences.

\[
E_1 : m_1(A) = 1, m_1(B) = 0, m_1(C) = 0,
\]

(9)

\[
E_2 : m_2(A) = 0, m_2(B) = 1, m_2(C) = 0.
\]

The conflicting factor in (7) is \( K = 1 \), which reports that evidences \( E_1 \) and \( E_2 \) are completely conflicting. Under such circumstances, the DS combination rule cannot be applied as the normalized index of (6) is \( 1/1 - K \rightarrow 0 \).

3.1.2. "One Ballot Veto" Paradox. In the multisensor system, assume that there are four evidences in the frame \( \Theta = \{A, B, C\} \) and proposition \( A \) is true.

\[
E_1 : m_1(A) = 0.7, m_1(B) = 0.2, m_1(C) = 0.1,
\]

\[
E_2 : m_2(B) = 0.9, m_2(C) = 0.1,
\]

\[
E_3 : m_3(A) = 0.75, m_3(B) = 0.15, m_3(C) = 0.1,
\]

\[
E_4 : m_4(A) = 0.8, m_4(B) = 0.1, m_4(C) = 0.1.
\]

(10)

In (10), evidences \( E_1, E_3, \) and \( E_4 \) all conformably allocate the biggest proposition to proposition \( A \), while, evidence
$E_2$ completely denies proposition $A$, which is opposite to the assumption.

Applying the DS combination rule, the fusing results and conflicting factor are calculated.

$$m(A) = 0, m(B) = 0.9643, m(C) = 0.0357,$$

$$K = 0.99.$$  

(11)

(12)

It’s clear in (11) that the fusing results are contrary to assumption. The counterintuitive situation is caused by inconsistent evidence $E_2 : m_2(A) = 0$, which is named the “one ballot veto” paradox.

3.1.3. “Total Trust” Paradox. In the multisensor system, assume that the frame is $\Theta = \{ A, B, C \}$, and there are two evidences.

$$E_1 : m_1(A) = 0.95, m_1(B) = 0.05, m_1(C) = 0,$$

$$E_2 : m_2(A) = 0, m_2(B) = 0.10, m_2(C) = 0.90.$$  

(13)

Equation (13) tells that evidences $E_1$ and $E_2$ are completely different. Using the DS combination rule, the fusing results and conflicting factor are calculated.

$$m(A) = 0, m(B) = 1, m(C) = 0,$$

$$K = 0.9950.$$  

(14)

In the “total trust” circumstance that $m_1(C) = 0, m_2(A) = 0$, the wrong proposition $B$ is identified to be true, and even both evidences $E_1$ and $E_2$ nearly vetoed proposition $B$.

$$E_1 : m_1(A) = 0.7, m_1(B) = 0.1, m_1(C) = 0.1, m_1(D) = 0, m_1(E) = 0.1.$$  

$$E_2 : m_2(A) = 0, m_2(B) = 0.5, m_2(C) = 0.2, m_2(D) = 0.1, m_2(E) = 0.2.$$  

$$E_3 : m_3(A) = 0.6, m_3(B) = 0.1, m_3(C) = 0.15, m_3(D) = 0, m_3(E) = 0.15.$$  

$$E_4 : m_4(A) = 0.55, m_4(B) = 0.1, m_4(C) = 0.1, m_4(D) = 0.15, m_4(E) = 0.1.$$  

$$E_5 : m_5(A) = 0.6, m_5(B) = 0.1, m_5(C) = 0.2, m_5(D) = 0, m_5(E) = 0.1.$$  

(15)

$$m(A) = 0, m(B) = 0.5556, m(C) = 0.2222, m(D) = 0, m(E) = 0.2222,$$

$$K = 0.9999.$$  

(16)

(17)

3.1.4. Highly Conflicting Paradox. In the multisensor system, assume that there are five evidences in the frame $\Theta = \{ A, B, C, D, E \}$, and proposition $A$ is true. The mass assignments are shown in (15).

After the DS combination, the fusing results and conflicting factor are separately acquired in (16) and (17).

In the situation of excessively high evidence conflict, the fusing results completely negate true proposition $A$.

These perverse application examples of DS evidence well illustrate the applicable limitation of the DS combination rule. Next, we will explore the sources of conflicts.

3.2. Conflicts Caused by Unreliable Sensor Sources. The former kind of methods considers that an unreliable sensor is the main conflict source. It is evident that the totally conflicting factor is deeply affected by all evidences. If an unreliable evidence is added, the totally conflicting factor will increase rapidly, which leads to wrong fusing results. For example, the emergence of conflicting evidence $E_2$ in (10) and (15) directly causes the “one ballot veto” paradox and highly conflicting paradox. The unreliable sensor sources mainly attribute to two aspects. Firstly, bad weather, dense noise, and man-made interference in a complex monitoring environment would reduce the acquiring accuracy of sensors and ulteriorly increase the difficulty of DS combination. Then, the inconsistency or poor anti-interference ability of sensors will lower the information reliability, which would cause conflicting situations for DS combination. Besides, the imprecise mass model and the increase of uncertain information in the DS theory will consequentially bring in evidence conflicts.

Typical methods for solving conflicts caused by unreliable sensor sources are presented in [25, 32–35]. This kind of methods modifies evidences before applying the DS combination rule. Firstly, an average procedure and a weighted procedure of conflicting evidences are separately raised in [32, 33], which show a modification direction for conflict management. Then, the Jousselme distance function, the KL (Kullback-Leibler) distance function, and the Mahalanobis distance function are, respectively, introduced in [25, 34, 35] to estimate the weighted coefficients. These methods dexterously solve the evidence conflicts.

3.3. Conflicts Caused by Compulsive Normalization Step. The latter kind of methods believes that the compulsive normalization of the DS combination directly leads to fusing paradoxes. On the one hand, we can see in (6) that the DS combination rule firstly sums the local conflicts between two propositions, and then directly normalizes them to total propositions. The global distribution of partial conflicts is not exactly reasonable. On the other hand, Figure 2 reveals that the fusing results are obtained by the direct sum operation of evidences, where the direct sum operation is sensitive to all mass assignments. For example, evidence $E_2$ has zero support to proposition $A$ as $m_2(A) = 0$ and the fusing results will totally deny proposition $A$ as $m(A) = 0$. No matter how many consistent evidences supporting proposition $A$ are added, the “one ballot veto” paradox will still exist. The compulsive normalization step exacerbates the conflicting situations.

To address the conflicting situations, [36–40] optimize the DS combination rule by assigning conflicting mass to certain subsets with different proportions. With a different understanding of the frame completeness, [36, 37] separately assigned the whole conflict mass to $\emptyset$ and $\Theta$, which only handled the evidence conflicts theoretically. Adopting the useful information of inconsistent evidences, Sun et al. [38] and Li et al. [39] separately presented two effective conflict redistribution principles. In addition, two local conflict redistribution schemes are proposed in [40].
4. Improved DS Combination Method

The DS theory can not only combine uncertain and imprecise information, but also strengthen the reliability and precision of fusing results. However, potential paradoxes hinder the development and application of the DS theory. Section III demonstrates the sources of conflicts: one is the unreliability of sensor sources, and the other is the compulsive normalization step. Thus, the aim of the innovative algorithm is to eliminate these two conflicting sources with effective processing.

The flow chart of the proposed algorithm is exhibited in Figure 3. Obviously, the improved DS combination method is accomplished by two steps:

Step 1. Evidence correction based on the reliability index and consistency index. By the introduction of the Matusita distance function and closeness degree function, the reliability degree and consistency degree of evidences are calculated. Based on the reliability and consistency analyses, potentially conflicting evidences are effectively modified. Accordingly, the former kind of conflicting sources is solved.

Step 2. Conflict redistribution based on weighted assignment. By normalizing the reliability index and consistency index, the credibility degree of corrected evidences is obtained. Applying the weighted mass assignment defined by the credibility degree, we raise a novel conflict allocation strategy, and further establish the conflict redistribution scheme. Consequently, the latter kind of conflicting sources is eliminated.

The specific implementation and theoretical analyses of the proposed algorithm are elaborated below.

4.1. Evidence Correction Based on Reliability Index and Consistency Index. To avoid the conflicts caused by unreliable sensor sources, we successively raise the reliability index and consistency index to revise the potential conflicting evidences.

4.1.1. Introduction of Reliability Index. The reliability degree of each evidence is diverse in a multisensor system. The conventional DS theory combines all evidences equally, which reduces the rationality and accuracy of synthesis results. Thus, we propose a reliability index to evaluate the different reliability degree of evidences.

In the multisensor system, assume that there are $N$ evidences in the frame $\Theta = \{H_1, H_2, \ldots, H_M\}$.

First of all, the Matusita distance function \[29\] is adopted to describe the relationship among $N$ evidences.

$$d_{ij} = \sqrt{\sum_{t=1}^{M} \left( \sqrt{m_i(H_t)} - \sqrt{m_j(H_t)} \right)^2},$$

(18)

where $i, j = 1, 2, \ldots, N, t = 1, 2, \ldots, M$. $N$ is the number of evidences, and $M$ is the number of propositions.

The Matusita distance function $d_{ij}$ in (18) effectively reflects the difference of evidences $m_i$ and $m_j$.

Then, the similarity degree $s_{ij}$ of evidences $m_i$ and $m_j$ is derived.

$$s_{ij} = 1 - d_{ij},$$

(19)

Similarity degree $s_{ij}$ in (19) represents the similarity degree of evidences $m_i$ and $m_j$. It’s obvious that $s_{ij}$ is an inverse function of $d_{ij}$. Hence, evidences $m_i$ and $m_j$ that have a bigger difference will own less similarity, and vice versa. Clearly, the deduced logic conforms to common sense.

Based on $s_{ij}$, the support degree of evidence $m_i$ is calculated.

$$\sup_i = \sum_{j=1, j\neq i}^N s_{ij},$$

(20)

Equation (20) reveals that $\sup_i$ describes its total similarity with other evidences. Thus, $\sup_i$ indirectly reflects the corresponding reliability degree of evidence $m_i$ in a multisensor system.

Accordingly, the proposed reliability index $\mu_i$, $0 \leq \mu_i \leq 1$ of evidence $m_i$ is derived.

$$\mu_i = \frac{\sup_i}{\max, \sup_i}.\quad (21)$$

For all evidences, we can calculate the reliability vector.

$$\overline{\mu} = [\mu_1, \mu_2, \ldots, \mu_N].\quad (22)$$

4.1.2. Introduction of Consistency Index. Due to possible errors and uncertainty of sensors, the consistency of evidences cannot be guaranteed. The traditional DS theory combines all evidences equally, which further depresses the validity and precision of fusing results. Therefore, we raise a consistency index to estimate the different consistency degrees of evidences.

Firstly, we averagely sum all evidences $m_1, m_2, \ldots, m_N$ to preliminarily evaluate the mass assignment of a multisensor fusion system.

$$\bar{m} = \frac{1}{N} \sum_{i=1}^{N} m_i,\quad (23)$$

Next, we define the closeness degree $c_i$ of evidence $m_i$ based on the average mass.

$$c_i = \sum_{j=1}^{M} \frac{m_i(H_j)}{m(H_j)}.\quad (24)$$

We can check in (24) that the closeness degree $c_i$ demonstrates the general difference between evidence $m_i$ and average mass $\bar{m}$. Hence, $c_i$ well indicates the consistency degree of evidence $m_i$. 
Consequently, the proposed consistency index $\gamma_i$ (0 ≤ $\gamma_i$ ≤ 1) of evidence $m_i$ is derived.

$$\gamma_i = \frac{c_i}{\max_i c_i}.$$  \hspace{0.5cm} (25)

For all evidences, we can calculate the consistency vector.

$$\vec{\gamma} = [\gamma_1, \gamma_2, \ldots , \gamma_N].$$  \hspace{0.5cm} (26)

4.1.3. Evidence Correction. Since the evidence reliability index and consistency index have been, respectively, addressed, we incorporated the reliability index $\mu_i$ and the consistency index $\gamma_i$ to modify the mass assignment.

$$\tilde{m}_i(H_j) = \mu_i \cdot \gamma_i \cdot m_i(H_j), \quad H_j \neq \Theta,$$

$$\tilde{m}_i(\Theta) = 1 - \sum_{H_j \neq \Theta} \mu_i \cdot \gamma_i \cdot m_i(H_j),$$  \hspace{0.5cm} (27)

where $i = 1, 2, \ldots, N$, and $N$ is the number of evidences.

It can be verified in (27) that the corrected mass $\tilde{m}_i(H_j)$ satisfied the nonnegativity and unity properties shown in (3) and (4). Equation (27) applies the reliability index and consistency index to effectively evaluate and correct the
potentially conflicting evidences. Thus, the corrected mass \( \tilde{m}_i(H_j) \) can accurately describe the support degree of evidence \( \tilde{m}_i \) to proposition \( H_j \). In addition, \( \tilde{m}_i(\emptyset) \) in (27) partially reflects the uncertainty degree of evidence \( \tilde{m}_i \), which can provide a reference basis for multisensor decision making.

Therefore, the accurate correction of evidences based on the reliability index and consistency index can effectively solve the conflicts caused by unreliable evidence sources.

4.2. Conflict Redistribution. Considering the conflicts caused by the straightforward normalization step, we primarily abolish the compulsive processing. Secondary, a weighted assignment strategy of conflicting mass is recommended in this paper.

At first, the conflicting factor of corrected evidences \( \tilde{m}_1, \tilde{m}_2, \ldots, \tilde{m}_N \) is calculated.

\[
\tilde{K} = \sum_{n \in H \cap H \subseteq M} \prod_{i \leq i \leq N} \tilde{m}_i(H_j).
\]  

(28)

The conflicting factor \( \tilde{K} \) in (28) represents the conflicting degree of the multisensor system, which needs to be readdressed effectively. Comparing with the conflicting factor \( K \) of original evidences, \( \tilde{K} \) describes the conflicting degree more accurately as the unreliability of original evidences has been analyzed and evaluated.

Instead of directly applying the DS combination rule, we next introduce an innovative conflict assignment scheme.

\[
M(H) = \sum_{n \in H \cap H \subseteq M} \prod_{i \leq i \leq N} \tilde{m}_i(H_j) + \Delta(H) \quad H \neq \emptyset,
\]

\[
M(\emptyset) = 0,
\]

(29)

where \( \Delta(H) \) is the efficient conflict allocation of conflicting factor \( \tilde{K} \).

\[
\Delta(H) = \tilde{K} \cdot Q(H).
\]  

(30)

\( Q(H) \) in (30) is the approximate mass assignment of a multisensor system based on the weighted sum processing.

\[
Q(H) = \sum_{i=1}^{N} \omega_i \cdot \tilde{m}_i(H),
\]

\[
\omega_i = \frac{\mu_i \cdot \gamma_i}{\sum_{i=1}^{N} \mu_i \cdot \gamma_i}.
\]  

(32)

Through the normalization of reliability index \( \mu_i \) and consistency index \( \gamma_i \), \( \omega_i \) in (32) synthetically describes the credibility degree of evidence \( \tilde{m}_i \). Then, based on the weighted sum processing in (31), \( Q(H) \) fleetly calculates the rough mass assignment of fusing results. Accordingly, \( Q(H) \) can be used as a weighted value in (30) to reasonably allocate conflicting mass \( \tilde{K} \).

Therefore, the weighted conflict assignment in (29) effectively solves the conflicts caused by the compulsive normalization step.

In conclusion, a robust DS combination method is formed. Firstly, the correction processing comprehensively takes evidences’ differences into account, and measures all evidences’ reliability degree and consistency degree. Thus, no matter how much conflicting evidences are, there will always be a correction processing to decrease the system conflict. Namely, the proposed evidence correction can tolerate certain evidence conflicts. Secondly, the conflict redistribution strategy based on weighted assignment overcomes the counterintuitive situations of the straightforward normalization step. Hence, system uncertainty is further solved. Accordingly, the proposed DS combination method is accurate and robust to realize combination.

It’s notable that we concentrate on the closed world of evidence and proposed a novel combination method in this paper. In practical application, the research about the open world of evidence also has great significance. As the open world of evidence is deduced based on the expansion of the closed world of evidence and the combination method of the open world of evidence is similar to those in the closed world of evidence [41], the strategies to avoid evidence conflicts in this paper can be developed to the open world of evidence.

4.3. Implementation Example. To better understand the implementation of the proposed DS combination method, its syncretic processing is exhibited where a general conflicting situation-highly conflicting paradox discussed in (15) is adopted.

For convenience, the mass assignments of a highly conflicting paradox are also shown in Figure 4:

\[
\mu = [\mu_1, \mu_2, \cdots, \mu_N]
\]

\[
= [0.9720, 0.4717, 1.0000, 0.8708, 0.9901],
\]

(33)

\[
\gamma = [\gamma_1, \gamma_2, \cdots, \gamma_N]
\]

\[
= [1.0000, 0.3882, 0.9100, 0.8303, 0.9126].
\]

(34)

After the correction processing in (27), the new evidences are expressed in Figure 5.

Apparently shown in Figure 4, evidences \( E_1, E_2, E_3, E_4 \), and \( E_5 \) are consistent to support proposition \( A \) with the biggest probability, while evidence \( E_2 \) is highly conflicting. As shown in Equation (17), the conflicting factor of this multisensor system is \( K = 0.9999 \). We can conclude from (16) that the DS combination rule will obtain counterintuitive results under a highly conflicting situation.

Comparing with Figure 4, evidences in Figure 5 have been effectively corrected. Firstly, (33) and (34) reveal the reliability degree and consistency degree of evidences. For example, the reliability degree and consistency degree of conflicting evidence \( E_2 \) are both the lowest. It reveals that the correction processing precisely analyzes the difference
among evidences, and further quantifies an evidence’s reliability degree and consistency degree. Thus, the correction processing based on the reliability index and consistency index adequately considers the system conflict. Then, as the multisensor system is highly conflicting, the corrected evidences assigned certain supports to $\Theta$ where the assignment of $\Theta$ reflects the uncertainty degree of evidence. For example, the conflicting evidence $E_2$ allocates most support to $\Theta$. It reveals that evidence $E_2$ has the biggest conflicting degree, and evidence $E_2$ is greatly modified. Hence, the correction evidences in Figure 5 exactly reflects the true sensor information.
Consequently, the usage of the correction processing has been elaborated:

Step 2. Based on (28), (32), and (31), \( K, \omega_i, \) and \( Q(H) \) are respectively derived.

\[
K = \sum_{i=1}^{M} H_j \prod_{1 \leq j \leq N} \tilde{m}_i(H_j) = 0.8360, \\
\omega_1 = 0.2633, \\
\omega_2 = 0.0495, \\
\omega_3 = 0.2465, \\
\omega_4 = 0.1959, \\
\omega_5 = 0.2448, \\
Q(A) = 0.5245, \\
Q(B) = 0.0888, \\
Q(C) = 0.1194, \\
Q(D) = 0.0221, \\
Q(E) = 0.0973, \\
Q(\Theta) = 0.1479. \\
\]

(35)

Then, the fusing results based the weighted conflict assignment \( \Delta(H) \) in (29) is obtained.

\[
M(A) = 0.5971, \\
M(B) = 0.0755, \\
M(C) = 0.1021, \\
M(D) = 0.0186, \\
M(E) = 0.0829, \\
M(\Theta) = 0.1238. \\
\]

(36)

Comparing with the DS combined results in (16), the proposed DS combination method recognizes the correct proposition \( A \) accurately. Simultaneously, the proposed algorithm solves the “one ballot veto” situation, which obtains rational and reliable fusing results. Moreover, \( M(\Theta) = 0.1238 \) in (36) indirectly reveals the uncertainty degree of fusing results, which outlines the uncertainty of the multisensor system.

From the above mentioned, the implementation steps of the proposed algorithm are clear and effective.

5. Numerical Experiments and Analyses

Three numerical simulations are presented to prove the validity and superiority of the proposed DS combination method. First of all, three multisensor data are adopted, where three data are, respectively, the consistent information, low conflicting information, and highly conflicting information.

Then, the DS combination rule and \([7, 25, 35, 38, 39]\) are used as the comparison methods.

5.1. Consistent Information. In the multisensor system, assume that there are 5 evidences in the frame \( \Theta = (A, B, C) \), and proposition \( A \) is the true; the consistent evidences are exhibited in Table 1.

Table 2 shows the fusing results of \([7, 25, 35, 38, 39]\) and the raised method.

Table 2 indicates that in the consistent situation, true proposition \( A \) is obtained by all methods with different solutions:

(1) Through the DS combination, proposition \( A \) has the biggest support, while the support to proposition \( B \) is 0. Although the DS combination produces correct results, the complete veto to proposition \( B \) causes possible errors for further decision making. Thus, the DS combination is not fully reliable even when handling consistent evidences.

(2) References \([38, 39]\) present two modified DS combination methods based on conflict reassignment. Reference \([38]\) assigns \( m(\Theta) = 0.0771 \) to describe the uncertainty of fusing results, while \([39]\) gives a bigger support to true proposition \( A \). Hence, \([38, 39]\) both produce correct results with different properties.

(3) References \([25, 35]\) are two modified DS combination methods based on evidence revision. Apparently, \([25]\) allocates a bigger mass to true proposition \( A \). Thus, \([25]\) owns a better fusing ability than \([35]\).

(4) Comparing with the fusing results of other methods, the proposed algorithm solves the “one ballot veto” conflict that will occur when using the DS combination, which precisely recognizes true proposition \( A \), and computes the system uncertainty as \( m(\Theta) \neq 0 \). Obviously, the proposed algorithm is effective and accurate for consistent evidence combination.

Hence, the proposed algorithm is more rational and precise when handling consistent evidences.

5.2. Low Conflicting Information. With the same assumption in Section 5.1, we can set the low conflicting evidences in Table 3.

Unlike Table 1, we adjust evidence \( E_3 \) into an inconsistent evidence in Table 3. Table 4 shows the fusing results of \([7, 25, 35, 38, 39]\) and the raised method.

Under a low conflicting condition, Table 4 reflects that the true proposition \( A \) is also identified by all methods:

(1) The “one ballot veto” situation still exists in the fusing results of the DS combination. Since \( m_1(B) = 0 \), the DS combination method totally denies proposition \( B \), which is not exactly accurate.

(2) Similar to the previous experiment, we can conclude that \([38, 39]\) have their own characteristics according to different modifications of the DS combination
Table 1: Mass assignments of consistent information.

<table>
<thead>
<tr>
<th>Sensors</th>
<th>Propositions</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1 : m_1(\cdot)$</td>
<td>$A$</td>
<td>$B$</td>
<td>$C$</td>
<td></td>
</tr>
<tr>
<td>$0.90$</td>
<td>$0$</td>
<td>$0.10$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_2 : m_2(\cdot)$</td>
<td>$0.88$</td>
<td>$0.01$</td>
<td>$0.11$</td>
<td></td>
</tr>
<tr>
<td>$E_3 : m_3(\cdot)$</td>
<td>$0.81$</td>
<td>$0.08$</td>
<td>$0.11$</td>
<td></td>
</tr>
<tr>
<td>$E_4 : m_4(\cdot)$</td>
<td>$0.98$</td>
<td>$0.01$</td>
<td>$0.01$</td>
<td></td>
</tr>
<tr>
<td>$E_5 : m_5(\cdot)$</td>
<td>$0.90$</td>
<td>$0.05$</td>
<td>$0.05$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Fusion results of different methods with consistent evidences ($K = 0.4342$).

<table>
<thead>
<tr>
<th>Methods</th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$\Theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS combination [7]</td>
<td>$0.9999$</td>
<td>$0.0001$</td>
<td>$0.0001$</td>
<td>$0$</td>
</tr>
<tr>
<td>Reference [38]</td>
<td>$0.8851$</td>
<td>$0.0107$</td>
<td>$0.0271$</td>
<td>$0.0771$</td>
</tr>
<tr>
<td>Reference [39]</td>
<td>$0.9540$</td>
<td>$0.0130$</td>
<td>$0.0330$</td>
<td>$0$</td>
</tr>
<tr>
<td>Reference [35]</td>
<td>$0.6149$</td>
<td>$0.0180$</td>
<td>$0.0484$</td>
<td>$0.3187$</td>
</tr>
<tr>
<td>Reference [25]</td>
<td>$0.8663$</td>
<td>$0.0174$</td>
<td>$0.0472$</td>
<td>$0.0691$</td>
</tr>
<tr>
<td>Proposed</td>
<td>$0.9244$</td>
<td>$0.0096$</td>
<td>$0.0254$</td>
<td>$0.0406$</td>
</tr>
</tbody>
</table>

Table 3: Mass assignments of low conflicting information.

<table>
<thead>
<tr>
<th>Sensors</th>
<th>Propositions</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1 : m_1(\cdot)$</td>
<td>$A$</td>
<td>$B$</td>
<td>$C$</td>
<td></td>
</tr>
<tr>
<td>$0.90$</td>
<td>$0$</td>
<td>$0.10$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_2 : m_2(\cdot)$</td>
<td>$0.88$</td>
<td>$0.01$</td>
<td>$0.11$</td>
<td></td>
</tr>
<tr>
<td>$E_3 : m_3(\cdot)$</td>
<td>$0.50$</td>
<td>$0.20$</td>
<td>$0.30$</td>
<td></td>
</tr>
<tr>
<td>$E_4 : m_4(\cdot)$</td>
<td>$0.98$</td>
<td>$0.01$</td>
<td>$0.01$</td>
<td></td>
</tr>
<tr>
<td>$E_5 : m_5(\cdot)$</td>
<td>$0.90$</td>
<td>$0.05$</td>
<td>$0.05$</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Fusion results of different methods with low conflicting information ($K = 0.6507$).

<table>
<thead>
<tr>
<th>Methods</th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$\Theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS combination [7]</td>
<td>$1$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Reference [38]</td>
<td>$0.7492$</td>
<td>$0.0260$</td>
<td>$0.0547$</td>
<td>$0.1701$</td>
</tr>
<tr>
<td>Reference [39]</td>
<td>$0.8907$</td>
<td>$0.0351$</td>
<td>$0.0742$</td>
<td>$0$</td>
</tr>
<tr>
<td>Reference [35]</td>
<td>$0.5799$</td>
<td>$0.0216$</td>
<td>$0.0562$</td>
<td>$0.3423$</td>
</tr>
<tr>
<td>Reference [25]</td>
<td>$0.7128$</td>
<td>$0.0236$</td>
<td>$0.0658$</td>
<td>$0.1978$</td>
</tr>
<tr>
<td>Proposed</td>
<td>$0.9012$</td>
<td>$0.0101$</td>
<td>$0.0295$</td>
<td>$0.0592$</td>
</tr>
</tbody>
</table>

rule, and [25] has better performance than [35] when fusing low conflicting information.

(3) The distinct advantage of the proposed DS combination method has been verified. Firstly, the proposed algorithm effectively handles the “one ballot veto” paradox that will occur with the DS combination. Then, the proposed algorithm gives the biggest mass to true proposition $A$ compared with other methods, which certifies its effectiveness and accuracy. Besides, the proposed method is beneficial to decision making as the system uncertainty is approximately evaluated ($m(\emptyset) \neq 0$).

5.3. Highly Conflicting Information. With the same assumption in the former two experiments, we can set the highly conflicting evidences in Table 5 where conflicting evidence $E_2$ appears when comparing to Table 3.

Table 6 shows the fusing results of [7, 25, 35, 38, 39] and the raised method.

We can see from Table 6 the following:

(1) The DS combination presents wrong results as the wrong proposition $C$ has the absolute support. Clearly, the DS combination is unreliable under highly conflicting conditions.

(2) Reference [38] cannot identify the true proposition $A$ as the mass to $\emptyset$ is the biggest. Reference [39] assigns the biggest support to proposition $A$ without an assignment for $\emptyset$, which is also not helpful to further decision making. Therefore, [38, 39] are not completely impactful under a highly conflicting situation.

(3) Reference [35] supports true proposition $A$ the most, and also gives a large mass to $\emptyset$. Hence, the fusing results of [35] are ambiguous for decision making. Contrarily, the fusing results of [25] are more reasonable.

(4) Comparing with other methods, the DS combination method allocates a bigger mass to true proposition $A$, and gives a reasonable mass to $\emptyset$. Thus, the proposed algorithm maintains rational, effective, and accurate fusing performance when combining highly conflicting information.

In this section, we use three different sensor sources with consistent information, low conflicting information, and highly conflicting information to demonstrate the robustness of the proposed algorithm. Comparing with Table 1, we firstly introduced conflicting evidence $E_3$ in Table 3 to verify the effectiveness of the proposed method. Experimental results in Table 4 indicate that the proposed algorithm can handle the conflicting situation more precisely and available. Then, we further introduced highly conflicting evidence $E_2$ in Table 5 to prove the robustness of the proposed method. Experiment results in Table 6 show that the proposed algorithm keeps its superiority to obtain accurate and stable fusion. Apparently, along with the increasing conflicting degree among evidences, the proposed method always has the best combination effects. Namely, the proposed method is valid and robust.

According to the above three experiments, we can conclude that comparing with other 5 fusing methods, the proposed DS combination method is predominant.

6. Conclusion

The DS evidence theory is a broadly applied uncertainty management method in a multisensor fusion system. Since the conflicting phenomena generally occur in the usage of
the DS theory, its practical application has certain limitations. First of all, through the overall discussions of typical conflicting paradoxes, the sources of conflicts are summarized to two categories. One is the unreliability of sensor sources, and the other is the peremptory normalization step. To eliminate these two conflicting sources, we then put forward a robust DS combination method based on evidence correction and conflict redistribution. In step 1, by the introductions of the Matusita distance function and closeness degree function, the reliability index and consistency index are separately proposed to synthetically revise the potentially conflicting evidences. In step 2, a weighted assignment of conflicting mass is raised according to corrected evidences where the weighted value is calculated based on the average sum processing of corrected evidences. Through the comprehensive processing of the proposed algorithm, sources of conflicts have been effectively solved. Finally, three experiments indicate the effectiveness and precision of the proposed algorithm when combining relatively consistent evidences and prove its robustness and superiority when combining highly conflicting evidences. Therefore, the robustness of the proposed DS combination method has been testified.

Notably, the research of the open world of evidence is also significant in real life. Considering the closed world of evidence in this paper, we derived the proposed DS combination method to solve the system uncertainty and potential conflict. As the open world of evidence is deduced based on the expansion of an exclusive and exhaustive frame of discernment, the ideas of evidence correction and conflict redistribution can be extended to the open world of evidence with some modifications. Consequently, we will develop the proposed DS combination method to the open world of evidence in a future study.

### Table 5: Mass assignments of highly conflicting information.

<table>
<thead>
<tr>
<th>Sensors</th>
<th>Propositions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>$E_1$ : $m_1(\cdot)$</td>
<td>0.90</td>
</tr>
<tr>
<td>$E_2$ : $m_2(\cdot)$</td>
<td>0</td>
</tr>
<tr>
<td>$E_3$ : $m_3(\cdot)$</td>
<td>0.50</td>
</tr>
<tr>
<td>$E_4$ : $m_4(\cdot)$</td>
<td>0.98</td>
</tr>
<tr>
<td>$E_5$ : $m_5(\cdot)$</td>
<td>0.90</td>
</tr>
</tbody>
</table>

### Table 6: Fusion results of different methods with highly conflicting information ($K = 1$).

<table>
<thead>
<tr>
<th>Methods</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>$\Theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS combination [7]</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Reference [38]</td>
<td>0.3773</td>
<td>0.0426</td>
<td>0.1553</td>
<td>0.4248</td>
</tr>
<tr>
<td>Reference [39]</td>
<td>0.6560</td>
<td>0.0740</td>
<td>0.2700</td>
<td>0</td>
</tr>
<tr>
<td>Reference [35]</td>
<td>0.4994</td>
<td>0.0547</td>
<td>0.1276</td>
<td>0.3182</td>
</tr>
<tr>
<td>Reference [25]</td>
<td>0.6077</td>
<td>0.0442</td>
<td>0.1606</td>
<td>0.1875</td>
</tr>
<tr>
<td>Proposed</td>
<td>0.8652</td>
<td>0.0179</td>
<td>0.0369</td>
<td>0.0800</td>
</tr>
</tbody>
</table>

### Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

### Acknowledgments

The paper is funded by the National Natural Science Foundation of China (no. 61701134), the National Key Research and Development Program of China (no. 2016YFF0102806), and the Natural Science Foundation of Heilongjiang Province, China (no. F2017004). Moreover, this work is supported by the Fundamental Research Funds for the Central Universities of China (nos. HEUCFM180801, HEUCFM180802), and the PhD Student Research and Innovation Fund of the Fundamental Research Funds for the Central Universities of China (no. HEUGIP201708).

### References


[12] G. Dong and G. Kuang, “Target recognition via information aggregation through Dempster-Shafer’s evidence theory,”

Journal of Sensors 11


